Discrete Pair of RVs

Joint CDF

$F_{X,Y}(x,y) = P[X \leq x, Y \leq y]$
$0 \leq F_{X,Y}(x,y) \leq 1$

$F_X(x) = F_{X,Y}(x,\infty) \rightarrow$ **Marginal CDF of X**

$F_Y(y) = F_{X,Y}(\infty,y) \rightarrow$ **Marginal CDF of Y**

$F_{X,Y}(x,-\infty) = F_{X,Y}(-\infty,y) = 0$

$x_1 \leq x_2 \textbf{and} y_1 \leq y_2 \rightarrow F_{X,Y}(x_1,y_1) \leq F_{X,Y}(x_2,x_2)$

$F_{X,Y}(\infty,\infty) = 1$

Joint PMF

$P_{X,Y}(x,y) = P[X = x, Y = y]$

$$\sum_x \sum_y P_{X,Y}(x,y) = 1$$

Marginal PMF

$P_Y(y) = \sum_x P_{X,Y}(x,y)$
$P_X(x) = \sum_y P_{X,Y}(x,y)$

Continuous Pair of RVs

Joint CDF

$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(u,v)dudv$

$$P[x_1 < X \leq x_2, y_1 < Y \leq y_2] = F_{X,Y}(x_1,y_1) - F_{X,Y}(x_1,y_2) - F_{X,Y}(x_2,y_1) + F_{X,Y}(x_2,y_2)$$

Joint PDF

$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$ $P_{X,Y}(x,y) = P[X = x, Y = y]$

$P[x < X \leq x + dx, y < Y \leq y + dy] = f_{X,Y}(x,y)dxdy$

$f_{X,Y}(x,y) \geq 0, \forall(x,y)$ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)dxdy = 1$

Marginal PDF

$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$
$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx$

Integration with Polar Coordination

$x = r\cos\theta$ and $y = r\sin\theta$
Then substitute and do it as normal