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Derived Discrete RV

Assume

$$Y = g(X) = aX + b$$

$P_Y = \sum_{x:g(x)=y} P_X(x)$
$E[Y] = \sum_{x \in S_X} g(x)P_X(x)$
$E[aX + b] = a(E[X]) + b$
$Var[aX + b] = a^2 Var[X]$

Continuous RV

PDF

$$f_X = \frac{dF_X(x)}{dx}$$

$f_X \geq 0, \forall x$
$\int_{-\infty}^{\infty} f_X(x)dx = 1$
$P[x_1 < X \leq x_2] = F_X(x_2 - F_X(x_1 = \int_{x_1}^{x_2} f_X(u)du$

CDF

$$F_X(x) = \int_{-\infty}^{x} f_X(u)du$$

Expected Value

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)dx$$

$E[X - E[X]] = 0$

Other RVs

Variance and Standard Deviation

For X is Bernoulli RV, then $Var[X] = p(1-p)$
For X is Geometric RV, then $Var[X] = \frac{(1-p)}{p^2}$
For X is Binomial RV, then $Var[X] = np(1-p)$
For X is Pascal RV, then $Var[X] = \frac{k(1-p)}{p^2}$
For X is Poisson RV, then $Var[X] = \alpha$
For X is Uniform RV, then $Var[X] = \frac{(l-k)(l-k+2)}{12}$
Else, then $Var[X] = E[X^2] - (E[X])^2$
Standard Deviation: $\sigma = \sqrt{Var[X]}$

Uniform

$$f_X(x) = \begin{cases} \frac{1}{(b-a)} & a \leq x < b \\ 0 & otherwise \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x \leq a \\ \frac{1}{(b-a)} & a < x \leq b \\ 1 & x > b \end{cases}$$

$$E[X] = \frac{(b+a)}{2}$$

$$Var[X] = \frac{(b-a)^2}{12}$$

Exponential

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & otherwise \end{cases}$$

$$E[X] = \frac{1}{\lambda}$$

$$Var[X] = \frac{1}{\lambda^2}$$

Gaussian

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{\frac{-u^2}{2}} du$$

$$\Phi(-z) = 1 - \Phi(z)$$

$z = \frac{x-u}{\sigma}$

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Discrete Pair of RVs

Joint CDF

$F_{X,Y}(x,y) = P[X \le x, Y \le y]$
$0 \le F_{X,Y}(x,y) \le 1$

$F_X(x) = F_{X,Y}(x,\infty) \rightarrow$ **Marginal CDF of** X

$F_Y(y) = F_{X,Y}(\infty,y) \rightarrow$ **Marginal CDF of** Y

$F_{X,Y}(x,-\infty) = F_{X,Y}(-\infty,y) = 0$

$x_1 \le x_2 \textbf{and} y_1 \le y_2 \rightarrow F_{X,Y}(x_1,y_1) \le F_{X,Y}(x_2,x_2)$

$F_{X,Y}(\infty,\infty) = 1$

Joint PMF

$P_{X,Y}(x,y) = P[X = x, Y = y]$

$$\sum_x \sum_y P_{X,Y}(x,y) = 1$$

Marginal PMF

$P_Y(y) = \sum_x P_{X,Y}(x,y)$
$P_X(x) = \sum_y P_{X,Y}(x,y)$

Continuous Pair of RVs

Joint CDF

$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(u,v)dudv$

$$P[x_1 < X \le x_2, y_1 < Y \le y_2] = F_{X,Y}(x_1,y_1) - F_{X,Y}(x_1,y_2) - F_{X,Y}(x_2,y_1) + F_{X,Y}(x_2,y_2)$$

Joint PDF

$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$ $P_{X,Y}(x,y) = P[X = x, Y = y]$

$P[x < X \le x + dx, y < Y \le y + dy] = f_{X,Y}(x,y)dxdy$

$f_{X,Y}(x,y) \ge 0, \forall (x,y)$ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)dxdy = 1$

Marginal PDF

$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$
$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx$

Integration with Polar Coordination

$x = r\cos\theta$ and $y = r\sin\theta$
Then substitute and do it as normal

Independent RVs

$$r_{X,Y} = E[XY]$$
$$Cov[X,Y] = r_{X,Y} - E[X]E[Y]$$
$$Var[X+Y] = Var[X] + Var[Y] + 2Cov[X,Y]$$
$$r_{X,Y} = E[XY]$$

Discrete

$$r_{X,Y} = \sum_{x \in S_X} \sum_{y \in S_Y} g(x,y)P_{X,Y}(x,y)$$

Continuous

$$r_{X,Y} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)f_{X,Y}(x,y)dxdy$$