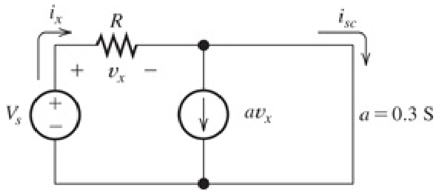


Figure T1.5

T1.6. We are given $i_4 = 2$ A for the circuit of **Figure TI.6**. Use Ohm’s law, KCL, and KVL to find the values of $i_1,\ i_2,\ i_3$ and v_s

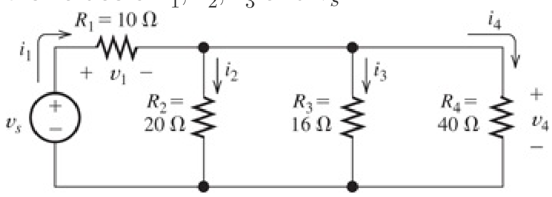
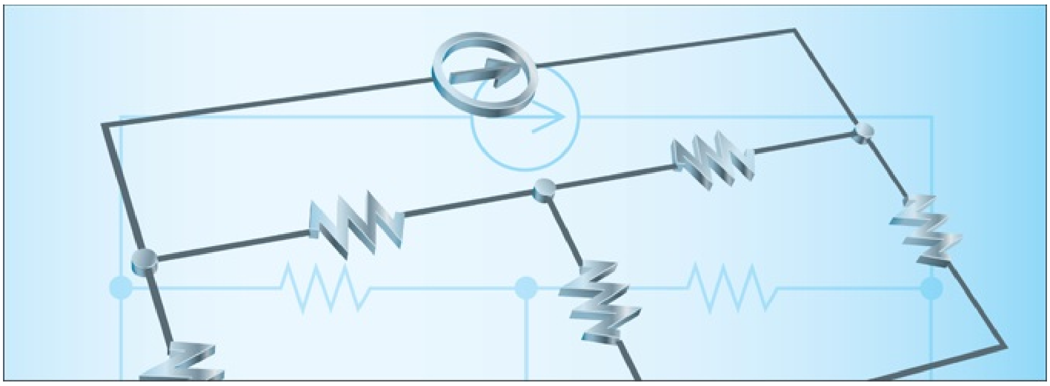


Figure T1.6

Chapter **2** Resistive Circuits



Study of this chapter will enable you to:

- Solve circuits (i.e., find currents and voltages of interest) by combining resistances in series and parallel.
- Apply the voltage-division and current-division principles.
- Solve circuits by the node-voltage technique.
- Solve circuits by the mesh-current technique.
- Find Thévenin and Norton equivalents and apply source transformations.
- Use MATLAB® to solve circuit equations numerically and symbolically.
- Understand and apply the superposition principle.
- Draw the circuit diagram and state the principles of operation for the Wheatstone bridge.

Introduction to this chapter:

In applications of electrical engineering, we often face circuit-analysis problems for which the structure of a circuit, including element values, is known and the currents, voltages, and powers need to be found. In this chapter, we examine techniques for analyzing circuits composed of resistances, voltage sources, and current sources. Later, we extend many of these concepts to circuits containing inductance and capacitance.

Over the years, you will meet many applications of electrical engineering in your field of engineering or science. This chapter will give you the skills needed to work effectively with the electronic instrumentation and other circuits that you will encounter. The material in this book will help you to answer questions on the Fundamentals of Engineering Examination and become a Registered Professional Engineer.

2.1 Resistances in Series and Parallel

In this section, we show how to replace series or parallel combinations of resistances by equivalent resistances. Then, we demonstrate how to use this knowledge in solving circuits.

Series Resistances

Consider the series combination of three resistances shown in **Figure 2.1(a)**. Recall that in a series circuit the elements are connected end to end and that the same current flows through all of the elements. By Ohm's law, we can write

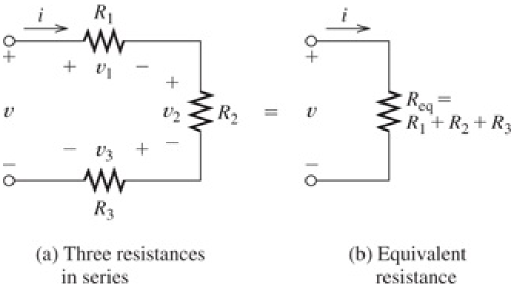


Figure 2.1
Series resistances can be combined into an equivalent resistance.

$$v_1 = R_1 i \tag{2.1}$$

$$v_2 = R_2 i \tag{2.2}$$

and

$$v_3 = R_3 i \tag{2.3}$$

Using KVL, we can write

$$v = v_1 + v_2 + v_3 \tag{2.4}$$

Substituting **Equations 2.1**, **2.2**, and **2.3** into **Equation 2.4**, we obtain

$$v = R_1 i + R_2 i + R_3 i \tag{2.5}$$

Factoring out the current *i*, we have

$$v = (R_1 + R_2 + R_3)\, i \tag{2.6}$$

Now, we define the equivalent resistance R_{eq} to be the sum of the resistances in series:

$$\boxed{R_{eq} = R_1 + R_2 + R_3} \tag{2.7}$$

Using this to substitute into **Equation 2.6**, we have

$$v = R_{eq} i \tag{2.8}$$

Thus, we conclude that the three resistances in series can be replaced by the equivalent resistance R_{eq} shown in **Figure 2.1(b)** with no change in the relationship between the voltage *v* and current *i*. If the three resistances are part of a larger circuit, replacing them by a single equivalent resistance would make no changes in the currents or voltages in other parts of the circuit.

A series combination of resistances has an equivalent resistance equal to the sum of the original resistances.

This analysis can be applied to any number of resistances. For example, two resistances in series can be replaced by a single resistance equal to the sum of the original two. To summarize, *a series combination of resistances has an equivalent resistance equal to the sum of the original resistances.*

Parallel Resistances

Figure 2.2(a) shows three resistances in parallel. In a parallel circuit, the voltage across each element is the same. Applying Ohm's law in **Figure 2.2(a)**, we can write

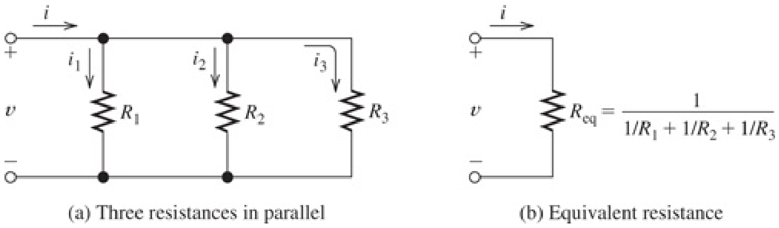


Figure 2.2
Parallel resistances can be combined into an equivalent resistance.

$$i_1 = \frac{v}{R_1} \tag{2.9}$$

$$i_2 = \frac{v}{R_2} \tag{2.10}$$

$$i_3 = \frac{v}{R_3} \tag{2.11}$$

The top ends of the resistors in **Figure 2.2(a)** are connected to a single node. (Recall that all points in a circuit that are connected by conductors constitute a node.) Thus, we can apply KCL to the top node of the circuit and obtain

$$i = i_1 + i_2 + i_3 \tag{2.12}$$

Now using **Equations 2.9**, **2.10**, and **2.11** to substitute into **Equation 2.12**, we have

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} \tag{2.13}$$

Factoring out the voltage, we obtain

$$i = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v \tag{2.14}$$

Now, we define the equivalent resistance as

$$\boxed{R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}} \tag{2.15}$$

In terms of the equivalent resistance, **Equation 2.14** becomes

$$i = \frac{1}{R_{eq}} v \tag{2.16}$$

Comparing **Equations 2.14** and **2.16**, we see that *i* and *v* are related in the same way by both equations provided that R_{eq} is given by **Equation 2.15**. Therefore, a parallel combination of resistances can be replaced by its equivalent resistance without changing the currents and voltages in other parts of the circuit. The equivalence is illustrated in **Figure 2.2(b)**.

> A parallel combination of resistances can be replaced by its equivalent resistance without changing the currents and voltages in other parts of the circuit.

This analysis can be applied to any number of resistances in parallel. For example, if four resistances are in parallel, the equivalent resistance is

$$\boxed{R_{\text{eq}} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4}} \tag{2.17}$$

Similarly, for two resistances, we have

$$\boxed{R_{\text{eq}} = \frac{1}{1/R_1 + 1/R_2}} \tag{2.18}$$

This can be put into the form

$$\boxed{R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}} \tag{2.19}$$

(Notice that **Equation 2.19** applies only for two resistances. The product over the sum does not apply for more than two resistances.)

> The product over the sum does not apply for more than two resistances.

Sometimes, resistive circuits can be reduced to a single equivalent resistance by repeatedly combining resistances that are in series or parallel.

Example 2.1 Combining Resistances in Series and Parallel

Find a single equivalent resistance for the network shown in **Figure 2.3(a)**.

Figure 2.3

Resistive network for **Example 2.1**.

Solution

First, we look for a combination of resistances that is in series or in parallel. In **Figure 2.3(a)**, R_3 and R_4 are in series. (In fact, as it stands, no other two resistances in this network are either in series or in parallel.) Thus, our first step is to combine R_3 and R_4, replacing them by their equivalent resistance. Recall that for a series combination, the equivalent resistance is the sum of the resistances in series:

$$R_{\mathrm{eq1}} = R_3 + R_4 = 5 + 15 = 20 \quad \Omega$$

1. Find a series or parallel combination of resistances.
2. Combine them.
3. Repeat until the network is reduced to a single resistance (if possible).

Figure 2.3(b) shows the network after replacing R_3 and R_4 by their equivalent resistance. Now we see that R_2 and R_{eq1} are in parallel. The equivalent resistance for this combination is

$$R_{\mathrm{eq2}} = \frac{1}{1/R_{\mathrm{eq1}} + 1/R_2} = \frac{1}{1/20 + 1/20} = 10 \quad \Omega$$

Making this replacement gives the equivalent network shown in **Figure 2.3(c)**.

Finally, we see that R_1 and R_{eq2} are in series. Thus, the equivalent resistance for the entire network is

$$R_{\mathrm{eq}} = R_1 + R_{\mathrm{eq2}} = 10 + 10 = 20 \quad \Omega$$

Exercise 2.1

Find the equivalent resistance for each of the networks shown in **Figure 2.4**. [*Hint for part (b):* R_3 and R_4 are in parallel.]

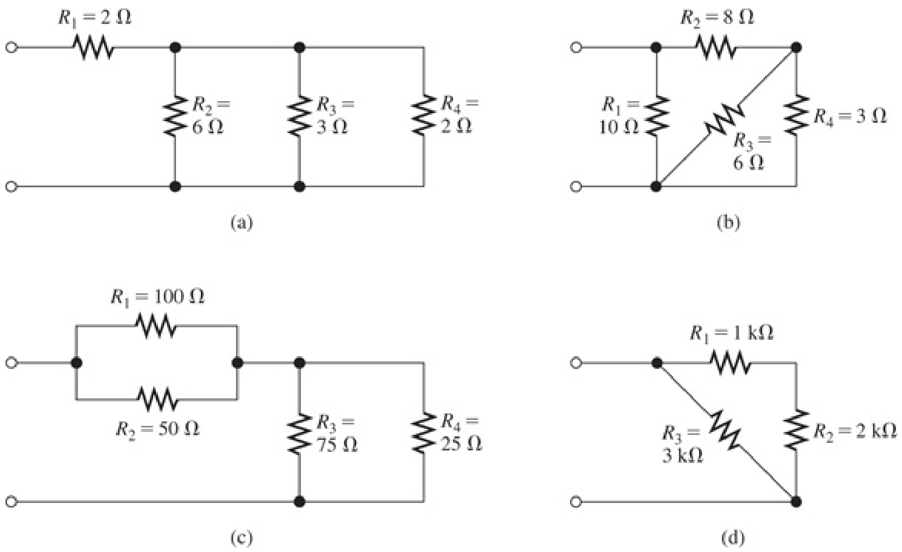


Figure 2.4

Resistive networks for **Exercise 2.1**.

Answer

a. $3\ \ \Omega$;

b. $5\ \ \Omega$;

c. $52.1\ \ \Omega$;

d. $1.5\ \mathrm{k}\,\Omega$.

Conductances in Series and Parallel

Recall that conductance is the reciprocal of resistance. Using this fact to change resistances to conductances for a series combination of *n* elements, we readily obtain:

$$\boxed{G_{\text{eq}} = \frac{1}{1/G_1 + 1/G_2 + \ldots + 1/G_n}} \tag{2.20}$$

Combine conductances in series as you would resistances in parallel. Combine conductances in parallel as you would resistances in series.

Thus, we see that conductances in series combine as do resistances in parallel. For two conductances in series, we have:

$$\boxed{G_{\text{eq}} = \frac{G_1 G_2}{G_1 + G_2}}$$

For *n* conductances in parallel, we can show that

$$\boxed{G_{\text{eq}} = G_1 + G_2 + \ldots + G_n} \tag{2.21}$$

Conductances in parallel combine as do resistances in series.

Series versus Parallel Circuits

An element such as a toaster or light bulb that absorbs power is called a **load**. When we want to distribute power from a single voltage source to various loads, we usually place the loads in parallel. A switch in series with each load can break the flow of current to that load without affecting the voltage supplied to the other loads.

> When we want to distribute power from a single voltage source to various loads, we usually place the loads in parallel.

Sometimes, to save wire, strings of Christmas lights consist of bulbs connected in series. The bulbs tend to fail or “burn out” by becoming open circuits. Then the entire string is dark and the defective bulb can be found only by trying each in turn. If several bulbs are burned out, it can be very tedious to locate the failed units. In a parallel connection, only the failed bulbs are dark.

2.2 Network Analysis by Using Series and Parallel Equivalents

An electrical **network** (or electrical circuit) consists of circuit elements, such as resistances, voltage sources, and current sources, connected together to form closed paths. **Network analysis** is the process of determining the current, voltage, and power for each element, given the circuit diagram and the element values. In this and the sections that follow, we study several useful techniques for network analysis.

> An electrical network consists of circuit elements such as resistances, voltage sources, and current sources, connected together to form closed paths.

Sometimes, we can determine the currents and voltages for each element in a resistive circuit by repeatedly replacing series and parallel combinations of resistances by their equivalent resistances. Eventually, this may reduce the circuit sufficiently that the equivalent circuit can be solved easily. The information gained from the simplified circuit is transferred to the previous steps in the chain of equivalent circuits. In the end, we gain enough information about the original circuit to determine all the currents and voltages.

Circuit Analysis Using Series/Parallel Equivalents

Here are the steps in solving circuits using series/parallel equivalents:

1. Begin by locating a combination of resistances that are in series or parallel. Often the place to start is farthest from the source.
2. Redraw the circuit with the equivalent resistance for the combination found in step 1.
3. Repeat steps 1 and 2 until the circuit is reduced as far as possible. Often (but not always) we end up with a single source and a single resistance.
4. Solve for the currents and voltages in the final equivalent circuit. Then, transfer results back one step and solve for additional unknown currents and voltages. Again transfer the results back one step and solve. Repeat until all of the currents and voltages are known in the original circuit.
5. Check your results to make sure that KCL is satisfied at each node, KVL is satisfied for each loop, and the powers add to zero.

> Some good advice for beginners: Don’t try to combine steps. Be very methodical and do one step at a time. Take the time to redraw each equivalent carefully and label unknown currents and voltages consistently in the various circuits. The slow methodical approach will be faster and more accurate when you are learning. Walk now—later you will be able to run.

Example 2.2 Circuit Analysis Using Series/Parallel Equivalents

Find the current, voltage, and power for each element of the circuit shown in **Figure 2.5(a)**.

Figure 2.5

A circuit and its simplified versions. See **Example 2.2**.

Steps 1, 2, and 3.

Solution

First, we combine resistances in series and parallel. For example, in the original circuit, R_2 and R_3 are in parallel. Replacing R_2 and R_3 by their parallel equivalent, we obtain the circuit shown in **Figure 2.5(b)**. Next, we see that R_1 and $R_{\text{eq}1}$ are in series. Replacing these resistances by their sum, we obtain the circuit shown in **Figure 2.5(c)**.

After we have reduced a network to an equivalent resistance connected across the source, we solve the simplified network. Then, we transfer results back through the chain of equivalent circuits. We illustrate this process in **Figure 2.6**. (**Figure 2.6** is identical to **Figure 2.5**, except for the currents and voltages shown in **Figure 2.6**. Usually, in solving a network by this technique, we first draw the chain of equivalent networks and then write results on the same drawings. However, this might be confusing in our first example.)

Figure 2.6

After reducing the circuit to a source and an equivalent resistance, we solve the simplified circuit. Then, we transfer results back to the original circuit. Notice that the logical flow in solving for currents and voltages starts from the simplified circuit in (c).

Step 4.

First, we solve the simplified network shown in **Figure 2.6(c)**. Because R_{eq} is in parallel with the 90-V voltage source, the voltage across R_{eq} must be 90 V, with its positive polarity at the top end. Thus, the current flowing through R_{eq} is given by

$$i_1 = \frac{v_s}{R_{\text{eq}}} = \frac{90\ \text{V}}{30\ \ \Omega} = 3\ \text{A}$$

We know that this current flows downward (from plus to minus) through R_{eq}. Since v_s and R_{eq} are in series in **Figure 2.6(c)**, the current must also flow upward through v_s. Thus, $i_1 = 3\ \text{A}$ flows clockwise around the circuit, as shown in **Figure 2.6(c)**.

Because R_{eq} is the equivalent resistance seen by the source in all three parts of **Figure 2.6**, the current through v_s must be $i_1 = 3\ \text{A}$, flowing upward in all three equivalent circuits. In **Figure 2.6(b)**, we see that i_1 flows clockwise through v_s, R_1, and R_{eq1}. The voltage across R_{eq1} is given by

$$v_2 = R_{\text{eq1}} i_1 = 20\ \Omega\ \times 3\ \text{A} = 60\ \text{V}$$

Because R_{eq1} is the equivalent resistance for the parallel combination of R_2 and R_3, the voltage v_2 also appears across R_2 and R_3 in the original network.

At this point, we have found that the current through v_s and R_1 is $i_1 = 3\ \text{A}$. Furthermore, the voltage across R_2 and R_3 is 60 V. This information is shown in **Figure 2.6(a)**. Now, we can compute the remaining values desired:

$$i_2 = \frac{v_2}{R_2} = \frac{60\ \text{V}}{30\ \ \Omega} = 2\ \text{A}$$

$$i_3 = \frac{v_2}{R_3} = \frac{60\ \text{V}}{60\ \ \Omega} = 1\ \text{A}$$

(As a check, we can use KCL to verify that $i_1 = i_2 + i_3$.)

Next, we can use Ohm's law to compute the value of v_1:

$$v_1 = R_1 i_1 = 10\ \Omega \times 3\ \text{A} = 30\ \text{V}$$

(As a check, we use KVL to verify that $v_s = v_1 + v_2$.)

Step 5.

Now, we compute the power for each element. For the voltage source, we have

$$p_s = -v_s i_1$$

We have included the minus sign because the references for v_s and i_1 are opposite to the passive configuration. Substituting values, we have

$$p_s = -(90\ \text{V}) \times 3\ \text{A} = -270\ \text{W}$$

Because the power for the source is negative, we know that the source is supplying energy to the other elements in the circuit.

The powers for the resistances are

$$p_1 = R_1 i_1^2 = 10\ \Omega \times (3\ \text{A})^2 = 90\ \text{W}$$
$$p_2 = \frac{v_2^2}{R_2} = \frac{(60\ \text{V})^2}{30\ \Omega} = 120\ \text{W}$$
$$p_3 = \frac{v_2^2}{R_3} = \frac{(60\ \text{V})^2}{60\ \Omega} = 60\ \text{W}$$

(As a check, we verify that $p_s + p_1 + p_2 + p_3 = 0,$ showing that power is conserved.) ■

Power Control by Using Heating Elements in Series or Parallel

Resistances are commonly used as heating elements for the reaction chamber of chemical processes. For example, the catalytic converter of an automobile is not effective until its operating temperature is achieved. Thus, during engine warm-up, large amounts of pollutants are emitted. Automotive engineers have proposed and studied the use of electrical heating elements to heat the converter more quickly, thereby reducing pollution. By using several heating elements that can be operated individually, in series, or in parallel, several power levels can be achieved. This is useful in controlling the temperature of a chemical process.

Exercise 2.2

Find the currents labeled in **Figure 2.7** by combining resistances in series and parallel.

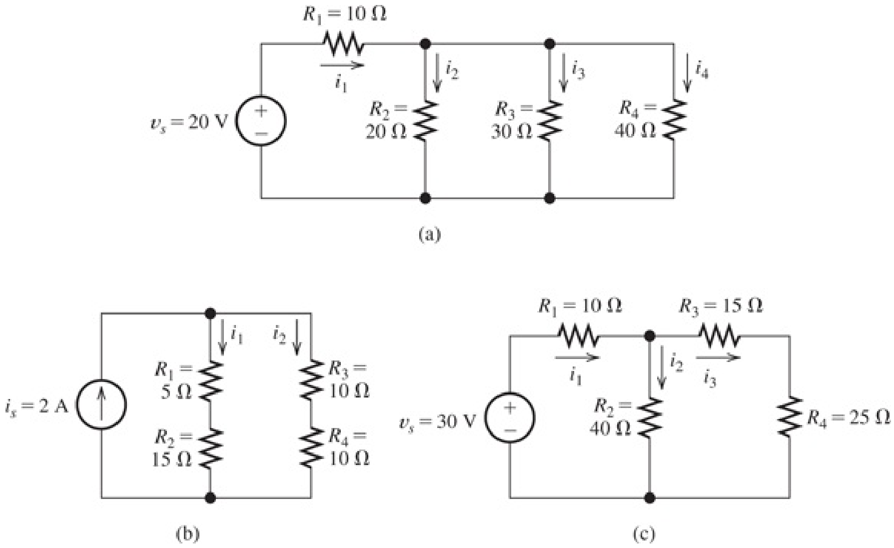


Figure 2.7
Circuits for **Exercise 2.2**.

Answer

a. $i_1 = 1.04\ \text{A},\ i_2 = 0.480\ \text{A},\ i_3 = 0.320\ \text{A},\ i_4 = 0.240\ \text{A};$

b. $i_1 = 1\ \text{A}, i_2 = 1\ \text{A};$

c. $i_1 = 1\ \text{A},\ i_2 = 0.5\ \text{A},\ i_3 = 0.5\ \text{A}\,.$

2.3 Voltage-Divider and Current-Divider Circuits

Voltage Division

When a voltage is applied to a series combination of resistances, a fraction of the voltage appears across each of the resistances. Consider the circuit shown in **Figure 2.8**. The equivalent resistance seen by the voltage source is

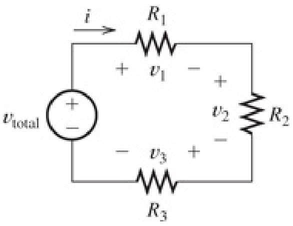


Figure 2.8
Circuit used to derive the voltage-division principle.

$$R_{\text{eq}} = R_1 + R_2 + R_3 \tag{2.22}$$

The current is the total voltage divided by the equivalent resistance:

$$i = \frac{v_{\text{total}}}{R_{\text{eq}}} = \frac{v_{\text{total}}}{R_1 + R_2 + R_3} \tag{2.23}$$

Furthermore, the voltage across R_1 is

$$\boxed{v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}}} \tag{2.24}$$

Similarly, we have

$$\boxed{v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}}} \tag{2.25}$$

and

$$\boxed{v_3 = R_3 i = \frac{R_3}{R_1 + R_2 + R_3} v_{\text{total}}} \tag{2.26}$$

We can summarize these results by the statement: *Of the total voltage, the fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.* This is known as the **voltage-division principle**.

> Of the total voltage, the fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.

We have derived the voltage-division principle for three resistances in series, but it applies for any number of resistances as long as they are connected in series.

Example 2.3 Application of the Voltage-Division Principle

Find the voltages v_1 and v_4 in **Figure 2.9**.

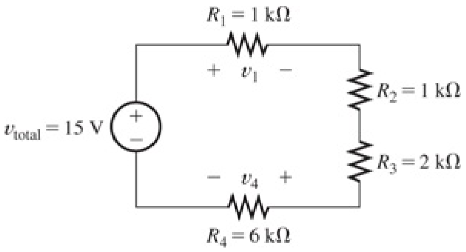


Figure 2.9
Circuit for **Example 2.3**.

Solution

Using the voltage-division principle, we find that v_1 is the total voltage times the ratio of R_1 to the total resistance:

$$\begin{aligned} v_1 &= \frac{R_1}{R_1 + R_2 + R_3 + R_4} v_{\text{total}} \\ &= \frac{1000}{1000 + 1000 + 2000 + 6000} \times 15 = 1.5\ \text{V} \end{aligned}$$

Similarly,

$$\begin{aligned} v_4 &= \frac{R_4}{R_1 + R_2 + R_3 + R_4} v_{\text{total}} \\ &= \frac{6000}{1000 + 1000 + 2000 + 6000} \times 15 = 9\ \text{V} \end{aligned}$$

Notice that the largest voltage appears across the largest resistance in a series circuit.■

Current Division

The total current flowing into a parallel combination of resistances divides, and a fraction of the total current flows through each resistance. Consider the circuit shown in **Figure 2.10**. The equivalent resistance is given by

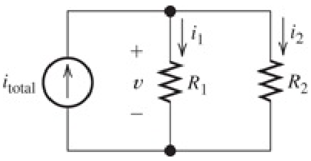


Figure 2.10
Circuit used to derive the current-division principle.

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} \tag{2.27}$$

The voltage across the resistances is given by

$$v = R_{\text{eq}} i_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} i_{\text{total}} \tag{2.28}$$

Now, we can find the current in each resistance:

$$\boxed{i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{\text{total}}} \tag{2.29}$$

and

$$\boxed{i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{\text{total}}} \tag{2.30}$$

We can summarize these results by stating the **current-division principle**: *For two resistances in parallel, the fraction of the total current flowing in a resistance is the ratio of the other resistance to the sum of the two resistances.* Notice that this principle applies only for two resistances. If we have more than two resistances in parallel, we should combine resistances so we only have two before applying the current-division principle.

> For two resistances in parallel, the fraction of the total current flowing in a resistance is the ratio of the other resistance to the sum of the two resistances.

An alternative approach is to work with conductances. For *n* conductances in parallel, it can be shown that

$$\boxed{i_1 = \frac{G_1}{G_1 + G_2 + \ldots + G_n} i_{\text{total}}}$$

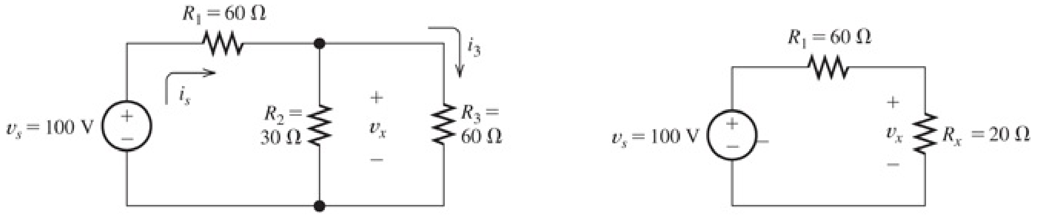
$$\boxed{i_2 = \frac{G_2}{G_1 + G_2 + \ldots + G_n} i_{\text{total}}}$$

and so forth. In other words, current division using conductances uses a formula with the same form as the formula for voltage division using resistances.

> Current division using conductances uses a formula with the same form as the formula for voltage division using resistances.

Example 2.4 Applying the Current- and Voltage-Division Principles

Use the voltage-division principle to find the voltage v_x in **Figure 2.11(a)**. Then find the source current i_s and use the current-division principle to compute the current i_3.



(a) Original circuit

(b) Equivalent circuit obtained by combining R_2 and R_3

Figure 2.11
Circuit for **Example 2.4**.

Solution

The voltage-division principle applies only for resistances in series. Therefore, we first must combine R_2 and R_3. The equivalent resistance for the parallel combination of R_2 and R_3 is

$$R_x = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\ \ \Omega$$

The equivalent network is shown in **Figure 2.11(b)**.

Now, we can apply the voltage-division principle to find v_x. The voltage v_x is equal to the total voltage times R_x divided by the total series resistance:

$$v_x = \frac{R_x}{R_1 + R_x} v_s = \frac{20}{60 + 20} \times 100 = 25\ \mathrm{V}$$

The source current i_s is given by

$$i_s = \frac{v_s}{R_1 + R_x} = \frac{100}{60 + 20} = 1.25\ \mathrm{A}$$

Now, we can use the current-division principle to find i_3. The fraction of the source current i_s that flows through R_3 is $R_2/(\ R_2 + R_3)$. Thus, we have

$$i_3 = \frac{R_2}{R_2 + R_3} i_s = \frac{30}{30 + 60} \times 1.25 = 0.417\ \mathrm{A}$$

As a check, we can also compute i_3 another way:

$$i_3 = \frac{v_x}{R_3} = \frac{25}{60} = 0.417\ \mathrm{A}$$

Example 2.5 Application of the Current-Division Principle

Use the current-division principle to find the current i_1 in **Figure 2.12(a)**.

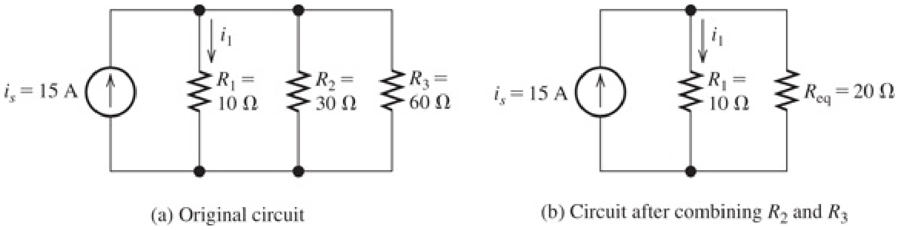


Figure 2.12
Circuit for **Example 2.5**.

The current-division principle applies for *two* resistances in parallel. Therefore, our first step is to combine R_2 and R_3

Solution

The current-division principle applies for two resistances in parallel. Therefore, our first step is to combine R_2 and R_3:

$$R_{\text{eq}} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20 \ \ \Omega$$

The resulting equivalent circuit is shown in **Figure 2.12(b)**. Applying the current-division principle, we have

$$i_1 = \frac{R_{\text{eq}}}{R_1 + R_{\text{eq}}} i_s = \frac{20}{10 + 20} 15 = 10 \text{ A}$$

Reworking the calculations using conductances, we have

$$G_1 = \frac{1}{R_1} = 100 \text{ mS}, \quad G_2 = \frac{1}{R_2} = 33.33 \text{ mS}, \quad \text{and} \quad G_3 = \frac{1}{R_3} = 16.67 \text{ mS}$$

Then, we compute the current

$$i_1 = \frac{G_1}{G_1 + G_2 + G_3} i_s = \frac{100}{100 + 33.33 + 16.67} 15 = 10 \text{ A}$$

which is the same value that we obtained working with resistances. ■

Position Transducers Based on the Voltage-Division Principle

Transducers are used to produce a voltage (or sometimes a current) that is proportional to a physical quantity of interest, such as distance, pressure, or temperature. For example, **Figure 2.13** shows how a voltage that is proportional to the rudder angle of a boat or aircraft can be obtained. As the rudder turns, a sliding contact moves along a resistance such that R_2 is proportional to the rudder angle θ. The total resistance $R_1 + R_2$ is fixed. Thus, the output voltage is

$$v_o = v_s \frac{R_2}{R_1 + R_2} = K\theta$$

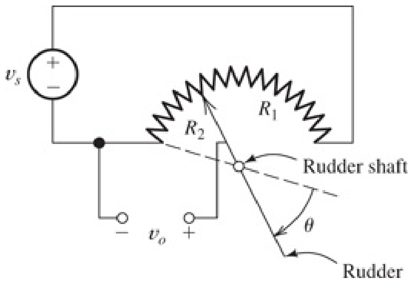


Figure 2.13
The voltage-division principle forms the basis for some position sensors. This figure shows a transducer that produces an output voltage v_o proportional to the rudder angle θ.

where *K* is a constant of proportionality that depends on the source voltage v_s and the construction details of the transducer. Many examples of transducers such as this are employed in all areas of science and engineering.

Exercise 2.3

Use the voltage-division principle to find the voltages labeled in **Figure 2.14**.

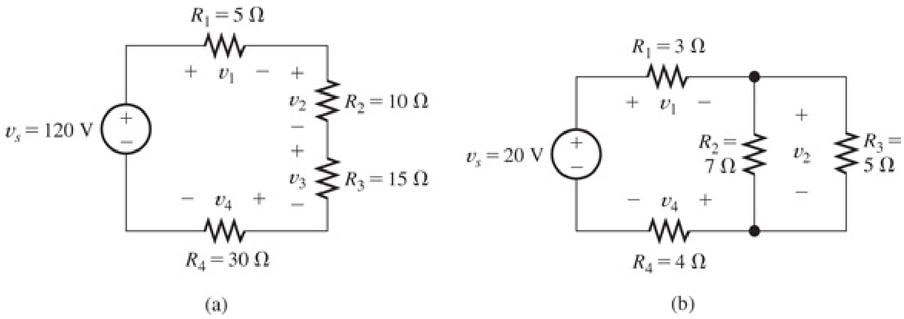


Figure 2.14
Circuits for **Exercise 2.3**.

Answer

a. $v_1 = 10\text{ V},\ v_2 = 20\text{ V},\ v_3 = 30\text{ V},\ v_4 = 60\text{ V};$

b. $v_1 = 6.05\text{ V}, v_2 = 5.88\text{ V}, v_4 = 8.07\text{ V}$.

Exercise 2.4

Use the current-division principle to find the currents labeled in **Figure 2.15**

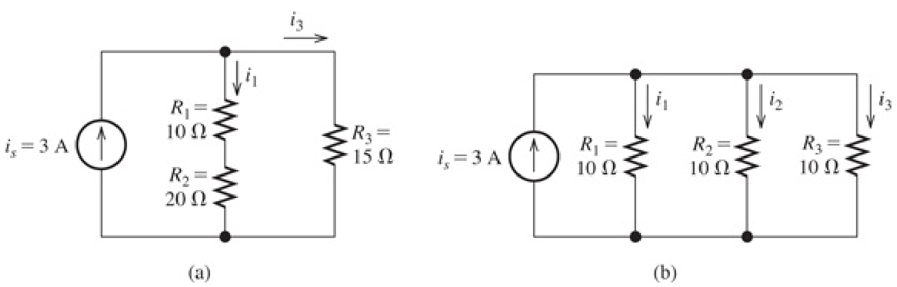


Figure 2.15
Circuits for **Exercise 2.4**.

Answer

a. $i_1 = 1\text{ A},\ i_3 = 2\text{ A};$

b. $i_1 = i_2 = i_3 = 1\text{ A}$.

2.4 Node-Voltage Analysis

> Although they are very important concepts, series/parallel equivalents and the current/voltage division principles are not sufficient to solve all circuits.

The network analysis methods that we have studied so far are useful, but they do not apply to all networks. For example, consider the circuit shown in **Figure 2.16**. We cannot solve this circuit by combining resistances in series and parallel because no series or parallel combination of resistances exists in the circuit. Furthermore, the voltage-division and current-division principles cannot be applied to this circuit. In this section, we learn **node-voltage analysis**, which is a general technique that can be applied to any circuit.

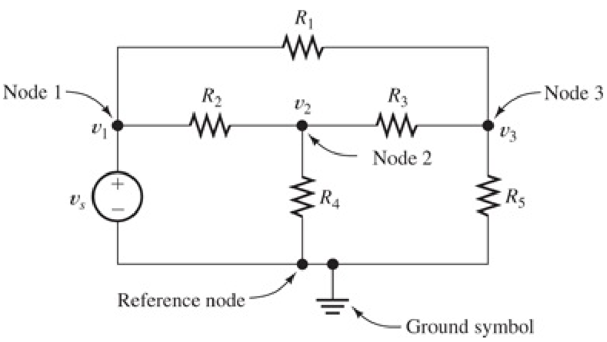


Figure 2.16
The first step in node analysis is to select a reference node and label the voltages at each of the other nodes.

Selecting the Reference Node

A **node** is a point at which two or more circuit elements are joined together. In node-voltage analysis, we first select one of the nodes as the **reference node**. In principle, any node can be picked to be the reference node. However, the solution is usually facilitated by selecting one end of a voltage source as the reference node. We will see why this is true as we proceed.

For example, the circuit shown in **Figure 2.16** has four nodes. Let us select the bottom node as the reference node. We mark the reference node by the **ground symbol**, as shown in the figure.

Assigning Node Voltages

Next, we label the voltages at each of the other nodes. For example, the voltages at the three nodes are labeled v_1, v_2, and v_3 in **Figure 2.16**. The voltage v_1 is the voltage between node 1 and the reference node. The reference polarity for v_1 is positive at node 1 and negative at the reference node. Similarly, v_2 is the voltage between node 2 and the reference node. The reference polarity for v_2 is positive at node 2 and negative at the reference node. *In fact, the negative reference polarity for each of the node voltages is at the reference node.* We say that v_1 is the voltage at node 1 with respect to the reference node.

> The negative reference polarity for each of the node voltages is at the reference node.

Finding Element Voltages in Terms of the Node Voltages

In node-voltage analysis, we write equations and eventually solve for the node voltages. Once the node voltages have been found, it is relatively easy to find the current, voltage, and power for each element in the circuit.

> Once the node voltages have been determined, it is relatively easy to determine other voltages and currents in the circuit.

For example, suppose that we know the values of the node voltages and we want to find the voltage across R_3 with its positive reference on the left-hand side. To avoid additional labels in **Figure 2.16**, we have made a second drawing of the circuit, which is shown in **Figure 2.17**. The node voltages and the voltage v_x across R_3 are shown in **Figure 2.17**. Notice that v_2, v_x, and v_3 are the voltages encountered in traveling around the closed path through R_4, R_3, and R_5. Thus, these voltages must obey Kirchhoff's voltage law. Traveling around the loop clockwise and summing voltages, we have

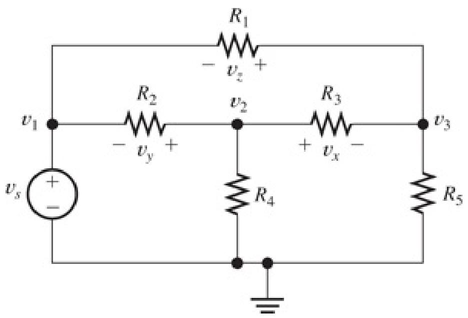


Figure 2.17

Assuming that we can determine the node voltages v_1, v_2, and v_3, we can use KVL to determine v_x, v_y, and v_z. Then using Ohm's law, we can find the current in each of the resistances. Thus, the key problem is in determining the node voltages.

> This is the same circuit shown in **Figure 2.16**. We have redrawn it simply to avoid cluttering the original diagram with the voltages v_x, v_y, and v_z that are not involved in the final node equations.

$$-v_2 + v_x + v_3 = 0$$

Solving for v_x, we obtain

$$v_x = v_2 - v_3$$

Thus, we can find the voltage across any element in the network as the difference between node voltages. (If one end of an element is connected to the reference node, the voltage across the element is a node voltage.)

After the voltages are found, Ohm's law and KCL can be used to find the current in each element. Then, power can be computed by taking the product of the voltage and current for each element.

Exercise 2.5

In the circuit of **Figure 2.17**, find expressions for v_y and v_z in terms of the node voltages v_1, v_2, and v_3.

Answer $v_y = v_2 - v_1$, $v_z = v_3 - v_1$.

Writing KCL Equations in Terms of the Node Voltages

After choosing the reference node and assigning the voltage variables, we write equations that can be solved for the node voltages. We demonstrate by continuing with the circuit of **Figure 2.16**.

> After choosing the reference node and assigning the voltage variables, we write equations that can be solved for the node voltages.

In **Figure 2.16**, the voltage v_1 is the same as the source voltage v_s:

$$v_1 = v_s$$

(In this case, one of the node voltages is known without any effort. This is the advantage in selecting the reference node at one end of an independent voltage source.)

Therefore, we need to determine the values of v_2 and v_3, and we must write two independent equations. We usually start by trying to write current equations at each of the nodes corresponding to an unknown node voltage. For example, at node 2 in **Figure 2.16**, the current leaving through R_4 is given by

$$\frac{v_2}{R_4}$$

This is true because v_2 is the voltage across R_4 with its positive reference at node 2. Thus, the current v_2/R_4 flows from node 2 toward the reference node, which is away from node 2.

Next, referring to **Figure 2.17**, we see that the current flowing out of node 2 through R_3 is given by v_x/R_3. However, we found earlier that $v_x = v_2 - v_3$. Thus, the current flowing out of node 2 through R_3 is given by

$$\frac{v_2 - v_3}{R_3}$$

> To find the current flowing out of node *n* through a resistance toward node *k*, we subtract the voltage at node *k* from the voltage at node *n* and divide the difference by the resistance.

At this point, we pause in our analysis to make a useful observation. *To find the current flowing out of node n through a resistance toward node k, we subtract the voltage at node k from the voltage at node n and divide the difference by the resistance.* Thus, if v_n and v_k are the node voltages and *R* is the resistance connected between the nodes, the current flowing from node *n* toward node *k* is given by

$$\frac{v_n - v_k}{R}$$

Applying this observation in **Figure 2.16** to find the current flowing out of node 2 through R_2, we have

$$\frac{v_2 - v_1}{R_2}$$

[In **Exercise 2.5**, we found that $v_y = v_2 - v_1$ (see **Figure 2.17**). The current flowing to the left through R_2 is v_y/R_2. Substitution yields the aforementioned expression.]

Of course, if the resistance is connected between node *n* and the reference node, the current away from node *n* toward the reference node is simply the node voltage v_n divided by the resistance. For example, as we noted previously, the current leaving node 2 through R_4 is given by v_2/R_4.

Now we apply KCL, adding all of the expressions for the currents leaving node 2 and setting the sum to zero. Thus, we obtain

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

Writing the current equation at node 3 is similar. We try to follow the same pattern in writing each equation. Then, the equations take a familiar form, and mistakes are less frequent. We usually write expressions for the currents leaving the node under consideration and set the sum to zero. Applying this approach at node 3 of **Figure 2.16**, we have

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

In many networks, we can obtain all of the equations needed to solve for the node voltages by applying KCL to the nodes at which the unknown voltages appear.

Example 2.6 Node-Voltage Analysis

Write equations that can be solved for the node voltages v_1, v_2, and v_3 shown in **Figure 2.18**.

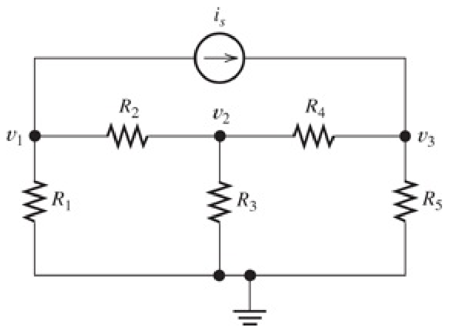


Figure 2.18
Circuit for **Example 2.6**.

Solution

We use KCL to write an equation at node 1:

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$

Each term on the left-hand side of this equation represents a current leaving node 1. Summing the currents leaving node 2, we have

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

Similarly, at node 3, we get

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} = i_s$$

Here, the currents leaving node 3 are on the left-hand side and the current entering is on the right-hand side. ■

Exercise 2.6

Use KCL to write equations at each node (except the reference node) for the circuit shown in **Figure 2.19**.

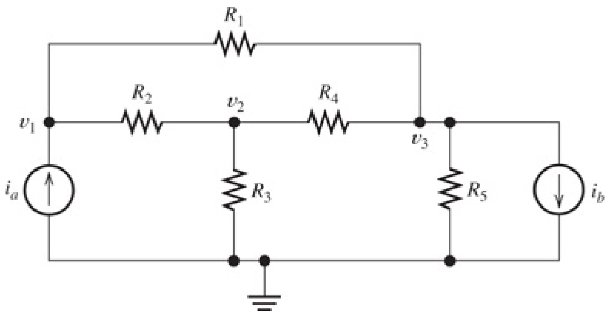


Figure 2.19
Circuit for **Exercise 2.6**.

Answer

$$\text{Node 1:} \quad \frac{v_1 - v_3}{R_1} + \frac{v_1 - v_2}{R_2} = i_a$$

$$\text{Node 2:} \quad \frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

$$\text{Node 3:} \quad \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} + \frac{v_3 - v_1}{R_1} + i_b = 0$$

Circuit Equations in Standard Form

Once we have written the equations needed to solve for the node voltages, we put the equations into standard form. We group the node-voltage variables on the left-hand sides of the equations and place terms that do not involve the node voltages on the right-hand sides. For two node voltages, this eventually puts the node-voltage equations into the following form:

$$g_{11}v_1 + g_{12}v_2 = i_1 \tag{2.31}$$

$$g_{21}v_1 + g_{22}v_2 = i_2 \tag{2.32}$$

If we have three unknown node voltages, the equations can be put into the form

$$g_{11}v_1 + g_{12}v_2 + g_{13}v_3 = i_1 \tag{2.33}$$

$$g_{21}v_1 + g_{22}v_2 + g_{23}v_3 = i_2 \tag{2.34}$$

$$g_{31}v_1 + g_{32}v_2 + g_{33}v_3 = i_3 \tag{2.35}$$

We have chosen the letter *g* for the node-voltage coefficients because they are often (but not always) conductances with units of siemens. Similarly, we have used *i* for the terms on the right-hand sides of the equations because they are often currents.

In matrix form, the equations can be written as

$$GV = I$$

in which we have

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \quad \text{or} \quad G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}$$

depending on whether we have two or three unknown node voltages. Also, **V** and **I** are column vectors:

$$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \text{or} \quad V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad \text{or} \quad I = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

As the number of nodes and node voltages increases, the dimensions of the matrices increase.

One way to solve for the node voltages is to find the inverse of **G** and then compute the solution vector as:

$$V = G^{-1}I$$

A Shortcut to Writing the Matrix Equations

If we put the node equations for the circuit of **Exercise 2.6** (**Figure 2.19**) into matrix form, we obtain

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_1} & -\frac{1}{R_4} & \frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_a \\ 0 \\ -i_b \end{bmatrix}$$

Let us take a moment to compare the circuit in **Figure 2.19** with the elements in this equation. First, look at the elements on the diagonal of the **G** matrix, which are

$$g_{11} = \frac{1}{R_1} + \frac{1}{R_2} \quad g_{22} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \quad \text{and} \quad g_{33} = \frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_5}$$

We see that the diagonal elements of **G** are equal to the sums of the conductances connected to the corresponding nodes. Next, notice the off diagonal terms:

$$g_{12} = -\frac{1}{R_2} \quad g_{13} = -\frac{1}{R_1} \quad g_{21} = -\frac{1}{R_2} \quad g_{23} = -\frac{1}{R_4} \quad g_{31} = -\frac{1}{R_1} \quad g_{32} = -\frac{1}{R_4}$$

In each case, g_{jk} is equal to the negative of the conductance connected between node *j* and *k*. The terms in the **I** matrix are the currents pushed into the corresponding nodes by the current sources. These observations hold whenever the network consists of resistances and independent current sources, assuming that we follow our usual pattern in writing the equations.

Thus, if a circuit consists of resistances and independent current sources, we can use the following steps to rapidly write the node equations directly in matrix form.

1. Make sure that the circuit contains only resistances and independent current sources.
2. The diagonal terms of **G** are the sums of the conductances connected to the corresponding nodes.
3. The off diagonal terms of **G** are the negatives of the conductances connected between the corresponding nodes.
4. The elements of **I** are the currents pushed into the corresponding nodes by the current sources.

> This is a shortcut way to write the node equations in matrix form, *provided that the circuit contains only resistances and independent current sources*.

Keep in mind that if the network contains voltage sources or controlled sources this pattern does not hold.

Exercise 2.7

Working directly from **Figure 2.18** on **page 63**, write its node-voltage equations in matrix form

Answer

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_4} \\ 0 & -\frac{1}{R_4} & \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_s \\ 0 \\ i_s \end{bmatrix}$$

Example 2.7 Node-Voltage Analysis

Write the node-voltage equations in matrix form for the circuit of **Figure 2.20**.

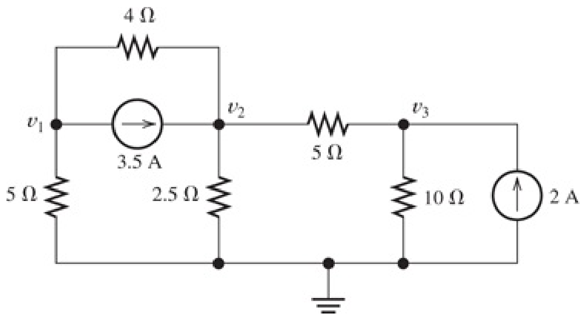


Figure 2.20
Circuit for **Example 2.7**.

Solution

Writing KCL at each node, we have

$$\frac{v_1}{5} + \frac{v_1 - v_2}{4} + 3.5 = 0$$
$$\frac{v_2 - v_1}{4} + \frac{v_2}{2.5} + \frac{v_2 - v_3}{5} = 3.5$$
$$\frac{v_3 - v_2}{5} + \frac{v_3}{10} = 2$$

Manipulating the equations into standard form, we have

$$0.45v_1 - 0.25v_2 = -3.5$$
$$-0.25v_1 + 0.85v_2 - 0.2v_3 = 3.5$$
$$-0.2v_2 + 0.35v_3 = 2$$

Then, in matrix form, we obtain

$$\begin{bmatrix} 0.45 & -0.25 & 0 \\ -0.25 & 0.85 & -0.20 \\ 0 & -0.20 & 0.30 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -3.5 \\ 3.5 \\ 2 \end{bmatrix} \tag{2.36}$$

Because the circuit contains no voltage sources or controlled sources, we could have used the shortcut method to write the matrix form directly. For example, $g_{11} = 0.45$ is the sum of the conductances connected to node 1, $g_{12} = -0.25$ is the negative of the conductance connected between nodes 1 and 2, $i_3 = 2$ is the current pushed into node 3 by the 2-A current source, and so forth. ■

Solving the Network Equations

After we have obtained the equations in standard form, we can solve them by a variety of methods, including substitution, Gaussian elimination, and determinants. As an engineering student, you may own a powerful calculator such as the TI-84 or TI-89 that has the ability to solve systems of linear equations. You should learn to do this by practicing on the exercises and the problems at the end of this chapter.

In some situations, you may not be allowed to use one of the more advanced calculators or a notebook computer. For example, only fairly simple scientific calculators are allowed on the Fundamentals of Engineering (FE) Examination, which is the first step in becoming a registered professional engineer in the United States. The calculator policy for the professional engineering examinations can be found at `http://ncees.org/`. Thus, even if you own an advanced calculator, you may wish to practice with one of those allowed in the FE Examination.

Exercise 2.8

Use your calculator to solve **Equation 2.36**.

Answer $v_1 = -5\ \mathrm{V},\ v_2 = 5\ \mathrm{V},\ v_3 = 10\ \mathrm{V}.$

Using MATLAB to Solve Network Equations

When you have access to a computer and MATLAB software, you have a very powerful system for engineering and scientific calculations. This software is available to students at many engineering schools and is very likely to be encountered in some of your other courses.

In this and the next several chapters, we illustrate the application of MATLAB to various aspects of circuit analysis, but we cannot possibly cover all of its many useful features in this book. If you are new to MATLAB, you can gain access to a variety of online interactive tutorials at http://www.mathworks.com/academia/student_center/tutorials/. If you have already used the program, the MATLAB commands we present may be familiar to you. In either case, you should be able to easily modify the examples we present to work out similar circuit problems.

Next, we illustrate the solution for **Equation 2.36** using MATLAB. Instead of using $V = G^{-1}I$ to compute node voltages, MATLAB documentation recommends using the command $V = G\backslash I$ which invokes a more accurate algorithm for computing solutions to systems of linear equations.

The comments following the % sign are ignored by MATLAB. For improved clarity, we use a **bold** font for the input commands, a regular font for comments, and a color font for the responses from MATLAB, otherwise the following has the appearance of the MATLAB command screen for this problem. (
$>>$
is the MATLAB command prompt.)

```
>> clear % First we clear the work space.
>> % Then, we enter the coefficient matrix of Equation 2.36 with
>> % spaces between elements in each row and semicolons between rows.
>> G = [0.45 -0.25 0; -0.25 0.85 -0.2; 0 -0.2 0.30]
G =
  0.4500 -0.2500        0
 -0.2500  0.8500 -0.2000
 0 -0.2000  0.3000
>> % Next, we enter the column vector for the right-hand side.
>> I = [-3.5; 3.5; 2]
I =
 -3.5000
 3.5000
 2.0000
>> % The MATLAB documentation recommends computing the node
>> % voltages using V = G\I instead of using V = inv(G)* I.
>> V = G\I
V =
 -5.0000
 5.0000
 10.0000
```

Thus, we have $v_1 = -5\ \text{V},\ v_2 = 5\ \text{V},$ and $v_3 = 10\ \text{V},$ as you found when working **Exercise 2.8** with your calculator.

Note: You can download m-files for some of the exercises and examples in this book that use MATLAB. See **Appendix E** for information on how to do this.

Example 2.8 Node-Voltage Analysis

Solve for the node voltages shown in **Figure 2.21** and determine the value of the current i_x.

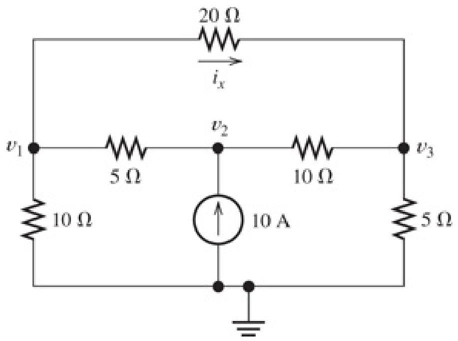


Figure 2.21
Circuit for **Example 2.8**.

Solution

Our first step in solving a circuit is to select the reference node and assign the node voltages. This has already been done, as shown in **Figure 2.21**.

Next, we write equations. In this case, we can write a current equation at each node. This yields

$$\begin{aligned}
&\text{Node 1:} \quad \frac{v_1}{10} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{20} = 0 \\
&\text{Node 2:} \quad \frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{10} = 10 \\
&\text{Node 3:} \quad \frac{v_3}{5} + \frac{v_3 - v_2}{10} + \frac{v_3 - v_1}{20} = 0
\end{aligned}$$

Next, we place these equations into standard form:

$$\begin{aligned}
0.35v_1 - 0.2v_2 - 0.05v_3 &= 0 \\
-0.2v_1 + 0.3v_2 - 0.10v_3 &= 10 \\
-0.05v_1 - 0.10v_2 + 0.35v_3 &= 0
\end{aligned}$$

In matrix form, the equations are

$$\begin{bmatrix} 0.35 & -0.2 & -0.05 \\ -0.2 & 0.3 & -0.1 \\ -0.05 & -0.1 & 0.35 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

or $GV = I$ in which **G** represents the coefficient matrix of conductances, **V** is the column vector of node voltages, and **I** is the column vector of currents on the right-hand side.

Here again, we could write the equations directly in standard or matrix form using the short cut method because the circuit contains only resistances and independent current sources.

The MATLAB solution is:

```
>> clear
>> G = [0.35 -0.2 -0.05; -0.2 0.3 -0.1; -0.05 -0.1 0.35];
>> % A semicolon at the end of a command suppresses the
>> % MATLAB response.
>> I = [0; 10; 0];
>> V = G\I
V =
   45.4545
   72.7273
   27.2727
>> % Finally, we calculate the current.
>> Ix = (V(1) - V(3))/20
Ix =
    0.9091■
```

Exercise 2.9

Repeat the analysis of the circuit of **Example 2.8**, using the reference node and node voltages shown in **Figure 2.22**.

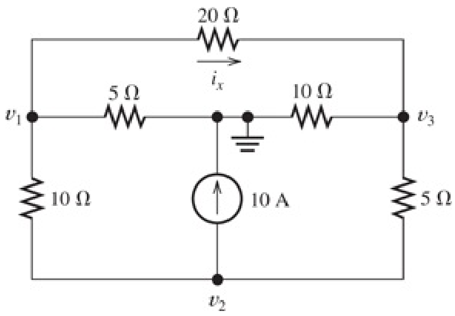


Figure 2.22
Circuit of **Example 2.8** with a different choice for the reference node. See **Exercise 2.9**.

a. First write the network equations.

b. Put the network equations into standard form.

c. Solve for v_1, v_2, and v_3. (The values will be different than those we found in **Example 2.8** because v_1, v_2, and v_3 are not the same voltages in the two figures.)

d. Find i_x. (Of course, i_x is the same in both figures, so it should have the same value.)

Answer

a.

$$\begin{aligned}
\frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} &= 0 \\
\frac{v_2 - v_1}{10} + 10 + \frac{v_2 - v_3}{5} &= 0 \\
\frac{v_3 - v_1}{20} + \frac{v_3}{10} + \frac{v_3 - v_2}{5} &= 0
\end{aligned}$$

b.

$$\begin{aligned}
0.35 v_1 - 0.10 v_2 - 0.05 v_3 &= \quad 0 \\
-0.10 v_1 + 0.30 v_2 - 0.20 v_3 &= -10 \\
-0.05 v_1 - 0.20 v_2 + 0.35 v_3 &= \quad 0
\end{aligned}$$

c. $v_1 = -27.27,\ v_2 = -72.73,\ v_3 = -45.45$

d. $i_x = 0.909\ \mathrm{A}$

Circuits with Voltage Sources

When a circuit contains a single voltage source, we can often pick the reference node at one end of the source, and then we have one less unknown node voltage for which to solve.

Example 2.9 Node-Voltage Analysis

Write the equations for the network shown in **Figure 2.23** and put them into standard form.

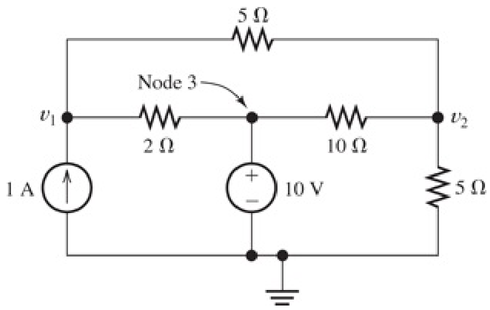


Figure 2.23
Circuit for **Example 2.9**.

Solution

Notice that we have selected the reference node at the bottom end of the voltage source. Thus, the voltage at node 3 is known to be 10 V, and we do not need to assign a variable for that node.

Writing current equations at nodes 1 and 2, we obtain

$$\frac{v_1 - v_2}{5} + \frac{v_1 - 10}{2} = 1$$
$$\frac{v_2}{5} + \frac{v_2 - 10}{10} + \frac{v_2 - v_1}{5} = 0$$

Now if we group terms and place the constants on the right-hand sides of the equations, we have

$$0.7v_1 - 0.2v_2 = 6$$
$$-0.2v_1 + 0.5v_2 = 1$$

Thus, we have obtained the equations needed to solve for v_1 and v_2 in standard form. ■

Exercise 2.10

Solve the equations of **Example 2.9** for v_1 and v_2.

Answer $v_1 = 10.32\text{ V},\ v_2 = 6.129\text{ V}$.

Exercise 2.11

Solve for the node voltages v_1 and v_2 in the circuit of **Figure 2.24**.

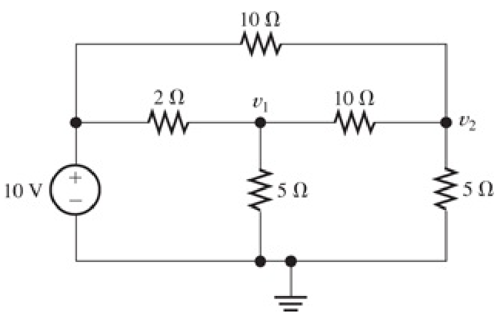


Figure 2.24
Circuit for **Exercise 2.11**.

Answer $v_1 = 6.77\text{ V},\ v_2 = 4.19\text{ V}$.

Sometimes, the pattern for writing node-voltage equations that we have illustrated so far must be modified. For example, consider the network and node voltages shown in **Figure 2.25**. Notice that $v_3 = -15\text{ V}$ because of the 15-V source connected between node 3 and the reference node. Therefore, we need two equations relating the unknowns v_1 and v_2.

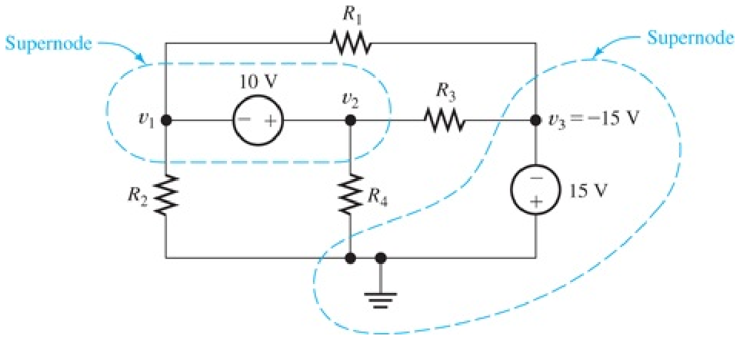


Figure 2.25
A supernode is formed by drawing a dashed line enclosing several nodes and any elements connected between them.

If we try to write a current equation at node 1, we must include a term for the current through the 10-V source. We could assign an unknown for this current, but then we would have a higher-order system of equations to solve. Especially if we are solving the equations manually, we want to minimize the number of unknowns. For this circuit, it is not possible to write a current equation in terms of the node voltages for any single node (even the reference node) because a voltage source is connected to each node.

Another way to obtain a current equation is to form a **supernode**. This is done by drawing a dashed line around several nodes, including the elements connected between them. This is shown in **Figure 2.25**. Two supernodes are indicated, one enclosing each of the voltage sources.

> Another way to state Kirchhoff's current law is that the net current flowing through any closed surface must equal zero.

We can state Kirchhoff's current law in a slightly more general form than we have previously: *The net current flowing through any closed surface must equal zero.* Thus, we can apply KCL to a supernode. For example, for the supernode enclosing the 10-V source, we sum currents leaving and obtain

$$\frac{v_1}{R_2} + \frac{v_1 - (-15)}{R_1} + \frac{v_2}{R_4} + \frac{v_2 - (-15)}{R_3} = 0 \tag{2.37}$$

Each term on the left-hand side of this equation represents a current leaving the supernode through one of the resistors. Thus, by enclosing the 10-V source within the supernode, we have obtained a current equation without introducing a new variable for the current in the source.

Next, we might be tempted to write another current equation for the other supernode. However, we would find that the equation is equivalent to the one already written. *In general, we obtain dependent equations if we use all of the nodes in writing current equations.* Nodes 1 and 2 were part of the first supernode, while node 3 and the reference node are part of the second supernode. Thus, in writing equations for both supernodes, we would have used all four nodes in the network.

> We obtain dependent equations if we use all of the nodes in a network to write KCL equations.

If we tried to solve for the node voltages by using substitution, at some point all of the terms would drop out of the equations and we would not be able to solve for those voltages. In MATLAB, you will receive a warning that the G matrix is singular, in other words, its determinant is zero. If this happens, we know that we should return to writing equations and find another equation to use in the solution. This will not happen if we avoid using all of the nodes in writing current equations.

There is a way to obtain an independent equation for the network under consideration. We can use KVL because v_1, the 10-V source, and v_2 form a closed loop. This is illustrated in **Figure 2.26**, where we have used arrows to indicate the polarities of v_1 and v_2. Traveling clockwise and summing the voltages around the loop, we obtain

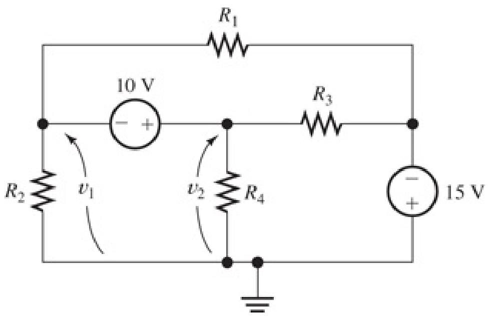


Figure 2.26

Node voltages v_1 and v_2 and the 10-V source form a closed loop to which KVL can be applied. (This is the same circuit as that of **Figure 2.25**.)

$$-v_1 - 10 + v_2 = 0 \tag{2.38}$$

Equations 2.37 and **2.38** form an independent set that can be used to solve for v_1 and v_2 (assuming that the resistance values are known).

> When a voltage source is connected between nodes so that current equations cannot be written at the individual nodes, first write a KVL equation, including the voltage source, and then enclose the voltage source in a supernode and write a KCL equation for the supernode.

Exercise 2.12

Write the current equation for the supernode that encloses the 15-V source in **Figure 2.25**. Show that your equation is equivalent to **Equation 2.37**.

Exercise 2.13

Write a set of independent equations for the node voltages shown in **Figure 2.27**.

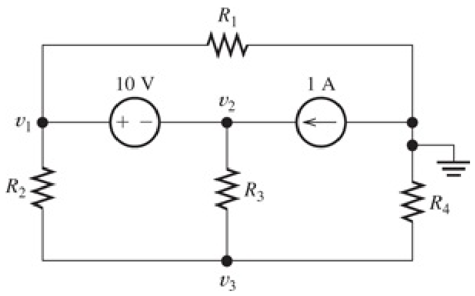


Figure 2.27

Circuit for **Exercise 2.13**.

Answer KVL:

$$-v_1 + 10 + v_2 = 0$$

KCL for the supernode enclosing the 10-V source:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

KCL for node 3:

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

KCL at the reference node:

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

For independence, the set must include the KVL equation. Any two of the three KCL equations can be used to complete the three-equation set. (The three KCL equations use all of the network nodes and, therefore, do not form an independent set.)

Circuits with Controlled Sources

Controlled sources present a slight additional complication of the node-voltage technique. (Recall that the value of a controlled source depends on a current or voltage elsewhere in the network.) In applying node-voltage analysis, first we write equations exactly as we have done for networks with independent sources. Then, we express the controlling variable in terms of the node-voltage variables and substitute into the network equations. We illustrate with two examples.

Example 2.10 Node-Voltage Analysis with a Dependent Source

Write an independent set of equations for the node voltages shown in **Figure 2.28**.

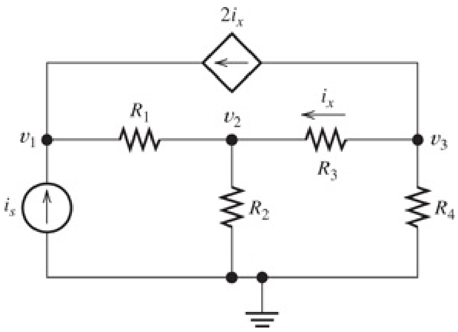


Figure 2.28
Circuit containing a current-controlled current source. See **Example 2.10**.

Solution

First, we write KCL equations at each node, including the current of the controlled source just as if it were an ordinary current source:

$$\frac{v_1 - v_2}{R_1} = i_s + 2i_x \tag{2.39}$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0 \tag{2.40}$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0 \tag{2.41}$$

Next, we find an expression for the controlling variable i_x in terms of the node voltages. Notice that i_x is the current flowing away from node 3 through R_3. Thus, we can write

$$i_x = \frac{v_3 - v_2}{R_3} \tag{2.42}$$

Finally, we use **Equation 2.42** to substitute into **Equations 2.39**, **2.40**, and **2.41**. Thus, we obtain the required equation set:

$$\frac{v_1 - v_2}{R_1} = i_s + 2\,\frac{v_3 - v_2}{R_3} \tag{2.43}$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0 \tag{2.44}$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2\,\frac{v_3 - v_2}{R_3} = 0 \tag{2.45}$$

Assuming that the value of i_s and the resistances are known, we could put this set of equations into standard form and solve for v_1, v_2, and v_3. ■

Example 2.11 Node-Voltage Analysis with a Dependent Source

Write an independent set of equations for the node voltages shown in **Figure 2.29**.

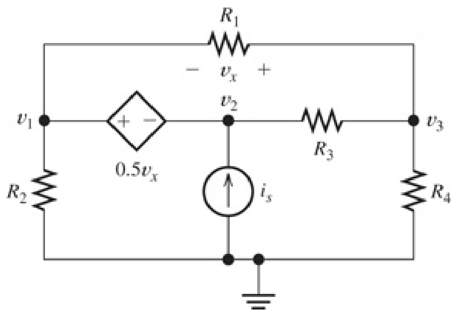


Figure 2.29

Circuit containing a voltage-controlled voltage source. See **Example 2.11**.

Solution

First, we ignore the fact that the voltage source is a dependent source and write equations just as we would for a circuit with independent sources. We cannot write a current equation at either node 1 or node 2, because of the voltage source connected between them. However, we can write a KVL equation:

$$-v_1 + 0.5v_x + v_2 = 0 \tag{2.46}$$

Then, we use KCL to write current equations. For a supernode enclosing the controlled voltage source,

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$$

For node 3,

$$\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0 \tag{2.47}$$

For the reference node,

$$\frac{v_1}{R_2} + \frac{v_3}{R_4} = i_s \tag{2.48}$$

Of course, these current equations are dependent because we have used all four nodes in writing them. We must use **Equation 2.46** and two of the KCL equations to form an independent set. However, **Equation 2.46** contains the controlling variable v_x, which must be eliminated before we have equations in terms of the node voltages.

Thus, our next step is to write an expression for the controlling variable v_x in terms of the node voltages. Notice that v_1, v_x, and v_3 form a closed loop. Traveling clockwise and summing voltages, we have

$$-v_1 - v_x + v_3 = 0$$

Solving for v_x, we obtain

$$v_x = v_3 - v_1$$

Now if we substitute into **Equation 2.46**, we get

$$v_1 = 0.5(v_3 - v_1) + v_2 \tag{2.49}$$

Equation 2.49 along with any two of the KCL equations forms an independent set that can be solved for the node voltages. ■

Using the principles we have discussed in this section, we can write node-voltage equations for any network consisting of sources and resistances. Thus, given a computer or calculator to help in solving the equations, we can compute the currents and voltages for any network.

Step-by-Step Node-Voltage Analysis

Next, we summarize the steps in analyzing circuits by the node-voltage technique:

1. First, combine any series resistances to reduce the number of nodes. Then, select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed.
2. Write network equations. First, use KCL to write current equations for nodes and supernodes. Write as many current equations as you can without using all of the nodes, including those within supernodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations.
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the network equations, and obtain equations having only the node voltages as unknowns.
4. Put the equations into standard form and solve for the node voltages.
5. Use the values found for the node voltages to calculate any other currents or voltages of interest.

Example 2.12 Node Voltage Analysis

Use node voltages to solve for the value of i_x in the circuit of **Figure 2.30(a)**. (This rather complex circuit has been contrived mainly to display all of the steps listed above.)

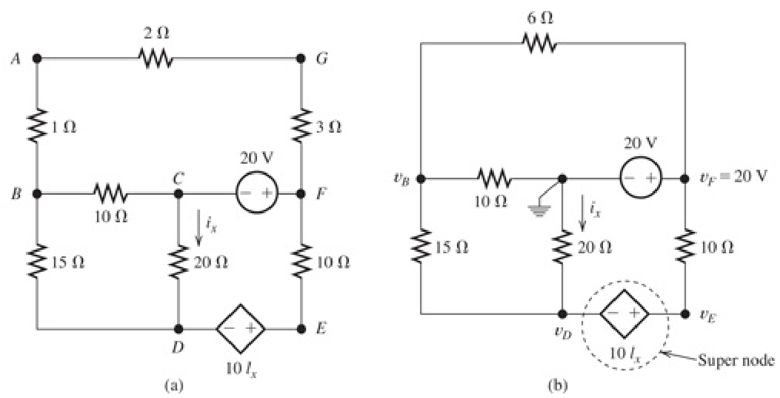


Figure 2.30
Circuit of **Example 2.12.**

Step 1

Solution

First, we combine the $1\ \Omega$, $2\ \Omega$, and $3\ \Omega$ resistances in series to eliminate nodes *A* and *G.* Then, we select node *C* at one end of the 20-V source as the reference node. Thus, we know that the voltage at node *F* is 20 V. (Of course, any node could be chosen for the reference node, but if we chose node *B,* for example, we would have one more variable in the equations.) The resulting circuit is shown in **Figure 2.30(b)**.

Step 2

We cannot write KCL equations at any single node, except node *B*, because each of the other nodes has a voltage source connected. The KCL equation at node *B* is

$$\frac{v_B - 20}{6} + \frac{v_B}{10} + \frac{v_B - v_D}{15} = 0$$

Multiplying all terms by 30 and rearranging, we have

$$10v_B - 2v_D = 100$$

Next, we form a super node enclosing the controlled voltage source as indicated in **Figure 2.30(b)**. This results in

$$\frac{v_E - 20}{10} + \frac{v_D}{20} + \frac{v_D - v_B}{15} = 0$$

(Another option would have been a super node enclosing the 20 V source.)

Multiplying all terms by 60 and rearranging, we have

$$-4v_B + 7v_D + 6v_E = 120$$

No options for another KCL equation exist without using all of the circuit nodes and producing dependent equations.

Thus, we write a KVL equation starting from the reference node to one end of the controlled voltage source, through the source, and back to the reference node. This results in $v_E = 10i_x + v_D$.

Step 3

Next, we note that i_x is the current through and v_D is the voltage across the $20-\ \Omega$ resistance. The current reference enters the negative end of the voltage, so we have $v_D = -20\ i_x$. Combining these two equations eventually results in

$$v_D - 2v_E = 0$$

Step 4

Thus, we have these three equations to solve for the node voltages:

$$\begin{aligned}
&10v_B - 2v_D = 100 \\
&-4v_B + 7v_D + 6v_E = 120 \\
&v_D - 2v_E = 0
\end{aligned}$$

Solving these three equations results in $v_D = 17.3913\ \text{V}$.

Then, we have $i_x = -v_D/20 = -0.8696\ \text{A}$. ■

Exercise 2.14

Use the node-voltage technique to solve for the currents labeled in the circuits shown in **Figure 2.31**.

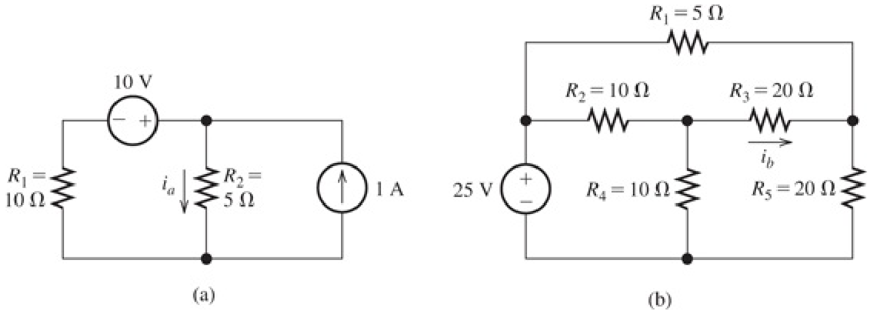


Figure 2.31
Circuits for **Exercise 2.14**.

Answer

a. $i_a = 1.33\ \text{A};$

b. $i_b = -0.259\ \text{A}.$

Exercise 2.15

Use the node-voltage technique to solve for the values of i_x and i_y in **Figure 2.32**.

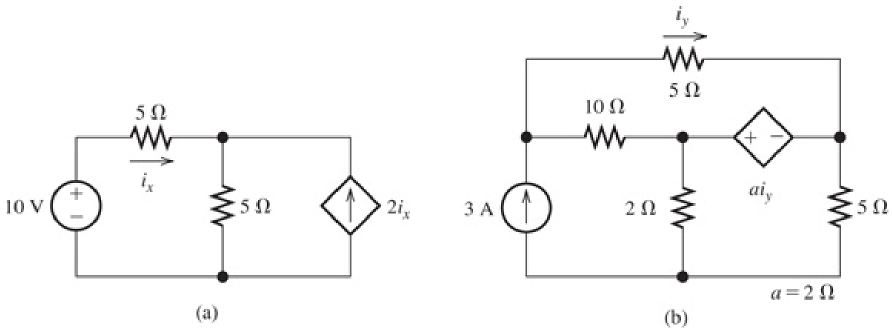


Figure 2.32
Circuits for **Exercise 2.15**.

Answer $i_x = 0.5\ \text{A},\ i_y = 2.31\ \text{A}.$

Using the MATLAB Symbolic Toolbox to Obtain Symbolic Solutions

If the Symbolic Toolbox is included with your version of MATLAB, you can use it to solve node voltage and other equations symbolically. We illustrate by solving **Equations 2.43**, **2.44**, and **2.45** from **Example 2.10** on **page 74**.

```
>> % First we clear the work space.
>> clear all
>> % Next, we identify the symbols used in the
>> % equations to be solved.
>> syms V1 V2 V3 R1 R2 R3 R4 Is
>> % Then, we enter the equations into the solve command
>> % followed by the variables for which we wish to solve.
>> [V1, V2, V3] = solve((V1 - V2)/R1 == Is + 2*(V3 - V2)/R3, ...
                     (V2 - V1)/R1 + V2/R2 + (V2 - V3)/R3 == 0, ...
                     (V3 - V2)/R3 + V3/R4 + 2*(V3 - V2)/R3 == 0, ...
                    V1, V2, V3)
V1 =
(Is*(R1*R2 + R1*R3 + 3*R1*R4 + R2*R3 + 3*R2*R4))/(3*R2 + R3 + 3*R4)
V2 =
(Is*R2*(R3 + 3*R4))/(3*R2 + R3 + 3*R4)
V3 = (3*Is*R2*R4)/(3*R2 + R3 +3*R4)
>> % The solve command gives the answers, but in a form that is
>> % somewhat difficult to read.
>> % A more readable version of the answers is obtained using the
>> % pretty command. We combine the three commands on one line
>> % by placing commas between them.
>> pretty(V1), pretty(V2), pretty(V3)
```

$$\frac{\text{Is R1 R2} + \text{Is R1 R3} + 3\ \text{Is R1 R4} + \text{Is R2 R3} + 3\ \text{Is R2 R4}}{3\ \text{R2} + \text{R3} + 3\ \text{R4}}$$

$$\frac{\text{Is R2 R3} + 3\ \text{Is R2 R4}}{3\ \text{R2} + \text{R3} + 3\ \text{R4}}$$

$$\frac{3\ \text{Is R2 R4}}{3\ \text{R2} + \text{R3} + 3\ \text{R4}}$$

(Here we have shown the results obtained using a particular version of MATLAB; other versions may give results different in appearance but equivalent mathematically.) In more standard mathematical format, the results are:

$$v_1 = \frac{i_s R_1 R_2 + i_s R_1 R_3 + 3 i_s R_1 R_4 + i_s R_2 R_3 + 3 i_s R_2 R_4}{3R_2 + R_3 + 3R_4}$$

$$v_2 = \frac{i_s R_2 R_3 + 3 i_s R_2 R_4}{3R_2 + R_3 + 3R_4}$$

$$\text{and } v_3 = \frac{3 i_s R_2 R_4}{3R_2 + R_3 + 3R_4}$$

Checking Answers

As usual, it is a good idea to apply some checks to the answers. First of all, make sure that the answers have proper units, which are volts in this case. If the units don't check, look to see if any of the numerical values entered in the equations have units. Referring to the circuit (**Figure 2.28** on **page 74**), we see that the only numerical parameter entered into the equations was the gain of the current-controlled current source, which has no units.

Again referring to the circuit diagram, we can see that we should have $v_2 = v_3$ for $R_3 = 0,$ and we check the results to see that this is the case. Another check is obtained by observing that we should have $v_3 = 0$ for $R_4 = 0.$ Still another check of the results comes from observing that, in the limit as R_3 approaches infinity, we should have $i_x = 0,$ (so the controlled current source becomes an open circuit), $v_3 = 0,\ v_1 = i_s(\ R_1 + R_2)$, and $v_2 = i_s R_2.$ Various other checks of a similar nature can be applied. This type of checking may not guarantee correct results, but it can find a lot of errors.

Exercise 2.16

Use the symbolic math features of MATLAB to solve **Equations 2.47**, **2.48**, and **2.49** for the node voltages in symbolic form.

Answer

$$v_1 = \frac{2i_s R_1\ R_2 R_3 + 3i_s R_1\ R_2 R_4 + 2i_s R_2\ R_3 R_4}{3\ R_1\ R_2 + 2\ R_1\ R_3 + 3\ R_1\ R_4 + 2\ R_2\ R_3 + 2\ R_3\ R_4}$$
$$v_2 = \frac{3i_s R_1\ R_2 R_3 + 3i_s R_1\ R_2 R_4 + 2i_s R_2\ R_3 R_4}{3\ R_1\ R_2 + 2\ R_1\ R_3 + 3\ R_1\ R_4 + 2\ R_2\ R_3 + 2\ R_3\ R_4}$$
$$v_3 = \frac{3i_s R_1\ R_2 R_4 + 2i_s R_2\ R_3 R_4}{3\ R_1\ R_2 + 2\ R_1\ R_3 + 3\ R_1\ R_4 + 2\ R_2\ R_3 + 2\ R_3\ R_4}$$

Depending on the version of MATLAB and the Symbolic Toolbox that you use, your answers may have a different appearance but should be algebraically equivalent to these.

2.5 Mesh-Current Analysis

In this section, we show how to analyze networks by using another general technique, known as mesh-current analysis. Networks that can be drawn on a plane without having one element (or conductor) crossing over another are called **planar networks**. On the other hand, circuits that must be drawn with one or more elements crossing others are said to be **nonplanar**. We consider only planar networks.

Let us start by considering the planar network shown in **Figure 2.33(a)**. Suppose that the source voltages and resistances are known and that we wish to solve for the currents. We first write equations for the currents shown in **Figure 2.33(a)**, which are called branch currents because a separate current is defined in each branch of the network. However, we will eventually see that using the mesh currents illustrated in **Figure 2.33(b)** makes the solution easier.

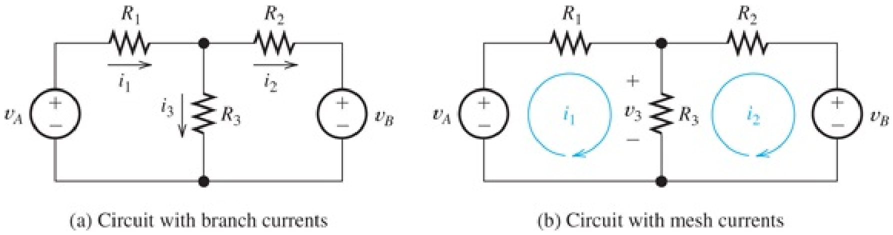


Figure 2.33

Circuit for illustrating the mesh-current method of circuit analysis.

Three independent equations are needed to solve for the three branch currents shown in **Figure 2.33(a)**. In general, the number of independent KVL equations that can be written for a planar network is equal to the number of open areas defined by the network layout. For example, the circuit of **Figure 2.33(a)** has two open areas: one defined by v_A, R_1, and R_3, while the other is defined by R_3, R_2, and v_B. Thus, for this network, we can write only two independent KVL equations. We must employ KCL to obtain the third equation.

Application of KVL to the loop consisting of v_A, R_1, and R_3 yields

$$R_1 i_1 + R_3 i_3 = v_A \tag{2.50}$$

Similarly, for the loop consisting of R_3, R_2, and v_B, we get

$$-R_3 i_3 + R_2 i_2 = -v_B \tag{2.51}$$

Applying KCL to the node at the top end of R_3, we have

$$i_1 = i_2 + i_3 \tag{2.52}$$

Next, we solve **Equation 2.52** for i_3 and substitute into **Equations 2.50** and **2.51**. This yields the following two equations:

$$R_1 i_1 + R_3 (i_1 - i_2) = v_A \tag{2.53}$$

$$-R_3 (i_1 - i_2) + R_2 i_2 = -v_B \tag{2.54}$$

Thus, we have used the KCL equation to reduce the KVL equations to two equations in two unknowns.

Now, consider the mesh currents i_1 and i_2 shown in **Figure 2.33(b)**. As indicated in the figure, mesh currents are considered to flow around closed paths. Hence, mesh currents automatically satisfy KCL. *When several mesh currents flow through one element, we consider the current in that element to be the* *algebraic sum of the mesh currents.* Thus, assuming a reference direction pointing downward, the current in R_3 is $(i_1 - i_2)$. Thus, $v_3 = R_3(i_1 - i_2)$. Now if we follow i_1 around its loop and apply KVL, we get **Equation 2.53** directly. Similarly, following i_2, we obtain **Equation 2.54** directly.

When several mesh currents flow through one element, we consider the current in that element to be the algebraic sum of the mesh currents.

Because mesh currents automatically satisfy KCL, some work is saved in writing and solving the network equations. The circuit of **Figure 2.33** is fairly simple, and the advantage of mesh currents is not great. However, for more complex networks, the advantage can be quite significant.

Choosing the Mesh Currents

For a planar circuit, we can choose the current variables to flow through the elements around the periphery of each of the open areas of the circuit diagram. For consistency, we usually define the mesh currents to flow clockwise.

Two networks and suitable choices for the mesh currents are shown in **Figure 2.34**. When a network is drawn with no crossing elements, it resembles a window, with each open area corresponding to a pane of glass. Sometimes it is said that the mesh currents are defined by "soaping the window panes."

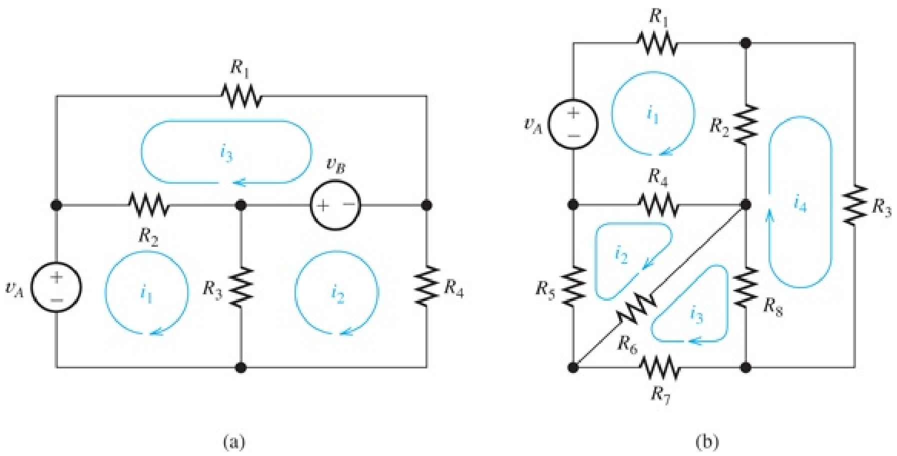


Figure 2.34

Two circuits and their mesh-current variables.

Keep in mind that, if two mesh currents flow through a circuit element, we consider the current in that element to be the algebraic sum of the mesh currents. For example, in **Figure 2.34(a)**, the current in R_2 referenced to the left is $i_3 - i_1$. Furthermore, the current referenced upward in R_3 is $i_2 - i_1$.

We usually choose the current variables to flow clockwise around the periphery of each of the open areas of the circuit diagram.

Exercise 2.17

Consider the circuit shown in **Figure 2.34(b)**. In terms of the mesh currents, find the current in

a. R_2 referenced upward;

b. R_4 referenced to the right;

c. R_8 referenced downward;

d. R_8 referenced upward.

Answer

a. $i_4 - i_1$;

b. $i_2 - i_1$;

c. $i_3 - i_4$;

d. $i_4 - i_3$. [Notice that the answer for part (d) is the negative of the answer for part (c).]

Writing Equations to Solve for Mesh Currents

If a network contains only resistances and independent voltage sources, we can write the required equations by following each current around its mesh and applying KVL. (We do not need to apply KCL because the mesh currents flow out of each node that they flow into.)

Example 2.13 Mesh-Current Analysis

Write the equations needed to solve for the mesh currents in **Figure 2.34(a)**.

Solution

Using a pattern in solving networks by the mesh-current method helps to avoid errors. Part of the pattern that we use is to select the mesh currents to flow clockwise. Then, we write a KVL equation for each mesh, going around the meshes clockwise. As usual, we add a voltage if its positive reference is encountered first in traveling around the mesh, and we subtract the voltage if the negative reference is encountered first. Our pattern is always to take the first end of each resistor encountered as the positive reference for its voltage. Thus, we are always adding the resistor voltages.

> If a network contains only resistances and independent voltage sources, we can write the required equations by following each current around its mesh and applying KVL.

For example, in mesh 1 of **Figure 2.34(a)**, we first encounter the left-hand end of R_2. The voltage across R_2 referenced positive on its left-hand end is $R_2(i_1 - i_3)$. Similarly, we encounter the top end of R_3 first, and the voltage across R_3 referenced positive at the top end is $R_3(i_1 - i_2)$. By using this pattern, we add a term for each resistor in the KVL equation, consisting of the resistance times the current in the mesh under consideration minus the current in the adjacent mesh (if any). Using this pattern for mesh 1 of **Figure 2.34(a)**, we have

$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$

Similarly, for mesh 2, we obtain

$$R_3(i_2 - i_1) + R_4 i_2 + v_B = 0$$

Finally, for mesh 3, we have

$$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$

Notice that we have taken the positive reference for the voltage across R_3 at the top in writing the equation for mesh 1 and at the bottom for mesh 3. This is not an error because the terms for R_3 in the two equations are opposite in sign.

In standard form, the equations become:

$$\begin{aligned}(R_2 + R_3) i_1 - R_3 i_2 - R_2 i_3 &= v_A \\ -R_3 i_1 + (R_3 + R_4) i_2 &= -v_B \\ -R_2 i_1 + (R_1 + R_2) i_3 &= v_B\end{aligned}$$

In matrix form, we have

$$\begin{bmatrix}(R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2)\end{bmatrix}\begin{bmatrix}i_1 \\ i_2 \\ i_3\end{bmatrix} = \begin{bmatrix}v_A \\ -v_B \\ v_B\end{bmatrix}$$

Often, we use **R** to represent the coefficient matrix, **I** to represent the column vector of mesh currents, and **V** to represent the column vector of the terms on the right-hand sides of the equations in standard form. Then, the mesh-current equations are represented as:

$$RI = V$$

We refer to the element of the *i*th row and *j*th column of **R** as r_{ij}.

Exercise 2.18

Write the equations for the mesh currents in **Figure 2.34(b)** and put them into matrix form.

Answer Following each mesh current in turn, we obtain

$$\begin{aligned} R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A &= 0 \\ R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) &= 0 \\ R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) &= 0 \\ R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) &= 0 \end{aligned}$$

$$\begin{bmatrix} (R_1 + R_2 + R_4) & -R_4 & 0 & -R_2 \\ -R_4 & (R_4 + R_5 + R_6) & -R_6 & 0 \\ 0 & -R_6 & (R_6 + R_7 + R_8) & -R_8 \\ -R_2 & 0 & -R_8 & (R_2 + R_3 + R_8) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} v_A \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.55}$$

Solving Mesh Equations

After we write the mesh-current equations, we can solve them by using the methods that we discussed in **Section 2.4** for the node-voltage approach. We illustrate with a simple example.

Example 2.14 Mesh-Current Analysis

Solve for the current in each element of the circuit shown in **Figure 2.35**.

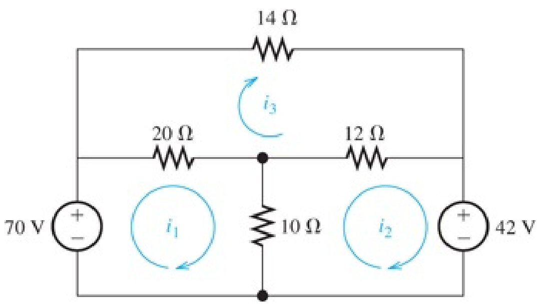


Figure 2.35
Circuit of **Example 2.14.**

Solution

First, we select the mesh currents. Following our standard pattern, we define the mesh currents to flow clockwise around each mesh of the circuit. Then, we write a KVL equation around mesh 1:

$$20(\ i_1 - i_3)\ + 10(\ i_1 - i_2)\ - 70 = 0 \tag{2.56}$$

For meshes 2 and 3, we have:

$$10(\ i_2 - i_1)\ + 12(\ i_2 - i_3)\ + 42 = 0 \tag{2.57}$$

$$20(\ i_3 - i_1)\ + 14 i_3 + 12(\ i_3 - i_2)\ = 0 \tag{2.58}$$

Putting the equations into standard form, we have:

$$30 i_1 - 10 i_2 - 20 i_3 = 70 \tag{2.59}$$

$$-10 i_1 + 22 i_2 - 12 i_3 = -42 \tag{2.60}$$

$$-20 i_1 - 12 i_2 + 46 i_3 = 0 \tag{2.61}$$

In matrix form, the equations become:

$$\begin{bmatrix} 30 & -10 & -20 \\ -10 & 22 & -12 \\ -20 & -12 & 46 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 70 \\ -42 \\ 0 \end{bmatrix}$$

These equations can be solved in a variety of ways. We will demonstrate using MATLAB. We use **R** for the coefficient matrix, because the coefficients often are resistances. Similarly, we use **V** for the column vector for the right-hand side of the equations and **I** for the column vector of the mesh currents. The commands and results are:

```
>> R = [30 -10 -20; -10 22 -12; -20 -12 46];
>> V = [70; -42; 0];
>> I = R\V % Try to avoid using i, which represents the square root of
>> % -1 in MATLAB.
I =
 4.0000
 1.0000
 2.0000
```

Thus, the values of the mesh currents are $i_1 = 4\ \mathrm{A},\ i_2 = 1\ \mathrm{A},$ and $i_3 = 2\ \mathrm{A}.$ Next, we can find the current in any element. For example, the current flowing downward in the $10 - \ \Omega$ resistance is $i_1 - i_2 = 3\ \mathrm{A}.$ ■

Exercise 2.19

Use mesh currents to solve for the current flowing through the $10 - \Omega$ resistance in **Figure 2.36**. Check your answer by combining resistances in series and parallel to solve the circuit. Check a second time by using node voltages.

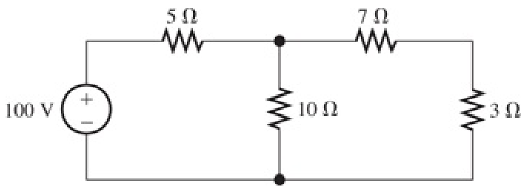


Figure 2.36
Circuit of **Exercise 2.19**.

Answer The current through the $10 - \Omega$ resistance is 5 A.

Exercise 2.20

Use mesh currents to solve for the current flowing through the $2 - \Omega$ resistance in **Figure 2.24** on **page 72**.

Answer The current is 1.613 A directed toward the right.

Writing Mesh Equations Directly in Matrix Form

This is a shortcut way to write the mesh equations in matrix form, *provided that the circuit contains only resistances and independent voltage sources*.

If a circuit contains only resistances and independent voltage sources, and if we select the mesh currents flowing clockwise, the mesh equations can be obtained directly in matrix form using these steps:

1. Make sure that the circuit contains only resistances and independent voltage sources. Select all of the mesh currents to flow in the clockwise direction.
2. Write the sum of the resistances contained in each mesh as the corresponding element on the main diagonal of **R**. In other words, r_{jj} equals the sum of the resistances encountered in going around mesh *j*.
3. Insert the negatives of the resistances common to the corresponding meshes as the off diagonal terms of **R**. Thus, for $i \neq j$, the elements r_{ij} and r_{ji} are the same and are equal to negative of the sum of the resistances common to meshes *i* and *j*.
4. For each element of the **V** matrix, go around the corresponding mesh clockwise, *subtracting* the values of voltage sources for which we encounter the positive reference first and *adding* the values of voltage sources for which we encounter the negative reference first. (We have reversed the rules for adding or subtracting the voltage source values from what we used when writing KVL equations because the elements of **V** correspond to terms on the opposite side of the KVL equations.)

Keep in mind that this procedure does not apply to circuits having current sources or controlled sources.

Example 2.15 Writing Mesh Equations Directly in Matrix Form

Write the mesh equations directly in matrix form for the circuit of **Figure 2.37**.

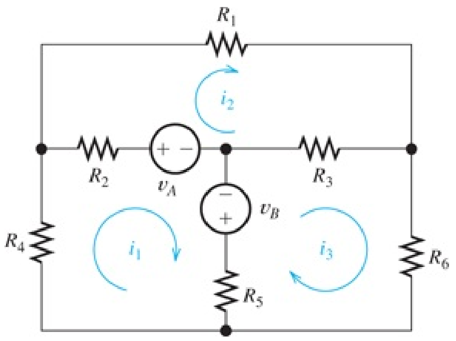


Figure 2.37
Circuit of **Example 2.15**.

Solution

The matrix equation is:

$$\begin{bmatrix} (R_2 + R_4 + R_5) & -R_2 & -R_5 \\ -R_2 & (R_1 + R_2 + R_3) & -R_3 \\ -R_5 & -R_3 & (R_3 + R_5 + R_6) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -v_A + v_B \\ v_A \\ -v_B \end{bmatrix}$$

Notice that mesh 1 includes $R_2,$ $R_4,$ and $R_5,$ so the r_{11} element of **R** is the sum of these resistances. Similarly, mesh 2 contains $R_1,$ $R_2,$ and $R_3,$ so r_{22} is the sum of these resistances. Because R_2 is common to meshes 1 and 2, we have $r_{12} = r_{21} = -R_2.$ Similar observations can be made for the other elements of **R**.

As we go around mesh 1 clockwise, we encounter the positive reference for v_A first and the negative reference for v_B first, so we have $v_1 = -v_A + v_B,$ and so forth.■

Exercise 2.21

Examine the circuit of **Figure 2.34(a)** on **page 82**, and write its mesh equations directly in matrix form.

Answer

$$\begin{bmatrix} (R_2 + R_3) & -R_3 & -R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ -R_2 & 0 & (R_1 + R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

Mesh Currents in Circuits Containing Current Sources

Recall that a current source forces a specified current to flow through its terminals, but the voltage across its terminals is not predetermined. Instead, the voltage across a current source depends on the circuit to which the source is connected. Often, it is not easy to write an expression for the voltage across a current source. *A common mistake made by beginning students is to assume that the voltages across current sources are zero.*

A common mistake made by beginning students is to assume that the voltages across current sources are zero.

Consequently, when a circuit contains a current source, we must depart from the pattern that we use for circuits consisting of voltage sources and resistances. First, consider the circuit of **Figure 2.38**. As usual, we have defined the mesh currents flowing clockwise. If we were to try to write a KVL equation for mesh 1, we would need to include an unknown for the voltage across the current source. Because we do not wish to increase the number of unknowns in our equations, we avoid writing KVL equations for loops that include current sources. In the circuit in **Figure 2.38**, we have defined the current in the current source as i_1. However, we know that this current is 2 A. Thus, we can write

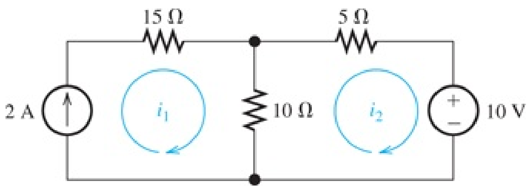


Figure 2.38
In this circuit, we have $i_1 = 2\ A$.

$$i_1 = 2\ \mathrm{A} \tag{2.62}$$

The second equation needed can be obtained by applying KVL to mesh 2, which yields

$$10(\ i_2 - i_1)\ + 5i_2 + 10 = 0 \tag{2.63}$$

Equations 2.62 and **2.63** can readily be solved for i_2. Notice that in this case the presence of a current source facilitates the solution.

Now let us consider the somewhat more complex situation shown in **Figure 2.39**. As usual, we have defined the mesh currents flowing clockwise. We cannot write a KVL equation around mesh 1 because the voltage across the 5-A current source is unknown (and we do not want to increase the number of unknowns in our equations). A solution is to combine meshes 1 and 2 into a **supermesh**. In other words, we write a KVL equation around the periphery of meshes 1 and 2 combined. This yields

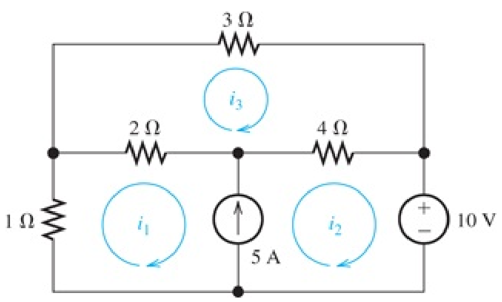


Figure 2.39
A circuit with a current source common to two meshes.

$$i_1 + 2(\ i_1 - i_3)\ + 4(\ i_2 - i_3)\ + 10 = 0 \tag{2.64}$$

Next, we can write a KVL equation for mesh 3:

$$3i_3 + 4(\ i_3 - i_2)\ + 2(\ i_3 - i_1)\ = 0 \tag{2.65}$$

Finally, we recognize that we have defined the current in the current source referenced upward as $i_2 - i_1$. However, we know that the current flowing upward through the current source is 5 A. Thus, we have

$$i_2 - i_1 = 5 \tag{2.66}$$

It is important to realize that **Equation 2.66** is not a KCL equation. Instead, it simply states that we have defined the current referenced upward through the current source in terms of the mesh currents as $i_2 - i_1$, but this current is known to be 5 A. **Equations 2.64**, **2.65**, and **2.66** can be solved for the mesh currents.

> It is important to realize that **Equation 2.66** is not a KCL equation.

Exercise 2.22

Write the equations needed to solve for the mesh currents in **Figure 2.40**.

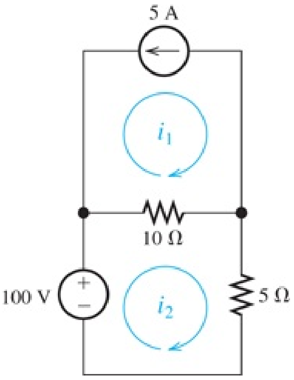


Figure 2.40
The circuit for **Exercise 2.22**.

Answer

$$\begin{aligned} i_1 &= -5\ \mathrm{A} \\ 10(i_2 - i_1) + 5i_2 - 100 &= 0 \end{aligned}$$

Exercise 2.23

Write the equations needed to solve for the mesh currents in **Figure 2.41**. Then solve for the currents.

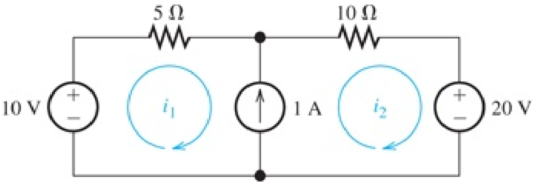


Figure 2.41
The circuit for **Exercise 2.23**.

Answer The equations are $i_2 - i_1 = 1$ and $5i_1 + 10i_2 + 20 - 10 = 0$. Solving, we have $i_1 = -4/3\ \mathrm{A}$ and $i_2 = -1/3\ \mathrm{A}$.

Circuits with Controlled Sources

Controlled sources present a slight additional complication to the mesh-current technique. First, we write equations exactly as we have done for networks with independent sources. Then, we express the controlling variables in terms of the mesh-current variables and substitute into the network equations. We illustrate with an example.

Example 2.16 Mesh-Current Analysis with Controlled Sources

Solve for the currents in the circuit of **Figure 2.42(a)**, which contains a voltage-controlled current source common to the two meshes.

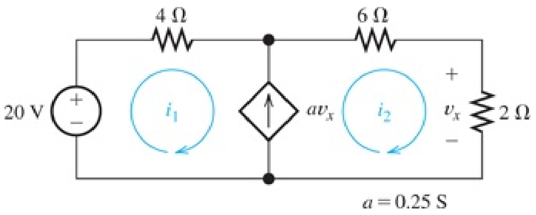


Figure 2.42
A circuit with a voltage-controlled current source. See **Example 2.16**.

Solution

First, we write equations for the mesh currents as we have done for independent sources. Since there is a current source common to mesh 1 and mesh 2, we start by combining the meshes to form a supermesh and write a voltage equation:

$$-20 + 4i_1 + 6i_2 + 2i_2 = 0 \tag{2.67}$$

Then, we write an expression for the source current in terms of the mesh currents:

$$a v_x = 0.25 v_x = i_2 - i_1 \tag{2.68}$$

Next, we see that the controlling voltage is

$$v_x = 2i_2 \tag{2.69}$$

Using **Equation 2.58** to substitute for v_x in **Equation 2.57**, we have

$$\frac{i_2}{2} = i_2 - i_1 \tag{2.70}$$

Finally, we put **Equations 2.67** and **2.70** into standard form, resulting in

$$4i_1 + 8i_2 = 20 \tag{2.71}$$

$$i_1 - \frac{i_2}{2} = 0 \tag{2.72}$$

Solving these equations yields $i_1 = 1\ \text{A}$ and $i_2 = 2\ \text{A}$. ■

Using the principles we have discussed in this section, we can write mesh-current equations for any planar network consisting of sources and resistances.

Step-by-Step Mesh-Current Analysis

Next, we summarize the steps in analyzing planar circuits by the mesh-current technique:

Here is a convenient step-by-step guide to mesh-current analysis.

1. If necessary, redraw the network without crossing conductors or elements. Consider combining resistances in parallel to reduce circuit complexity. Then, define the mesh currents flowing around each of the open areas defined by the network. For consistency, we usually select a clockwise direction for each of the mesh currents, but this is not a requirement.
2. Write network equations, stopping after the number of equations is equal to the number of mesh currents. First, use KVL to write voltage equations for meshes that do not contain current sources. Next, if any current sources are present, write expressions for their currents in terms of the mesh currents. Finally, if a current source is common to two meshes, write a KVL equation for the supermesh.
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the mesh currents. Substitute into the network equations, and obtain equations having only the mesh currents as unknowns.
4. Put the equations into standard form. Solve for the mesh currents by use of determinants or other means.
5. Use the values found for the mesh currents to calculate any other currents or voltages of interest.

Example 2.17 Mesh Current Analysis

Use mesh currents to solve for the value of v_x in the circuit of **Figure 2.43(a)**. (This rather complex circuit has been contrived mainly to illustrate the steps listed above.)

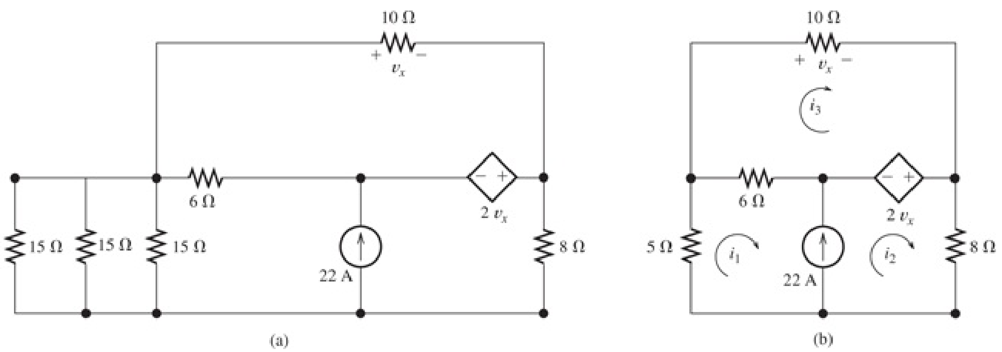


Figure 2.43
Circuit of **Example 2.17.**

Solution

Step 1

First, we combine the $15-\ \Omega$ resistances in parallel to eliminate two meshes. The resulting circuit is shown in **Figure 2.30(b)**. As usual, we select the mesh currents flowing clockwise around the open areas.

Step 2

We cannot write KVL equations for meshes 1 or 2 because we do not know the voltage across the 22-A current source, and we do not want to introduce another unknown. Thus, we write a KVL equation for mesh 3:

$$10i_3 + 2v_x + 6(i_3 - i_1) = 0$$

Next, in terms of the mesh currents the current flowing upward through the current source is $i_2 - i_1$. However, we know that this current is 22 A. Thus, we have:

$$i_2 - i_1 = 22$$

Next, we write a KVL equation for the super mesh formed by combining meshes 1 and 2:

$$5i_1 + 6(i_1 - i_3) - 2v_x + 8i_2 = 0$$

Next, Ohm's law gives

$$v_x = 10i_3$$

Substituting this into the previous equations and putting them into a standard form produces:

$$\begin{aligned} -6i_1 + 36i_3 &= 0 \\ -i_1 + i_2 &= 22 \\ 11i_1 + 8i_2 - 26i_3 &= 0 \end{aligned}$$

Solving these equations produces $i_1 = -12\ \mathrm{A},\ i_2 = 10\ \mathrm{A},$ and $i_3 = -2\ \mathrm{A}$. Then, we have $v_x = 10i_3 = -20\ \mathrm{V}$. ■

Exercise 2.24

Use the mesh-current technique to solve for the currents labeled in the circuits shown in **Figure 2.31** on **page 78**.

Answer

a. $i_a = 1.33\ \mathrm{A}$;

b. $i_b = -0.259\ \mathrm{A}$.

Exercise 2.25

Use the mesh-current technique to solve for the values of i_x and i_y in **Figure 2.32** on **page 78**.

Answer $i_x = 0.5\ \mathrm{A},\ i_y = 2.31\ \mathrm{A}$.

2.6 Thévenin and Norton Equivalent Circuits

In this section, we learn how to replace two-terminal circuits containing resistances and sources by simple equivalent circuits. By a two-terminal circuit, we mean that the original circuit has only two points that can be connected to other circuits. The original circuit can be any complex interconnection of resistances and sources. However, a restriction is that the controlling variables for any controlled sources must appear inside the original circuit.

Thévenin Equivalent Circuits

One type of equivalent circuit is the **Thévenin equivalent**, which consists of an independent voltage source in series with a resistance. This is illustrated in **Figure 2.44**.

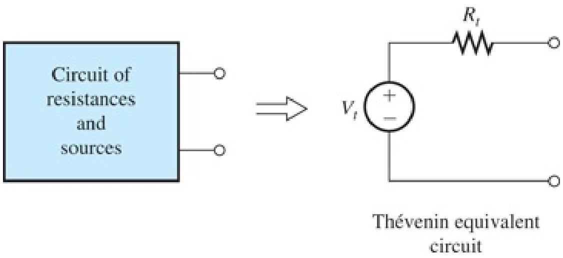


Figure 2.44
A two-terminal circuit consisting of resistances and sources can be replaced by a Thévenin equivalent circuit.

> The Thévenin equivalent circuit consists of an independent voltage source in series with a resistance.

Consider the Thévenin equivalent with open-circuited terminals as shown in **Figure 2.45**. By definition, no current can flow through an open circuit. Therefore, no current flows through the Thévenin resistance, and the voltage across the resistance is zero. Applying KVL, we conclude that

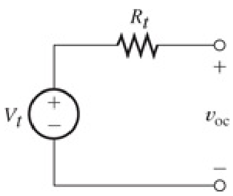


Figure 2.45
Thévenin equivalent circuit with open-circuited terminals. The open-circuit voltage v_{oc} is equal to the Thévenin voltage V_t.

$$\boxed{V_t = v_{oc}}$$

Both the original circuit and the equivalent circuit are required to have the same open-circuit voltage. *Thus, the Thévenin source voltage V_t is equal to the open-circuit voltage of the original network.*

Now, consider the Thévenin equivalent with a short circuit connected across its terminals as shown in **Figure 2.46**. The current flowing in this circuit is

$$i_{\mathrm{sc}} = \frac{V_t}{R_t}$$

Figure 2.46
Thévenin equivalent circuit with short-circuited terminals. The short-circuit current is $i_{sc} = V_t / R_t$.

The Thévenin voltage v_t is equal to the open-circuit voltage of the original network.

The short-circuit current i_{sc} is the same for the original circuit as for the Thévenin equivalent. Solving for the Thévenin resistance, we have

$$R_t = \frac{V_t}{i_{\mathrm{sc}}} \tag{2.73}$$

Using the fact that the Thévenin voltage is equal to the open-circuit voltage of the network, we have

$$\boxed{R_t = \frac{v_{\mathrm{oc}}}{i_{\mathrm{sc}}}} \tag{2.74}$$

Thus, to determine the Thévenin equivalent circuit, we can start by analyzing the original network for its open-circuit voltage and its short-circuit current. The Thévenin voltage equals the open-circuit voltage, and the Thévenin resistance is given by **Equation 2.74**.

The Thévenin resistance is equal to the open-circuit voltage divided by the short-circuit current.

Example 2.18 Determining the Thévenin Equivalent Circuit

Find the Thévenin equivalent for the circuit shown in **Figure 2.47(a)**.

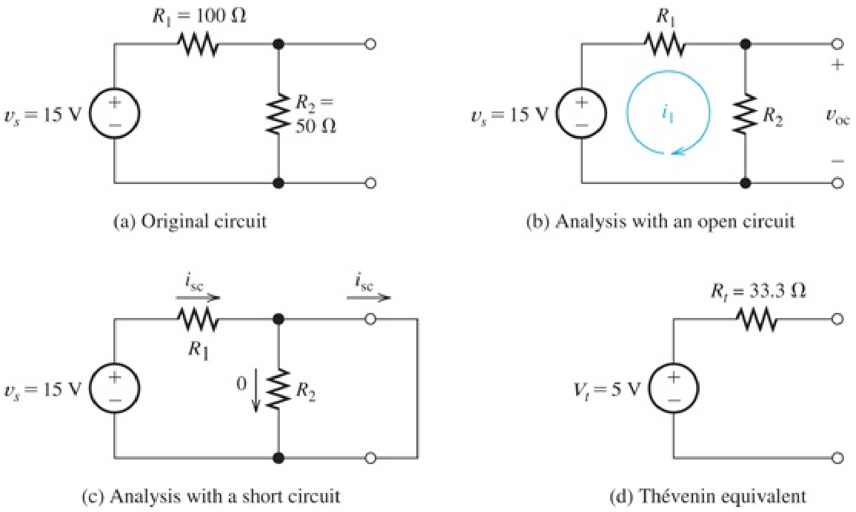


Figure 2.47
Circuit for **Example 2.18**.

Solution

First, we analyze the circuit with open-circuited terminals. This is shown in **Figure 2.47(b)**. The resistances R_1 and R_2 are in series and have an equivalent resistance of $R_1 + R_2$. Therefore, the current circulating is

$$i_1 = \frac{v_s}{R_1 + R_2} = \frac{15}{100 + 50} = 0.10 \text{ A}$$

The open-circuit voltage is the voltage across R_2:

$$v_{\text{oc}} = R_2 i_1 = 50 \times 0.10 = 5 \text{ V}$$

Thus, the Thévenin voltage is $V_t = 5$ V.

Now, we consider the circuit with a short circuit connected across its terminals as shown in **Figure 2.47(c)**. By definition, the voltage across a short circuit is zero. Hence, the voltage across R_2 is zero, and the current through it is zero, as shown in the figure. Therefore, the short-circuit current i_{sc} flows through R_1. The source voltage v_s appears across R_1, so we can write

$$i_{\text{sc}} = \frac{v_s}{R_1} = \frac{15}{100} = 0.15 \text{ A}$$

Now, we can use **Equation 2.74** to determine the Thévenin resistance:

$$R_t = \frac{v_{\text{oc}}}{i_{\text{sc}}} = \frac{5 \text{ V}}{0.15 \text{ A}} = 33.3\ \ \Omega$$

The Thévenin equivalent circuit is shown in **Figure 2.47(d)**. ■

Exercise 2.26

Find the Thévenin equivalent circuit for the circuit shown in **Figure 2.48**.

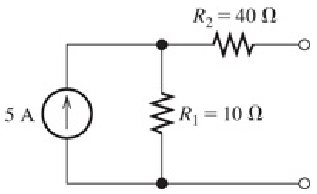


Figure 2.48
Circuit for **Exercise 2.26**.

Answer $V_t = 50\ \mathrm{V},\ R_t = 50\ \ \Omega$.

Finding the Thévenin Resistance Directly.

If a network contains no dependent sources, there is an alternative way to find the Thévenin resistance. First, we *zero* the sources in the network. In zeroing a voltage source, we reduce its voltage to zero. A voltage source with zero voltage is equivalent to a short circuit.

> When zeroing a current source, it becomes an open circuit. When zeroing a voltage source, it becomes a short circuit.

In zeroing a current source, we reduce its current to zero. By definition, an element that always carries zero current is an open circuit. *Thus, to zero the independent sources, we replace voltage sources with short circuits and replace current sources with open circuits.*

> We can find the Thévenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.

Figure 2.49 shows a Thévenin equivalent before and after zeroing its voltage source. Looking back into the terminals after the source is zeroed, we see the Thévenin resistance. *Thus, we can find the Thévenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.*

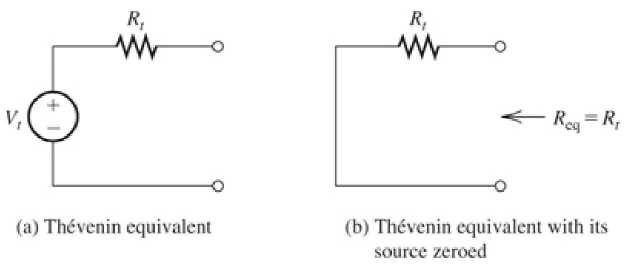


Figure 2.49
When the source is zeroed, the resistance seen from the circuit terminals is equal to the Thévenin resistance.

Example 2.19 Zeroing Sources to Find Thévenin Resistance

Find the Thévenin resistance for the circuit shown in **Figure 2.50(a)** by zeroing the sources. Then, find the short-circuit current and the Thévenin equivalent circuit.

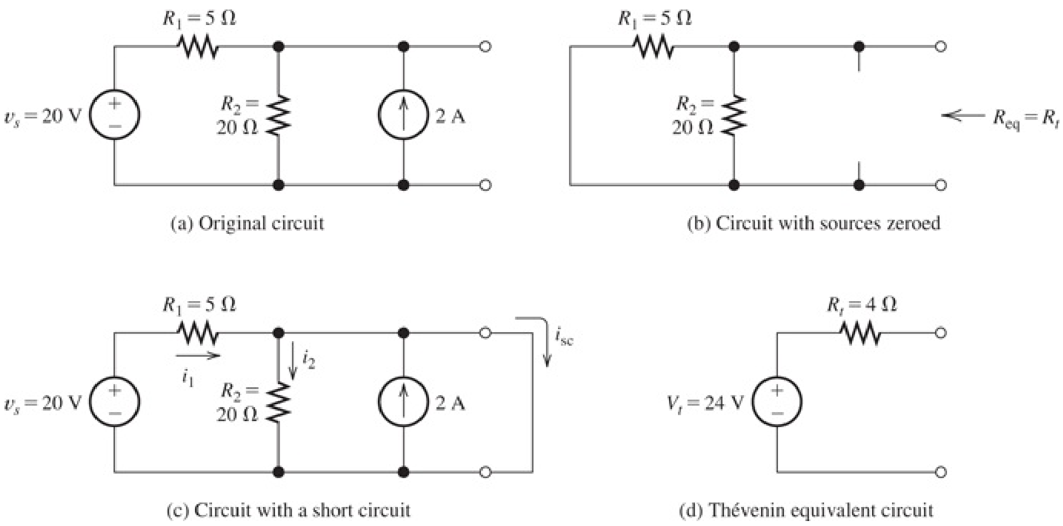


Figure 2.50
Circuit for **Example 2.19**.

Solution
To zero the sources, we replace the voltage source by a short circuit and replace the current source by an open circuit. The resulting circuit is shown in **Figure 2.50(b)**.

The Thévenin resistance is the equivalent resistance between the terminals. This is the parallel combination of R_1 and R_2, which is given by

$$R_t = R_{\text{eq}} = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/5 + 1/20} = 4\ \ \Omega$$

Next, we find the short-circuit current for the circuit. The circuit is shown in **Figure 2.50(c)**. In this circuit, the voltage across R_2 is zero because of the short circuit. Thus, the current through R_2 is zero:

$$i_2 = 0$$

Furthermore, the voltage across R_1 is equal to 20 V. Thus, the current is

$$i_1 = \frac{v_s}{R_1} = \frac{20}{5} = 4\ \text{A}$$

Finally, we write a current equation for the node joining the top ends of R_2 and the 2-A source. Setting the sum of the currents entering equal to the sum of the currents leaving, we have

$$i_1 + 2 = i_2 + i_{\text{sc}}$$

This yields $i_{\text{sc}} = 6\ \text{A}$.

Now, the Thévenin voltage can be found. Applying **Equation 2.74**, we get

$$V_t = R_t i_{\text{sc}} = 4 \times 6 = 24\ \text{V}$$

The Thévenin equivalent circuit is shown in **Figure 2.50(d)**. ■

Exercise 2.27

Use node-voltage analysis of the circuit shown in **Figure 2.50(a)** to show that the open-circuit voltage is equal to the Thévenin voltage found in **Example 2.19**.

Exercise 2.28

Find the Thévenin resistance for each of the circuits shown in **Figure 2.51** by zeroing the sources.

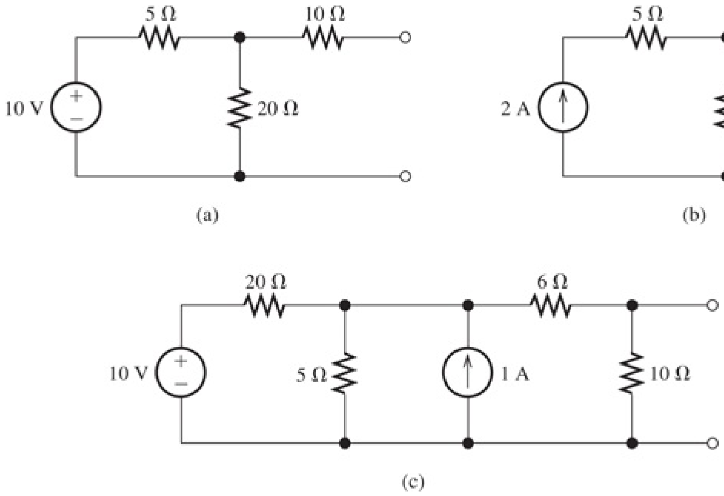


Figure 2.51
Circuits for **Exercise 2.28**.

Answer

a. $R_t = 14\ \ \Omega$;

b. $R_t = 30\ \ \Omega$;

c. $R_t = 5\ \ \Omega$.

We complete our discussion of Thévenin equivalent circuits with one more example.

Example 2.20 Thévenin Equivalent of a Circuit with a Dependent Source

Find the Thévenin equivalent for the circuit shown in **Figure 2.52(a)**.

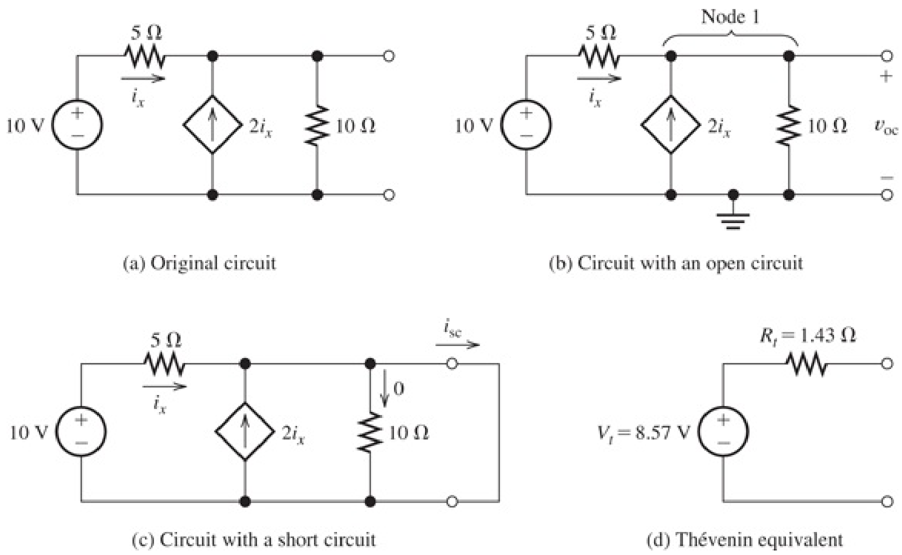


Figure 2.52
Circuit for **Example 2.20**.

Solution

Because this circuit contains a dependent source, we cannot find the Thévenin resistance by zeroing the sources and combining resistances in series and parallel. Thus, we must analyze the circuit to find the open-circuit voltage and the short-circuit current.

> If a circuit contains a dependent source, we cannot find the Thévenin resistance by zeroing the sources and combining resistances in series and parallel.

We start with the open-circuit voltage. Consider **Figure 2.52(b)**. We use node-voltage analysis, picking the reference node at the bottom of the circuit. Then, v_{oc} is the unknown node-voltage variable. First, we write a current equation at node 1.

$$i_x + 2i_x = \frac{v_{oc}}{10} \tag{2.75}$$

Next, we write an expression for the controlling variable i_x in terms of the node voltage v_{oc}:

$$i_x = \frac{10 - v_{oc}}{5}$$

Substituting this into **Equation 2.75**, we have

$$3\frac{10 - v_{oc}}{5} = \frac{v_{oc}}{10}$$

Solving, we find that $v_{oc} = 8.57\ \text{V}$.

Now, we consider short-circuit conditions as shown in **Figure 2.52(c)**. In this case, the current through the $10 - \ \Omega$ resistance is zero. Furthermore, we get

$$i_x = \frac{10\ \text{V}}{5\ \ \Omega} = 2\ \text{A}$$

and

$$i_{sc} = 3i_x = 6\ \text{A}$$

Next, we use **Equation 2.74** to compute the Thévenin resistance:

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{8.57\ \text{V}}{6\ \text{A}} = 1.43\ \ \Omega$$

Finally, the Thévenin equivalent circuit is shown in **Figure 2.52(d)**. ■

Norton Equivalent Circuit

Another type of equivalent, known as the **Norton equivalent circuit**, is shown in **Figure 2.53**. It consists of an independent current source I_n in parallel with the Thévenin resistance. Notice that if we zero the Norton current source, replacing it by an open circuit, the Norton equivalent becomes a resistance of R_t. This also happens if we zero the voltage source in the Thévenin equivalent by replacing the voltage source by a short circuit. Thus, the resistance in the Norton equivalent is the same as the Thévenin resistance.

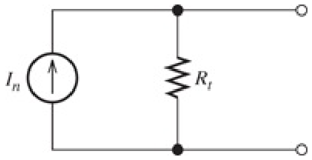


Figure 2.53
The Norton equivalent circuit consists of an independent current source I_n in parallel with the Thévenin resistance R_t.

Consider placing a short circuit across the Norton equivalent as shown in **Figure 2.54**. In this case, the current through R_t is zero. *Therefore, the Norton current is equal to the short-circuit current*:

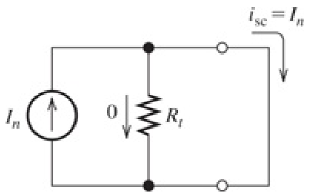


Figure 2.54
The Norton equivalent circuit with a short circuit across its terminals.

$$\boxed{I_n = i_{sc}}$$

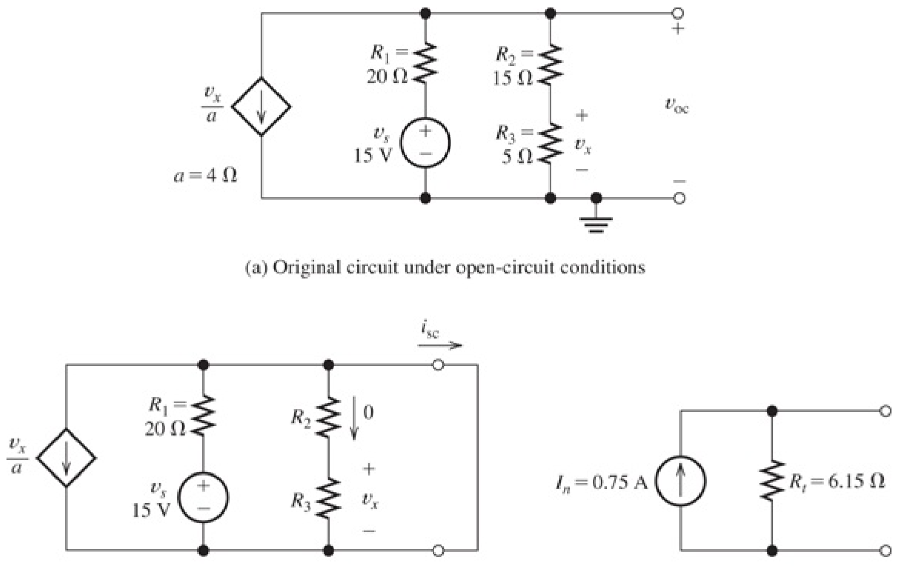
We can find the Norton equivalent by using the same techniques as we used for the Thévenin equivalent.

Step-by-Step Thévenin/Norton-Equivalent-Circuit Analysis

1. Perform two of these:
    a. Determine the open-circuit voltage $V_t = v_{oc}$.
    b. Determine the short-circuit current $I_n = i_{sc}$.
    c. Zero the independent sources and find the Thévenin resistance R_t looking back into the terminals. Do not zero dependent sources.
2. Use the equation $V_t = R_t I_n$ to compute the remaining value.
3. The Thévenin equivalent consists of a voltage source V_t in series with R_t.
4. The Norton equivalent consists of a current source I_n in parallel with R_t.

Example 2.21 Norton Equivalent Circuit

Find the Norton equivalent for the circuit shown in **Figure 2.55(a)**.



(a) Original circuit under open-circuit conditions

(b) Circuit with a short circuit

(c) Norton equivalent circuit

Figure 2.55

Circuit of **Example 2.21.**

Solution

Because the circuit contains a controlled source, we cannot zero the sources and combine resistances to find the Thévenin resistance. First, we consider the circuit with an open circuit as shown in **Figure 2.53(a)**. We treat v_{oc} as a node-voltage variable. Writing a current equation at the top of the circuit, we have

$$\frac{v_x}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0 \tag{2.76}$$

Next, we use the voltage-divider principle to write an expression for v_x in terms of resistances and v_{oc}:

$$v_x = \frac{R_3}{R_2 + R_3} v_{oc} = 0.25 v_{oc}$$

Substituting into **Equation 2.76**, we find that

$$\frac{0.25 v_{oc}}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

Substituting resistance values and solving, we observe that $v_{oc} = 4.62\ \mathrm{V}$.

Next, we consider short-circuit conditions as shown in **Figure 2.55(b)**. In this case, the current through R_2 and R_3 is zero. Thus, $v_x = 0$, and the controlled current source appears as an open circuit. The short-circuit current is given by

$$i_{sc} = \frac{v_s}{R_1} = \frac{15\ \mathrm{V}}{20\ \ \Omega} = 0.75\ \mathrm{A}$$

Now, we can find the Thévenin resistance:

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{4.62}{0.75} = 6.15\ \ \Omega$$

The Norton equivalent circuit is shown in **Figure 2.55(c)**. ■

Exercise 2.29

Find the Norton equivalent for each of the circuits shown in **Figure 2.56**.

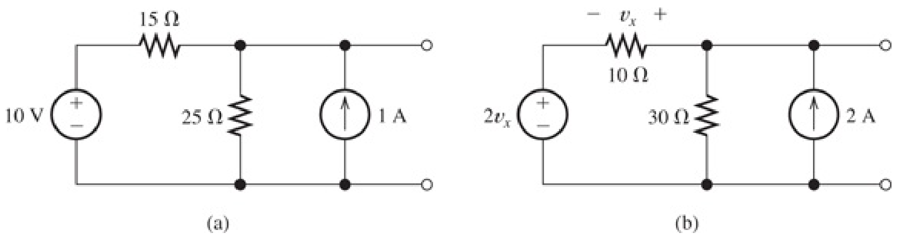


Figure 2.56
Circuits for **Exercise 2.29**.

Answer

a. $I_n = 1.67\ \mathrm{A},\ R_t = 9.375\ \ \Omega\,;$

b. $I_n = 2A,\ R_t = 15\ \ \Omega\ .$

Source Transformations

We can replace a voltage source in series with a resistance by a Norton equivalent circuit, which consists of a current source in parallel with the resistance. This is called a **source transformation** and is illustrated in **Figure 2.57**. The two circuits are identical in terms of their external behavior. In other words, the voltages and currents at terminals *a* and *b* remain the same after the transformation is made. However, in general, the current flowing through R_t is different for the two circuits. For example, suppose that the two circuits shown in **Figure 2.57** are open circuited. Then no current flows through the resistor in series with the voltage source, but the current I_n flows through the resistance in parallel with the current source.

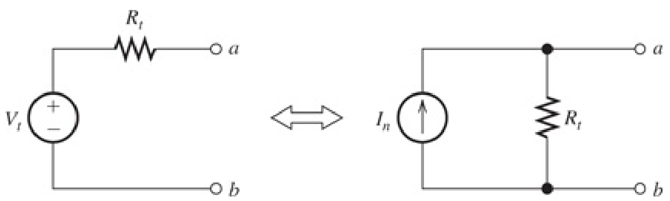


Figure 2.57
A voltage source in series with a resistance is externally equivalent to a current source in parallel with the resistance, provided that $I_n = V_t/R_t$.

> Here is a "trick" question that you might have some fun with: Suppose that the circuits of **Figure 2.57** are placed in identical black boxes with the terminals accessible from outside the box. How could you determine which box contains the Norton equivalent? An answer can be found at the end of the chapter summary on the top of **page 111**.

In making source transformations, it is very important to maintain the proper relationship between the reference direction for the current source and the polarity of the voltage source. If the positive polarity is closest to terminal *a*, the current reference must point toward terminal *a*, as shown in **Figure 2.57**.

Sometimes, we can simplify the solution of a circuit by source transformations. This is similar to solving circuits by combining resistances in series or parallel. We illustrate with an example.

Example 2.22 Using Source Transformations

Use source transformations to aid in solving for the currents i_1 and i_2 shown in **Figure 2.58(a)**.

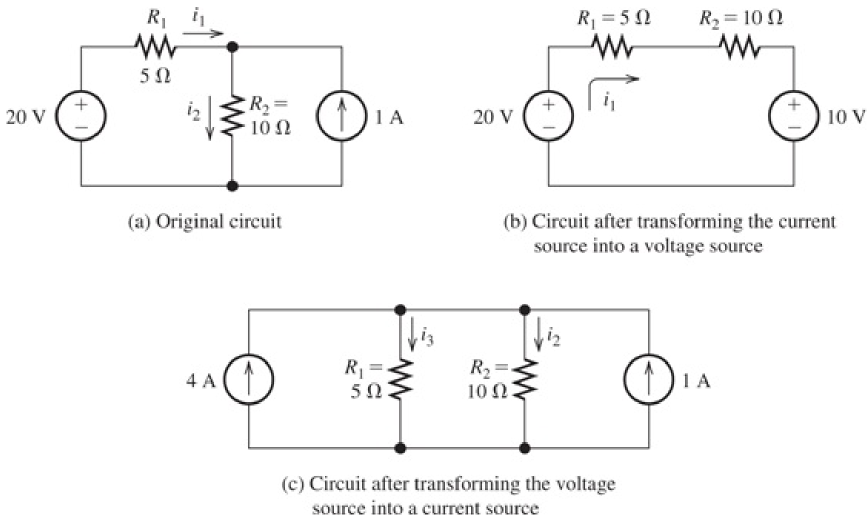


Figure 2.58
Circuit for **Example 2.22**.

Solution

Several approaches are possible. One is to transform the 1-A current source and R_2 into a voltage source in series with R_2. This is shown in **Figure 2.58(b)**. Notice that the positive polarity of the 10-V source is at the top, because the 1-A source reference points upward. The single-loop circuit of **Figure 2.58(b)** can be solved by writing a KVL equation. Traveling clockwise and summing voltages, we have

$$R_1 i_1 + R_2 i_1 + 10 - 20 = 0$$

Solving and substituting values, we get

$$i_1 = \frac{10}{R_1 + R_2} = 0.667 \text{ A}$$

Then in the original circuit, we can write a current equation at the top node and solve for i_2:

$$i_2 = i_1 + 1 = 1.667 \text{ A}$$

Another approach is to transform the voltage source and R_1 into a current source in parallel with R_1. Making this change to the original circuit yields the circuit shown in **Figure 2.58(c)**. Notice that we have labeled the current through R_1 as i_3 rather than i_1. This is because the current in the resistance of the transformed source is not the same as in the original circuit. Now, in **Figure 2.58(c)**, we see that a total current of 5 A flows into the parallel combination of R_1 and R_2. Using the current-division principle, we find the current through R_2:

$$i_2 = \frac{R_1}{R_1 + R_2} i_{\text{total}} = \frac{5}{5 + 10}(5) = 1.667 \text{ A}$$

This agrees with our previous result. ■

Exercise 2.30

Use two different approaches employing source transformations to solve for the values of i_1 and i_2 in **Figure 2.59**.

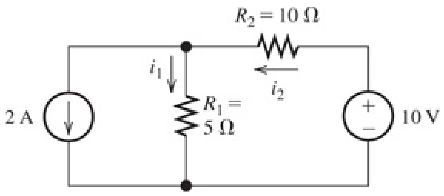


Figure 2.59
Circuit for **Exercise 2.30**.

In the first approach, transform the current source and R_1 into a voltage source in series with R_1. (Make sure in making the transformation that the polarity of the voltage source bears the correct relationship to the current reference direction.) Then solve the transformed circuit and determine the values of i_1 and i_2.

In the second approach, starting with the original circuit, transform the 10-V source and R_2 into a current source in parallel with R_2. Then solve the transformed circuit and determine the values of i_1 and i_2. Of course, the answers should be the same for both approaches.

Answer $i_1 = -0.667\ \text{A},\ i_2 = 1.333\ \text{A}.$

Maximum Power Transfer

Suppose that we have a two-terminal circuit and we want to connect a load resistance R_L such that the maximum possible power is delivered to the load. This is illustrated in **Figure 2.60(a)**. To analyze this problem, we replace the original circuit by its Thévenin equivalent as shown in **Figure 2.60(b)**. The current flowing through the load resistance is given by

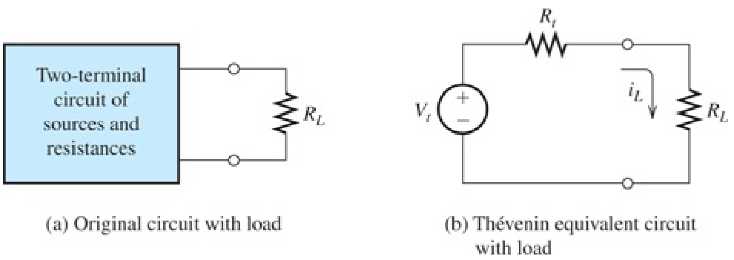


Figure 2.60
Circuits for analysis of maximum power transfer.

$$i_L = \frac{V_t}{R_t + R_L}$$

The power delivered to the load is

$$p_L = i_L^2 R_L$$

Substituting for the current, we have

$$p_L = \frac{V_t^2 R_L}{\left(R_t + R_L\right)^2} \tag{2.77}$$

To find the value of the load resistance that maximizes the power delivered to the load, we set the derivative of p_L with respect to R_L equal to zero:

$$\frac{dp_L}{dR_L} = \frac{V_t^2 \left(R_t + R_L \right)^2 - 2V_t^2 R_L \left(R_t + R_L \right)}{\left(R_t + R_L \right)^4} = 0$$

Solving for the load resistance, we have

$$\boxed{R_L = R_t}$$

The load resistance that absorbs the maximum power from a two-terminal circuit is equal to the Thévenin resistance.

Thus, the load resistance that absorbs the maximum power from a two-terminal circuit is equal to the Thévenin resistance. The maximum power is found by substituting $R_L = R_t$ into **Equation 2.77**. The result is

$$P_{L\,\max} = \frac{V_t^2}{4R_t} \tag{2.78}$$

An All-Too-Common Example.

You may have had difficulty in starting your car on a frigid morning. The battery in your car can be represented by a Thévenin equivalent circuit. It turns out that the Thévenin voltage of the battery does not change greatly with temperature. However, when the battery is very cold, the chemical reactions occur much more slowly and its Thévenin resistance is much higher. Thus, the power that the battery can deliver to the starter motor is greatly reduced.

Example 2.23 Determining Maximum Power Transfer

Find the load resistance for maximum power transfer from the circuit shown in **Figure 2.61**. Also, find the maximum power.

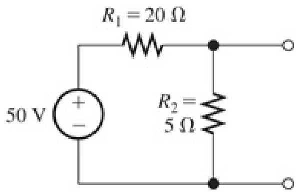


Figure 2.61
Circuit for **Example 2.23**.

Solution
First, we must find the Thévenin equivalent circuit. Zeroing the voltage source, we find that the resistances R_1 and R_2 are in parallel. Thus, the Thévenin resistance is

$$R_t = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/20 + 1/5} = 4\ \ \Omega$$

The Thévenin voltage is equal to the open-circuit voltage. Using the voltage-division principle, we find that

$$V_t = v_{\text{oc}} = \frac{R_2}{R_1 + R_2}\left(50\right) = \frac{5}{5 + 20}\left(50\right) = 10\ \text{V}$$

Hence, the load resistance that receives maximum power is

$$R_L = R_t = 4\ \ \Omega$$

and the maximum power is given by **Equation 2.78**:

$$P_{L\ \max} = \frac{V_t^2}{4R_t} = \frac{10^2}{4 \times 4} = 6.25\ \text{W}$$

PRACTICAL APPLICATION

2.1 An Important Engineering Problem: Energy-Storage Systems for Electric Vehicles

Imagine pollution-free electric vehicles with exciting performance and 500-mile range. They do not exist, but they are the target of an ongoing large-scale engineering effort to which you may contribute. Such electric vehicles (EVs) are a worthwhile goal because they can be very efficient in their use of energy, particularly in stop-and-go traffic. Kinetic energy can be recovered during braking and saved for later use during acceleration. Furthermore, EVs emit little pollution into crowded urban environments.

So far, EV range and performance remains less than ideal. The availability of suitable energy-storage devices is the key stumbling block in achieving better EVs (and a multitude of other highly desirable devices, such as smart phones that do not need recharging for a week).

In **Chapter 3**, we will see that capacitors and inductors are capable of storing electrical energy. However, it turns out that their energy content per unit volume is too small to make them a practical solution for EVs. The energy content of modern rechargeable batteries is better but still not on a par with the energy content of gasoline, which is approximately 10,000 watt-hours/liter (Wh/L). In contrast, the energy content of nickel-metal hydride batteries used in current EVs is about 175 Wh/L. Lithium-ion batteries under current development are expected to increase this to about 300 Wh/L. Thus, even allowing for the relative inefficiency of the internal combustion engine in converting chemical energy to mechanical energy, much more usable energy can be obtained from gasoline than from current batteries of comparable volume.

Although EVs do not emit pollutants at the point of use, the mining, refining, and disposal of metals pose grave environmental dangers. We must always consider the entire environmental (as well as economic) impact of the systems we design. As an engineer, you can do a great service to humanity by accepting the challenge to develop safe, clean systems for storing energy in forms that are readily converted to and from electrical form.

Naturally, one possibility currently under intense development is improved electrochemical batteries based on nontoxic chemicals. Another option is a mechanical flywheel system that would be coupled through an electrical generator to electric drive motors. Still another solution is a hybrid vehicle that uses a small internal combustion engine, an electrical generator, an energy-storage system, and electrical drive motors. The engine achieves low pollution levels by being optimized to run at a constant load while charging a relatively small energy-storage system. When the storage capacity becomes full, the engine shuts down automatically and the vehicle runs on stored energy. The engine is just large enough to keep up with energy demands under high-speed highway conditions.

Whatever form the ultimate solution to vehicle pollution may take, we can anticipate that it will include elements from mechanical, chemical, manufacturing, and civil engineering in close combination with electrical-engineering principles.

Application of Maximum Power Transfer.

When a load resistance equals the internal Thévenin resistance of the source, half of the power is dissipated in the source resistance and half is delivered to the load. In higher power applications for which efficiency is important, we do not usually design for maximum power transfer. For example, in designing an electric vehicle, we would want to deliver the energy stored in the batteries mainly to the drive motors and minimize the power loss in the resistance of the battery and wiring. This system would approach maximum power transfer rarely when maximum acceleration is needed.

On the other hand, when small amounts of power are involved, we would design for maximum power transfer. For example, we would design a radio receiver to extract the maximum signal power from the receiving antenna. In this application, the power is very small, typically much less than one microwatt, and efficiency is not a consideration.

2.7 Superposition Principle

Suppose that we have a circuit composed of resistances, linear dependent sources, and *n* independent sources. (We will explain the term *linear* dependent source shortly.) The current flowing through a given element (or the voltage across it) is called a **response**, because the currents and voltages appear in response to the independent sources.

Recall that we zeroed the independent sources as a method for finding the Thévenin resistance of a two-terminal circuit. To zero a source, we reduce its value to zero. Then, current sources become open circuits, and voltage sources become short circuits.

Now, consider zeroing all of the independent sources except the first, observe a particular response (a current or voltage), and denote the value of that response as r_1. (We use the symbol *r* rather than *i* or *v* because the response could be either a current or a voltage.) Similarly, with only source 2 activated, the response is denoted as r_2, and so on. The response with all the sources activated is called the total response, denoted as r_T. The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually. In equation form, this is

$$\boxed{r_T = r_1 + r_2 + \ldots + r_n} \tag{2.79}$$

The superposition principle states that any response in a linear circuit is the sum of the responses for each independent source acting alone with the other independent sources zeroed. When zeroed, current sources become open circuits and voltage sources become short circuits.

Next, we illustrate the validity of superposition for the example circuit shown in **Figure 2.62**. In this circuit, there are two independent sources: the first is the voltage source v_{s1}, and the second is the current source i_{s2}. Suppose that the response of interest is the voltage across the resistance R_2.

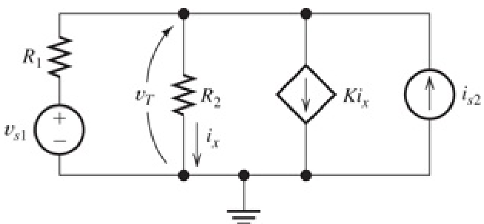


Figure 2.62
Circuit used to illustrate the superposition principle.

First, we solve for the total response v_T by solving the circuit with both sources in place. Writing a current equation at the top node, we obtain

$$\frac{v_T - v_{s1}}{R_1} + \frac{v_T}{R_2} + Ki_x = i_{s2} \tag{2.80}$$

The control variable i_x is given by

$$i_x = \frac{v_T}{R_2} \tag{2.81}$$

Substituting **Equation 2.81** into **Equation 2.80** and solving for the total response, we get

$$v_T = \frac{R_2}{R_1 + R_2 + KR_1} v_{s1} + \frac{R_1 R_2}{R_1 + R_2 + KR_1} i_{s2} \tag{2.82}$$

If we set i_{s2} to zero, we obtain the response to v_{s1} acting alone:

$$v_1 = \frac{R_2}{R_1 + R_2 + KR_1} v_{s1} \tag{2.83}$$

Similarly, if we set v_{s1} equal to zero in **Equation 2.82**, the response due to i_{s2} is given by

$$v_2 = \frac{R_1 R_2}{R_1 + R_2 + KR_1} i_{s2} \tag{2.84}$$

Comparing **Equations 2.82**, **2.83**, and **2.84**, we see that

$$v_T = v_1 + v_2$$

Thus, as expected from the superposition principle, the total response is equal to the sum of the responses for each of the independent sources acting individually.

Notice that if we zero both of the independent sources ($v_{s1} = 0$ and $i_{s2} = 0$), the response becomes zero. Hence, the dependent source does not contribute to the total response. However, the dependent source affects the contributions of the two independent sources. This is evident because the gain parameter *K* of the dependent source appears in the expressions for both v_1 and v_2. *In general, dependent sources do not contribute a separate term to the total response, and we must not zero dependent sources in applying superposition.*

Dependent sources do not contribute a separate term to the total response, and we must not zero dependent sources in applying superposition.

Linearity

If we plot voltage versus current for a resistance, we have a straight line. This is illustrated in **Figure 2.63**. Thus, we say that Ohm's law is a **linear equation**. Similarly, the current in the controlled source shown in **Figure 2.62** is given by $i_{\text{cs}} = Ki_x,$ which is also a linear equation. In this book, the term **linear controlled source** means a source whose value is a constant times a control variable that is a current or a voltage appearing in the network.

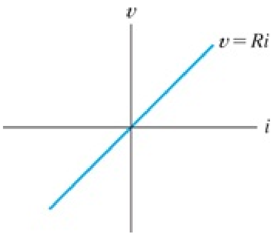


Figure 2.63
A resistance that obeys Ohm's law is linear.

Some examples of nonlinear equations are

$$\begin{aligned} v &= 10i^2 \\ i_{\text{cs}} &= K\cos(i_x) \end{aligned}$$

and

$$i = e^v$$

The superposition principle does not apply to any circuit that has element(s) described by nonlinear equation(s). We will encounter nonlinear elements later in our study of electronic circuits.

> The superposition principle does not apply to any circuit that has element(s) described by nonlinear equation(s).

Furthermore, superposition does not apply for power in resistances, because $P = v^2/R$ and $P = i^2R$ are nonlinear equations.

Using Superposition to Solve Circuits

We can apply superposition in circuit analysis by analyzing the circuit for each source separately. Then, we add the individual responses to find the total response. Sometimes, the analysis of a circuit is simplified by considering each independent source separately. We illustrate with an example.

Example 2.24 Circuit Analysis Using Superposition

Use superposition in solving the circuit shown in **Figure 2.64(a)** for the voltage v_T.

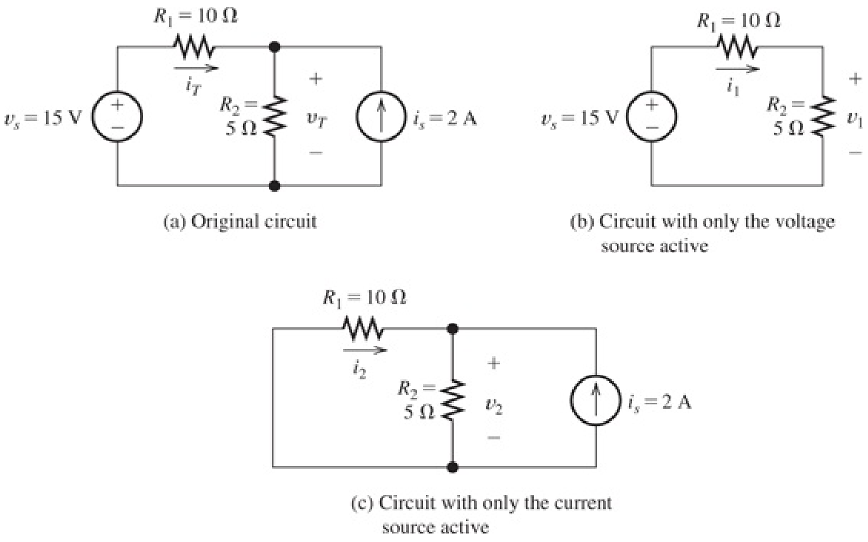


Figure 2.64
Circuit for **Example 2.24** and **Exercise 2.31**.

Solution

We analyze the circuit with only one source activated at a time and add the responses. **Figure 2.64(b)** shows the circuit with only the voltage source active. The response can be found by applying the voltage-division principle:

$$v_1 = \frac{R_2}{R_1 + R_2} v_s = \frac{5}{5 + 10}(15) = 5\ \text{V}$$

Next, we analyze the circuit with only the current source active. The circuit is shown in **Figure 2.64(c)**. In this case, the resistances R_1 and R_2 are in parallel, and the equivalent resistance is

$$R_{\text{eq}} = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/10 + 1/5} = 3.33\ \ \Omega$$

The voltage due to the current source is given by

$$v_2 = i_s R_{\text{eq}} = 2 \times 3.33 = 6.66\ \text{V}$$

Finally, we obtain the total response by adding the individual responses:

$$v_T = v_1 + v_2 = 5 + 6.66 = 11.66\ \text{V}$$

Exercise 2.31

Find the responses i_1, i_2, and i_T for the circuit of **Figure 2.64**.

Answer $i_1 = 1\ \text{A},\ i_2 = -0.667\ \text{A},\ i_T = 0.333\ \text{A}$.

Exercise 2.32

Use superposition to find the responses v_T and i_T for the circuit shown in **Figure 2.65**.

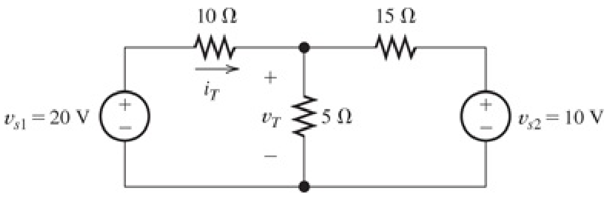


Figure 2.65
Circuit for **Exercise 2.32**.

Answer $v_1 = 5.45$ V, $v_2 = 1.82$ V, $v_T = 7.27$ V, $i_1 = 1.45$ A, $i_2 = -0.181$ A, $i_T = 1.27$ A.

2.8 Wheatstone Bridge

The **Wheatstone bridge** is a circuit used to measure unknown resistances. For example, it is used by mechanical and civil engineers to measure the resistances of strain gauges in experimental stress studies of machines and buildings. The circuit is shown in **Figure 2.66**. The circuit consists of a dc voltage source v_s, a detector, the unknown resistance to be measured R_x, and three precision resistors, R_1, R_2, and R_3. Usually, R_2 and R_3 are adjustable resistances, which is indicated in the figure by the arrow drawn through the resistance symbols.

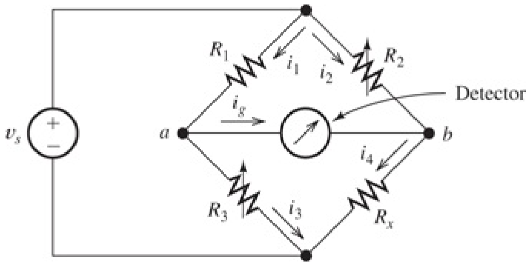


Figure 2.66

The Wheatstone bridge. When the Wheatstone bridge is balanced, $i_g = 0$ and $v_{ab} = 0$.

The Wheatstone bridge is used by mechanical and civil engineers to measure the resistances of strain gauges in experimental stress studies of machines and buildings.

The detector is capable of responding to very small currents (less than one microampere). However, it is not necessary for the detector to be calibrated. It is only necessary for the detector to indicate whether or not current is flowing through it. Often, the detector has a pointer that deflects one way or the other, depending on the direction of the current through it.

In operation, the resistors R_2 and R_3 are adjusted in value until the detector indicates zero current. In this condition, we say that the bridge is **balanced**. Then, the current i_g and the voltage across the detector v_{ab} are zero.

Applying KCL at node *a* (**Figure 2.66**) and using the fact that $i_g = 0$, we have

$$i_1 = i_3 \tag{2.85}$$

Similarly, at node *b*, we get

$$i_2 = i_4 \tag{2.86}$$

Writing a KVL equation around the loop formed by R_1, R_2, and the detector, we obtain

$$R_1 i_1 + v_{ab} = R_2 i_2 \tag{2.87}$$

However, when the bridge is balanced, $v_{ab} = 0$, so that

$$R_1 i_1 = R_2 i_2 \tag{2.88}$$

Similarly, for the loop consisting of R_3, R_4, and the detector, we have

$$R_3 i_3 = R_x i_4 \tag{2.89}$$

Using Equations 2.85 and 2.86 to substitute into Equation 2.89, we obtain

$$R_3 i_1 = R_x i_2 \tag{2.90}$$

Dividing each side of Equation 2.90 by the respective side of Equation 2.88, we find that

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

Finally, solving for the unknown resistance, we have

$$\boxed{R_x = \frac{R_2}{R_1} R_3} \tag{2.91}$$

Often, in commercial bridges, a multiposition switch selects an order-of-magnitude scale factor R_2/R_1 by changing the value of R_2. Then, R_3 is adjusted by means of calibrated switches until balance is achieved. Finally, the unknown resistance R_x is the scale factor times the value of R_3.

Example 2.25 Using a Wheatstone Bridge to Measure Resistance

In a certain commercial Wheatstone bridge, R_1 is a fixed $1-\mathrm{k}\,\Omega$ resistor, R_3 can be adjusted in $1-\ \Omega$ steps from 0 to $1100\ \ \Omega$, and R_2 can be selected to be $1\ \mathrm{k}\,\Omega$, $10\ \mathrm{k}\,\Omega$, $100\ \mathrm{k}\,\Omega$, or $1\ \mathrm{M}\,\Omega$.

a. Suppose that the bridge is balanced with $R_3 = 732\ \ \Omega$ and $R_2 = 10\ \mathrm{k}\,\Omega$. What is the value of R_x?
b. What is the largest value of R_x for which the bridge can be balanced?
c. Suppose that $R_2 = 1\ \mathrm{M}\,\Omega$. What is the increment between values of R_x for which the bridge can be precisely balanced?

Solution

a. From Equation 2.91, we have

$$R_x = \frac{R_2}{R_1} R_3 = \frac{10\ \mathrm{k}\,\Omega}{1\ \mathrm{k}\,\Omega} \times 732\ \ \Omega\ = 7320\ \ \Omega$$

Notice that R_2/R_1 is a scale factor that can be set at 1, 10, 100, or 1000, depending on the value selected for R_2. The unknown resistance is the scale factor times the value of R_3 needed to balance the bridge.

b. The maximum resistance for which the bridge can be balanced is determined by the largest values available for R_2 and R_3. Thus,

$$R_{x\ \max} = \frac{R_{2\ \max}}{R_1} R_{3\ \max} = \frac{1\ \mathrm{M}\,\Omega}{1\ \mathrm{k}\,\Omega} \times 1100\ \ \Omega\ = 1.1\ \mathrm{M}\,\Omega$$

c. The increment between values of R_x for which the bridge can be precisely balanced is the scale factor times the increment in R_3:

$$R_{x\mathrm{inc}} = \frac{R_2}{R_1} R_{3\mathrm{inc}} = \frac{1\ \mathrm{M}\,\Omega}{1\ \mathrm{k}\,\Omega} \times 1\ \ \Omega\ = 1\ \mathrm{k}\,\Omega$$

Strain Measurements

The Wheatstone bridge circuit configuration is often employed with strain gauges in measuring strains of beams and other mechanical structures. (See the Practical Application on page 30 for more information about strain gauges.)

For example, consider the cantilevered beam subject to a downward load force at its outer end as shown in Figure 2.67(a). Two strain gauges are attached to the top of the beam where they are stretched, increasing their resistance by ΔR when the load is applied. The change in resistance is given by

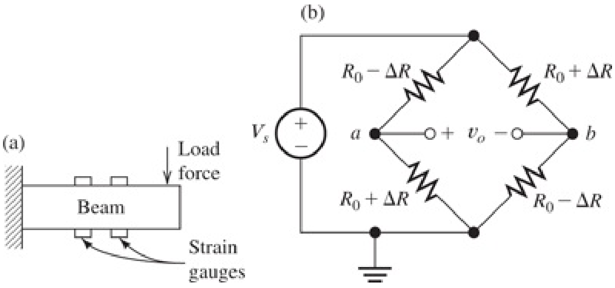


Figure 2.67
Strain measurements using the Wheatstone bridge.

$$\Delta R = R_0 G \frac{\Delta L}{L} \tag{2.92}$$

in which $\Delta L/L$ is the strain for the surface of the beam to which the gauge is attached, R_0 is the gauge resistance before strain is applied, and *G* is the **gauge factor** which is typically about 2. Similarly, two gauges on the bottom of the beam are compressed, reducing their resistance by ΔR with load. (For simplicity, we have assumed that the strain magnitude is the same for all four gauges.)

The four gauges are connected in a Wheatstone bridge as shown in Figure 2.67(b). The resistances labeled $R_0 + \Delta R$ are the gauges on the top of the beam and are being stretched, and those labeled $R_0 - \Delta R$ are those on the bottom and are being compressed. Before the load is applied, all four resistances have a value of $R_0,$ the Wheatstone bridge is balanced, and the output voltage v_o is zero.

It can be shown that the output voltage v_o from the bridge is given by

$$v_o = V_s \frac{\Delta R}{R_0} = V_s G \frac{\Delta L}{L} \tag{2.93}$$

Thus, the output voltage is proportional to the strain of the beam.

In principle, the resistance of one of the gauges could be measured and the strain determined from the resistance measurements. However, the changes in resistance are very small, and the measurements would need to be very precise. Furthermore, gauge resistance changes slightly with temperature. In the bridge arrangement with the gauges attached to the beam, the temperature changes tend to track very closely and have very little effect on v_o.

Usually, v_o is amplified by an instrumentation-quality differential amplifier such as that discussed in Section 13.8 which starts on page 676. The amplified voltage can be converted to digital form and input to a computer or relayed wirelessly to a remote location for monitoring.

Summary

1. Series resistances have an equivalent resistance equal to their sum. For *n* resistances in series, we have

$$R_{\mathrm{eq}} = R_1 + R_2 + \ldots + R_n$$

2. Parallel resistances have an equivalent resistance equal to the reciprocal of the sum of their reciprocals. For *n* resistances in parallel, we get

$$R_{\mathrm{eq}} = \frac{1}{1/R_1 + 1/R_2 + \ldots + 1/R_n}$$

3. Some resistive networks can be solved by repeatedly combining resistances in series or parallel. The simplified network is solved, and results are transferred back through the chain of equivalent circuits. Eventually, the currents and voltages of interest in the original circuit are found.
4. The voltage-division principle applies when a voltage is applied to several resistances in series. A fraction of the total voltage appears across each resistance. The fraction that appears across a given resistance is the ratio of the given resistance to the total series resistance.
5. The current-division principle applies when current flows through two resistances in parallel. A fraction of the total current flows through each resistance. The fraction of the total current flowing through R_1 is equal to $R_2/(R_1 + R_2)$.
6. The node-voltage method can be used to solve for the voltages in any resistive network. A step-by-step summary of the method is given starting on **page 76**.
7. A step-by-step procedure to write the node-voltage equations directly in matrix form for circuits consisting of resistances and independent current sources appears on **page 66**.
8. The mesh-current method can be used to solve for the currents in any planar resistive network. A step-by-step summary of the method is given on **page 89**.
9. A step-by-step procedure to write the mesh-current equations directly in matrix form for circuits consisting of resistances and independent voltage sources appears on **page 85**. For this method to apply, all of the mesh currents must flow in the clockwise direction.
10. A two-terminal network of resistances and sources has a Thévenin equivalent that consists of a voltage source in series with a resistance. The Thévenin voltage is equal to the open-circuit voltage of the original network. The Thévenin resistance is the open-circuit voltage divided by the short-circuit current of the original network. Sometimes, the Thévenin resistance can be found by zeroing the independent sources in the original network and combining resistances in series and parallel. When independent voltage sources are zeroed, they are replaced by short circuits. Independent current sources are replaced by open circuits. Dependent sources must not be zeroed.
11. A two-terminal network of resistances and sources has a Norton equivalent that consists of a current source in parallel with a resistance. The Norton current is equal to the short-circuit current of the original network. The Norton resistance is the same as the Thévenin resistance. A step-by-step procedure for determining Thévenin and Norton equivalent circuits is given on **page 97**.
12. Sometimes source transformations (i.e., replacing a Thévenin equivalent with a Norton equivalent or vice versa) are useful in solving networks.
13. For maximum power from a two-terminal network, the load resistance should equal the Thévenin resistance.
14. The superposition principle states that the total response in a resistive circuit is the sum of the responses to each of the independent sources acting individually. The superposition principle does not apply to any circuit that has element(s) described by nonlinear equation(s).
15. The Wheatstone bridge is a circuit used to measure unknown resistances. The circuit consists of a voltage source, a detector, three precision calibrated resistors, of which two are adjustable, and the

unknown resistance. The resistors are adjusted until the bridge is balanced, and then the unknown resistance is given in terms of the three known resistances.

Here's the answer to the trick question on **page 97**: Suppose that we open circuit the terminals. Then, no current flows through the Thévenin equivalent, but a current I_n circulates in the Norton equivalent. Thus, the box containing the Norton equivalent will become warm because of power dissipation in the resistance. The point of this question is that the circuits are equivalent in terms of their terminal voltage and current, not in terms of their internal behavior.

Problems

Section 2.1: Resistances in Series and Parallel

***P2.1.** Reduce each of the networks shown in **Figure P2.1** to a single equivalent resistance by combining resistances in series and parallel.

* Denotes that answers are contained in the Student Solutions files. See **Appendix E** for more information about accessing the Student Solutions.

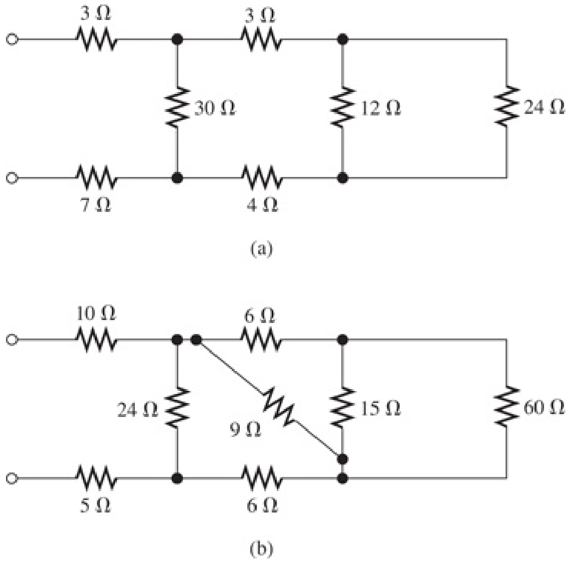


Figure P2.1

***P2.2.** A $4-\ \Omega$ resistance is in series with the parallel combination of a $20-\ \Omega$ resistance and an unknown resistance R_x. The equivalent resistance for the network is $8\ \ \Omega$. Determine the value of R_x.

***P2.3.** Find the equivalent resistance looking into terminals *a* and *b* in **Figure P2.3**.

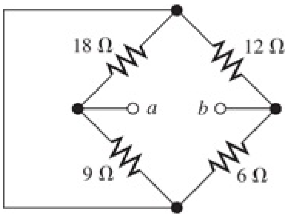


Figure P2.3

***P2.4.** Suppose that we need a resistance of $1.5\ \mathrm{k}\ \Omega$ and you have a box of $1-\mathrm{k}\ \Omega$ resistors. Devise a network of $1-\mathrm{k}\ \Omega$ resistors so the equivalent resistance is $1.5\ \mathrm{k}\ \Omega$. Repeat for an equivalent resistance of $2.2\ \mathrm{k}\ \Omega$.

***P2.5.** Find the equivalent resistance between terminals *a* and *b* in **Figure P2.5**.

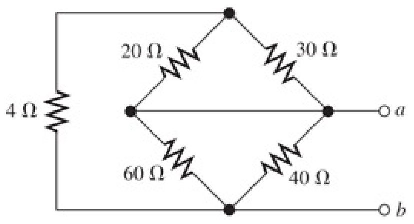


Figure P2.5

P2.6. Find the equivalent resistance between terminals *a* and *b* for each of the networks shown in **Figure P2.6**.

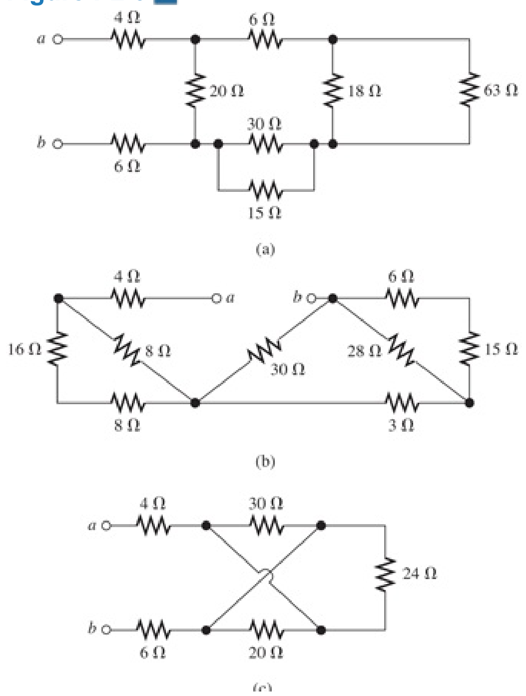


Figure P2.6

P2.7. What resistance in parallel with $120\ \Omega$ results in an equivalent resistance of $48\ \Omega$?

P2.8.

a. Determine the resistance between terminals *a* and *b* for the network shown in **Figure P2.8**.

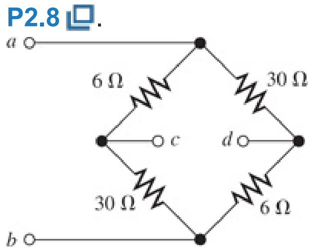


Figure P2.8

b. Repeat after connecting *c* and *d* with a short circuit.

P2.9. Two resistances having values of *R* and 2*R* are in parallel. *R* and the equivalent resistance are both positive integers. What are the possible values for *R*?

P2.10. A network connected between terminals *a* and *b* consists of two parallel combinations that are in series. The first parallel combination is composed of a $16 - \ \Omega$ resistor and a $48 - \ \Omega$

resistor. The second parallel combination is composed of a $12-\Omega$ resistor and a $24-\Omega$ resistor. Draw the network and determine its equivalent resistance.

P2.11. Two resistances R_1 and R_2 are connected in parallel. We know that $R_1 = 90\ \Omega$ and that the current through R_2 is three times the value of the current through R_1. Determine the value of R_2.

P2.12. Find the equivalent resistance for the infinite network shown in **Figure P2.12(a)**. Because of its form, this network is called a semi-infinite ladder. [*Hint:* If another section is added to the ladder as shown in **Figure P2.12(b)**, the equivalent resistance is the same. Thus, working from **Figure P2.12(b)**, we can write an expression for R_{eq} in terms of R_{eq}. Then, we can solve for R_{eq}.]

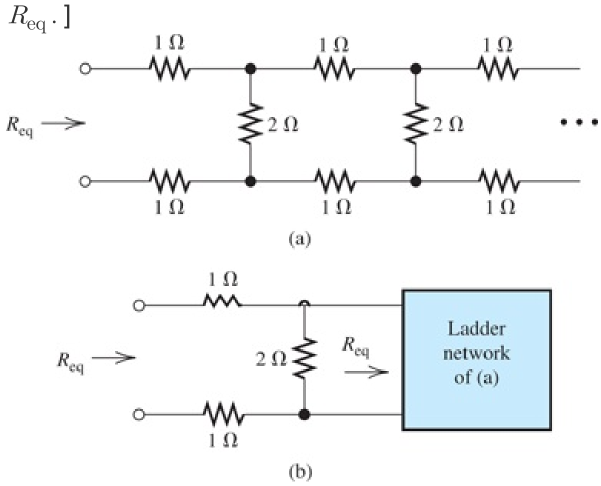


Figure P2.12

P2.13. If we connect *n* $1000-\Omega$ resistances in parallel, what value is the equivalent resistance?

P2.14. The heating element of an electric cook top has two resistive elements, $R_1 = 57.6\ \Omega$ and $R_2 = 115.2\ \Omega$, that can be operated separately, in series, or in parallel from voltages of either 120 V or 240 V. For the lowest power, R_1 is in series with R_2, and the combination is operated from 120 V. What is the lowest power? For the highest power, how should the elements be operated? What power results? List three more modes of operation and the resulting power for each.

P2.15. We are designing an electric space heater to operate from 120 V. Two heating elements with resistances R_1 and R_2 are to be used that can be operated in parallel, separately, or in series. The highest power is to be 1280 W, and the lowest power is to be 240 W. What values are needed for R_1 and R_2? What intermediate power settings are available?

P2.16. Sometimes, we can use symmetry considerations to find the resistance of a circuit that cannot be reduced by series or parallel combinations. A classic problem of this type is illustrated in **Figure P2.16**. Twelve $1-\Omega$ resistors are arranged on the edges of a cube, and terminals *a* and *b* are connected to diagonally opposite corners of the cube. The problem is to find the resistance between the terminals. Approach the problem this way: Assume that 1 A of current enters terminal *a* and exits through terminal *b*. Then, the voltage between terminals *a* and *b* is equal to the unknown resistance. By symmetry considerations, we can find the current in each resistor. Then, using KVL, we can find the voltage between *a* and *b*.

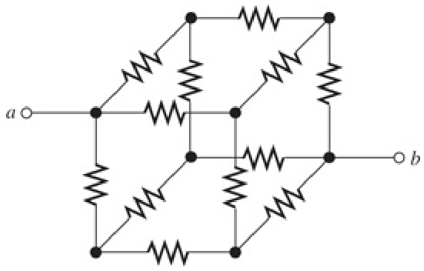


Figure P2.16
Each resistor has a value of $1\ \Omega$.

P2.17. The equivalent resistance between terminals *a* and *b* in **Figure P2.17** is $R_{ab} = 23\ \Omega$. Determine the value of *R*.

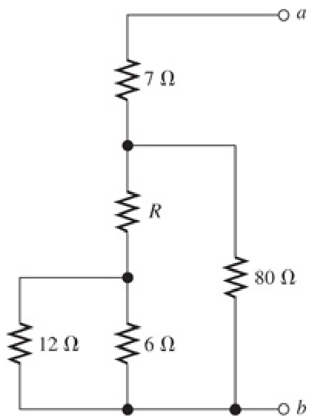


Figure P2.17

P2.18.

a. Three conductances G_1, G_2, and G_3 are in series. Write an expression for the equivalent conductance $G_{\text{eq}} = 1/R_{\text{eq}}$ in terms of G_1, G_2, and G_3.
b. Repeat part (a) with the conductances in parallel.

P2.19. Most sources of electrical power behave as (approximately) ideal voltage sources. In this case, if we have several loads that we want to operate independently, we place the loads in parallel with a switch in series with each load. Thereupon, we can switch each load on or off without affecting the power delivered to the other loads.
How would we connect the loads and switches if the source is an ideal independent current source? Draw the diagram of the current source and three loads with on–off switches such that each load can be switched on or off without affecting the power supplied to the other loads. To turn a load off, should the corresponding switch be opened or closed? Explain.
P2.20. The resistance for the network shown in **Figure P2.20** between terminals *a* and *b* with *c* open circuited is $R_{ab} = 50\ \Omega$. Similarly, the resistance between terminals *b* and *c* with *a* open is $R_{bc} = 100\ \Omega$, and between *c* and *a* with *b* open is $R_{ca} = 70\ \Omega$. Now, suppose that a short circuit is connected from terminal *b* to terminal *c*, and determine the resistance between terminal *a* and the shorted terminals *b*–*c*.

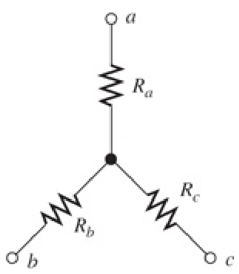


Figure P2.20

P2.21. Often, we encounter delta-connected loads, such as that illustrated in **Figure P2.21**, in three-phase power distribution systems (which are treated in **Section 5.7**). If we only have access to the three terminals, a method for determining the resistances is to repeatedly short two terminals together and measure the resistance between the shorted terminals and the third terminal. Then, the resistances can be calculated from the three measurements. Suppose that the measurements are $R_{as} = 12\ \Omega$, $R_{bs} = 20\ \Omega$, and $R_{cs} = 15\ \Omega$. Where R_{as} is the resistance between terminal *a* and the short between *b* and *c*, etc. Determine the values of R_a, R_b, and R_c. (*Hint:* You may find the equations easier to deal with if you work in terms of conductances rather than resistances. Once the conductances are known, you can easily invert their values to find the resistances.)

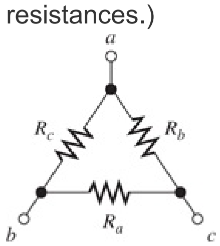


Figure P2.21

Section 2.2: Network Analysis by Using Series and Parallel Equivalents

P2.22. What are the steps in solving a circuit by network reduction (series/parallel combinations)? Does this method always provide the solution? Explain.

***P2.23.** Find the values of i_1 and i_2 in **Figure P2.23**.

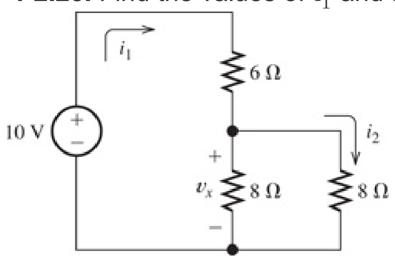


Figure P2.23

***P2.24.** Find the voltages v_1 and v_2 for the circuit shown in **Figure P2.24** by combining resistances in series and parallel.

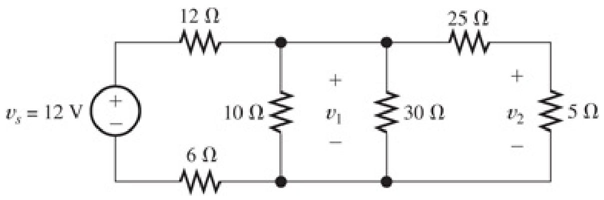


Figure P2.24

***P2.25.** Find the values of *v* and *i* in **Figure P2.25**.

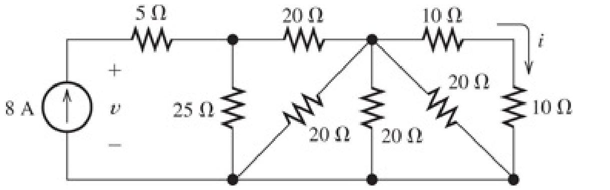


Figure P2.25

P2.26. Consider the circuit shown in **Figure P2.24**. Suppose that the value of v_s is adjusted until $v_2 = 5\ \mathrm{V}$. Determine the new value of v_s. [*Hint:* Start at the right-hand side of the circuit and compute currents and voltages, moving to the left until you reach the source.]

P2.27. Find the voltage *v* and the currents i_1 and i_2 for the circuit shown in **Figure P2.27**.

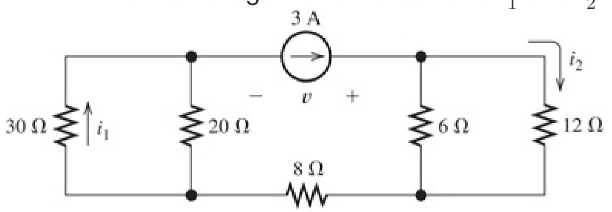


Figure P2.27

P2.28. Find the values of v_s, v_1, and i_2 in **Figure P2.28**.

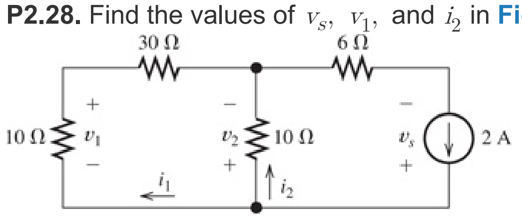


Figure P2.28

P2.29. Find the values of i_1 and i_2 in **Figure P2.29**.

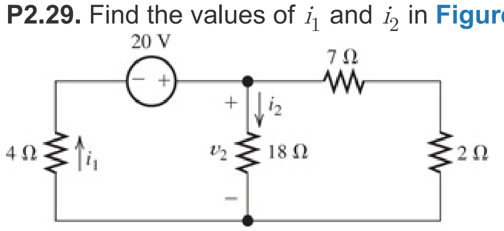


Figure P2.29

P2.30. Consider the circuit shown in **Figure P2.30**. Find the values of v_1, v_2, and v_{ab}.

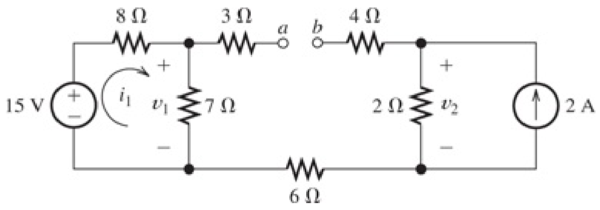


Figure P2.30

P2.31. Solve for the values of i_1, i_2, and the powers for the sources in **Figure P2.31**. Is the current source absorbing energy or delivering energy? Is the voltage source absorbing energy or delivering it?

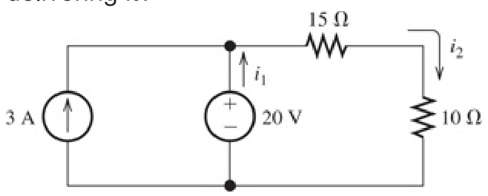


Figure P2.31

P2.32. The 12-V source in **Figure P2.32** is delivering 36 mW of power. All four resistors have the same value *R*. Find the value of *R*.

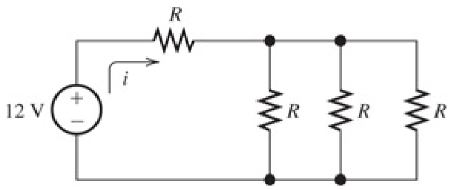


Figure P2.32

P2.33. Refer to the circuit shown in **Figure P2.33**. With the switch open, we have $v_2 = 8\ \mathrm{V}$. On the other hand, with the switch closed, we have $v_2 = 6\ \mathrm{V}$. Determine the values of R_2 and R_L.

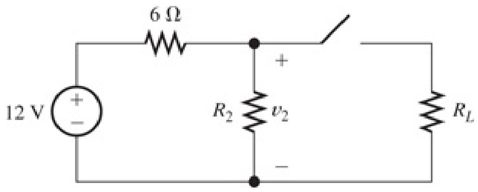


Figure P2.33

***P2.34.** Find the values of i_1 and i_2 in **Figure P2.34**. Find the power for each element in the circuit, and state whether each is absorbing or delivering energy. Verify that the total power absorbed equals the total power delivered.

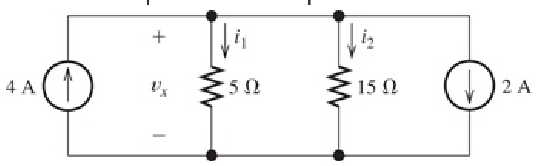


Figure P2.34

***P2.35.** Find the values of i_1 and i_2 in **Figure P2.35**.

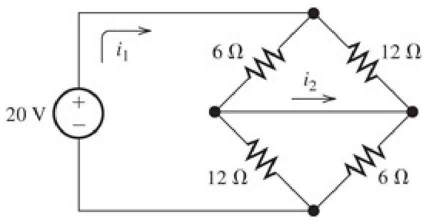


Figure P2.35

Section2.3: Voltage-Divider and Current-Divider Circuits

***P2.36.** Use the voltage-division principle to calculate v_1, v_2, and v_3 in **Figure P2.36**.

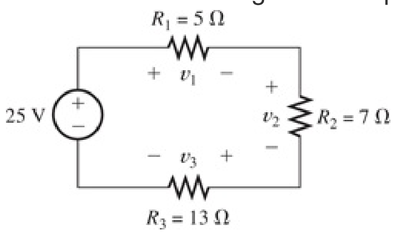


Figure P2.36

***P2.37.** Use the current-division principle to calculate i_1 and i_2 in **Figure P2.37**.

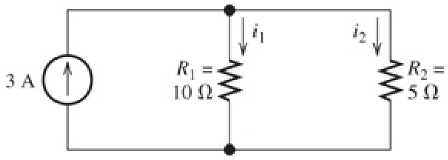


Figure P2.37

***P2.38.** Use the voltage-division principle to calculate *v* in **Figure P2.38**.

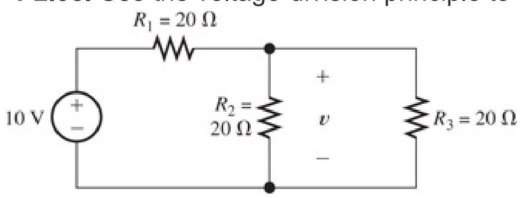


Figure P2.38

P2.39. Use the current-division principle to calculate the value of i_3 in **Figure P2.39**.

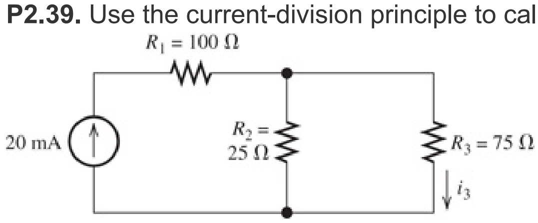


Figure P2.39

P2.40. Suppose we need to design a voltage-divider circuit to provide an output voltage $v_o = 5\ \text{V}$ from a 15-V source as shown in **Figure P2.40**. The current taken from the 15-V source is to be 200 mA.

a. Find the values of R_1 and R_2.

b. Now suppose that a load resistance of $200\ \Omega$ is connected across the output terminals (i.e., in parallel with R_2). Find the value of v_o.

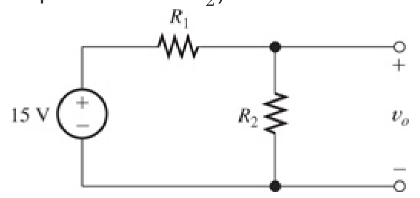


Figure P2.40

P2.41. A source supplies 120 V to the series combination of a $10-\ \Omega$ resistance, a $5-\ \Omega$ resistance, and an unknown resistance R_x. The voltage across the $5-\ \Omega$ resistance is 20 V. Determine the value of the unknown resistance.

P2.42. We have a $60-\ \Omega$ resistance, a $20-\ \Omega$ resistance, and an unknown resistance R_x in parallel with a 15 mA current source. The current through the unknown resistance is 10 mA. Determine the value of R_x.

***P2.43.** A worker is standing on a wet concrete floor, holding an electric drill having a metallic case. The metallic case is connected through the ground wire of a three-terminal power outlet to power-system ground. The resistance of the ground wire is R_g. The resistance of the worker's body is $R_w = 500\ \ \Omega$. Due to faulty insulation in the drill, a current of 2 A flows into its metallic case. The circuit diagram for this situation is shown in **Figure P2.43**. Find the maximum value of R_g so that the current through the worker does not exceed 0.1 mA.

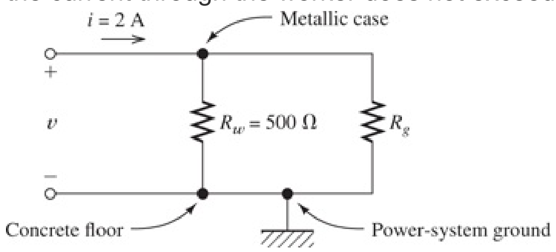


Figure P2.43

P2.44. Suppose we have a load that absorbs power and requires a current varying between 0 and 50 mA. The voltage across the load must remain between 4.7 and 5.0 V. A 15-V source is available. Design a voltage-divider network to supply the load. You may assume that resistors of any value desired are available. Also, determine the maximum power for each resistor.

P2.45. We have a load resistance of $50\ \ \Omega$ that we wish to supply with 5 V. A 12.6-V voltage source and resistors of any value needed are available. Draw a suitable circuit consisting of the voltage source, the load, and one additional resistor. Specify the value of the resistor.

P2.46. We have a load resistance of $1\ \mathrm{k}\,\Omega$ that we wish to supply with 25 mW. A 20-mA current source and resistors of any value needed are available. Draw a suitable circuit consisting of the current source, the load, and one additional resistor. Specify the value of the resistor.

P2.47. The circuit of **Figure P2.47** is similar to networks used in digital-to-analog converters. For this problem, assume that the circuit continues indefinitely to the right. Find the values of $i_1,\ i_2,\ i_3,$ and i_4. How is i_{n+2} related to i_n? What is the value of i_{18}? (*Hint:* See **Problem P2.12**.)

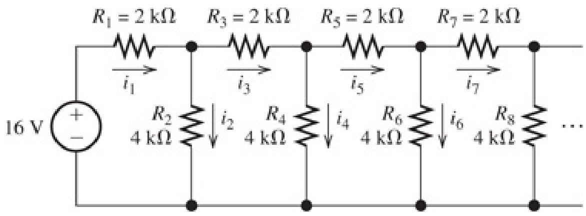


Figure P2.47

Section 2.4: Node-Voltage Analysis

***P2.48.** Write equations and solve for the node voltages shown in **Figure P2.48**. Then, find the value of i_1.

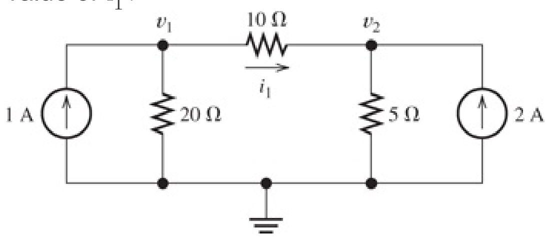


Figure P2.48

***P2.49.** Solve for the node voltages shown in **Figure P2.49**. Then, find the value of i_s.

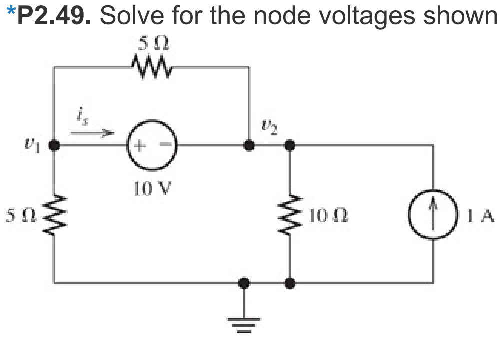


Figure P2.49

P2.50. Solve for the node voltages shown in **Figure P2.50**. What are the new values of the node voltages after the direction of the current source is reversed? How are the values related?

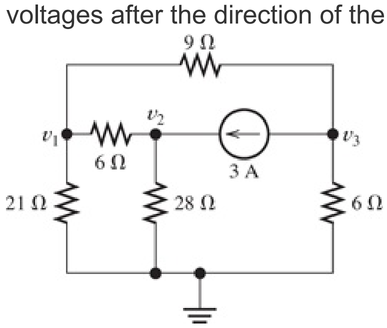


Figure P2.50

P2.51. Given $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 8\ \Omega$, $R_4 = 10\ \Omega$, $R_5 = 2\ \Omega$, and $I_s = 2\ \text{A}$, solve for the node voltages shown in **Figure P2.51**.

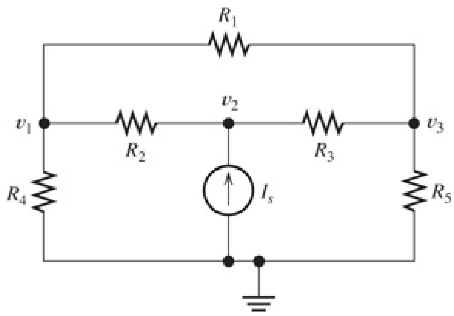


Figure P2.51

P2.52. Determine the value of i_1 in **Figure P2.52** using node voltages to solve the circuit. Select the location of the reference node to minimize the number of unknown node voltages. What effect does the $20 - \Omega$ resistance have on the answer? Explain.

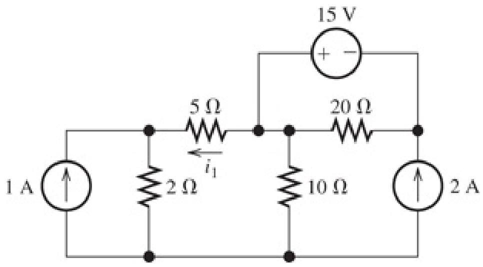


Figure P2.52

P2.53. Given $R_1 = 15\ \ \Omega$, $R_2 = 5\ \ \Omega$, $R_3 = 20\ \ \Omega$, $R_4 = 10\ \ \Omega$, $R_5 = 8\ \ \Omega$, $R_6 = 4\ \ \Omega$, and $I_s = 5\ \mathrm{A}$, solve for the node voltages shown in **Figure P2.53**.

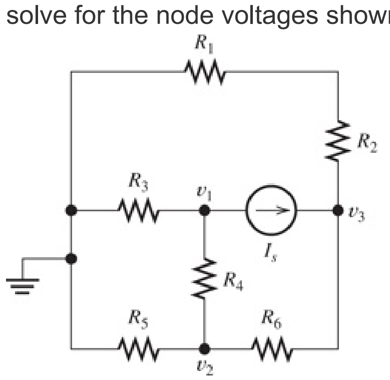


Figure P2.53

P2.54. In solving a network, what rule must you observe when writing KCL equations? Why?

P2.55. Use the symbolic features of MATLAB to find an expression for the equivalent resistance for the network shown in **Figure P2.55**. [*Hint:* First, connect a 1-A current source across terminals *a* and *b*. Then, solve the network by the node-voltage technique. The voltage across the current source is equal in value to the equivalent resistance.] Finally, use the subs command to evaluate for $R_1 = 15\ \ \Omega$, $R_2 = 5\ \ \Omega$, $R_3 = 20\ \ \Omega$, $R_4 = 10\ \ \Omega$, and $R_5 = 8\ \ \Omega$.

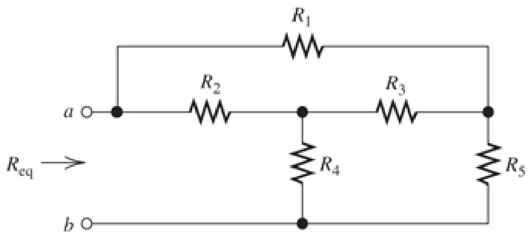


Figure P2.55

***P2.56.** Solve for the values of the node voltages shown in **Figure P2.56**. Then, find the value of i_x.

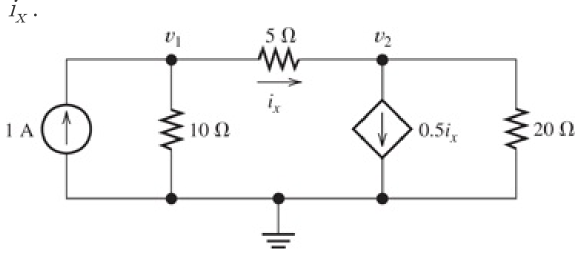


Figure P2.56

***P2.57.** Solve for the node voltages shown in **Figure P2.57**.

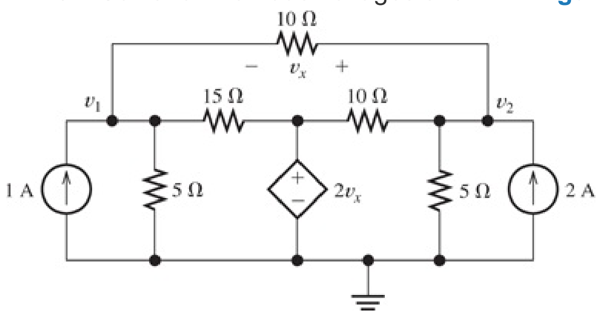


Figure P2.57

P2.58 Solve for the power delivered to the $8 - \Omega$ resistance and for the node voltages shown in **Figure P2.58**.

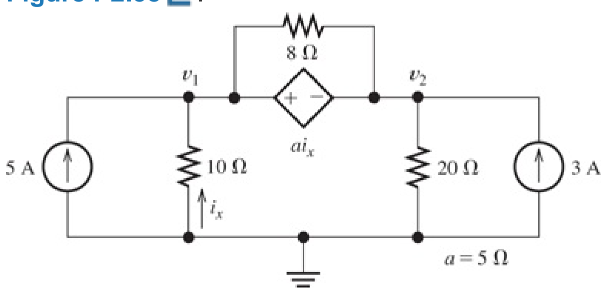


Figure P2.58

P2.59. Solve for the node voltages shown in **Figure P2.59**.

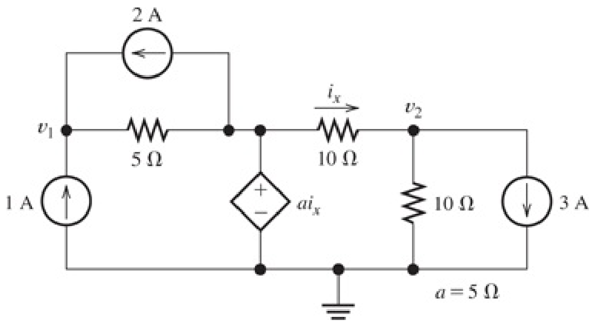


Figure P2.59

P2.60. Find the equivalent resistance looking into terminals for the network shown in **Figure P2.60**. [*Hint:* First, connect a 1-A current source across terminals *a* and *b*. Then, solve the network by the node-voltage technique. The voltage across the current source is equal in value to the equivalent resistance.]

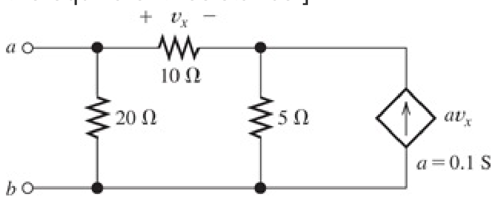


Figure P2.60

P2.61. Find the equivalent resistance looking into terminals for the network shown in **Figure P2.61**. [*Hint:* First, connect a 1-A current source across terminals *a* and *b*. Then, solve the network by the node-voltage technique. The voltage across the current source is equal in value to the equivalent resistance.]

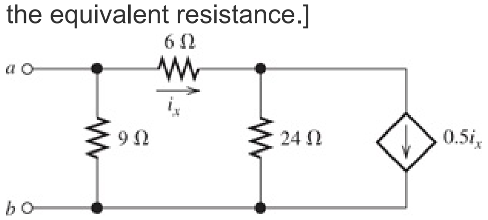


Figure P2.61

P2.62. **Figure P2.62** shows an unusual voltage-divider circuit. Use node-voltage analysis and the symbolic math commands in MATLAB to solve for the voltage division ratio V_{out}/V_{in} in terms of the resistances. Notice that the node-voltage variables are V_1, V_2, and V_{out}.

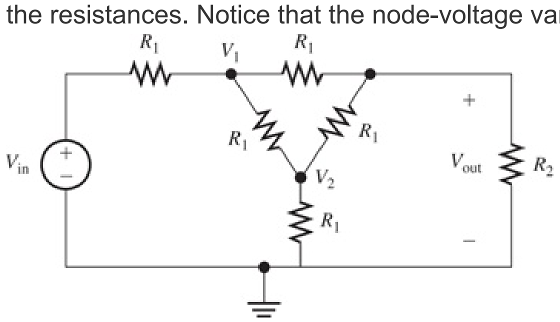


Figure P2.62

P2.63. Solve for the node voltages in the circuit of **Figure P2.63**. Disregard the mesh currents, i_1, i_2, i_3, and i_4 when working with the node voltages.

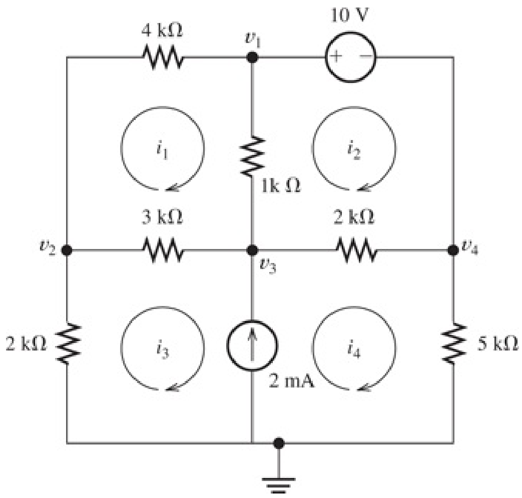


Figure P2.63

P2.64. We have a cube with $1-\ \Omega$ resistances along each edge as illustrated in **Figure P2.64** in which we are looking into the front face which has corners at nodes 1, 2, 7, and the reference node. Nodes 3, 4, 5, and 6 are the corners on the rear face of the cube. (Alternatively, you can consider it to be a planar network.) We want to find the resistance between adjacent nodes, such as node 1 and the reference node. We do this by connecting a 1-A current source as shown and solving for v_1, which is equal in value to the resistance between any two adjacent nodes.

a. Use MATLAB to solve the matrix equation $GV = I$ for the node voltages and determine the resistance.
b. Modify your work to determine the resistance between nodes at the ends of a diagonal across a face, such as node 2 and the reference node.
c. Finally, find the resistance between opposite corners of the cube. [*Comment:* Part (c) is the same as **Problem 2.16** in which we suggested using symmetry to solve for the resistance. Parts (a) and (b) can also be solved by use of symmetry and the fact that nodes having the same value of voltage can be connected by short circuits without changing the currents and voltages. With the shorts in place, the resistances can be combined in series and parallel to obtain the answers. Of course, if the resistors have arbitrary values, the MATLAB approach will still work, but considerations of symmetry will not.]

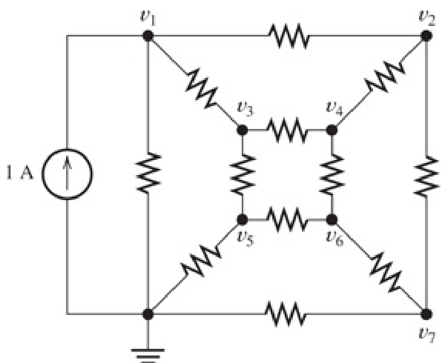


Figure P2.64

Section 2.5: Mesh-Current Analysis

*__P2.65.__ Solve for the power delivered to the $15-\ \Omega$ resistor and for the mesh currents shown in **Figure P2.65**.

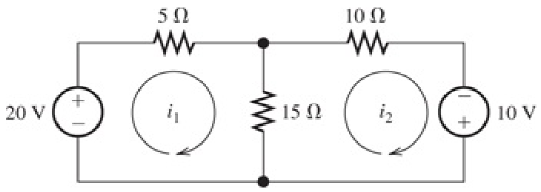


Figure P2.65

*P2.66. Determine the value of v_2 and the power delivered by the source in the circuit of Figure P2.24 by using mesh-current analysis.

*P2.67. Use mesh-current analysis to find the value of i_1 in the circuit of Figure P2.48.

P2.68. Solve for the power delivered by the voltage source in Figure P2.68, using the mesh-current method.

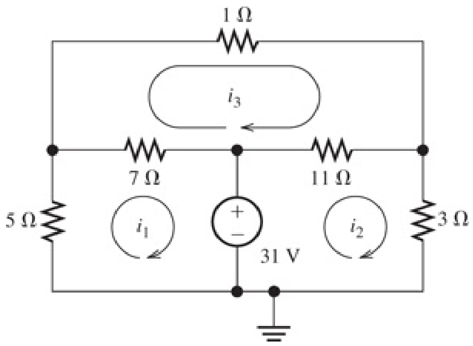


Figure P2.68

P2.69. Use mesh-current analysis to find the value of *v* in the circuit of Figure P2.38.

P2.70. Use mesh-current analysis to find the value of i_3 in the circuit of Figure P2.39.

P2.71. Use mesh-current analysis to find the values of i_1 and i_2 in Figure P2.27. Select i_1 clockwise around the left-hand mesh, i_2 clockwise around the right-hand mesh, and i_3 clockwise around the center mesh.

P2.72. Find the power delivered by the source and the values of i_1 and i_2 in the circuit of Figure P2.23, using mesh-current analysis.

P2.73. Use mesh-current analysis to find the values of i_1 and i_2 in Figure P2.29. First, select i_A clockwise around the left-hand mesh and i_B clockwise around the right-hand mesh. After solving for the mesh currents, i_A and i_B, determine the values of i_1 and i_2.

P2.74. Use mesh-current analysis to find the values of i_1 and i_2 in Figure P2.28. First, select i_A clockwise around the left-hand mesh and i_B clockwise around the right-hand mesh. After solving for the mesh currents, i_A and i_B, determine the values of i_1 and i_2.

P2.75. The circuit shown in Figure P2.75 is the dc equivalent of a simple residential power distribution system. Each of the resistances labeled R_1 and R_2 represents various parallel-connected loads, such as lights or devices plugged into outlets that nominally operate at 120 V, while R_3 represents a load, such as the heating element in an oven that nominally operates at 240 V. The resistances labeled R_w represent the resistances of wires. R_n represents the “neutral” wire.

a. Use mesh-current analysis to determine the voltage magnitude for each load.
b. Now suppose that due to a fault in the wiring at the distribution panel, the neutral wire becomes an open circuit. Again compute the voltages across the loads and comment on the probable outcome for a sensitive device such as a computer or plasma television that is part of the $15-\ \Omega$ load

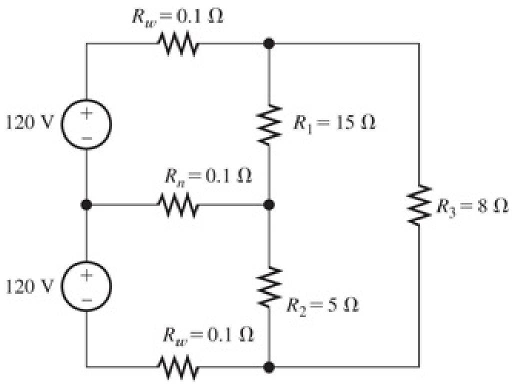


Figure P2.75

P2.76. Use MATLAB and mesh-current analysis to determine the value of v_3 in the circuit of **Figure P2.51**. The component values are $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 8\ \Omega$, $R_4 = 10\ \Omega$, $R_5 = 2\ \Omega$, and $I_s = 2\ \text{A}$.

P2.77. Connect a 1-V voltage source across terminals *a* and *b* of the network shown in **Figure P2.55**. Then, solve the network by the mesh-current technique to find the current through the source. Finally, divide the source voltage by the current to determine the equivalent resistance looking into terminals *a* and *b*. The resistance values are $R_1 = 6\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 4\ \Omega$, $R_4 = 8\ \Omega$, and $R_5 = 2\ \Omega$.

P2.78. Connect a 1-V voltage source across the terminals of the network shown in **Figure P2.1(a)**. Then, solve the network by the mesh-current technique to find the current through the source. Finally, divide the source voltage by the current to determine the equivalent resistance looking into the terminals. Check your answer by combining resistances in series and parallel.

P2.79. Use MATLAB to solve for the mesh currents in **Figure P2.63**.

Section 2.6: Thévenin and Norton Equivalent Circuits

***P2.80.** Find the Thévenin and Norton equivalent circuits for the two-terminal circuit shown in **Figure P2.80**.

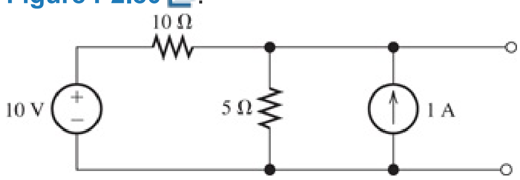


Figure P2.80

***P2.81.** We can model a certain battery as a voltage source in series with a resistance. The open-circuit voltage of the battery is 9 V. When a $100-\ \Omega$ resistor is placed across the terminals of the battery, the voltage drops to 6 V. Determine the internal resistance (Thévenin resistance) of the battery.

P2.82. Find the Thévenin and Norton equivalent circuits for the circuit shown in **Figure P2.82**.

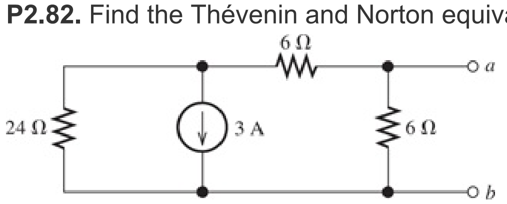


Figure P2.82

P2.83. Find the Thévenin and Norton equivalent circuits for the two-terminal circuit shown in **Figure P2.83**.

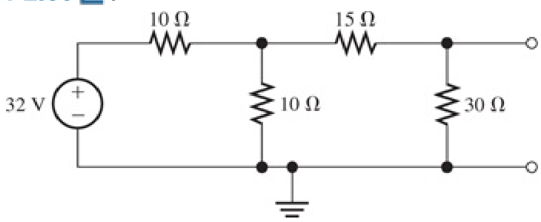


Figure P2.83

P2.84. Find the Thévenin and Norton equivalent circuits for the circuit shown in **Figure P2.84**. Take care that you orient the polarity of the voltage source and the direction of the current source correctly relative to terminals *a* and *b*. What effect does the $7-\ \Omega$ resistor have on the equivalent circuits? Explain your answer.

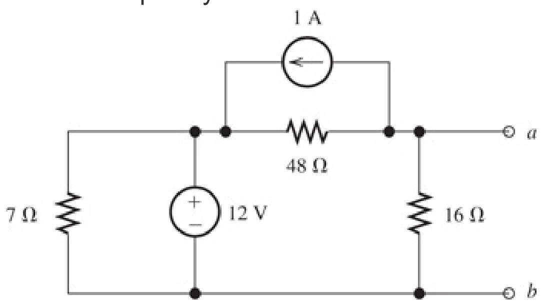


Figure P2.84

P2.85. An automotive battery has an open-circuit voltage of 12.6 V and supplies 100 A when a $0.1-\ \Omega$ resistance is connected across the battery terminals. Draw the Thévenin and Norton equivalent circuits, including values for the circuit parameters. What current can this battery deliver to a short circuit? Considering that the energy stored in the battery remains constant under open-circuit conditions, which of these equivalent circuits seems more realistic? Explain.

P2.86. A certain two-terminal circuit has an open-circuit voltage of 15 V. When a $2-\mathrm{k}\ \Omega$ load is attached, the voltage across the load is 10 V. Determine the Thévenin resistance for the circuit.

P2.87. If we measure the voltage at the terminals of a two-terminal network with two known (and different) resistive loads attached, we can determine the Thévenin and Norton equivalent circuits. When a $2.2-\mathrm{k}\ \Omega$ load is attached to a two-terminal circuit, the load voltage is 4.4 V. When the load is increased to $10\ \mathrm{k}\ \Omega$, the load voltage becomes 5 V. Find the Thévenin voltage and resistance for this circuit.

P2.88. Find the Thévenin and Norton equivalent circuits for the circuit shown in **Figure P2.88**.

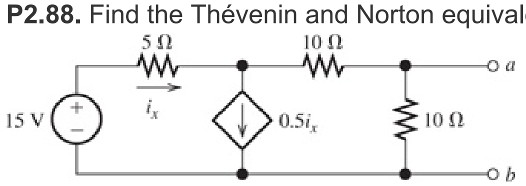


Figure P2.88

P2.89. Find the maximum power that can be delivered to a resistive load by the circuit shown in **Figure P2.80**. For what value of load resistance is the power maximum?

P2.90. Find the maximum power that can be delivered to a resistive load by the circuit shown in **Figure P2.82**. For what value of load resistance is the power maximum?

***P2.91.** **Figure P2.91** shows a resistive load R_L connected to a Thévenin equivalent circuit. For what value of Thévenin resistance is the power delivered to the load maximized? Find the maximum power delivered to the load. [*Hint:* Be careful; this is a trick question if you don't stop to think about it.]

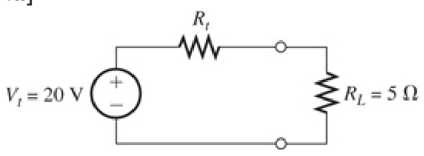


Figure P2.91

P2.92. Starting from the Norton equivalent circuit with a resistive load R_L attached, find an expression for the power delivered to the load in terms of I_n, R_t, and R_L. Assuming that I_n and R_t are fixed values and that R_L is variable, show that maximum power is delivered for $R_L = R_t$. Find an expression for maximum power delivered to the load in terms of I_n and R_t.

P2.93. A battery can be modeled by a voltage source V_t in series with a resistance R_t. Assuming that the load resistance is selected to maximize the power delivered, what percentage of the power taken from the voltage source V_t is actually delivered to the load? Suppose that $R_L = 9R_t$; what percentage of the power taken from V_t is delivered to the load? Usually, we want to design battery-operated systems so that nearly all of the energy stored in the battery is delivered to the load. Should we design for maximum power transfer?

Section 2.7: Superposition Principle

***P2.94.** Use superposition to find the current *i* in **Figure P2.94**. First, zero the current source and find the value i_v caused by the voltage source alone. Then, zero the voltage source and find the value i_c caused by the current source alone. Finally, add the results algebraically.

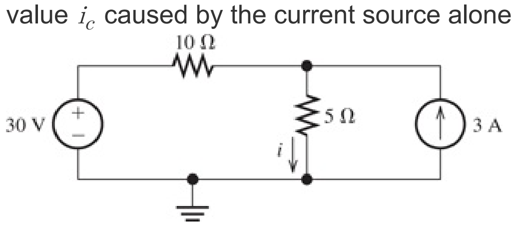


Figure P2.94

***P2.95.** Solve for i_s in **Figure P2.49** by using superposition.

P2.96. Solve the circuit shown in **Figure P2.48** by using superposition. First, zero the 1-A source and find the value of i_1 with only the 2-A source activated. Then, zero the 2-A source and find the value of i_1 with only the 1-A source activated. Finally, find the total value of i_1 with both sources activated by algebraically adding the previous results.

P2.97. Solve for i_1 in **Figure P2.34** by using superposition.

P2.98. Another method of solving the circuit of **Figure P2.24** is to start by assuming that $v_2 = 1\ \mathrm{V}$. Accordingly, we work backward toward the source, using Ohm's law, KCL, and KVL to find the value of v_s. Since we know that v_2 is proportional to the value of v_s, and since we have found the value of v_s that produces $v_2 = 1\ \mathrm{V}$, we can calculate the value of v_2 that results when $v_s = 12\ \mathrm{V}$. Solve for v_2 by using this method.

P2.99. Use the method of **Problem P2.98** for the circuit of **Figure P2.23**, starting with the assumption that $i_2 = 1\ \mathrm{A}$.

P2.100. Solve for the actual value of i_6 for the circuit of **Figure P2.100**, starting with the assumption that $i_6 = 1\ \mathrm{A}$. Work back through the circuit to find the value of I_s that results in $i_6 = 1\ \mathrm{A}$. Then, use proportionality to determine the value of i_6 that results for $I_s = 10\ \mathrm{A}$.

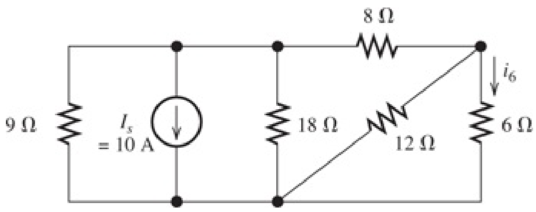


Figure P2.100

P2.101. Device *A* shown in **Figure P2.101** has $v = 3i^2$ for $i \geq 0$ and $v = 0$ for $i < 0$.

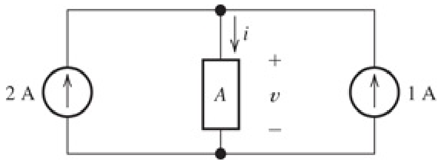


Figure P2.101

a. Solve for *v* with the 2-A source active and the 1-A source zeroed.
b. Solve for *v* with the 1-A source active and the 2-A source zeroed.
c. Solve for *v* with both sources active. Why doesn't superposition apply?

Section 2.8: Wheatstone Bridge

P2.102.

a. The Wheatstone bridge shown in **Figure 2.66** is balanced with $R_1 = 10\ \mathrm{k}\,\Omega$, $R_3 = 3419\ \ \Omega$, and $R_2 = 1\ \mathrm{k}\,\Omega$. Find R_x.
b. Repeat if R_2 is $100\ \mathrm{k}\,\Omega$ and the other values are unchanged.

***P2.103.** The Wheatstone bridge shown in **Figure 2.66** has $v_s = 10\ \mathrm{V}, R_1 = 10\ \mathrm{k}\,\Omega$, $R_2 = 10\ \mathrm{k}\,\Omega$, and $R_x = 5932\ \ \Omega$. The detector can be modeled as a $5 - \mathrm{k}\,\Omega$ resistance.

a. What value of R_3 is required to balance the bridge?
b. Suppose that R_3 is $1\ \ \Omega$ higher than the value found in part (a). Find the current through the detector. [*Hint:* Find the Thévenin equivalent for the circuit with the detector removed. Then, place the detector across the Thévenin equivalent and solve for the current.] Comment.

P2.104. In theory, any values can be used for R_1 and R_3 in the Wheatstone bridge of **Figure 2.66**. For the bridge to balance, it is only the *ratio* R_3/R_1 that is important. What practical problems might occur if the values are very small? What practical problems might occur if the values are very large?

P2.105. Derive expressions for the Thévenin voltage and resistance “seen” by the detector in the Wheatstone bridge in **Figure 2.66**. (In other words, remove the detector from the circuit and determine the Thévenin resistance for the remaining two-terminal circuit.) What is the value of the Thévenin voltage when the bridge is balanced?

P2.106. Derive **Equation 2.93** for the bridge circuit of **Figure 2.67** on **page 109**.

P2.107. Consider a strain gauge in the form of a long thin wire having a length *L* and a cross-sectional area *A* before strain is applied. After the strain is applied, the length increases slightly to $L + \Delta L$ and the area is reduced so the volume occupied by the wire is constant. Assume that $\Delta L / L \ll 1$ and that the resistivity ρ of the wire material is constant. Determine the gauge factor

$$G = \frac{\Delta R / R_0}{\Delta L / L}$$

[*Hint:* Make use of **Equation 1.10** on **page 28**.]

P2.108. Explain what would happen if, in wiring the bridge circuit of **Figure 2.67** on **page 109**Explain what would happen if, in wiring the bridge circuit of , the gauges in tension (i.e., those labeled $R + \Delta R$) were both placed on the top of the bridge circuit diagram, shown in part (b) of the figure, and those in compression were both placed at the bottom of the bridge circuit diagram.

Practice Test

Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in **Appendix D** and complete solutions are included in the Student Solutions files. See **Appendix E** for more information about the Student Solutions.

T2.1. Match each entry in **Table T2.1(a)** with the best choice from the list given in **Table T2.1(b)** for circuits composed of sources and resistances. [Items in **Table T2.1(b)** may be used more than once or not at all.]

Table T2.1

Item	Best Match
(a)	
a. The equivalent resistance of parallel-connected resistances...	
b. Resistances in parallel combine as do...	
c. Loads in power distribution systems are most often connected...	
d. Solving a circuit by series/parallel combinations applies to...	
e. The voltage-division principle applies to...	
f. The current-division principle applies to...	
g. The superposition principle applies to...	
h. Node-voltage analysis can be applied to...	
i. In this book, mesh-current analysis is applied to...	
j. The Thévenin resistance of a two-terminal circuit equals...	
k. The Norton current source value of a two-terminal circuit equals...	
l. A voltage source in parallel with a resistance is equivalent to...	
(b)	
1. conductances in parallel	
2. in parallel	
3. all circuits	
4. resistances or conductances in parallel	
5. is obtained by summing the resistances	
6. is the reciprocal of the sum of the reciprocals of the resistances	
7. some circuits	
8. planar circuits	
9. a current source in series with a resistance	
10. conductances in series	
11. circuits composed of linear elements	
12. in series	
13. resistances or conductances in series	
14. a voltage source	
15. the open-circuit voltage divided by the short-circuit current	
16. a current source	
17. the short-circuit current	

T2.2. Consider the circuit of **Figure T2.2** with $v_s = 96\ \mathrm{V}$, $R_1 = 6\ \ \Omega$, $R_2 = 48\ \ \Omega$, $R_3 = 16\ \ \Omega$, and $R_4 = 60\ \ \Omega$. Determine the values of i_s and i_4.

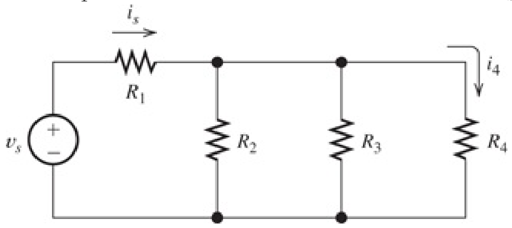


Figure T2.2

T2.3. Write MATLAB code to solve for the node voltages for the circuit of **Figure T2.3**.

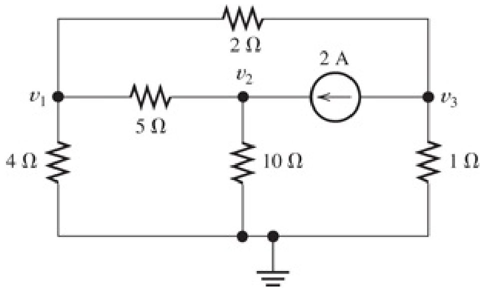


Figure T2.3

T2.4. Write a set of equations that can be used to solve for the mesh currents of **Figure T2.4**. Be sure to indicate which of the equations you write form the set.

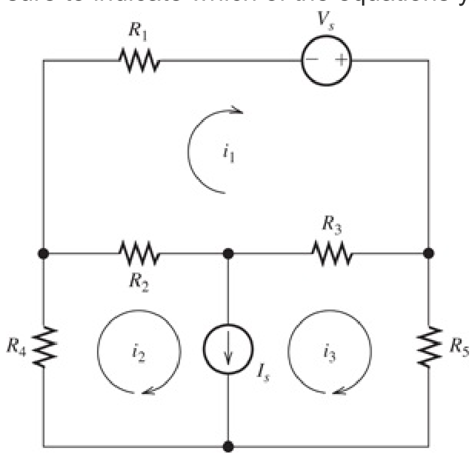


Figure T2.4

T2.5. Determine the Thévenin and Norton equivalent circuits for the circuit of **Figure T2.5**. Draw the equivalent circuits labeling the terminals to correspond with the original circuit.

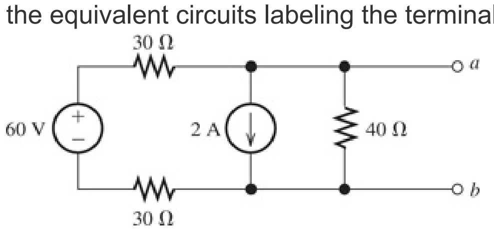


Figure T2.5

T2.6. According to the superposition principle, what percentage of the total current flowing through the $5-\ \Omega$ resistance in the circuit of **Figure T2.6** results from the 5-V source? What percentage of the power supplied to the $5-\Omega$ resistance is supplied by the 5-V source? Assume that both sources are active when answering both questions.

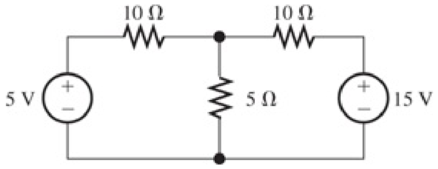


Figure T2.6

T2.7. Determine the equivalent resistance between terminals *a* and *b* in **Figure T2.7**.

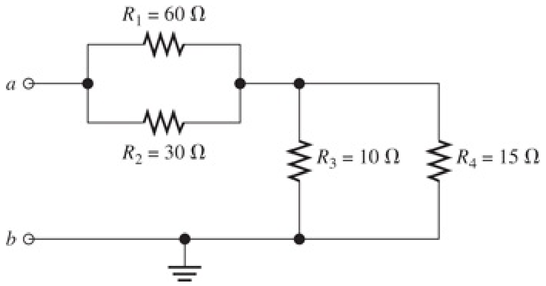


Figure T2.7

T2.8. Transform the 2-A current source and $6-\Omega$ resistance in **Figure T2.8** into an equivalent series combination. Then, combine the series voltage sources and resistances. Draw the circuit after each step.

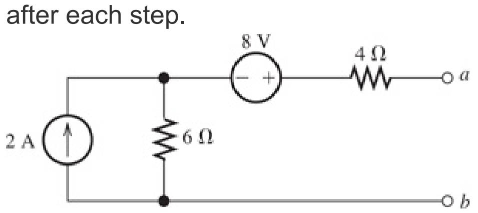


Figure T2.8