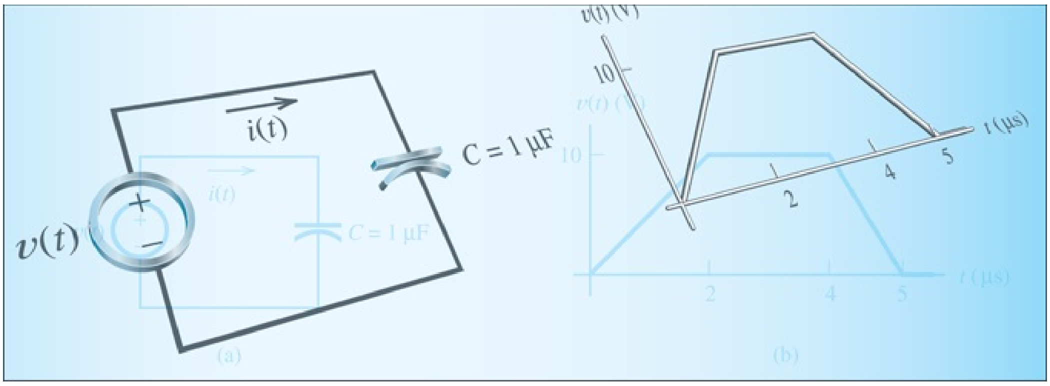
# Chapter 3 Inductance and Capacitance




**Study of this chapter will enable you to:**

- Find the current (voltage) for a capacitance or inductance given the voltage (current) as a function of time.
- Compute the capacitances of parallel-plate capacitors.
- Compute the energies stored in capacitances or inductances.
- Describe typical physical construction of capacitors and inductors and identify parasitic effects.
- Find the voltages across mutually coupled inductances in terms of the currents.
- Apply the MATLAB Symbolic Toolbox to the current–voltage relationships for capacitances and inductances.

## Introduction to this chapter:

*Previously, we studied circuits composed of resistances and sources. In this chapter, we discuss two additional circuit elements: inductors and capacitors. Whereas resistors convert electrical energy into heat, inductors and capacitors are **energy-storage elements**. They can store energy and later return it to the circuit. Capacitors and inductors do not generate energy—only the energy that has been put into these elements can be extracted. Thus, like resistors, they are said to be **passive** elements.*

*Electromagnetic field theory is the basic approach to the study of the effects of electrical charge. However, circuit theory is a simplification of field theory that is much easier to apply. Capacitance is the circuit property that accounts for energy stored in electric fields. Inductance accounts for energy stored in magnetic fields.*

*We will learn that the voltage across an ideal inductor is proportional to the time derivative of the current. On the other hand, the voltage across an ideal capacitor is proportional to the time integral of the current.*

*We will also study mutual inductance, a circuit property that accounts for magnetic fields that are mutual to several inductors. In **Chapter 14**, we will see that mutual inductance forms the basis for transformers, which are critical to the transmission of electrical power over long distances.*

*Several types of transducers are based on inductance and capacitance. For example, one type of microphone is basically a capacitor in which the capacitance changes with sound pressure. An application of mutual inductance is the linear variable differential transformer in which position of a moving iron core is converted into a voltage.*

*Sometimes an electrical signal that represents a physical variable such as -displacement is noisy. For example, in an active (electronically controlled) suspension for an automobile, the position sensors are affected by road roughness as well as by the loading of the vehicle. To obtain an electrical signal representing the displacement of each wheel, the rapid fluctuations due to road roughness must be eliminated. Later, we will see that this can be accomplished using inductance and capacitance in circuits known as filters.*

*After studying this chapter, we will be ready to extend the basic circuit-analysis techniques learned in **Chapter 2** to circuits having inductance and capacitance.*

## 3.1 Capacitance

Capacitors are constructed by separating two sheets of conductor, which is usually metallic, by a thin layer of insulating material. In a parallel-plate capacitor, the sheets are flat and parallel as shown in **Figure 3.1**. The insulating material between the plates, called a **dielectric**, can be air, Mylar®, polyester, polypropylene, mica, or a variety of other materials.

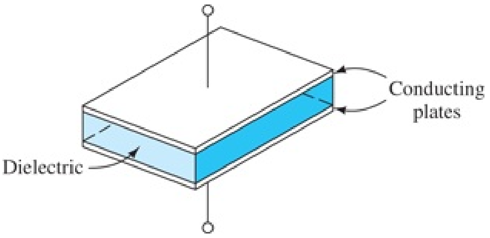



**Figure 3.1**
A parallel-plate capacitor consists of two conductive plates separated by a dielectric layer.

> Capacitors are constructed by separating two conducting plates, which are usually metallic, by a thin layer of insulating material.

Let us consider what happens as current flows through a capacitor. Suppose that current flows downward, as shown in **Figure 3.2(a)**. In most metals, current consists of electrons moving, and conventional current flowing downward represents electrons actually moving upward. As electrons move upward, they collect on the lower plate of the capacitor. Thus, the lower plate accumulates a net negative charge that produces an electric field in the dielectric. This electric field forces electrons to leave the upper plate at the same rate that they accumulate on the lower plate. Therefore, current appears to flow through the capacitor. As the charge builds up, voltage appears across the capacitor.

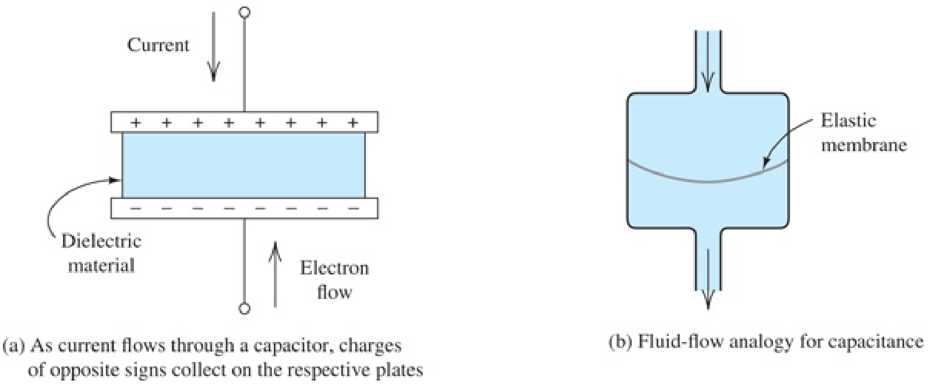



**Figure 3.2**
A capacitor and its fluid-flow analogy.

We say that the charge accumulated on one plate is stored in the capacitor. However, the total charge on both plates is always zero, because positive charge on one plate is balanced by negative charge of equal magnitude on the other plate.

> Positive charge on one plate is balanced by negative charge of equal magnitude on the other plate.

## Fluid-Flow Analogy

In terms of the fluid-flow analogy, a capacitor represents a reservoir with an elastic membrane separating the inlet and outlet as shown in **Figure 3.2(b)**. As the fluid flows into the inlet, the membrane is stretched, creating a force (analogous to capacitor voltage) that opposes further flow. The displaced fluid volume starting from the unstretched membrane position is analogous to the charge stored on one plate of the capacitor.

> In terms of the fluid-flow analogy, a capacitor represents a reservoir with an elastic membrane separating the inlet and outlet.

## Stored Charge in Terms of Voltage

In an ideal capacitor, the stored charge *q* is proportional to the voltage between the plates:

$$q = Cv \tag{3.1}$$

The constant of proportionality is the capacitance *C*, which has units of farads (F). Farads are equivalent to coulombs per volt.

> In an ideal capacitor, the stored charge *q* is proportional to the voltage between the plates.

To be more precise, the charge *q* is the net charge on the plate corresponding to the positive reference for *v*. Thus, if *v* is positive, there is positive charge on the plate corresponding to the positive reference for *v*. On the other hand, if *v* is negative, there is negative charge on the plate corresponding to the positive reference.

A farad is a very large amount of capacitance. In most applications, we deal with capacitances in the range from a few picofarads $\left(1\ \mathrm{pF} = 10^{-12}\ \mathrm{F}\right)$ up to perhaps 0.01 F. Capacitances in the femtofarad $\left(1\ \mathrm{fF} = 10^{-15}\ \mathrm{F}\right)$ range are responsible for limiting the performance of computer chips.

> In most applications, we deal with capacitances in the range from a few picofarads up to perhaps 0.01 F.

## Current in Terms of Voltage

Recall that current is the time rate of flow of charge. Taking the derivative of each side of **Equation 3.1** with respect to time, we have

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv) \tag{3.2}$$

Ordinarily, capacitance is not a function of time. (An exception is the capacitor microphone mentioned earlier.) Thus, the relationship between current and voltage becomes

$$i = C\frac{dv}{dt} \tag{3.3}$$

**Equations 3.1** and **3.3** show that as voltage increases, current flows through the capacitance and charge accumulates on each plate. If the voltage remains constant, the charge is constant and the current is zero. Thus, a capacitor appears to be an open circuit for a steady dc voltage.

> Capacitors act as open circuits for steady dc voltages.

The circuit symbol for capacitance and the references for *v* and *i* are shown in **Figure 3.3**. Notice that the references for the voltage and current have the passive configuration. In other words, the current reference direction points into the positive reference polarity. If the references were opposite to the passive configuration, **Equation 3.3** would have a minus sign:

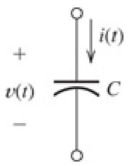



**Figure 3.3**
The circuit symbol for capacitance, including references for the current *i*(*t*) and voltage *v*(*t*).

$$i = -C\frac{dv}{dt} \tag{3.4}$$

Sometimes, we emphasize the fact that in general the voltage and current are functions of time by denoting them as *v*(*t*) and *i*(*t*).

***Example 3.1 Determining Current for a Capacitance Given Voltage***

Suppose that the voltage *v*(*t*) shown in **Figure 3.4(b)** is applied to a $1-\mu\mathrm{F}$ capacitance. Plot the stored charge and the current through the capacitance versus time.

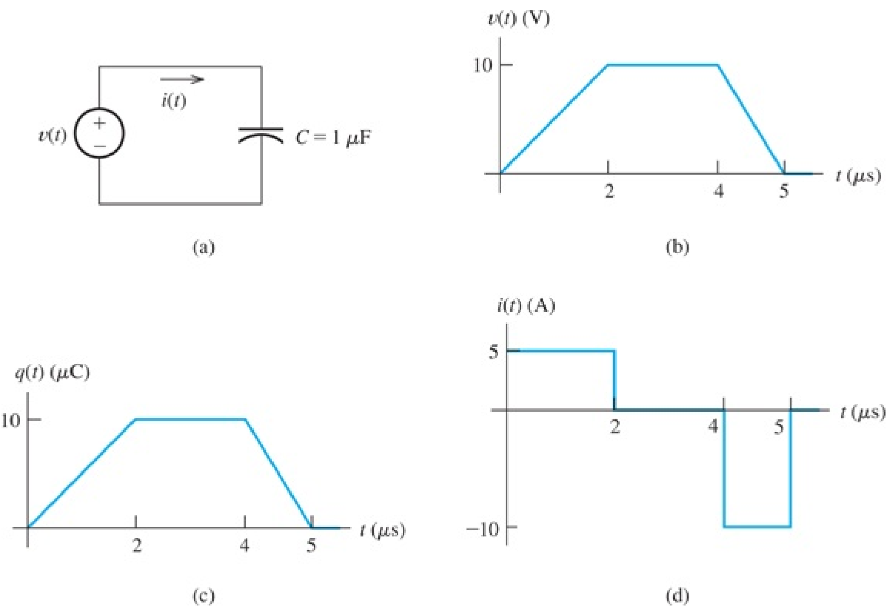



**Figure 3.4**
Circuit and waveforms for **Example 3.1**.

Solution

The charge stored on the top plate of the capacitor is given by **Equation 3.1**. [We know that *q*(*t*) represents the charge on the top plate because that is the plate corresponding to the positive reference for *v*(*t*).] Thus,

$$q(t) = Cv(t) = 10^{-6}v(t)$$

This is shown in **Figure 3.4(c)**.

The current flowing through the capacitor is given by **Equation 3.3**:

$$i(t) = C\frac{dv(t)}{dt} = 10^{-6}\frac{dv(t)}{dt}$$

Of course, the derivative of the voltage is the slope of the voltage versus time plot. Hence, for *t* between 0 and $2\ \mu\text{s},$ we have

$$\frac{dv(t)}{dt} = \frac{10\ \text{V}}{2\times10^{-6}\ \text{s}} = 5\times10^{6}\ \text{V/s}$$

and

$$i(t) = C\frac{dv(t)}{dt} = 10^{-6}\times5\times10^{6} = 5\ \text{A}$$

Between $t = 2$ and $4\ \mu\text{s},$ the voltage is constant $(dv/dt = 0)$ and the current is zero. Finally, between $t = 4$ and $5\ \mu\text{s},$ we get

$$\frac{dv(t)}{dt} = \frac{-10\ \text{V}}{10^{-6}\ \text{s}} = -10^{7}\ \text{V/s}$$

and

$$i(t) = C\frac{dv(t)}{dt} = 10^{-6}\times\left(-10^{7}\right) = -10\ \text{A}$$

A plot of *i*(*t*) is shown in **Figure 3.4(d)**.

Notice that as the voltage increases, current flows through the capacitor and charges accumulate on the plates. For constant voltage, the current is zero and the charge is constant. When the voltage decreases, the direction of the current reverses, and the stored charge is removed from the capacitor.

*Exercise 3.1*

The charge on a $2$-$\mu\text{F}$ capacitor is given by

$$q(t) = 10^{-6}\sin\left(10^{5}t\right)\ \text{C}$$

Find expressions for the voltage and for the current. (The angle is in radians.)

**Answer** $v(t) = 0.5\sin\left(10^{5}t\right)\ \text{V},\ i(t) = 0.1\cos\left(10^{5}t\right)\ \text{A}.$

## Voltage in Terms of Current

Suppose that we know the current *i*(*t*) flowing through a capacitance *C* and we want to compute the charge and voltage. Since current is the time rate of charge flow, we must integrate the current to compute charge. Often in circuit analysis problems, action starts at some initial time $t_0,$ and the initial charge $q(t_0)$ is known. Then, charge as a function of time is given by

$$q(t) = \int_{t_0}^{t} i(t)\, dt + q(t_0) \tag{3.5}$$

Setting the right-hand sides of **Equations 3.1** and **3.5** equal to each other and solving for the voltage *v*(*t*), we have

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt + \frac{q(t_0)}{C} \tag{3.6}$$

However, the initial voltage across the capacitance is given by

$$v(t_0) = \frac{q(t_0)}{C} \tag{3.7}$$

Substituting this into **Equation 3.6**, we have

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt + v(t_0) \tag{3.8}$$

Usually, we take the initial time to be $t_0 = 0$.

*Example 3.2 Determining Voltage for a Capacitance Given Current*

After $t_0 = 0,$ the current in a $0.1\text{-}\mu\text{F}$ capacitor is given by

$$i(t) = 0.5 \sin\left(10^4 t\right) \text{ A}$$

(The argument of the sin function is in radians.) The initial charge on the capacitor is $q(0) = 0.$ Plot *i*(*t*), *q*(*t*), and *v*(*t*) to scale versus time.

Solution

First, we use **Equation 3.5** to find an expression for the charge:

$$\begin{aligned} q(t) &= \int_0^t i(t)\, dt + q(0) \\ &= \int_0^t 0.5 \sin\left(10^4 t\right) dt \\ &= -0.5 \times 10^{-4} \cos\left(10^4 t\right) \Big|_0^t \\ &= 0.5 \times 10^{-4} [1 - \cos\left(10^4 t\right)] C \end{aligned}$$

Solving **Equation 3.1** for voltage, we have

$$\begin{aligned} v(t) &= \frac{q(t)}{C} = \frac{q(t)}{10^{-7}} \\ &= 500\left[1 - \cos\left(10^4 t\right)\right] \text{ V} \end{aligned}$$

Plots of *i*(*t*), *q*(*t*), and *v*(*t*) are shown in **Figure 3.5**. Immediately after $t = 0,$ the current is positive and *q*(*t*) increases. After the first half-cycle, *i*(*t*) becomes negative and *q*(*t*) decreases. At the completion of one cycle, the charge and voltage have returned to zero.

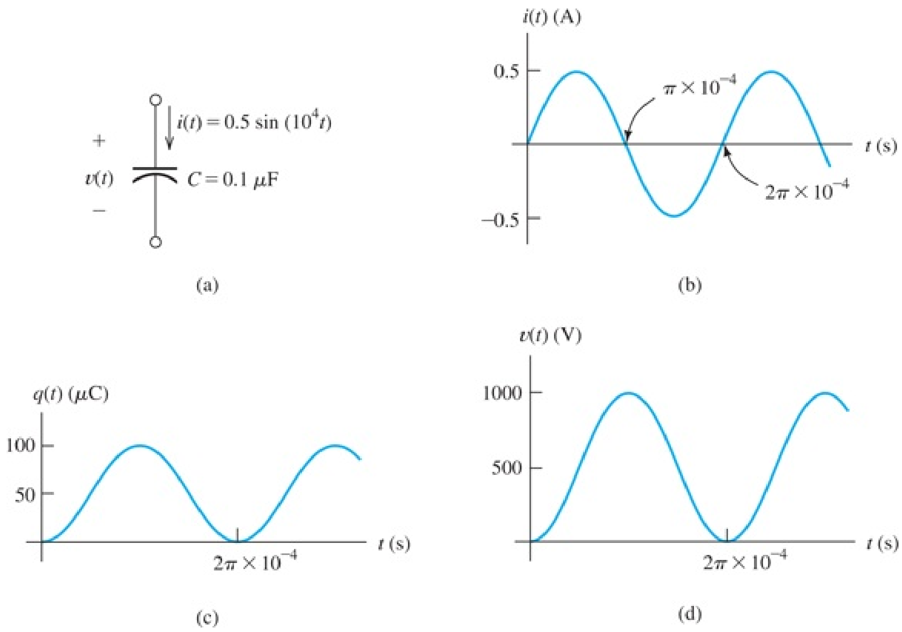



**Figure 3.5**
Waveforms for **Example 3.2**.

## Stored Energy

The power delivered to a circuit element is the product of the current and the voltage (provided that the references have the passive configuration):

$$p(t) = v(t)\, i(t) \tag{3.9}$$

Using **Equation 3.3** to substitute for the current, we have

$$p(t) = Cv\frac{dv}{dt} \tag{3.10}$$

Suppose we have a capacitor that initially has $v(t_0) = 0$. Then the initial stored electrical energy is zero, and we say that the capacitor is uncharged. Furthermore, suppose that between time $t_0$ and some later time *t* the voltage changes from 0 to *v*(*t*) volts. As the voltage magnitude increases, energy is delivered to the capacitor, where it is stored in the electric field between the plates.

If we integrate the power delivered from $t_0$ to *t*, we find the energy delivered:

$$w(t) = \int_{t_0}^{t} p(t)\, dt \tag{3.11}$$

Using **Equation 3.10** to substitute for power, we find that

$$w(t) = \int_{t_0}^{t} Cv\frac{dv}{dt}dt \tag{3.12}$$

Canceling differential time and changing the limits to the corresponding voltages, we have

$$w(t) = \int_{0}^{v(t)} Cv\, dv \tag{3.13}$$

Integrating and evaluating, we get

$$\boxed{w(t) = \frac{1}{2}Cv^2(t)} \tag{3.14}$$

This represents energy stored in the capacitance that can be returned to the circuit.

Solving **Equation 3.1** for *v*(*t*) and substituting into **Equation 3.14**, we can obtain two alternative expressions for the stored energy:

$$\boxed{w(t) = \frac{1}{2}v(t)\, q(t)} \tag{3.15}$$

$$\boxed{w(t) = \frac{q^2(t)}{2C}} \tag{3.16}$$

***Example 3.3 Current, Power, and Energy for a Capacitance***

Suppose that the voltage waveform shown in **Figure 3.6(a)** is applied to a $10\text{-}\mu\text{F}$ capacitance. Find and plot the current, the power delivered, and the energy stored for time between 0 and 5 s.

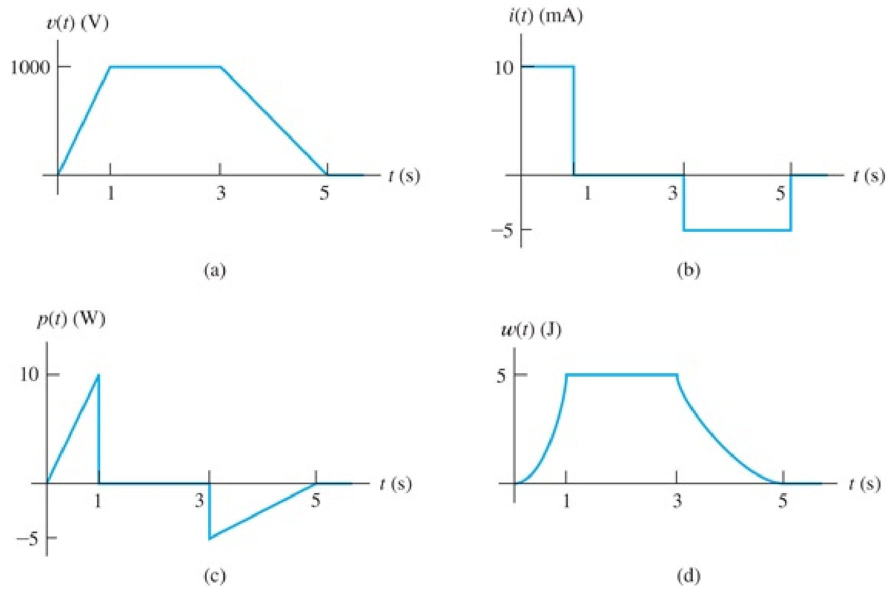



**Figure 3.6**

Waveforms for **Example 3.3**.

Solution

First, we write expressions for the voltage as a function of time:

$$v(t) = \begin{cases} 1000t \text{ V} & \text{for } 0 < t < 1 \\ 1000 \text{ V} & \text{for } 1 < t < 3 \\ 500(5 - t) \text{ V} & \text{for } 3 < t < 5 \end{cases}$$

Using **Equation 3.3**, we obtain expressions for the current:

$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) = \begin{cases} 10 \times 10^{-3} \text{ A} & \text{for } 0 < t < 1 \\ 0 \text{ A} & \text{for } 1 < t < 3 \\ -5 \times 10^{-3} \text{ A} & \text{for } 3 < t < 5 \end{cases}$$

The plot of *i*(*t*) is shown in **Figure 3.6(b)**.

Next, we find expressions for power by multiplying the voltage by the current:

$$p(t) = v(t)\, i(t)$$

$$p(t) = \begin{cases} 10t \text{ W} & \text{for } 0 < t < 1 \\ 0 \text{ W} & \text{for } 1 < t < 3 \\ 2.5(t - 5) \text{ W} & \text{for } 3 < t < 5 \end{cases}$$

The plot of *p*(*t*) is shown in **Figure 3.6(c)**. Notice that between $t = 0$ and $t = 1$ power is positive, showing that energy is being delivered to the capacitance. Between $t = 3$ and $t = 5,$ energy flows out of the capacitance back into the rest of the circuit.

Next, we use **Equation 3.14** to find expressions for the stored energy:

$$w(t) = \frac{1}{2}Cv^2(t)$$

$$w(t) = \begin{cases} 5t^2 \text{ J} & \text{for } 0 < t < 1 \\ 5 \text{ J} & \text{for } 1 < t < 3 \\ 1.25(5 - t)^2 \text{ J} & \text{for } 3 < t < 5 \end{cases}$$

The plot of *w*(*t*) is shown in **Figure 3.6(d)**.

*Exercise 3.2*

The current through a $0.1\text{-}\mu\mathrm{F}$ capacitor is shown in **Figure 3.7**. At $t_0 = 0,$ the voltage across the capacitor is zero. Find the charge, voltage, power, and stored energy as functions of time and plot them to scale versus time.

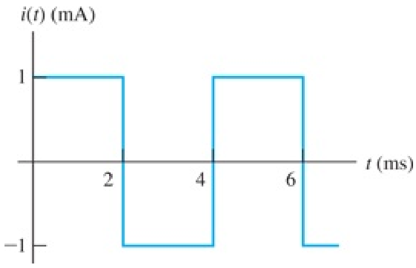



**Figure 3.7**
Square-wave current for **Exercise 3.2**.

**Answer** The plots are shown in **Figure 3.8**.

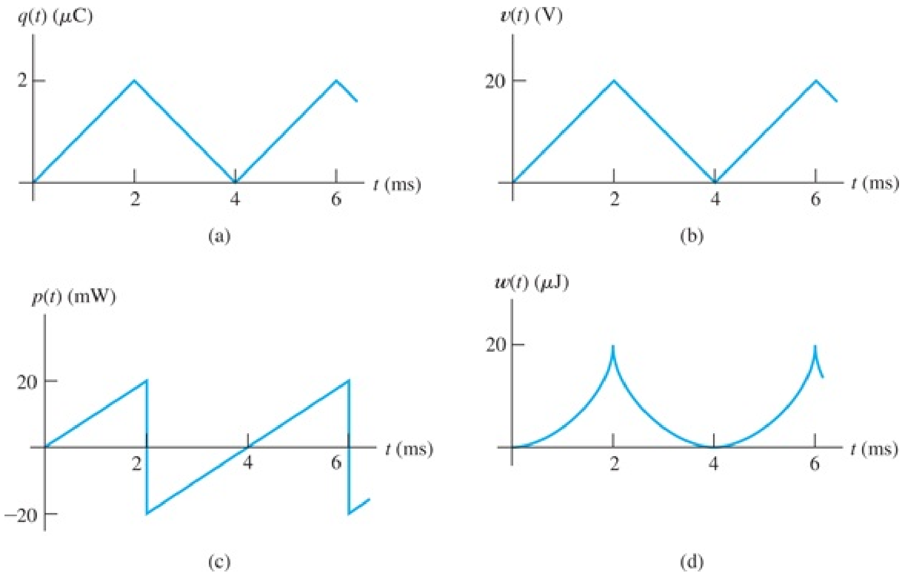



**Figure 3.8**
Answers for **Exercise 3.2.**
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## 3.2 Capacitances in Series and Parallel

### Capacitances in Parallel

Suppose that we have three capacitances in parallel as shown in **Figure 3.9**. Of course, the same voltage appears across each of the elements in a parallel circuit. The currents are related to the voltage by **Equation 3.3**. Thus, we can write

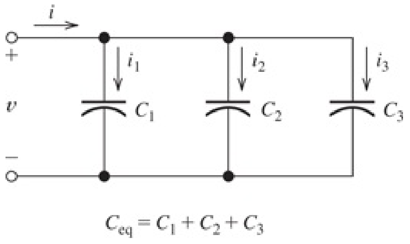



**Figure 3.9**
Three capacitances in parallel.

$$i_1 = C_1 \frac{dv}{dt} \tag{3.17}$$

$$i_2 = C_2 \frac{dv}{dt} \tag{3.18}$$

$$i_3 = C_3 \frac{dv}{dt} \tag{3.19}$$

Applying KCL at the top node of the circuit, we have

$$i = i_1 + i_2 + i_3 \tag{3.20}$$

Using **Equations 3.17**, **3.18**, and **3.19** to substitute into **Equation 3.20**, we obtain

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} \tag{3.21}$$

This can be written as

$$i = (C_1 + C_2 + C_3) \frac{dv}{dt} \tag{3.22}$$

Now, we define the equivalent capacitance as the sum of the capacitances in parallel:

> We add parallel capacitances to find the equivalent capacitance.

$$C_{eq} = C_1 + C_2 + C_3 \tag{3.23}$$

Using this definition in **Equation 3.22**, we find that

$$i = C_{eq} \frac{dv}{dt} \tag{3.24}$$

Thus, the current in the equivalent capacitance is the same as the total current flowing through the parallel circuit.

In sum, we add parallel capacitances to find the equivalent capacitance. Recall that for resistances, the resistances are added if they are in *series* rather than parallel. Thus, we say that capacitances in parallel are combined like resistances in series.

> Capacitances in parallel are combined like resistances in series.

### Capacitances in Series

By a similar development, it can be shown that the equivalent capacitance for three series capacitances is

$$C_{\text{eq}} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3} \tag{3.25}$$

We conclude that capacitances in series are combined like resistances in parallel.

> Capacitances in series are combined like resistances in parallel.

A technique for obtaining high voltages from low-voltage sources is to charge *n* capacitors in parallel with the source, and then to switch them to a series combination. The resulting voltage across the series combination is *n* times the source voltage. For example, in some cardiac pacemakers, a 2.5-V battery is used, but 5 V need to be applied to the heart muscle to initiate a beat. This is accomplished by charging two capacitors from the 2.5-V battery. The capacitors are then connected in series to deliver a brief 5-V pulse to the heart.

*Example 3.4 Capacitances in Series and Parallel*

Determine the equivalent capacitance between terminals *a* and *b* in **Figure 3.10(a)**.

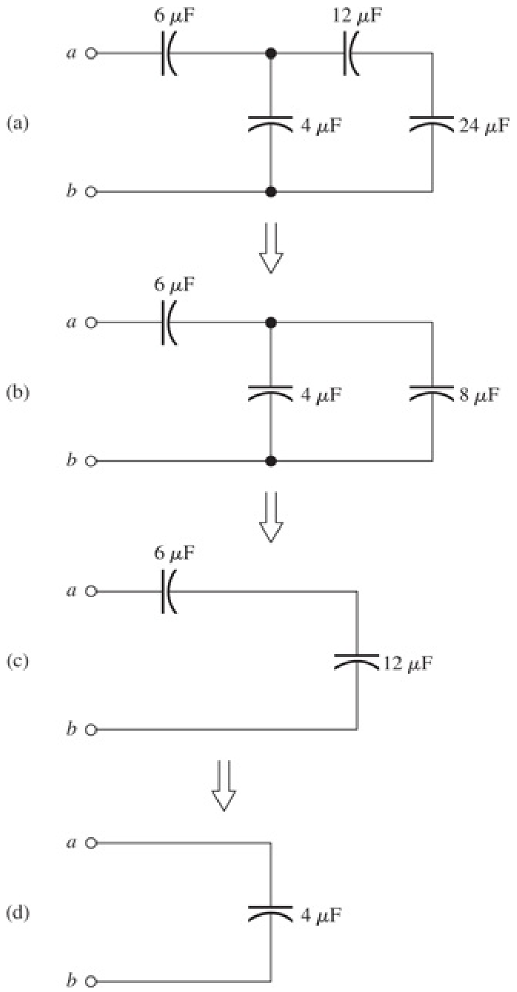



**Figure 3.10**
Circuit of **Example 3.4.**

Solution

First, notice that the $12\text{-}\mu\mathrm{F}$ and $24\text{-}\mu\mathrm{F}$ capacitances are in series.

Thus, their equivalent capacitance is:

$$\frac{1}{1/12+1/24}=8\ \mu\mathrm{F}$$

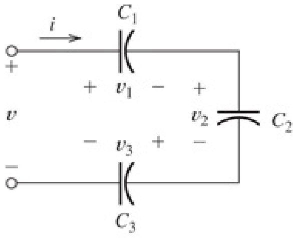
The resulting equivalent is shown in **Figure 3.10(b)**.

Then, the $8\text{-}\mu\mathrm{F}$ and $4\text{-}\mu\mathrm{F}$ capacitances are in parallel. Their equivalent is $12\text{-}\mu\mathrm{F}$ as shown in **Figure 3.10(c)**.

Finally we combine the $6\text{-}\mu\mathrm{F}$ and $12\text{-}\mu\mathrm{F}$ capacitances in series resulting in $4\text{-}\mu\mathrm{F}$ as shown in **Figure 3.10(d)**.

*Exercise 3.3*

Derive **Equation 3.25** for the three capacitances shown in **Figure 3.11**.




$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

**Figure 3.11**
Three capacitances in series.

*Exercise 3.4*

a. Two capacitances of $2\ \mu\mathrm{F}$ and $1\ \mu\mathrm{F}$ are in series. Find the equivalent capacitance.

b. Repeat if the capacitances are in parallel.

**Answer**

a. $2/3\ \mu\mathrm{F};$

b. $3\ \mu\mathrm{F}\,.$

# 3.3 Physical Characteristics of Capacitors

## Capacitance of the Parallel-Plate Capacitor

A parallel-plate capacitor is shown in **Figure 3.12**, including dimensions. The area of each plate is denoted as *A*. (Actually, *A* is the area of one side of the plate.) The rectangular plate shown has a width *W*, length *L*, and area $A = W \times L$. The plates are parallel, and the distance between them is denoted as *d*.

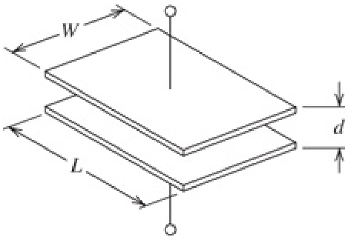



**Figure 3.12**
A parallel-plate capacitor, including dimensions.

If the distance *d* between the plates is much smaller than both the width and the length of the plates, the capacitance is approximately given by

$$C = \frac{\epsilon A}{d} \tag{3.26}$$

in which $\epsilon$ is the **dielectric constant** of the material between the plates. For vacuum, the dielectric constant is

Dielectric constant of vacuum.

$$\epsilon = \epsilon_0 \cong 8.85 \times 10^{-12}\ \mathrm{F/m}$$

For other materials, the dielectric constant is

$$\epsilon = \epsilon_r \epsilon_0 \tag{3.27}$$

where $\epsilon_r$ is the **relative dielectric constant** which has no physical units. Values of the relative dielectric constant for selected materials are given in **Table 3.1**.

**Table 3.1 Relative Dielectric Constants for Selected Materials**

| | |
|---|---|
| Air | 1.0 |
| Diamond | 5.5 |
| Mica | 7.0 |
| Polyester | 3.4 |
| Quartz | 4.3 |
| Silicon dioxide | 3.9 |
| Water | 78.5 |

***Example 3.5 Calculating Capacitance Given Physical Parameters***

Compute the capacitance of a parallel-plate capacitor having rectangular plates 10 cm by 20 cm separated by a distance of 0.1 mm. The dielectric is air. Repeat if the dielectric is mica.

Solution

First, we compute the area of a plate:

$$A = L \times W = \left(10 \times 10^{-2}\right) \times \left(20 \times 10^{-2}\right) = 0.02\ \mathrm{m}^2$$

From **Table 3.1**, we see that the relative dielectric constant of air is 1.00. Thus, the dielectric constant is

$$\in\ =\ \in_r \in_0 = 1.00 \times 8.85 \times 10^{-12}\ \mathrm{F/m}$$

Then, the capacitance is

$$C = \frac{\in A}{d} = \frac{8.85 \times 10^{-12} \times 0.02}{10^{-4}} = 1770 \times 10^{-12}\ \mathrm{F}$$

For a mica dielectric, the relative dielectric constant is 7.0. Thus, the capacitance is seven times larger than for air or vacuum:

$$C = 12,390 \times 10^{-12}\ \mathrm{F}$$

*Exercise 3.5*

We want to design a $1\text{-}\mu\mathrm{F}$ capacitor. Compute the length required for rectangular plates of 2-cm width if the dielectric is polyester of $15\text{-}\mu\mathrm{m}$ thickness.

**Answer** $L = 24.93\ \mathrm{m}$.

### Practical Capacitors

To achieve capacitances on the order of a microfarad, the dimensions of parallel-plate capacitors are too large for compact electronic circuits such as portable computers or cellular telephones. Frequently, capacitors are constructed by alternating the plates with two layers of dielectric, which are then rolled to fit in a smaller area. By staggering the plates before rolling, electrical contact can be made with the plates from the ends of the roll. This type of construction is illustrated in **Figure 3.13**.

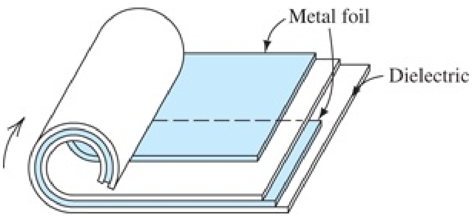



**Figure 3.13**

Practical capacitors can be constructed by interleaving the plates with two dielectric layers and rolling them up. By staggering the plates, connection can be made to one plate at each end of the roll.

To achieve small-volume capacitors, a very thin dielectric having a high dielectric constant is desirable. However, dielectric materials break down and become conductors when the electric field intensity (volts per meter) is too high. Thus, real capacitors have maximum voltage ratings. For a given voltage, the electric field intensity becomes higher as the dielectric layer becomes thinner. Clearly, an engineering trade-off exists between compact size and voltage rating.

Real capacitors have maximum voltage ratings.

An engineering trade-off exists between compact size and high voltage rating.

### Electrolytic Capacitors

In **electrolytic capacitors**, one of the plates is metallic aluminum or tantalum, the dielectric is an oxide layer on the surface of the metal, and the other “plate” is an electrolytic solution. The oxide-coated metallic plate is immersed in the electrolytic solution.

This type of construction results in high capacitance per unit volume. However, only one polarity of voltage should be applied to electrolytic capacitors. For the opposite polarity, the dielectric layer is chemically attacked, and a conductive path appears between the plates. (Usually, the allowed polarity is marked on the outer case.) On the other hand, capacitors constructed with polyethylene, Mylar®, and so on can be used in applications where the voltage polarity reverses. When the application results in voltages of only one polarity and a large-value capacitance is required, designers frequently use electrolytic capacitors.

Only voltages of the proper polarity should be applied to electrolytic capacitors.

### Parasitic Effects

Real capacitors are not always well modeled simply as a capacitance. A more complete circuit model for a capacitor is shown in **Figure 3.14**. In addition to the capacitance *C*, series resistance $R_s$ appears because of the resistivity of the material composing the plates. A series inductance $L_s$ (we discuss inductance later in this chapter) occurs because the current flowing through the capacitor creates a magnetic field. Finally, no practical material is a perfect insulator, and the resistance $R_p$ represents conduction through the dielectric.

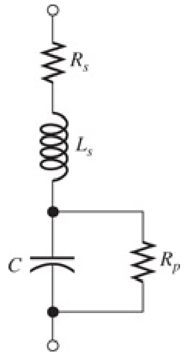



**Figure 3.14**
The circuit model for a capacitor, including the parasitic elements $R_s,\ L_s,$ and $R_p$.

We call $R_s,\ L_s,$ and $R_p$ **parasitic elements**. We design capacitors to minimize the effects of parasitic circuit elements consistent with other requirements such as physical size and voltage rating. However, parasitics are always present to some degree. In designing circuits, care must be used to select components for which the parasitic effects do not prevent proper operation of the circuit.

***Example 3.6 What Happened to the Missing Energy?***

Consider the situation shown in **Figure 3.15**. Prior to $t = 0,$ the capacitor $C_1$ is charged to a voltage of $v_1 = 100\ \mathrm{V}$ and the other capacitor has no charge (i.e., $v_2 = 0$ ). At $t = 0,$ the switch closes. Compute the total energy stored by both capacitors before and after the switch closes.

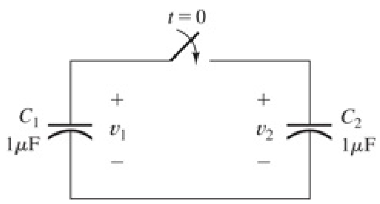



**Figure 3.15**
See **Example 3.6**.

Solution

The initial stored energy for each capacitor is

$$\begin{aligned} w_1 &= \frac{1}{2} C_1 v_1^{\ 2} = \frac{1}{2}\left(10^{-6}\right)(100)^{\ 2} = 5\ \mathrm{mJ} \\ w_2 &= 0 \end{aligned}$$

and the total energy is

$$w_{\text{total}} = w_1 + w_2 = 5\ \mathrm{mJ}$$

To find the voltage and stored energy after the switch closes, we make use of the fact that the total charge on the top plates cannot change when the switch closes. This is true because there is no path for electrons to enter or leave the upper part of the circuit.

The charge stored on the top plate of $C_1$ prior to $t = 0$ is given by

$$q_1 = C_1 v_1 = 1 \times 10^{-6} \times 100 = 100\ \mu\mathrm{C}$$

Furthermore, the initial charge on $C_2$ is zero:

$$q_2 = 0$$

Thus, after the switch closes, the charge on the equivalent capacitance is

$$q_{\text{eq}} = q_1 + q_2 = 100\ \mu\text{C}$$

Also, notice that after the switch is closed, the capacitors are in parallel and have an equivalent capacitance of

$$C_{\text{eq}} = C_1 + C_2 = 2\ \mu\text{F}$$

The voltage across the equivalent capacitance is

$$v_{\text{eq}} = \frac{q_{\text{eq}}}{C_{\text{eq}}} = \frac{100\ \mu\text{C}}{2\ \mu\text{F}} = 50\ \text{V}$$

Of course, after the switch is closed, $v_1 = v_2 = v_{\text{eq}}$.

Now, we compute the stored energy with the switch closed:

$$\begin{aligned} w_1 &= \frac{1}{2} C_1 v^2{}_{\text{eq}} = \frac{1}{2}\left(10^{-6}\right)(50)^2 = 1.25\ \text{mJ} \\ w_2 &= \frac{1}{2} C_2 v^2{}_{\text{eq}} = \frac{1}{2}\left(10^{-6}\right)(50)^2 = 1.25\ \text{mJ} \end{aligned}$$

The total stored energy with the switch closed is

$$w_{\text{total}} = w_1 + w_2 = 2.5\ \text{mJ}$$

Thus, we see that the stored energy after the switch is closed is half of the value before the switch is closed. What happened to the missing energy?

Usually, the answer to this question is that it is absorbed in the parasitic resistances. It is impossible to construct capacitors that do not have some parasitic effects. Even if we use superconductors for the wires and capacitor plates, there would be parasitic inductance. If we included the parasitic inductance in the circuit model, we would not have missing energy. (We study circuits with time-varying voltages and currents in **Chapter 4**.)

> Usually, the missing energy is absorbed in the parasitic resistances.

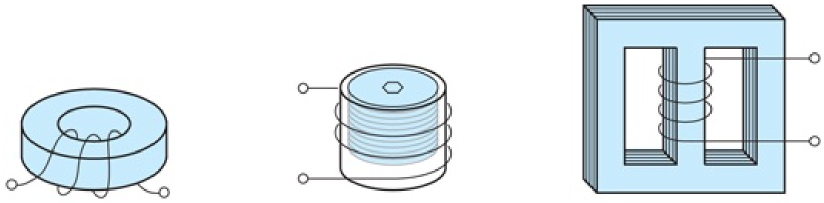
To put it another way, a physical circuit that is modeled exactly by **Figure 3.15** does not exist. Invariably, if we use a realistic model for an actual circuit, we can account for all of the energy.

> A physical circuit that is modeled exactly by **Figure 3.15** does not exist.

# 3.4 Inductance

Inductors are usually constructed by coiling wire around a form.

An inductor is usually constructed by coiling a wire around some type of form. Several examples of practical construction are illustrated in **Figure 3.16**. Current flowing through the coil creates a magnetic field or flux that links the coil. Frequently, the coil form is composed of a magnetic material such as iron or iron oxides that increases the magnetic flux for a given current. (Iron cores are often composed of thin sheets called **laminations**. We discuss the reason for this construction technique in **Chapter 14**.)



(a) Toroidal inductor

(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

(c) Inductor with a laminated iron core

**Figure 3.16**
An inductor is constructed by coiling a wire around some type of form.

When the current changes in value, the resulting magnetic flux changes. According to Faraday's law of electromagnetic induction, time-varying magnetic flux linking a coil induces voltage across the coil. For an ideal inductor, the voltage is proportional to the time rate of change of the current. Furthermore, the polarity of the voltage is such as to oppose the change in current. The constant of proportionality is called inductance, usually denoted by the letter *L*.

The circuit symbol for inductance is shown in **Figure 3.17**. In equation form, the voltage and current are related by

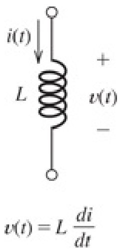



**Figure 3.17**
Circuit symbol and the $v-i$ relationship for inductance.

$$v(t) = L\frac{di}{dt} \tag{3.28}$$

As usual, we have assumed the passive reference configuration. In case the references are opposite to the passive configuration, **Equation 3.28** becomes

$$v(t) = -L\frac{di}{dt} \tag{3.29}$$

Inductance has units of henries (H), which are equivalent to volt seconds per ampere. Typically, we deal with inductances ranging from a fraction of a microhenry $(\mu\mathrm{H})$ to several tens of henries.

Inductance has units of henries (H), which are equivalent to volt seconds per ampere.

### Fluid-Flow Analogy

The fluid-flow analogy for inductance is the inertia of the fluid flowing through a *frictionless* pipe of constant diameter. The pressure differential between the ends of the pipe is analogous to voltage, and the flow rate or velocity is analogous to current. Thus, the acceleration of the fluid is analogous to rate of change of current. A pressure differential exists between the ends of the pipe only when the flow rate is increasing or decreasing.

The fluid-flow analogy for inductance is the inertia of the fluid flowing through a frictionless pipe of constant diameter.

One place where the inertia of flowing fluid is encountered is when a valve (typically operated by an electrical solenoid) closes suddenly, cutting off the flow. For example, in a washing machine, the sudden change in velocity of the water flow can cause high pressure, resulting in a bang and vibration of the plumbing. This is similar to electrical effects that occur when current in an inductor is suddenly interrupted. An application for the high voltage that appears when current is suddenly interrupted is in the ignition system for a gasoline-powered internal combustion engine.

### Current in Terms of Voltage

Suppose that we know the initial current $i(t_0)$ and the voltage *v*(*t*) across an inductance. Furthermore, suppose that we need to compute the current for $t > t_0$. Rearranging **Equation 3.28**, we have

$$di = \frac{1}{L} v(t)\, dt \tag{3.30}$$

Integrating both sides, we find that

$$\int_{i(t_0)}^{i(t)} di = \frac{1}{L} \int_{t_0}^{t} v(t)\, dt \tag{3.31}$$

Notice that the integral on the right-hand side of **Equation 3.31** is with respect to time. Furthermore, the limits are the initial time $t_0$ and the time variable *t*. The integral on the left-hand side is with respect to current with limits that correspond to the time limits on the right-hand side. Integrating, evaluating, and rearranging, we have

$$i(t) = \frac{1}{L} \int_{t_0}^{t} v(t)\, dt + i(t_0) \tag{3.32}$$

Notice that as long as *v*(*t*) is finite, *i*(*t*) can change only by an incremental amount in a time increment. Thus, *i*(*t*) must be continuous with no instantaneous jumps in value (i.e., discontinuities). (Later, we encounter idealized circuits in which infinite voltages appear briefly, and then the current in an inductance can change instantaneously.)

### Stored Energy

Assuming that the references have the passive configuration, we compute the power delivered to a circuit element by taking the product of the current and the voltage:

$$p(t) = v(t)\, i(t) \tag{3.33}$$

Using **Equation 3.28** to substitute for the voltage, we obtain

$$p(t) = Li(t) \frac{di}{dt} \tag{3.34}$$

Consider an inductor having an initial current $i(t_0) = 0$. Then, the initial electrical energy stored is zero. Furthermore, assume that between time $t_0$ and some later time *t*, the current changes from 0 to *i*(*t*). As the current magnitude increases, energy is delivered to the inductor, where it is stored in the magnetic field.

Integrating the power from $t_0$ to *t*, we find the energy delivered:

$$w(t) = \int_{t_0}^{t} p(t)\, dt \tag{3.35}$$

Using **Equation 3.34** to substitute for power, we have

$$w(t) = \int_{t_0}^{t} Li \frac{di}{dt}\, dt \tag{3.36}$$

Canceling differential time and changing the limits to the corresponding currents, we get

$$w(t) = \int_{0}^{i(t)} Li\, di \tag{3.37}$$

Integrating and evaluating, we obtain

$$w(t) = \frac{1}{2} Li^2(t) \tag{3.38}$$

This represents energy stored in the inductance that is returned to the circuit if the current changes back to zero.

***Example 3.7 Voltage, Power, and Energy for an Inductance***

The current through a 5-H inductance is shown in **Figure 3.18(a)**. Plot the voltage, power, and stored energy to scale versus time for *t* between 0 and 5 s.

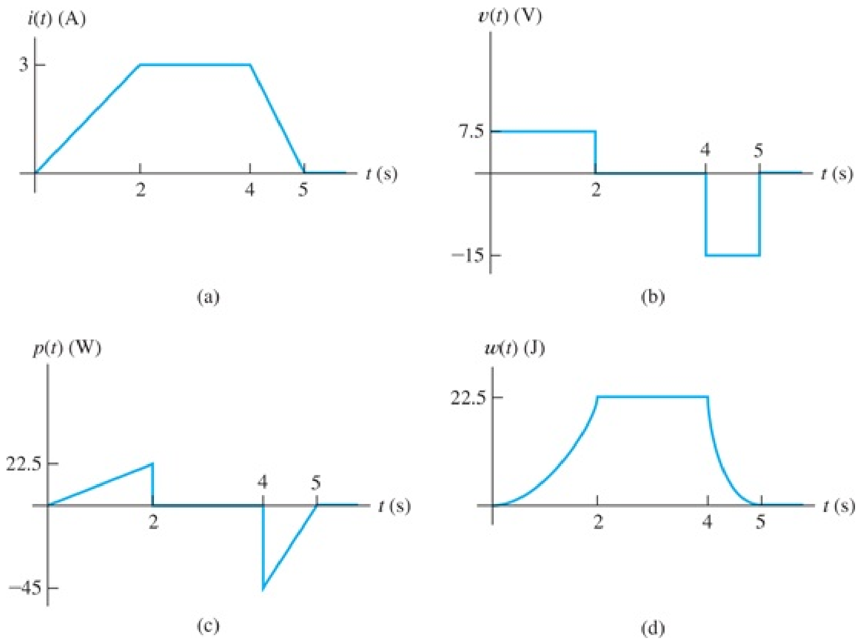



**Figure 3.18**

Waveforms for **Example 3.7**.

Solution

We use **Equation 3.28** to compute voltages:

$$v(t) = L\frac{di}{dt}$$

The time derivative of the current is the slope (rise over run) of the current versus time plot. For *t* between 0 and 2 s, we have $di/dt = 1.5\ \mathrm{A/s}$ and thus, $v = 7.5\ \mathrm{V}$. For *t* between 2 and 4 s, $di/dt = 0$, and therefore, $v = 0$. Finally, between 4 and 5 s, $di/dt = -3\ \mathrm{A/s}$ and $v = -15\ \mathrm{V}$. A plot of the voltage versus time is shown in **Figure 3.18(b)**.

Next, we obtain power by taking the product of current and voltage at each point in time. The resulting plot is shown in **Figure 3.18(c)**.

Finally, we use **Equation 3.38** to compute the stored energy as a function of time:

$$w(t) = \frac{1}{2}Li^2(t)$$

The resulting plot is shown in **Figure 3.18(d)**.

Notice in **Figure 3.18** that as current magnitude increases, power is positive and stored energy accumulates. When the current is constant, the voltage is zero, the power is zero, and the stored energy is constant. When the current magnitude falls toward zero, the power is negative, showing that energy is being returned to the other parts of the circuit.

*Example 3.8 Inductor Current with Constant Applied Voltage*

Consider the circuit shown in **Figure 3.19(a)**. In this circuit, we have a switch that closes at $t = 0,$ connecting a 10-V source to a 2-H inductance. Find the current as a function of time.

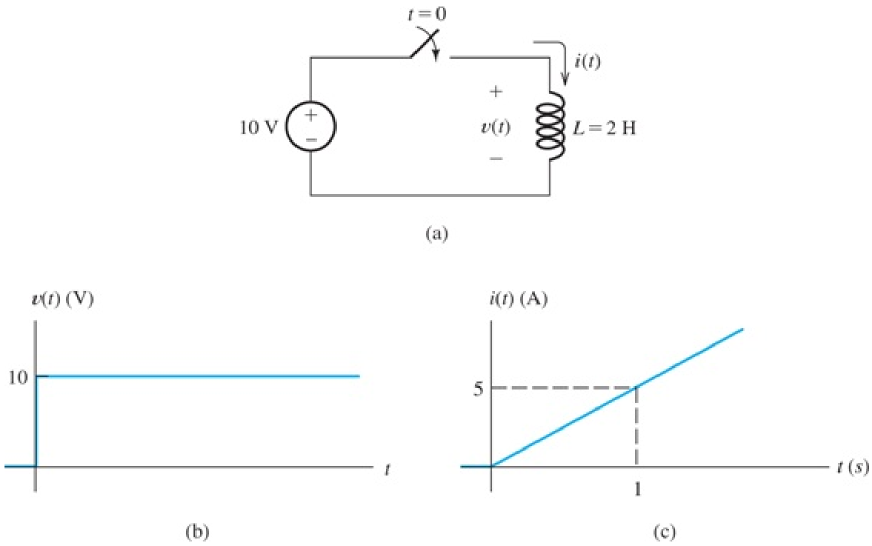



**Figure 3.19**
Circuit and waveforms for **Example 3.8**.

Solution

Notice that because the voltage applied to the inductance is finite, the current must be continuous. Prior to $t = 0,$ the current must be zero. (Current cannot flow through an open switch.) Thus, the current must also be zero immediately after $t = 0.$

The voltage across the inductance is shown in **Figure 3.19(b)**. To find the current, we employ **Equation 3.32**:

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\ dt + i(t_0)$$

In this case, we take $t_0 = 0,$ and we have $i(t_0) = i(0) = 0.$ Substituting values, we get

$$i(t) = \frac{1}{2}\int_{0}^{t} 10\ dt$$

where we have assumed that *t* is greater than zero. Integrating and evaluating, we obtain

$$i(t) = 5t\ \text{A} \qquad \text{for } t > 0$$

A plot of the current is shown in **Figure 3.19(c)**.

Notice that the current in the inductor gradually increases after the switch is closed. Because a constant voltage is applied after $t = 0,$ the current increases at a steady rate as predicted by **Equation 3.28**, which is repeated here for convenience:

$$v(t) = L\frac{di}{dt}$$

If *v*(*t*) is constant, the rate of change of the current *di*/*dt* is constant.

Suppose that at $t = 1\ \text{s},$ we open the switch in the circuit of **Figure 3.19**. Ideally, current cannot flow through an open switch. Hence, we expect the current to fall abruptly to zero at $t = 1\ \text{s}.$ However, the voltage across the inductor is proportional to the time rate of change of the current. For an abrupt change in current, this principle predicts infinite voltage across the inductor. This infinite voltage would last for only the instant at which the current falls. Later, we introduce the concept of an impulse function to describe this situation (and similar ones). For now, we simply point out that very large voltages can appear when we switch circuits that contain inductances.

If we set up a real circuit corresponding to **Figure 3.19(a)** and open the switch at $t = 1\ \mathrm{s}$, we will probably find that the high voltage causes an arc across the switch contacts. The arc persists until the energy in the inductor is used up. If this is repeated, the switch will soon be destroyed.

*Exercise 3.6*

The current through a 10-mH inductance is $i(t) = 0.1\cos(10^4 t)\ \mathrm{A}$. Find the voltage and stored energy as functions of time. Assume that the references for *v*(*t*) and *i*(*t*) have the passive configuration. (The angle is in radians.)

**Answer** $v(t) = -10\sin(10^4 t)\ \mathrm{V}$, $w(t) = 50\cos^2(10^4 t)\ \mu\mathrm{J}$.

*Exercise 3.7*

The voltage across a $150\text{-}\mu\mathrm{H}$ inductance is shown in **Figure 3.20(a)**. The initial current is $i(0) = 0$. Find and plot the current *i*(*t*) to scale versus time. Assume that the references for *v*(*t*) and *i*(*t*) have the passive configuration.

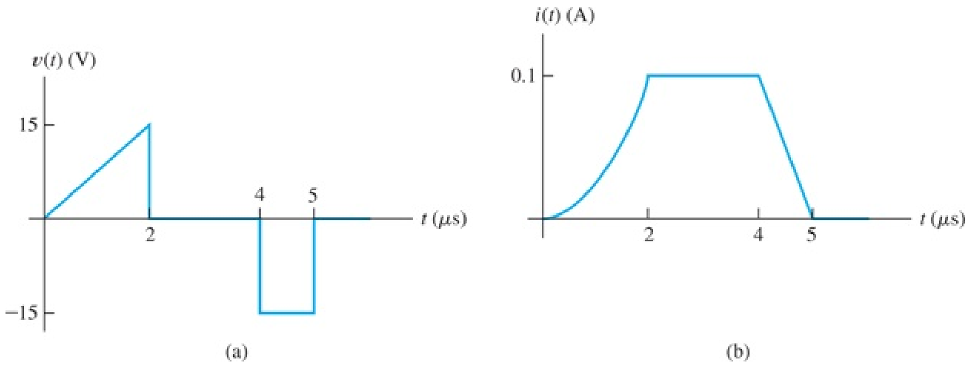



**Figure 3.20**
See **Exercise 3.7**.

**Answer** The current is shown in **Figure 3.20(b)**.

## 3.5 Inductances in Series and Parallel

It can be shown that the equivalent inductance for a series circuit is equal to the sum of the inductances connected in series. On the other hand, for inductances in parallel, we find the equivalent inductance by taking the reciprocal of the sum of the reciprocals of the parallel inductances. Series and parallel equivalents for inductances are illustrated in **Figure 3.21**. Notice that inductances are combined in exactly the same way as are resistances. These facts can be proven by following the pattern used earlier in this chapter to derive the equivalents for series capacitances.

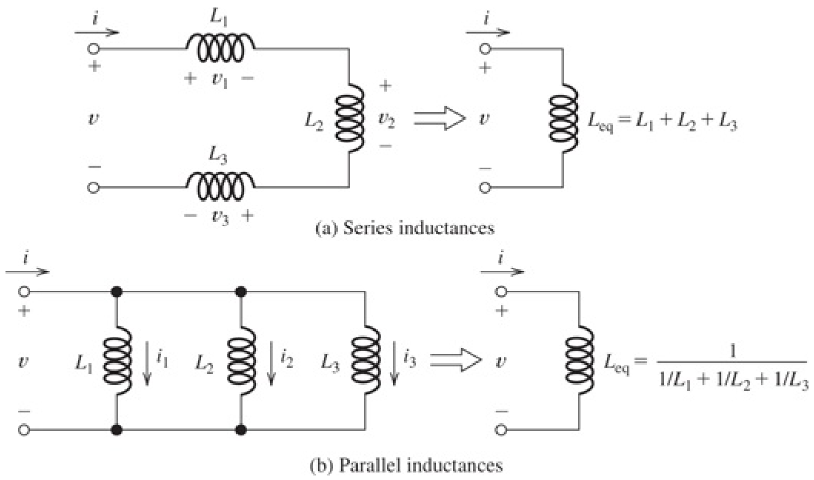



**Figure 3.21**
Inductances in series and parallel are combined in the same manner as resistances.

> Inductances in series and parallel are combined by using the same rules as for resistances: series inductances are added; parallel inductances are combined by taking the reciprocal of the sum of the reciprocals of the individual inductances.

**Example 3.9 Inductances in Series and Parallel**

Determine the equivalent inductance between terminals *a* and *b* in **Figure 3.22(a)**.

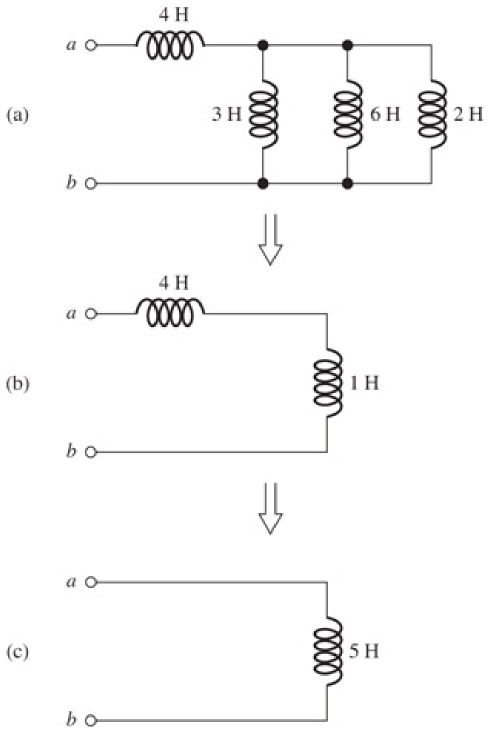



**Figure 3.22**
Circuit of **Example 3.9**.

*Solution*

First, notice that the 3-H, 6-H, and 2-H inductances are in parallel.

Thus, their equivalent inductance is:

$$\frac{1}{1/3 + 1/6 + 1/2} = 1\ \mathrm{H}$$

The resulting equivalent is shown in **Figure 3.22(b)**.

Finally, we combine the 4-H and 1-H inductances in series resulting in 5 H as shown in **Figure 3.22(c)**.

## Exercise 3.8

Prove that inductances in series are added to find the equivalent inductance.

## Exercise 3.9

Prove that inductances in parallel are combined according to the formula given in **Figure 3.21(b)**.

## Exercise 3.10

Find the equivalent inductance for each of the circuits shown in **Figure 3.23**.

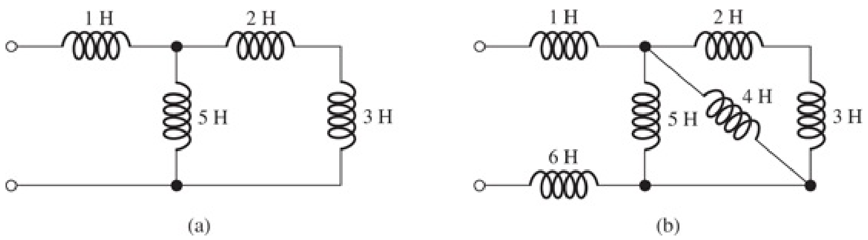



**Figure 3.23**
See **Exercise 3.10**.

### Answer

a. 3.5 H;

b. 8.54 H.

# 3.6 Practical Inductors

Real inductors take a variety of appearances, depending on their inductance and the application. (Some examples were shown earlier in **Figure 3.16**.) For example, a $1\text{-}\mu\text{H}$ inductor could consist of 25 turns of fine (say, number 28) wire wound on an iron oxide toroidal (doughnut-shaped) core having an outside diameter of 1/2 cm. On the other hand, a typical 5-H inductor consists of several hundred turns of number 18 wire on an iron form having a mass of 1 kg.

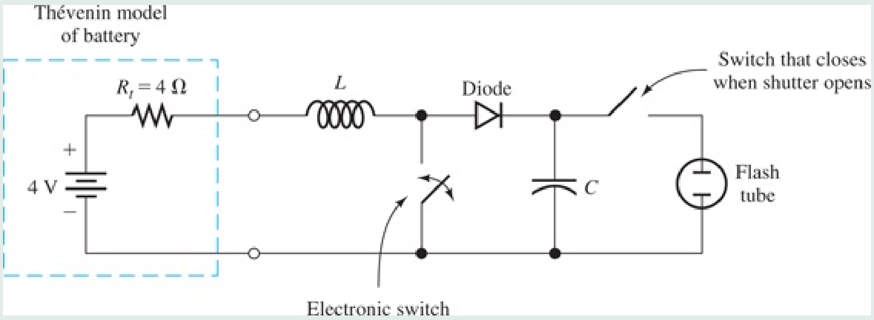
Usually, metallic iron forms, also called *cores*, are made of thin sheets called *laminations*. [See **Figure 3.16(c)** for an example.] This is necessary because voltages are induced in the core by the changing magnetic field. These voltages cause **eddy currents** to flow in the core, dissipating energy. Usually, this **core loss** is undesirable. Using laminations that are insulated from one another helps to reduce eddy-current loss. The laminations are arranged perpendicular to the expected current direction.

Another way to defeat eddy currents is to use a core composed of **ferrites**, which are oxides of iron that are electrical insulators. Still another approach is to combine powdered iron with an insulating binder.

PRACTICAL APPLICATION

3.1  Electronic Photo Flash

**Figure PA3.1** shows the electrical circuit of an electronic photo flash such as you may have seen on a camera. The objective of the unit is to produce a bright flash of light by passing a high current through the flash tube while the camera shutter is open. As much as 1000 W is supplied to the flash tube during the flash, which lasts for less than a millisecond. Although the power level is quite high, the total energy delivered is not great because of the short duration of the flash. (The energy is on the order of a joule.)



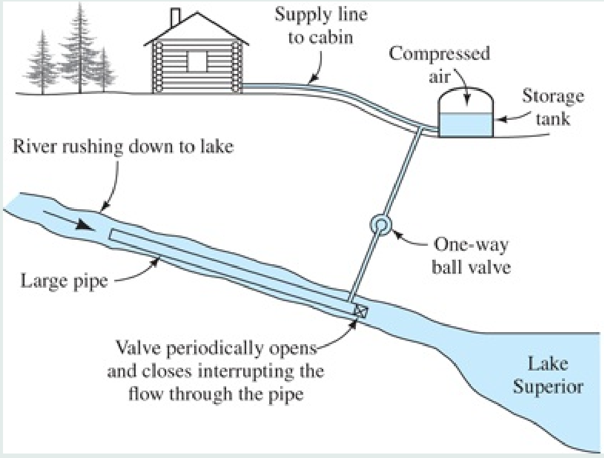

FIGURE

**PA3.1**

It is not possible to deliver the power directly from the battery to the flash tube for several reasons. First, practical batteries supply a few tens of volts at most, while several hundred volts are needed to operate the flash tube. Second, applying the principle of maximum power transfer, the maximum power available from the battery is limited to 1 W by its internal Thévenin resistance. (See **Equation 2.78** and the related discussion.) This does not nearly meet the needs of the flash tube. Instead, energy is delivered by the battery over a period of several seconds and stored in the capacitor. The stored energy can be quickly extracted from the capacitor because the parasitic resistance in series with the capacitor is very low.

The electronic switch alternates between open and closed approximately 10,000 times per second. (In some units, you can hear a high-pitched whistle resulting from incidental conversion of some of the energy to acoustic form.) While the electronic switch is closed, the battery causes the current in the inductor to build up. Then when the switch opens, the inductor forces current to flow through the diode, charging the capacitor. (Recall that the current in an inductor cannot change instantaneously.) Current can flow through the diode only in the direction of the arrow. Thus, the diode allows charge to flow into the capacitor when the electronic switch is open and prevents charge from flowing off the capacitor when the electronic switch is closed. Thus, the charge stored on the capacitor increases each time the electronic switch opens. Eventually, the voltage on the capacitor reaches several hundred volts. When the camera shutter is opened, another switch is closed, allowing the capacitor to discharge through the flash tube.

A friend of the author has a remote cabin on the north shore of Lake Superior that has an unusual water system (illustrated in **Figure PA3.2**) analogous to the electronic flash circuit. Water flows through a large pipe immersed in the river. Periodically, a valve on the bottom end of the pipe suddenly closes, stopping the flow. The inertia of the flowing water creates a pulse of high pressure when the valve closes. This high pressure forces water through a one-way ball valve into a storage tank. Air trapped in the storage tank is compressed and forces water to flow to the cabin as needed.




**FIGURE**

**PA3.2**

Can you identify the features in **Figure PA3.2** that are analogous to each of the circuit elements in **Figure PA3.1**?

## Parasitic Effects for Real Inductors

Real inductors have parasitic effects in addition to the desired inductance. A circuit model for a real inductor is shown in **Figure 3.24**. The series resistance $R_s$ is caused by the resistivity of the material composing the wire. (This parasitic effect can be avoided by using wire composed of a superconducting material, which has zero resistivity.) The parallel capacitance is associated with the electric field in the dielectric (insulation) between the coils of wire. It is called **interwinding capacitance**. The parallel resistance $R_p$ represents core loss due, in part, to eddy currents in the core.

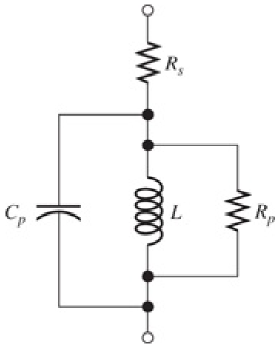



**Figure 3.24**

Circuit model for real inductors including several parasitic elements.

Actually, the circuit model for a real inductor shown in **Figure 3.24** is an approximation. The series resistance is distributed along the length of the wire, as is the interwinding capacitance. A more accurate model for a real inductor would break each of the parasitic effects into many segments (possibly, an infinite number). Ultimately, we could abandon circuit models altogether and use electromagnetic field theory directly.

Rarely is this degree of detail necessary. Usually, modeling a real inductor as an inductance, including at most a few parasitic effects, is sufficiently accurate. Of course, computer-aided circuit analysis allows us to use more complex models and achieve more accurate results than traditional mathematical analysis.

## 3.7 Mutual Inductance

Sometimes, several coils are wound on the same form so that magnetic flux produced by one coil links the others. Then a time-varying current flowing through one coil induces voltages in the other coils. The circuit symbols for two mutually coupled inductances are shown in **Figure 3.25**. The **self inductances** of the two coils are denoted as $L_1$ and $L_2,$ respectively. The **mutual inductance** is denoted as *M*, which also has units of henries. Notice that we have selected the passive reference configuration for each coil in **Figure 3.25**.

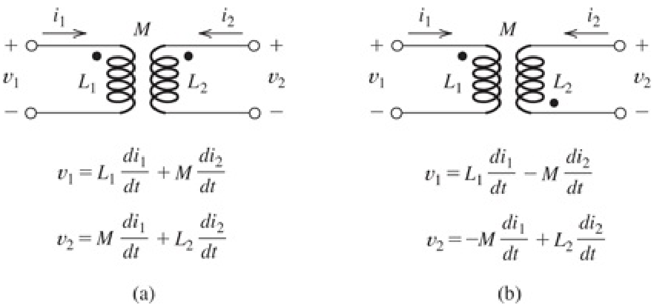



**Figure 3.25**
Circuit symbols and $v - i$ relationships for mutually coupled inductances.

The equations relating the voltages to the currents are also shown in **Figure 3.25**. The mutual terms, $M\ di_1/dt$ and $M\ di_2/dt,$ appear because of the mutual coupling of the coils. The self terms, $L_1\ di_1/dt$ and $L_2\ di_2/dt,$ are the voltages induced in each coil due to its own current.

The magnetic flux produced by one coil can either aid or oppose the flux produced by the other coil. The dots on the ends of the coils indicate whether the fields are aiding or opposing. If one current enters a dotted terminal and the other leaves, the fields oppose one another. For example, if both $i_1$ and $i_2$ have positive values in **Figure 3.25(b)**, the fields are opposing. If both currents enter the respective dots (or if both leave), the fields aid. Thus, if both $i_1$ and $i_2$ have positive values in **Figure 3.25(a)**, the fields are aiding.

> The magnetic flux produced by one coil can either aid or oppose the flux produced by the other coil.

The signs of the mutual terms in the equations for the voltages depend on how the currents are referenced with respect to the dots. If both currents are referenced into (or if both are referenced out of) the dotted terminals, as in **Figure 3.25(a)**, the mutual term is positive. If one current is referenced into a dot and the other out, as in **Figure 3.25(b)**, the mutual term carries a negative sign.

## Linear Variable Differential Transformer

An application of mutual inductance can be found in a position transducer known as the linear variable differential transformer (LVDT), illustrated in **Figure 3.26**. An ac source connected to the center coil sets up a magnetic field that links both halves of the secondary coil. When the iron core is centered in the coils, the voltages induced in the two halves of the secondary cancel so that $v_o(t) = 0$. (Notice that the two halves of the secondary winding are wound in opposite directions.) As the core moves up or down, the couplings between the primary and the halves of the secondary change. The voltage across one half of the coil becomes smaller, and the voltage across the other half becomes greater. Ideally, the output voltage is given by

$$v_o(t) = Kx \cos(\omega t)$$

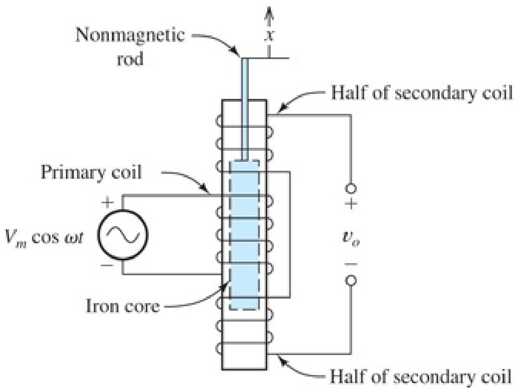



**Figure 3.26**
A linear variable differential transformer used as a position transducer.

where *x* is the displacement of the core. LVDTs are used in applications such as automated manufacturing operations to measure displacements.

## 3.8 Symbolic Integration and Differentiation Using MATLAB

As we have seen, finding the current given the voltage (or vice versa) for an energy storage element involves integration or differentiation. Thus, we may sometimes need to find symbolic answers for integrals or derivatives of complex functions, which can be very difficult by traditional methods. Then, we can resort to using symbolic mathematics software. Several programs are available including Maple™ from Maplesoft Corporation, Mathematica™ from Wolfram Research, and the Symbolic Toolbox which is an optional part of MATLAB from Mathsoft. Each of these programs has its strengths and weaknesses, and when a difficult problem warrants the effort, all of them should be tried. Because MATLAB is widely used in Electrical Engineering, we confine our brief discussion to the Symbolic Toolbox.

**One note of caution:** We have checked the examples, exercises, and problems using MATLAB version R2015b. Keep in mind that if you use versions other than R2015b, you may not be able to reproduce our results. Try running our example m-files before sinking a lot of time into solving the problems. Hopefully, your instructor can give you some guidance on what to expect with the MATLAB versions available to you.

In the following, we assume that you have some familiarity with MATLAB. A variety of online interactive tutorials are available at `https://www.mathworks.com/`. However, you may find it easier to write MATLAB instructions for the exercises and problems in this chapter by modeling your solutions after the code in our examples.

#### Example 3.10 Integration and Differentiation Using the MATLAB Symbolic Toolbox

Use MATLAB to find expressions for the three voltages shown in **Figure 3.27** given $v_C(0) = 0$ and

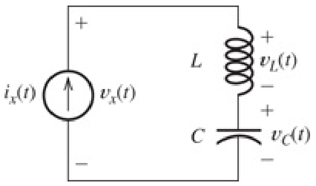



**Figure 3.27**
Circuit of **Example 3.10**.

$$\begin{aligned} i_x(t) &= kt^2 \exp(-at) \sin(\omega t) \text{ for } t \geq 0 \\ &= 0 \text{ for } t < 0 \end{aligned} \tag{3.39}$$

Also, plot the current and the voltages for $k = 3,\ a = 2,\ \omega = 1,\ L = 0.5\ \text{H},\ C = 1\ \text{F},$ and $t \geq 0.$ (These values have been chosen mainly to facilitate the demonstration of MATLAB capabilities.) The currents are in amperes, voltages are in volts, $\omega t$ is in radians, and time *t* is in seconds.

*Solution*

At first, we use symbols to represent the various parameters (*k*, *a*, $\omega$, $L$, and *C*), denoting the current and the voltages as ix, vx, vL, and vC. Then, we substitute the numerical values for the symbols and denote the results as ixn, vxn, vLn, and vCn. (The letter "n" is selected to suggest that the "numerical" values of the parameters have been substituted into the expressions.)

We show the commands in **boldface**, comments in regular font, and MATLAB responses in color. Comments (starting with the % sign) are ignored by MATLAB. We present the work as if we were entering the commands and comments one at a time in the MATLAB command window, however, it is usually more convenient to place all of the commands in an m-file and execute them as a group.

To start, we define the various symbols as symbolic objects in MATLAB, define the current ix, and substitute the numerical values of the parameters to obtain ixn.

```
>> clear all % Clear work area of previous work.
>> syms vx ix vC vL vxn ixn vCn vLn k a w t L C
>> % Names for symbolic objects must start with a letter and
>> % contain only alpha-numeric characters.
>> % Next, we define ix.
>> ix=k*t^2*exp(-a*t)*sin(w*t)
 ix =
 (k*t^2*sin(w*t))/exp(a*t)
>> % Next, we substitute k=3, a=2, and w=1
>> % into ix and denote the result as ixn.
>> ixn = subs(ix,[k a w],[3 2 1])
 ixn =
 (3*t^2*sin(t))/exp(2*t)
```

Next, we want to plot the current versus time. We need to consider what range of *t* should be used for the plot. In standard mathematical typesetting, the expression we need to plot is

$$\begin{aligned} i_x(t) &= 3t^2 \exp(-2t) \sin(t) \quad \text{for } t \geq 0 \\ &= 0 \text{ for } t < 0 \end{aligned}$$

Thoughtful examination of this expression (perhaps supplemented with a little work with a calculator) reveals that the current is zero at $t = 0,$ builds up quickly after $t = 0$ because of the $t^2$ term, and decays to relatively small values after about $t = 10 \text{ s}$ because of the exponential term. Thus, we select the range from $t = 0$ to $t = 10 \text{ s}$ for the plot. Continuing in MATLAB, we have

```
>> % Next, we plot ixn for t ranging from 0 to 10 s.
>> ezplot(ixn,[0,10])
```

This opens a window with a plot of the current versus time as shown in **Figure 3.28**. As expected, the current increases rapidly after $t = 0$ and decays to insignificant values by $t = 10 \text{ s}$. (We have used various Edit menu commands to improve the appearance of the plot for inclusion in this book.)

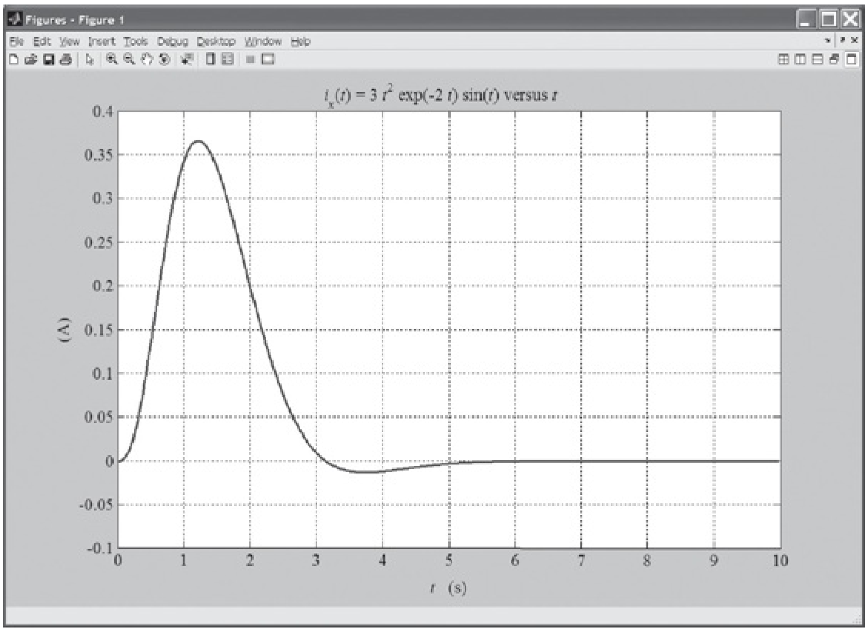



**Figure 3.28**

Plot of $i_x(t)$ produced by MATLAB.

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Next, we determine the inductance voltage, which is given by

$$v_L(t) = L\frac{di_x(t)}{dt}$$

in which the parameters, *a*, *k*, and $\omega$ are treated as constants. The corresponding MATLAB command and the result are:

```
>> vL=L*diff(ix,t)  % L × the derivative of ix with respect to t.
 vL =
 L*((2*k*t*sin(t*w))/exp(a*t) - (a*k*t^2*sin(t*w))/exp(a*t) +
   (k*t^2*w*cos(t*w))/exp(a*t))
>> % A nicer display for vL is produced with the command:
>> pretty(vL)
 L k t (2 sin(t w) - a t sin(t w) + t w cos(t w))
 -------------------------------------------------
   exp(a t)
```

In more standard mathematical typesetting, this becomes

$$v_L(t) = Lkt\exp(-at)\,[2\sin(\omega t) - at\sin(\omega t) + \omega t\cos(\omega t)]$$

which we can verify by manually differentiating the right-hand side of **Equation 3.39** and multiplying by *L*. Next, we determine the voltage across the capacitance.

$$v_C(t) = \frac{1}{C}\int_0^t i_x(t)\,dt + v_C(0)\ \text{for}\ t \ge 0$$

Substituting the expressions for the current and initial voltage we obtain,

$$v_C(t) = \frac{1}{C}\int_0^t kt^2\exp(-at)\sin(\omega t)\,dt\ \text{for}\ t \ge 0$$

This is not a simple integration to perform by hand, but we can accomplish it easily with MATLAB:

```
>> % Integrate ix with respect to t with limits from 0 to t.
>> vC=(1/C)*int(ix,t,0,t);
>> % We included the semicolon to suppress the output, which is
>> % much too complex for easy interpretation.
>> % Next, we find the total voltage vx.
>> vx = vC + vL;
>> % Now we substitute numerical values for the parameters.
>> vLn=subs(vL,[k a w L C],[3 2 1 0.5 1]);
>> vCn=subs(vC,[k a w L C],[3 2 1 0.5 1]);
>> vxn=subs(vx,[k a w L C],[3 2 1 0.5 1]);
>> % Finally, we plot all three voltages in the same window.
>> figure % Open a new figure for this plot.
>> ezplot(vLn,[0,10])
>> hold on % Hold so the following two plots are on the same axes.
>> ezplot(vCn,[0,10])
>> ezplot(vxn,[0,10])
```

The resulting plot is shown in **Figure 3.29**. (Here again, we have used various items on the Edit menu to change the scale of the vertical axis and dress up the plot for inclusion in this book.)

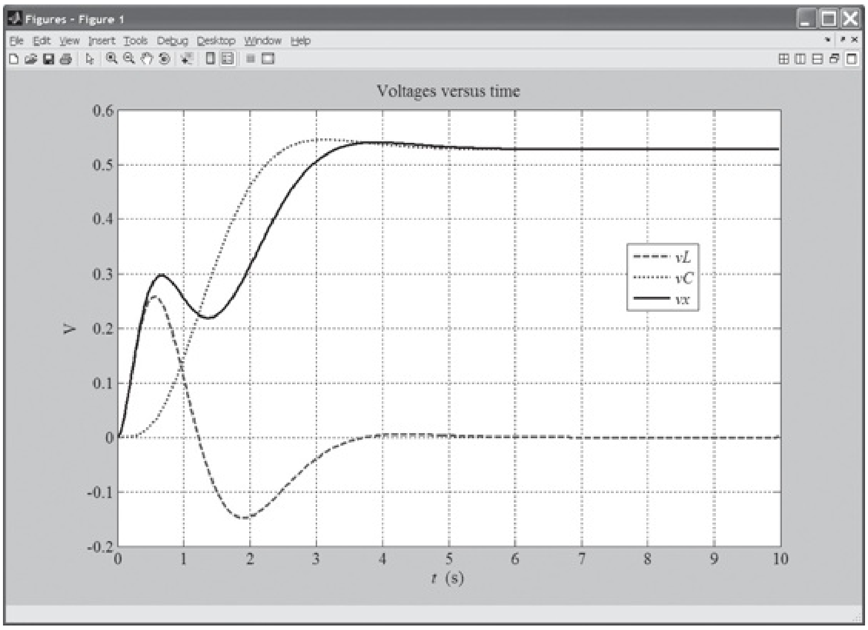


**Figure 3.29**
Plots of the voltages for **Example 3.10**.

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The commands for this example are included as an m-file named **Example_3_10** in the MATLAB files. (See **Appendix E** for information about accessing these MATLAB files.) If you copy the file and place it in a folder in the MATLAB path for your computer, you can run the file and experiment with it. For example, after running the m-file, if you enter the command

```
>> vC
```

you will see the rather complicated symbolic mathematical expression for the voltage across the capacitance.

## Exercise 3.11

Use MATLAB to work **Example 3.2** on page **132** resulting in plots like those in **Figure 3.5**.

**Answer** The MATLAB commands including some explanatory comments are:

```
clear  % Clear the work area.
% We avoid using i alone as a symbol for current because
% we reserve i for the square root of -1 in MATLAB. Thus, we
% will use iC for the capacitor current.
syms t iC qC vC % Define t, iC, qC and vC as symbolic objects.
iC = 0.5*sin((1e4)*t);
ezplot(iC, [0 3*pi*1e-4])
qC=int(iC,t,0,t);  % qC equals the integral of iC.
figure  % Plot the charge in a new window.
ezplot(qC, [0 3*pi*1e-4])
vC = 1e7*qC;
figure % Plot the voltage in a new window.
ezplot(vC, [0 3*pi*1e-4])
```

The plots are very similar to those of **Figure 3.5** on page **133**. An m-file (named Exercise_3_11) can be found in the MATLAB folder.n

## Summary

1. Capacitance is the circuit property that accounts for electric-field effects. The units of capacitance are farads (F), which are equivalent to coulombs per volt.
2. The charge stored by a capacitance is given by $q = Cv$.
3. The relationships between current and voltage for a capacitance are

$$i = C\frac{dv}{dt}$$

and

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt + v(t_0)$$

4. The energy stored by a capacitance is given by

$$w(t) = \frac{1}{2}Cv^2(t)$$

5. Capacitances in series are combined in the same manner as resistances in parallel.
6. Capacitances in parallel are combined in the same manner as resistances in series.
7. The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\in A}{d}$$

For vacuum, the dielectric constant is $\in = \in_0 \cong 8.85 \times 10^{-12}\ \mathrm{F/m}$. For other materials, the dielectric constant is $\in = \in_r \in_0$, where $\in_r$ is the relative dielectric constant.

8. Real capacitors have several parasitic effects.
9. Inductance accounts for magnetic-field effects. The units of inductance are henries (H).
10. The relationships between current and voltage for an inductance are

$$v(t) = L\frac{di}{dt}$$

and

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\, dt + i(t_0)$$

11. The energy stored in an inductance is given by

$$w(t) = \frac{1}{2}Li^2(t)$$

12. Inductances in series or parallel are combined in the same manner as resistances.
13. Real inductors have several parasitic effects.
14. Mutual inductance accounts for mutual coupling of magnetic fields between coils.
15. MATLAB is a powerful tool for symbolic integration, differentiation, and plotting of functions.

# Problems

## Section 3.1: Capacitance

**P3.1.** What is a dielectric material? Give two examples.

**P3.2.** Briefly discuss how current can flow "through" a capacitor even though a nonconducting layer separates the metallic parts.

**P3.3.** What current flows through an ideal capacitor if the voltage across the capacitor is constant with time? To what circuit element is an ideal capacitor equivalent in circuits for which the currents and voltages are constant with time?

**P3.4.** Describe the internal construction of capacitors.

**P3.5.** A voltage of 50 V appears across a $10\text{-}\mu\mathrm{F}$ capacitor. Determine the magnitude of the net charge stored on each plate and the total net charge on both plates.

***P3.6.** A $2000\text{-}\mu\mathrm{F}$ capacitor, initially charged to 100 V, is discharged by a steady current of $100\ \mu\mathrm{A}$. How long does it take to discharge the capacitor to 0 V?

* Denotes that answers are contained in the Student Solutions files.See **Appendix E** for more information about accessing the Student Solutions.

**P3.7.** A $5\text{-}\mu\mathrm{F}$ capacitor is charged to 1000 V. Determine the initial stored charge and energy. If this capacitor is discharged to 0 V in a time interval of $1\ \mu\mathrm{s}$, find the average power delivered by the capacitor during the discharge interval.

***P3.8.** The voltage across a $10\text{-}\mu\mathrm{F}$ capacitor is given by $v(t) = 100\sin(1000t)$. Find expressions for the current, power, and stored energy. Sketch the waveforms to scale versus time.

**P3.9.** The voltage across a $1\text{-}\mu\mathrm{F}$ capacitor is given by $v(t) = 100e^{-100t}$. Find expressions for the current, power, and stored energy. Sketch the waveforms to scale versus time.

**P3.10.** Prior to $t = 0$, a $100\text{-}\mu\mathrm{F}$ capacitance is uncharged. Starting at $t = 0$, the voltage across the capacitor is increased linearly with time to 100 V in 2 s. Then, the voltage remains constant at 100 V. Sketch the voltage, current, power, and stored energy to scale versus time.

**P3.11.** The current through a $0.5\text{-}\mu\mathrm{F}$ capacitor is shown in **Figure P3.11**. At $t = 0$, the voltage is zero. Sketch the voltage, power, and stored energy to scale versus time.

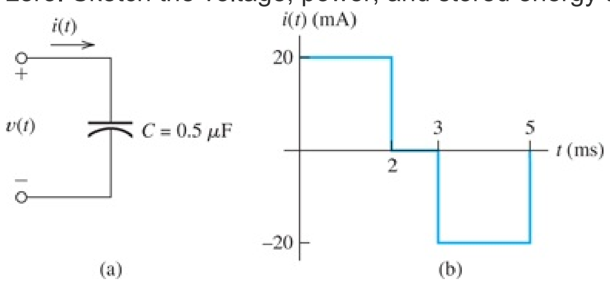


**Figure P3.11**

**P3.12.** Determine the capacitor voltage, power, and stored energy at $t = 20\ \mathrm{ms}$ in the circuit of **Figure P3.12**.

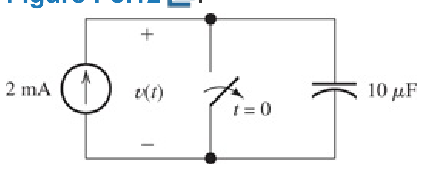


**Figure P3.12**

**P3.13.** A current given by $i(t) = I_m \cos(\omega t)$ flows through a capacitance *C*. The voltage is zero at $t = 0$. Suppose that $\omega$ is very large, ideally, approaching infinity. For this current does the capacitance approximate either an open or a short circuit? Explain.

**P3.14.** The current through a $3\text{-}\mu\text{F}$ capacitor is shown in **Figure P3.14**. At $t = 0$, the voltage is $v(0) = 10\ \text{V}$. Sketch the voltage, power, and stored energy to scale versus time.

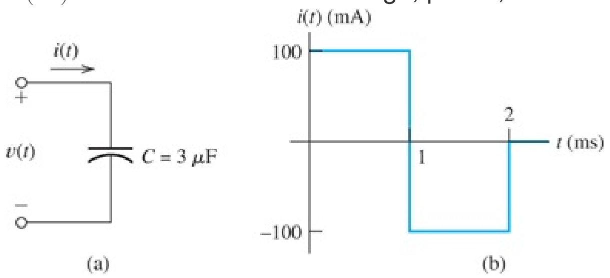


**Figure P3.14**

***P3.15.** A constant (dc) current $i(t) = 3\ \text{mA}$ flows into a $50\text{-}\mu\text{F}$ capacitor. The voltage at $t = 0$ is $v(0) = -20\ \text{V}$. The references for *v*(*t*) and *i*(*t*) have the passive configuration. Find the power at $t = 0$ and state whether the power flow is into or out of the capacitor. Repeat for $t = 1\ \text{s}$.

**P3.16.** The energy stored in a $20\text{-}\mu\text{F}$ capacitor is 200 J and is increasing at 500 J/s at $t = 3\ \text{s}$. Determine the voltage magnitude and current magnitude at $t = 3\ \text{s}$. Does the current enter or leave the positive terminal of the capacitor?

**P3.17.** At $t = t_0$ the voltage across a certain capacitance is zero. A pulse of current flows through the capacitance between $t_0$ and $t_0 + \Delta t$, and the voltage across the capacitance increases to $V_f$. What can you say about the peak amplitude $I_m$ and area under the pulse waveform (i.e., current versus time)? What are the units and physical significance of the area under the pulse? What must happen to the peak amplitude and area under the pulse as $\Delta t$ approaches zero, assuming that $V_f$ remains the same?

**P3.18.** An unusual capacitor has a capacitance that is a function of time given by

$$C = 2 + \cos(2000t)\ \mu\text{F}$$

in which the argument of the cosine function is in radians. A constant voltage of 50 V is applied to this capacitor. Determine the current as a function of time.

**P3.19.** For a resistor, what resistance corresponds to a short circuit? For an uncharged capacitor, what value of capacitance corresponds to a short circuit? Explain your answers. Repeat for an open circuit.

**P3.20.** Suppose we have a very large capacitance (ideally, infinite) charged to 10 V. What other circuit element has the same current-voltage relationship? Explain your answer.

***P3.21.** We want to store sufficient energy in a 0.01-F capacitor to supply 5 horsepower (hp) for 1 hour. To what voltage must the capacitor be charged? (*Note:* One horsepower is equivalent to 745.7 watts.) Does this seem to be a practical method for storing this amount of energy? Do you think that an electric automobile design based on capacitive energy storage is feasible?

**P3.22.** A $100\text{-}\mu\text{F}$ capacitor has a voltage given by $v(t) = 10 - 10\exp(-2t)\ \text{V}$. Find the power at $t = 0$ and state whether the power flow is into or out of the capacitor. Repeat for $t_2 = 0.5\ \text{s}$.

## Section 3.2: Capacitances in Series and Parallel

**P3.23.** How are capacitances combined in series and in parallel? Compare with how resistances are combined.

***P3.24.** Find the equivalent capacitance for each of the circuits shown in **Figure P3.24**.

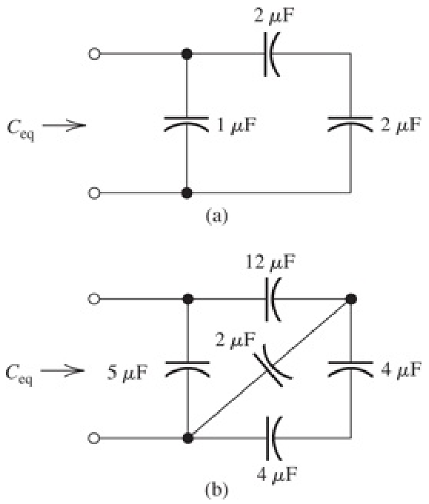


**Figure P3.24**

**P3.25.** Find the equivalent capacitance between terminals *x* and *y* for each of the circuits shown in **Figure P3.25**.

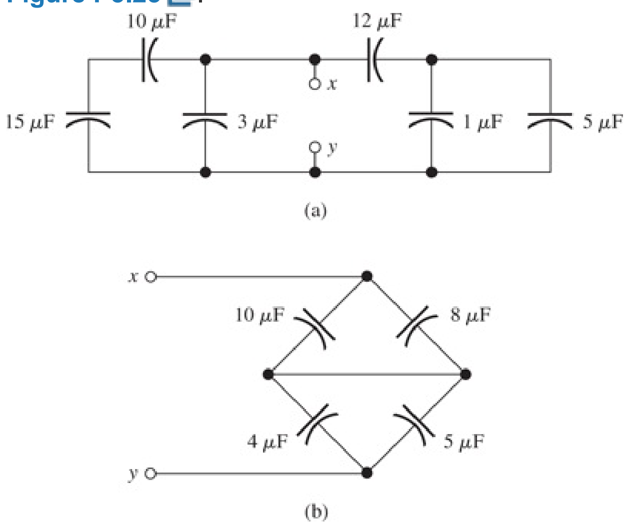


**Figure P3.25**

**P3.26.** A network has a $5\text{-}\mu\text{F}$ capacitance in series with the parallel combination of a $12\text{-}\mu\text{F}$ capacitance and an $8\text{-}\mu\text{F}$ capacitance. Sketch the circuit diagram and determine the equivalent capacitance of the combination.

**P3.27.** What are the minimum and maximum values of capacitance that can be obtained by connecting four $2\text{-}\mu\text{F}$ capacitors in series and/or parallel? How should the capacitors be connected?

**P3.28.** Two initially uncharged capacitors $C_1 = 15\ \mu\text{F}$ and $C_2 = 10\ \mu\text{F}$ are connected in series. Then, a 10-V source is connected to the series combination, as shown in **Figure P3.28**. Find the voltages $v_1$ and $v_2$ after the source is applied. [*Hint:* The charges stored on the two capacitors must be equal, because the current is the same for both capacitors.]

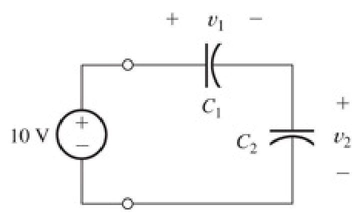


**Figure P3.28**

***P3.29.** Suppose that we are designing a cardiac pacemaker circuit. The circuit is required to deliver pulses of 1-ms duration to the heart, which can be modeled as a $500\text{-}\Omega$ resistance. The peak amplitude of the pulses is required to be 5 V. However, the battery delivers only 2.5 V. Therefore, we decide to charge two equal-value capacitors in parallel from the 2.5-V battery and then switch the capacitors in series with the heart during the 1-ms pulse. What is the minimum value of the capacitances required so the output pulse amplitude remains between 4.9 V and 5.0 V throughout its 1-ms duration? If the pulses occur once every second, what is the average current drain from the battery? Use approximate calculations, assuming constant current during the output pulse. Find the ampere-hour rating of the battery so it lasts for five years.

**P3.30.** Suppose that we have two $100\text{-}\mu\text{F}$ capacitors. One is charged to an initial voltage of 50 V, and the other is charged to 100 V. If they are placed in series with the positive terminal of the first connected to the negative terminal of the second, determine the equivalent capacitance and its initial voltage. Now compute the total energy stored in the two capacitors. Compute the energy stored in the equivalent capacitance. Why is it less than the total energy stored in the original capacitors?

## Section 3.3: Physical Characteristics of Capacitors

***P3.31.** Determine the capacitance of a parallel-plate capacitor having plates 10 cm by 30 cm separated by 0.01 mm. The dielectric has $\epsilon_r = 15.$

**P3.32.** A 100-pF capacitor is constructed of parallel plates of metal, each having a width *W* and a length *L*. The plates are separated by air with a distance *d*. Assume that *L* and *W* are both much larger than *d*. What is the new capacitance if

a. both *L* and *W* are doubled and the other parameters are unchanged?
b. the separation *d* is doubled and the other parameters are unchanged from their initial values?
c. the air dielectric is replaced with oil having a relative dielectric constant of 25 and the other parameters are unchanged from their initial values?

**P3.33.** We have a parallel-plate capacitor with plates of metal each having a width *W* and a length *L*. The plates are separated by the distance *d*. Assume that *L* and *W* are both much larger than *d*. The maximum voltage that can be applied is limited to $V_{\max} = K\,d,$ in which *K* is called the breakdown strength of the dielectric. Derive an expression for the maximum energy that can be stored in the capacitor in terms of *K* and the volume of the dielectric. If we want to store the maximum energy per unit volume, does it matter what values are chosen for *L*, *W*, and *d*? What parameters are important?

***P3.34.** Suppose that we have a 1000-pF parallel-plate capacitor with air dielectric charged to 1000 V. The capacitor terminals are open circuited. Find the stored energy. If the plates are moved farther apart so that *d* is doubled, determine the new voltage on the capacitor and the new stored energy. Where did the extra energy come from?

**P3.35.** Two $1\text{-}\mu\text{F}$ capacitors have an initial voltage of 100 V (before the switch is closed), as shown in **Figure P3.35**. Find the total stored energy before the switch is closed. Find the voltage across each capacitor and the total stored energy after the switch is closed. What could have happened to the energy?

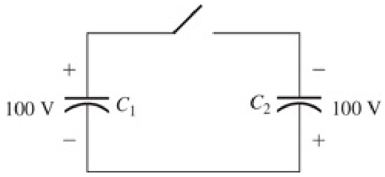


**Figure P3.35**

**P3.36.** A liquid-level transducer consists of two parallel plates of conductor immersed in an insulating liquid, as illustrated in **Figure P3.36**. When the tank is empty (i.e., $x = 0$ ), the capacitance of the plates is 200 pF. The relative dielectric constant of the liquid is 25. Determine an expression for the capacitance *C* as a function of the height *x* of the liquid.

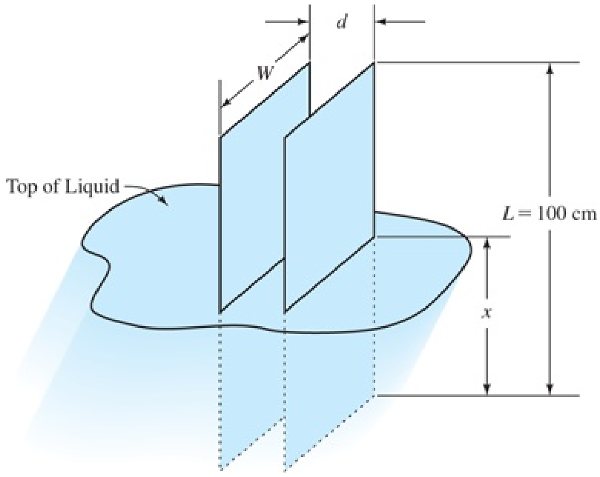


**Figure P3.36**

**P3.37.** A parallel-plate capacitor like that shown in **Figure P3.36** has a capacitance of 2000 pF when the tank is full so the plates are totally immersed in the insulating liquid. (The dielectric constant of the fluid is different for this problem than for **Problem P3.36**.) The capacitance is 200 pF when the tank is empty and the space between the plates is filled with air. Suppose that the tank is full and the capacitance is charged to 1000 V. Then, the capacitance is open circuited so the charge on the plates cannot change, and the tank is drained. Compute the electrical energy stored in the capacitor before and after the tank is drained. With the plates open circuited, there is no electrical source for the extra energy. Where could it have come from?

**P3.38.** A parallel-plate capacitor is used as a vibration sensor. The plates have an area of $100\ \mathrm{cm}^2$, the dielectric is air, and the distance between the plates is a function of time given by

$$d(t) = 1 + 0.01 \sin(200t) \quad \mathrm{mm}$$

A constant voltage of 200 V is applied to the sensor. Determine the current through the sensor as a function of time by using the approximation $1/(1+x) \cong 1 - x$ for $x << 1$. (The argument of the sinusoid is in radians.)

**P3.39.** A $0.1\text{-}\mu\mathrm{F}$ capacitor has a parasitic series resistance of $10\ \Omega$, as shown in **Figure P3.39**. Suppose that the voltage across the capacitance is $v_c(t) = 10 \cos(100t)$ ; find the voltage across the resistance. In this situation, to find the total voltage $v(t) = v_r(t) + v_c(t)$ to within 1 percent accuracy, is it necessary to include the parasitic resistance? Repeat if $v_c(t) = 0.1 \cos\left(10^7 t\right)$ .

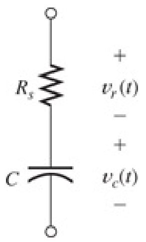


**Figure P3.39**

***P3.40.** Suppose that a parallel-plate capacitor has a dielectric that breaks down if the electric field exceeds *K* V/m. Thus, the maximum voltage rating of the capacitor is $V_{\max} = K\,d,$ where *d* is the separation between the plates. In working **Problem P3.33**, we find that the maximum energy that can be stored is $w_{\max} = \frac{1}{2} \in_r \in_0 K^2$ (Vol) in which Vol is the volume of the dielectric. Given that $K = 32 \times 10^5\ \mathrm{V/m}$ and that $\in_r = 1$ (the approximate values for air), find the dimensions of a parallel-plate capacitor having square plates if it is desired to store 1 mJ at a voltage of 1000 V in the least possible volume.

## Section 3.4: Inductance

**P3.41.** Briefly discuss how inductors are constructed.

**P3.42.** The current flowing through an inductor is increasing in magnitude. Is energy flowing into or out of the inductor?

**P3.43.** If the current through an ideal inductor is constant with time, what is the value of the voltage across the inductor? Comment. To what circuit element is an ideal inductor equivalent for circuits with constant currents and voltages?

**P3.44.** Briefly discuss the fluid-flow analogy for an inductor.

***P3.45.** The current flowing through a 2-H inductance is shown in **Figure P3.45**. Sketch the voltage, power, and stored energy versus time.

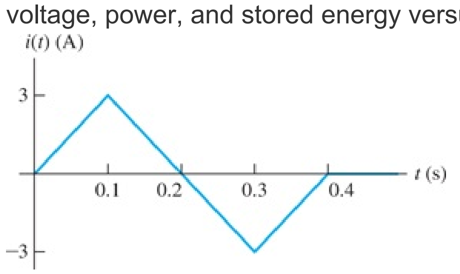


**Figure P3.45**

**P3.46.** The current flowing through a 100-mH inductance is given by 0.5 sin (1000*t*) A, in which the angle is in radians. Find expressions and sketch the waveforms to scale for the voltage, power, and stored energy.

**P3.47.** The current flowing through a 2-H inductance is given by $5\exp(-20t)\ \mathrm{A}.$ Find expressions for the voltage, power, and stored energy. Sketch the waveforms to scale for $0 < t < 100\ \mathrm{ms}.$

**P3.48.** The voltage across a 2-H inductance is shown in **Figure P3.48**. The initial current in the inductance is $i(0) = 0.$ Sketch the current, power, and stored energy to scale versus time.

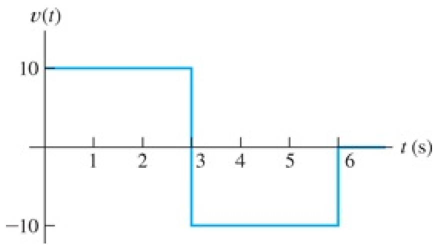


**Figure P3.48**

**P3.49.** The voltage across a $10\text{-}\mu\ \mathrm{H}$ inductance is given by $v(t) = 5\sin(10^6 t)\ \mathrm{V}$. The initial current is $i(0) = -0.5\ \mathrm{A}$. Find expressions for the current, power, and stored energy for $t > 0$. Sketch the waveforms to scale versus time.

**P3.50.** A 2-H inductance has $i(0) = 0$ and $v(t) = t\exp(-t)$ for $0 \le t$. Find an expression for *i*(*t*). Then, using the computer program of your choice, plot *v*(*t*) and *i*(*t*) for $0 \le t \le 10\ \mathrm{s}$.

***P3.51.** A constant voltage of 10 V is applied to a $50\text{-}\mu\mathrm{H}$ inductance, as shown in **Figure P3.51**.

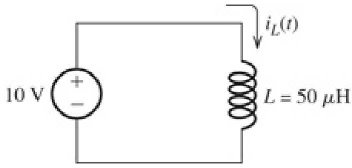


**Figure P3.51**

The current in the inductance at $t = 0$ is $-100\ \mathrm{mA}$. At what time $t_x$ does the current reach $+100\ \mathrm{mA}$?

***P3.52.** At $t = 0$, the current flowing in a 0.5-H inductance is 4 A. What constant voltage must be applied to reduce the current to 0 at $t = 0.2\ \mathrm{s}$?

**P3.53.** The current through a 100-mH inductance is given by $i(t) = \exp(-t)\ \sin(10t)$ in which the angle is in radians. Determine the voltage across the inductance. Then, use the computer program of your choice to plot both the current and the voltage for $0 \le t \le 3\ \mathrm{s}$.

**P3.54.** Prior to $t = 0$, the current in a 2-H inductance is zero. Starting at $t = 0$, the current is increased linearly with time to 10 A in 5 s. Then, the current remains constant at 10 A. Sketch the voltage, current, power, and stored energy to scale versus time.

**P3.55.** At $t = 0$, a constant 5-V voltage source is applied to a 3-H inductor. Assume an initial current of zero for the inductor. Determine the current, power, and stored energy at $t = 2\ \mathrm{s}$.

**P3.56.** At $t = t_0$ the current through a certain inductance is zero. A voltage pulse is applied to the inductance between $t_0$ and $t_0 + \Delta t$, and the current through the inductance increases to $I_f$. What can you say about the peak amplitude $V_m$ and area under the pulse waveform (i.e., voltage versus time)? What are the units of the area under the pulse? What must happen to the peak amplitude and area under the pulse as $\Delta t$ approaches zero, assuming that $I_f$ remains the same?

**P3.57.** At $t = 5\ \mathrm{s}$, the energy stored in a 2-H inductor is 200 J and is increasing at 100 J/s. Determine the voltage magnitude and current magnitude at $t = 5\ \mathrm{s}$. Does the current enter or leave the positive terminal of the inductor?

**P3.58.** What value of inductance (having zero initial current) corresponds to an open circuit? Explain your answer. Repeat for a short circuit.

**P3.59.** To what circuit element does a very large (ideally, infinite) inductance having an initial current of 10 A correspond? Explain your answer.

**P3.60.** The voltage across an inductance *L* is given by $v(t) = V_m \cos(\omega t)$. The current is zero at $t = 0$. Suppose that $\omega$ is very large ideally, approaching infinity. For this voltage, does the inductance approximate either an open or a short circuit? Explain.

## Section 3.5: Inductances in Series and Parallel

**P3.61.** Discuss how inductances are combined in series and in parallel. Compare with how resistances are combined.

***P3.62.** Determine the equivalent inductance for each of the series and parallel combinations shown in **Figure P3.62**.

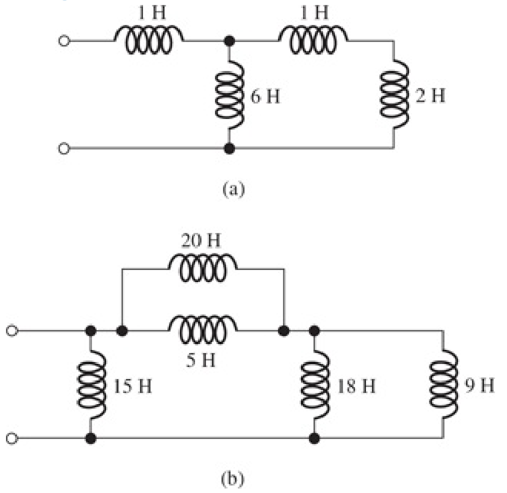


**Figure P3.62**

**P3.63.** Find the equivalent inductance for each of the series and parallel combinations shown in **Figure P3.63**.

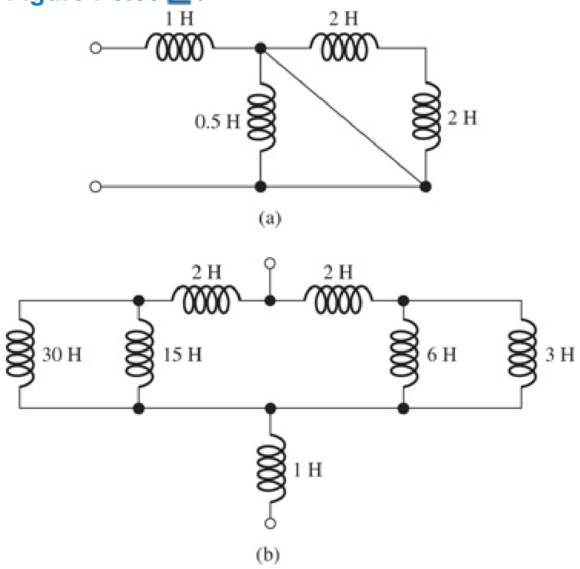


**Figure P3.63**

**P3.64.** What is the maximum inductance that can be obtained by connecting four 2-H inductors in series and/or parallel? What is the minimum inductance?

**P3.65.** Suppose we want to combine (in series or in parallel) an inductance *L* with a 6-H inductance to attain an equivalent inductance of 2 H. Should *L* be placed in series or in parallel with the original inductance? What value is required for *L*?

**P3.66.** Repeat **Problem P3.65** for an equivalent inductance of 8 H.

***P3.67.** Two inductances $L_1 = 1\ \mathrm{H}$ and $L_2 = 2\ \mathrm{H}$ are connected in parallel, as shown in **Figure P3.67**. The initial currents are $i_1(0) = 0$ and $i_2(0) = 0$. Find an expression for $i_1(t)$ in terms of *i*(*t*), $L_1$, and $L_2$. Repeat for $i_2(t)$. Comment.

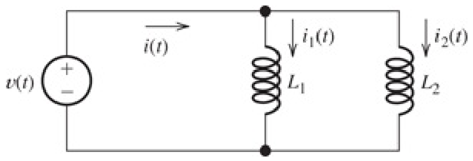


**Figure P3.67**

## Section 3.6: Practical Inductors

**P3.68.** A 10-mH inductor has a parasitic series resistance of $R_s = 1\ \Omega$, as shown in **Figure P3.68**.

a. The current is given by $i(t) = 0.1\cos(10^5 t)$. Find $v_R(t)$, $v_L(t)$, and *v*(*t*). In this case, for 1-percent accuracy in computing *v*(*t*), could the resistance be neglected?

b. Repeat if $i(t) = 0.1\cos(10t)$.

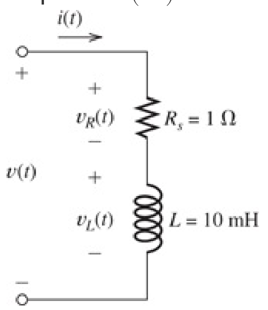


**Figure P3.68**

**P3.69.** Draw the equivalent circuit for a real inductor, including three parasitic effects.

**P3.70.** Suppose that the equivalent circuit shown in **Figure 3.24** accurately represents a real inductor. A constant current of 100 mA flows through the inductor, and the voltage across its external terminals is 500 mV. Which of the circuit parameters can be deduced from this information and what is its value?

**P3.71.** Consider the circuit shown in **Figure P3.71**, in which $v_C(t) = 10\sin(1000t)\ \mathrm{V}$, with the argument of the sine function in radians. Find *i*(*t*), $v_L(t)$, *v*(*t*), the energy stored in the capacitance, the energy stored in the inductance, and the total stored energy. Show that the total stored energy is constant with time. Comment on the results.

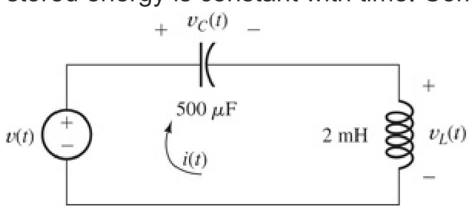


**Figure P3.71**

**P3.72.** The circuit shown in **Figure P3.72** has $i_L(t) = 0.1\cos(5000t)\ \mathrm{A}$ in which the argument of the cos function is in radians. Find *v*(*t*), $i_C(t)$, *i*(*t*), the energy stored in the capacitance, the energy stored in the inductance, and the total stored energy. Show that the total stored energy is constant with time. Comment on the results.

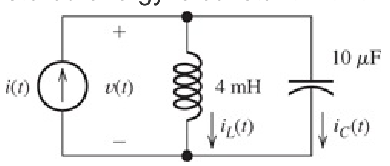


**Figure P3.72**

## Section 3.7: Mutual Inductance

**P3.73.** Describe briefly the physical basis for mutual inductance.

**P3.74.** The mutually coupled inductances in **Figure P3.74** have $L_1 = 1\ \text{H},\ L_2 = 2\ \text{H},$ and $M = 1\ \text{H}$. Furthermore, $i_1(t) = \sin(10t)$ and $i_2(t) = 0.5\sin(10t)$. Find expressions for $v_1(t)$ and $v_2(t)$. The arguments of the sine functions are in radians.

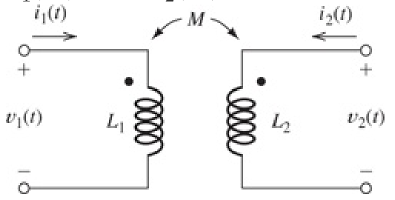


**Figure P3.74**

***P3.75.** Repeat **Problem P3.74** with the dot placed at the bottom of $L_2$.

***P3.76.**

a. Derive an expression for the equivalent inductance for the circuit shown in **Figure P3.76**.
b. Repeat if the dot for $L_2$ is moved to the bottom end.

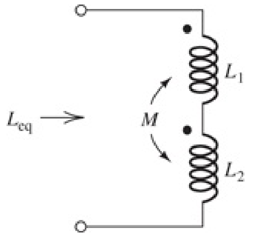


**Figure P3.76**

**P3.77.** Consider the parallel inductors shown in **Figure P3.67**, with mutual coupling and the dots at the top ends of $L_1$ and $L_2$. Derive an expression for the equivalent inductance seen by the source in terms of $L_1,\ L_2,$ and *M*. [*Hint*: Write the circuit equations and manipulate them to obtain an expression of the form $v(t) = L_{eq}di(t)/dt$ in which $L_{eq}$ is a function of $L_1,\ L_2,$ and *M*.]

**P3.78.** Consider the mutually coupled inductors shown in **Figure 3.25(a)**, with a short connected across the terminals of $L_2$. Derive an expression for the equivalent inductance seen looking into the terminals of $L_1$.

**P3.79.** Mutually coupled inductances have

$$
\begin{aligned}
L_1 &= 2\ \text{H} \\
L_2 &= 1\text{H} \\
i_1 &= 10\cos(1000t) \\
i_2 &= 0 \\
v_2 &= 10^4\sin(1000t)
\end{aligned}
$$

Find $v_1(t)$ and the magnitude of the mutual inductance. The angles are in radians.

## Section 3.8: Symbolic Integration and Differentiation Using MATLAB

**P3.80.** The current through a 200-mH inductance is given by $i_L(t) = \exp(-2t)\sin(4\pi t)$ A in which the angle is in radians. Using your knowledge of calculus, find an expression for the voltage across the inductance. Then, use MATLAB to verify your answer for the voltage and to plot both the current and the voltage for $0 \le t \le 2$ s.

**P3.81.** A 1-H inductance has $i_L(0) = 0$ and $v_L(t) = t\exp(-t)$ for $0 \le t$. Using your calculus skills, find and an expression for $i_L(t)$. Then, use MATLAB to verify your answer for $i_L(t)$ and to plot $v_L(t)$ and $i_L(t)$ for $0 \le t \le 10$ s.

## Practice Test

Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in **Appendix D** and complete solutions are included in the Student Solutions files. See **Appendix E** for more information about the Student Solutions.

**T3.1.** The current flowing through a $10\text{-}\mu\text{F}$ capacitor having terminals labeled *a* and *b* is $i_{ab} = 0.3\exp(-2000t)$ A for $t \geq 0$. Given that $v_{ab}(0) = 0$, find an expression for $v_{ab}(t)$ for $t \geq 0$. Then, find the energy stored in the capacitor for $t = \infty$.

**T3.2.** Determine the equivalent capacitance $C_{\text{eq}}$ for **Figure T3.2**.

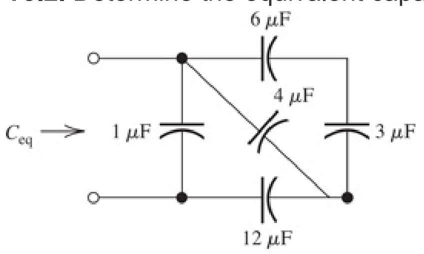


**Figure T3.2**

**T3.3.** A certain parallel-plate capacitor has plate length of 2 cm and width of 3 cm. The dielectric has a thickness of 0.1 mm and a relative dielectric constant of 80. Determine the capacitance.

**T3.4.** A 2-mH inductance has $i_{ab} = 0.3\sin(2000t)$ A. Find an expression for $v_{ab}(t)$. Then, find the peak energy stored in the inductance.

**T3.5.** Determine the equivalent inductance $L_{\text{eq}}$ between terminals *a* and *b* in **Figure T3.5**.

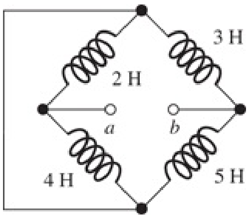


**Figure T3.5**

**T3.6.** Given that $v_c(t) = 10\sin(1000t)$ V, find $v_s(t)$ in the circuit of **Figure T3.6**. The argument of the sine function is in radians.

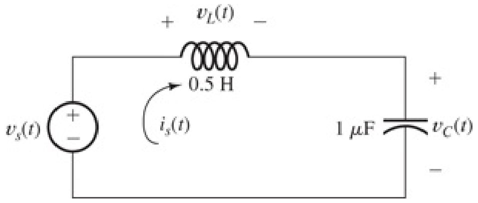


**Figure T3.6**

**T3.7.** **Figure T3.7** has $L_1 = 40$ mH, $M = 20$ mH, and $L_2 = 30$ mH. Find expressions for $v_1(t)$ and $v_2(t)$.

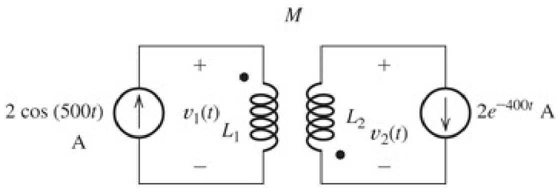


**Figure T3.7**

**T3.8.** The current flowing through a $20\text{-}\mu\text{F}$ capacitor having terminals labeled *a* and *b* is $i_{ab} = 3 \times 10^5 t^2 \exp(-2000t)$ A for $t \geq 0$. Given that $v_{ab}(0) = 5$ V, write a sequence of MATLAB commands to find the expression for $v_{ab}(t)$ for $t \geq 0$ and to produce plots of the current and voltage for $0 \leq t \leq 5$ ms.