Week 10 Homework

Probability Model and Data Analysis

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Homework of Continuous RVs

Question 1:

The random variable X has a probability density function

$$f_X(x) = \begin{cases} cx & 0 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

Use the PDF to find

(a) the constant c
(b) $P[0 \leq X \leq 1]$
(c) $P[-\frac{1}{2} \leq X \leq \frac{1}{2}]$
(d) the CDF $F_X(x)$

Solution

(a)

$$\begin{aligned} \int_0^2 cx\,dx &= 1 \\ c\int_0^2 x\,dx &= 1 \\ c\frac{x^2}{2}\bigg|_0^2 &= 1 \\ 2c - 0c &= 1 \\ 2c &= 1 \\ \therefore c &= \frac{1}{2} \end{aligned}$$

(b)

$$
\begin{aligned}
P[0 \le X \le 1] &= \int_0^1 \frac{1}{2} x \, dx \\
&= \left. \frac{x^2}{4} \right|_0^1 \\
\therefore P[0 \le X \le 1] &= \frac{1}{4} - \frac{0}{4} = \frac{1}{4}
\end{aligned}
$$

(c)

$$
\begin{aligned}
P[-\frac{1}{2} \le X \le \frac{1}{2}] &= \int_0^{\frac{1}{2}} \frac{1}{2} x \, dx \text{ ; since } P[X \le 0] = 0 \\
&= \left. \frac{x^2}{4} \right|_0^{\frac{1}{2}} \\
\therefore P[-\frac{1}{2} \le X \le \frac{1}{2}] &= \frac{1}{16} - \frac{0}{4} = \frac{1}{16}
\end{aligned}
$$

(d)

From $F_X(x) = \int_0^x f_X(u) \, du$

$$
F_X(x) = \int_0^{\frac{1}{2}} \frac{1}{2} x \, dx = \left. \frac{x^2}{4} \right|_0^x
$$

$$
F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}
$$

Question 2:

The cumulative distribution function of random variable X is

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{2} & -1 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Find the PDF $f_X(x)$ of X

Solution

$$\begin{aligned} f_X(x) &= \frac{dF_X(x)}{dx} \\ &= \frac{d[\frac{x+1}{2}]}{dx} \\ &= \frac{1}{2} \end{aligned}$$

$$\therefore F_X(x) = \begin{cases} \frac{1}{2} & -1 \leq x < 1 \\ 0 & otherwise \end{cases}$$

Homework of Continuous RVs-Expected Value and $Var[X]$

Question

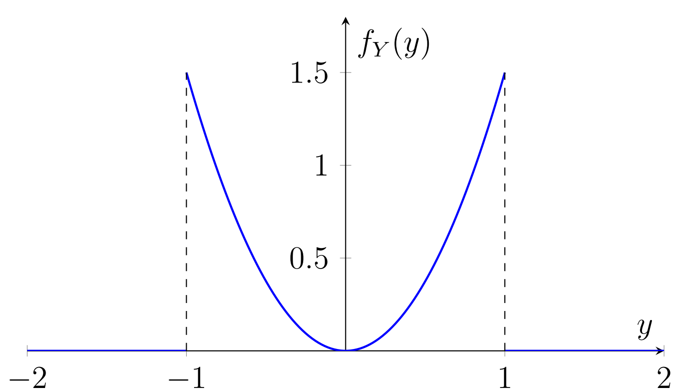
The probability density of the random variable Y is

$$f_Y(y) = \begin{cases} \frac{3y^2}{2} & -1 \leq y \leq 1 \\ 0 & otherwise \end{cases}$$

Sketch the PDF and find the following:

(1)	the expected value $E[Y]$	(2)	the second moment $E[Y^2]$
(3)	the variance $Var[Y]$	(4)	the standard deviation σ_Y

Solution



(1)

$$
\begin{aligned}
E[Y] &= \int_{-1}^{1} y(\frac{3y^2}{2})\,dy \\
&= \frac{3y^4}{8}\bigg|_{0}^{\frac{-1}{1}} \\
\therefore E[Y] &= \frac{3}{8} - \frac{3}{8} = 0
\end{aligned}
$$

(2)

$$
\begin{aligned}
E[Y^2] &= \int_{-1}^{1} y^2(\frac{3y^2}{2})\,dy \\
&= \frac{3y^5}{10}\bigg|_{0}^{\frac{-1}{1}} \\
&= \frac{3}{10} - (-\frac{3}{10}) \\
\therefore E[Y^2] &= \frac{6}{10} = \frac{3}{5}
\end{aligned}
$$

(3)

$$
\begin{aligned}
Var[Y] &= E[YY^2] - (E[Y])^2 \\
\therefore Var[Y] &= \frac{3}{5} - 0 = \frac{3}{5}
\end{aligned}
$$

(4)

$$
\begin{aligned}
\sigma_Y &= \sqrt{Var[Y]} \\
\therefore \sigma_Y &= \sqrt{\frac{3}{5}}
\end{aligned}
$$