Week 9 Homework

Probability Model and Data Analysis

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Homework of Functtions of RVs

Question 1:

Suppose that a random variable, X, has an expected value of 8 and a variance of 1.8. What is the expected value and variance of $-0.4 + 3X$?

Solution

We know that $E[X] = 8$.
Let $Y = -0.4 + 3X$.

$$\begin{aligned} \therefore Y &= -0.4 + 3X \\ \therefore E[Y] &= -0.4 + 3E[X] \\ &= -0.4 + 3(8) \\ &= -0.4 + 24 \\ \therefore E[Y] &= 23.6 \end{aligned}$$

Now, we know that $E[Y] = 23.6$.

$$\begin{aligned} \therefore Var[Y] &= Var[-0.4 + 3E[X]]; \text{a} = 3 \\ \therefore Var[Y] &= a^2 Var[X] \\ &= 3^2(1.8) \\ &= 9(1.8) \\ \therefore Var[Y] &= 16.2 \end{aligned}$$

Answer

$\therefore$ Expected value and the variance of $-0.4 + 3X$ are 23.6 and 16.2, respectively.

Question 2:

Monitor three customer purchasing smartphones at the Apple IT store and observe whether each buys an iPhone 12 Pro Max for 40,000 THB or Samsung Galaxy S20 for 30,000 THB. The random variable N is the number of customers purchasing an iPhone 12 Pro Max. Assume N has PMF

$$P_N[n] = \begin{cases} 0.4, n = 0 \\ 0.2, n = 1, 2, 3 \\ 0, n = otherwise \end{cases}$$

M THB is the amount of money paid by three customers.
1. Express M as a function of N
2. Find PMF of N
3. Find $E[M]$
4. Find $Var[M]$

Solution

1.

$$\begin{aligned} M &= 40000N + 30000(3 - N) \\ &= (40000 - 30000)N + 3(30000) \\ \therefore M &= 10000N + 90000 \end{aligned}$$

2.

$$P_M[m] = \begin{cases} 0.4, m = 90000 \\ 0.2, m = 100000, 110000, 120000 \\ 0, m = otherwise \end{cases}$$

3.

We know that $E[M] = 10000E[N] + 90000$ and $E[M] = \sum_m m \cdot P_M(m)$

$$\begin{aligned} E[M] &= 90000(0.4) + 100000(0.2) + 110000(0.2) + 120000(0.2) \\ &= 36000 + 20000 + 22000 + 24000 \\ \therefore E[M] &= 102000 \text{ THB} \end{aligned}$$

4.

To find $Var[M]$, we need to find $E[M^2]$.
We know that $E[M^2] = \sum_m m^2 \cdot P_M(m)$

$$\begin{aligned} E[M^2] &= 90000^2(0.4) + 100000^2(0.2) + 110000^2(0.2) + 120000^2(0.2) \\ \therefore E[M^2] &= 10.54 \times 10^9 = 10540000000 \end{aligned}$$

Now, we can find $Var[M]$

$$\begin{aligned} Var[M] &= E[M^2] - (E[M])^2 \\ \therefore Var[M] &= 10540000000 - 10404000000 = 136000000 \text{ THB}^3 \end{aligned}$$