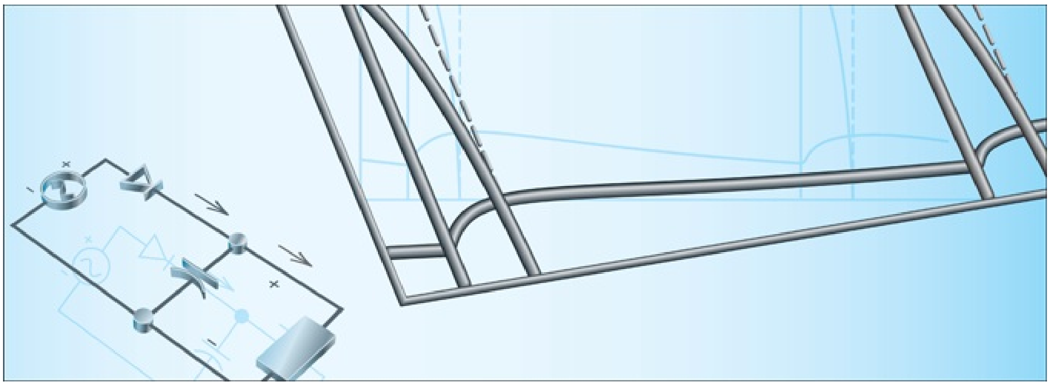
# Chapter 9 Diodes



**Study of this chapter will enable you to:**

- Understand diode operation and select diodes for various applications.
- Use the graphical load-line technique to analyze nonlinear circuits.
- Analyze and design simple voltage-regulator circuits.
- Use the ideal-diode model and piecewise-linear models to solve circuits.
- Understand various rectifier and wave-shaping circuits.
- Understand small-signal equivalent circuits.

## Introduction to this chapter:

*Electronic circuits are useful for processing information and controlling energy. Some applications of electronic circuits are computers, radio, television, navigation systems, light dimmers, calculators, appliances, controls for machines, motion sensors, and surveying equipment. A basic understanding of electronic circuits will help you in working with instrumentation in any field of engineering. In the next several chapters, we introduce the most important electronic devices, their basic circuit applications, and several important analysis techniques. In this chapter, we discuss the diode.*

## 9.1 Basic Diode Concepts

The diode is a basic but very important device that has two terminals, the **anode** and the **cathode**. The circuit symbol for a diode is shown in **Figure 9.1(a)**, and a typical volt–ampere characteristic is shown in **Figure 9.1(b)**. As shown in **Figure 9.1(a)**, the voltage $v_D$ across the diode is referenced positive at the anode and negative at the cathode. Similarly, the diode current $i_D$ is referenced positive from anode to cathode.

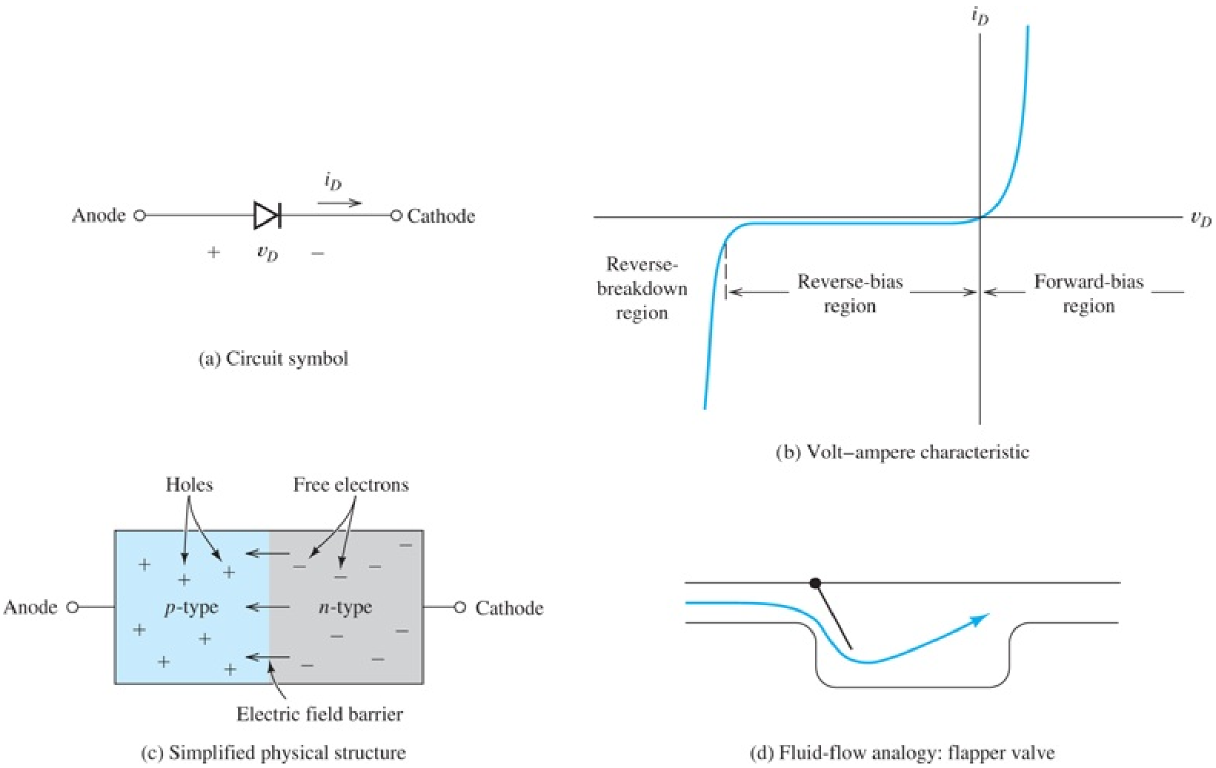



**Figure 9.1**
Semiconductor diode.

Notice in the characteristic that if the voltage $v_D$ applied to the diode is positive, relatively large amounts of current flow for small voltages. This condition is called **forward bias**. Thus, current flows easily through the diode in the direction of the arrowhead of the circuit symbol.

> Diodes readily conduct current from anode to cathode (in the direction of the arrow), but do not readily allow current to flow in the opposite direction.

On the other hand, for moderate negative values of $v_D$, the current $i_D$ is very small in magnitude. This is called the **reverse-bias region**, as shown on the diode characteristic. In many applications, the ability of the diode to conduct current easily in one direction, but not in the reverse direction, is very useful. For example, in an automobile, diodes allow current from the alternator to charge the battery when the engine is running. However, when the engine stops, the diodes prevent the battery from discharging through the alternator. In these applications, the diode is analogous to a one-way valve in a fluid-flow system, as illustrated in **Figure 9.1(d)**.

If a sufficiently large reverse-bias voltage is applied to the diode, operation enters the reverse-breakdown region of the characteristic, and currents of large magnitude flow.

If a sufficiently large reverse-bias voltage is applied to the diode, operation enters the **reverse-breakdown region** of the characteristic, and currents of large magnitude flow. Provided that the power dissipated in the diode does not raise its temperature too high, operation in reverse breakdown is not destructive to the device. In fact, we will see that diodes are sometimes deliberately operated in the reverse-breakdown region.

## Brief Sketch of Diode Physics

We concentrate our discussion on the external behavior of diodes and some of their circuit applications. However, at this point, we give a thumbnail sketch of the internal physics of the diode.

The diodes that we consider consist of a junction between two types of semiconducting material (usually, silicon with carefully selected impurities). On one side of the junction, the impurities create ***n*-type material**, in which large numbers of electrons move freely. On the other side of the junction, different impurities are employed to create (in effect) positively charged particles known as **holes**. Semiconductor material in which holes predominate is called ***p*-type material**. Most diodes consist of a junction between *n*-type material and *p*-type material, as shown in **Figure 9.1(c)**.

Even with no external applied voltage, an electric-field **barrier** appears naturally at the *pn* junction. This barrier holds the free electrons on the *n*-side and the holes on the *p*-side of the junction. If an external voltage is applied with positive polarity on the *n*-side, the barrier is enhanced and the charge carriers cannot cross the junction. Thus, virtually no current flows. On the other hand, if a voltage is applied with positive polarity on the *p*-side, the barrier is reduced and large currents cross the junction. Thus, the diode conducts very little current for one polarity and large current for the other polarity of applied voltage. The anode corresponds to the *p*-type material and the cathode is the *n*-side.

## Small-Signal Diodes

Various materials and structures are used to fabricate diodes. For now, we confine our discussion to small-signal silicon diodes, which are the most common type found in low- and medium-power electronic circuits.

The characteristic curve of a typical small-signal silicon diode operated at a temperature of 300 K is shown in **Figure 9.2**. Notice that the voltage and current scales for the forward-bias region are different than for the reverse-bias region. This is necessary for displaying details of the characteristic, because the current magnitudes are much smaller in the reverse-bias region than in the forward-bias region. Furthermore, the forward-bias voltage magnitudes are much less than typical breakdown voltages.

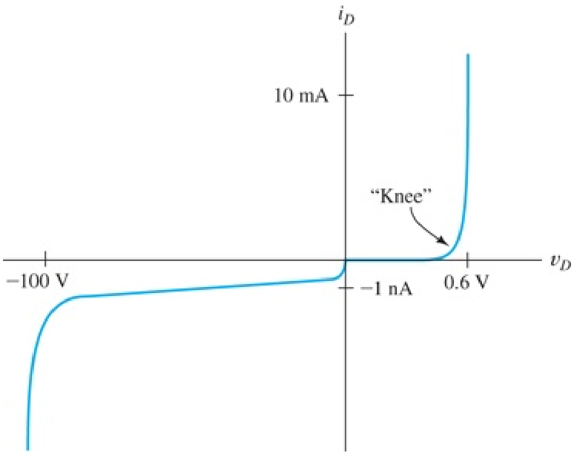



**Figure 9.2**

Volt–ampere characteristic for a typical small-signal silicon diode at a temperature of 300 K. Notice the change of scale for negative current and voltage.

In the forward-bias region, small-signal silicon diodes conduct very little current (much less than 1 mA) until a forward voltage of about 0.6 V is applied (assuming that the diode is at a temperature of about 300 K). Then, current increases very rapidly as the voltage is increased. We say that the forward-bias characteristic displays a *knee* in the forward bias characteristic at about 0.6 V. (The exact value of the knee voltage depends on the device, its temperature, and the current magnitude. Typical values are 0.6 or 0.7 V.) As temperature is increased, the knee voltage decreases by about 2 mV/K. (Because of the linear change in voltage with temperature, diodes are useful as temperature sensors. The diode is operated at a fixed current, and the voltage across the diode depends on its temperature. Electronic thermometers used by physicians contain a diode sensor, amplifiers, and other electronic circuits that drive the liquid-crystal temperature display.)

In the reverse-bias region, a typical current is about 1 nA for small-signal silicon diodes at room temperature. As temperature increases, reverse current increases in magnitude. A rule of thumb is that the reverse current doubles for each 10-K increase in temperature.

When reverse breakdown is reached, current increases in magnitude very rapidly. The voltage for which this occurs is called the **breakdown voltage**. For example, the breakdown voltage of the diode characteristic shown in Figure 9.2 is approximately $-100\text{ V}$. Breakdown-voltage magnitudes range from several volts to several hundred volts. Some applications call for diodes that operate in the forward-bias and nonconducting reverse-bias regions without entering the breakdown region. Diodes intended for these applications have a specification for the minimum magnitude of the breakdown voltage.

## Shockley Equation

Under certain simplifying assumptions, theoretical considerations result in the following relationship between current and voltage for a junction diode:

$$i_D = I_s \left[ \exp\left( \frac{v_D}{n V_T} \right) - 1 \right] \tag{9.1}$$

This is known as the **Shockley equation**. The **saturation current** $I_s$, has a value on the order of $10^{-14}$ A for small-signal junction diodes at 300 K. ($I_s$ depends on temperature, doubling for each 5-K increase in temperature for silicon devices.) The parameter *n*, known as the **emission coefficient**, takes values between 1 and 2, depending on details of the device structure. The voltage $V_T$ is given by

$$V_T = \frac{kT}{q} \tag{9.2}$$

and is called the **thermal voltage**. The temperature of the junction in kelvin is represented by *T*. Furthermore, $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, and $q = 1.60 \times 10^{-19}$ C is the magnitude of the electrical charge of an electron. At a temperature of 300 K, we have $V_T \cong 0.026$ V.

If we solve the Shockley equation for the diode voltage, we find that

$$v_D = n V_T \ln\left[ \left( \frac{i_D}{I_s} \right) + 1 \right] \tag{9.3}$$

For small-signal junction diodes operated at forward currents between $0.01\ \mu$A and 10 mA, the Shockley equation with *n* taken as unity is usually very accurate. Because the derivation of the Shockley equation ignores several phenomena, the equation is not accurate for smaller or larger currents. For example, under reverse bias, the Shockley equation predicts $i_D \cong -I_s$, but we usually find that the reverse current is much larger in magnitude than $I_s$ (although still small). Furthermore, the Shockley equation does not account for reverse breakdown.

With forward bias of at least several tenths of a volt, the exponential in the Shockley equation is much larger than unity; with good accuracy, we have

$$i_D \cong I_s \exp\left( \frac{v_D}{n V_T} \right) \tag{9.4}$$

This approximate form of the equation is often easier to use.

Occasionally, we are able to derive useful analytical results for electronic circuits by use of the Shockley equation, but much simpler models for diodes are usually more useful.

## Zener Diodes

Diodes that are intended to operate in the breakdown region are called **Zener diodes**. Zener diodes are useful in applications for which a constant voltage in breakdown is desirable. Therefore, manufacturers try to optimize Zener diodes for a nearly vertical characteristic in the breakdown region. The modified diode symbol shown in **Figure 9.3** is used for Zener diodes. Zener diodes are available with breakdown voltages that are specified to a tolerance of $\pm 5\%$ .

**Figure 9.3**
Zener-diode symbol.

*Exercise 9.1*

At a temperature of 300 K, a certain junction diode has $i_D = 0.1 \text{ mA}$ for $v_D = 0.6 \text{ V}$. Assume that *n* is unity and use $V_T = 0.026 \text{ V}$. Find the value of the saturation current $I_s$. Then, compute the diode current at $v_D = 0.65 \text{ V}$ and at 0.70 V.

**Answer**

$I_s = 9.50 \times 10^{-15} \text{ A}, \; i_D = 0.684 \text{ mA}, \; i_D = 4.68 \text{ mA}$.

*Exercise 9.2*

Consider a diode under forward bias so that the approximate form of the Shockley equation (**Equation 9.4**) applies. Assume that $V_T = 0.026 \text{ V}$ and $n = 1$.

a. By what increment must $v_D$ increase to double the current?

b. To increase the current by a factor of 10?

**Answer**

a. $\Delta \, v_D = 18 \text{ mV};$

b. $\Delta \, v_D = 59.9 \text{ mV}$.

## 9.2 Load-Line Analysis of Diode Circuits

In **Section 9.1**, we learned that the volt–ampere characteristics of diodes are nonlinear. We will see shortly that other electronic devices are also nonlinear. On the other hand, resistors have linear volt–ampere characteristics, as shown in **Figure 9.4**. Because of this nonlinearity, many of the techniques that we have studied for linear circuits in **Chapters 1** through **6** do not apply to circuits involving diodes. In fact, much of the study of electronics is concerned with techniques for analysis of circuits containing nonlinear elements.

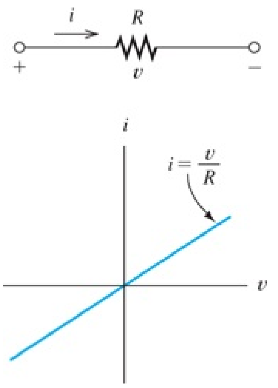



**Figure 9.4**
In contrast to diodes, resistors have linear volt–ampere characteristics.

Graphical methods provide one approach to analysis of nonlinear circuits. For example, consider the circuit shown in **Figure 9.5**. By application of Kirchhoff's voltage law, we can write the equation

$$V_{SS} = Ri_D + v_D \tag{9.5}$$

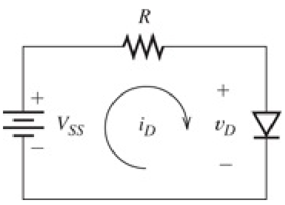



**Figure 9.5**
Circuit for load-line analysis.

We assume that the values of $V_{SS}$ and $R$ are known and that we wish to find $i_D$ and $v_D$. Thus, **Equation 9.5** has two unknowns, and another equation (or its equivalent) is needed before a solution can be found. This is available in graphical form in **Figure 9.6**, which shows the volt–ampere characteristic of the diode.

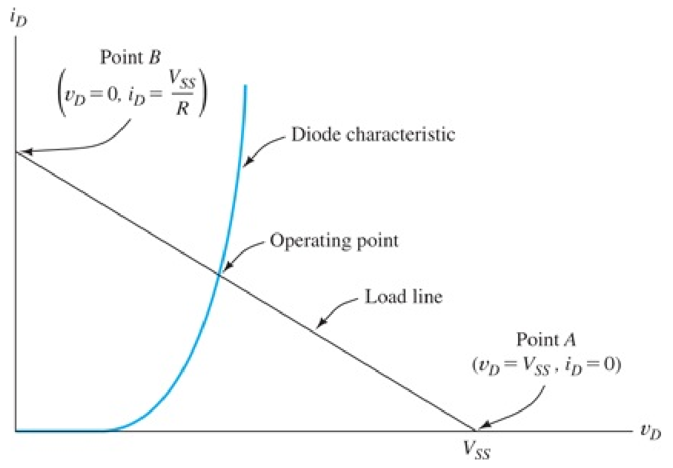



**Figure 9.6**
Load-line analysis of the circuit of **Figure 9.5**.

We can obtain a solution by plotting **Equation 9.5** on the same set of axes used for the diode characteristic. Since **Equation 9.5** is linear, it plots as a straight line, which can be drawn if two points satisfying the equation are located. A simple method is to assume that $i_D = 0$, and then **Equation 9.5** yields $v_D = V_{SS}$. This pair of values is shown as point *A* in **Figure 9.6**. A second point results if we assume that $v_D = 0$, for which the equation yields $i_D = V_{SS}/R$. The pair of values is shown as point *B* in **Figure 9.6**. Then, connecting points *A* and *B* result in a plot called the **load line**. The **operating point** is the intersection of the load line and the diode characteristic. This point represents the simultaneous solution of **Equation 9.5** and the diode characteristic.

### Example 9.1 Load-Line Analysis

If the circuit of **Figure 9.5** has $V_{SS} = 2\ \mathrm{V}$, $R = 1\ \mathrm{k}\,\Omega$, and a diode with the characteristic shown in **Figure 9.7**, find the diode voltage and current at the operating point.

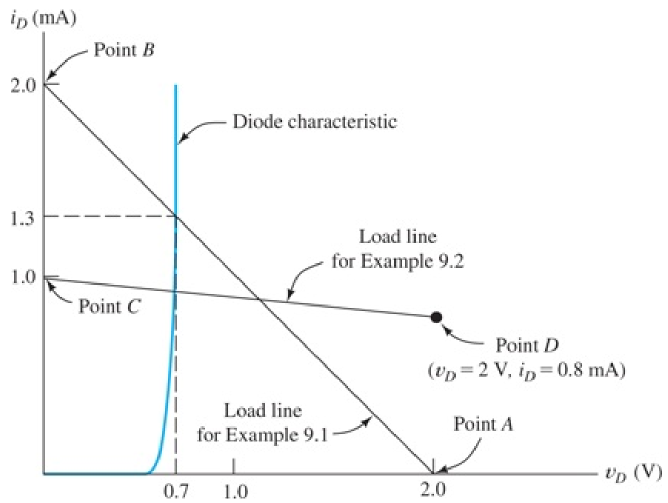



**Figure 9.7**
Load-line analysis for **Examples 9.1** and **9.2**.

*Solution*

First, we locate the ends of the load line. Substituting $v_D = 0$ and the values given for $V_{SS}$ and *R* into **Equation 9.5** yields $i_D = 2\ \mathrm{mA}$. These values plot as point *B* in **Figure 9.7**. Substitution of $i_D = 0$ and circuit values results in $v_D = 2\ \mathrm{V}$. These values plot as point *A* in the figure. Constructing the load line results in an operating point of $V_{DQ} \cong 0.7\ \mathrm{V}$ and $I_{DQ} \cong 1.3\ \mathrm{mA}$, as shown in the figure.

**Example 9.2 Load-Line Analysis**

Repeat **Example 9.1** if $V_{SS} = 10\ \mathrm{V}$ and $R = 10\ \mathrm{k}\,\Omega$ .

*Solution*

If we let $v_D = 0$ and substitute values into **Equation 9.5**, we find that $i_D = 1\ \mathrm{mA}$ . This is plotted as point *C* in **Figure 9.7**.

> When an intercept of the load line falls off the page, we select a point at the edge of the page.

If we proceed as before by assuming that $i_D = 0,$ we find that $v_D = 10\ \mathrm{V}$ . This is a perfectly valid point on the load line, but it plots at a point far off the page. Of course, we can use any other point satisfying **Equation 9.5** to locate the load line. Since we already have point *C* on the $i_D$ axis, a good point to use would be on the right-hand edge of **Figure 9.7**. Thus, we assume that $v_D = 2\ \mathrm{V}$ and substitute values into **Equation 9.5**, resulting in $i_D = 0.8\ \mathrm{mA}$ . These values plot as point *D*. Then, we can draw the load line and find that the operating-point values are $V_{DQ} \cong 0.68\ \mathrm{V}$ and $I_{DQ} \cong 0.93\ \mathrm{mA}$ .

## Exercise 9.3

Find the operating point for the circuit of **Figure 9.5** if the diode characteristic is shown in **Figure 9.8** and:

a. $V_{SS} = 2\ \mathrm{V}$ and $R = 100\ \ \Omega$ ;

b. $V_{SS} = 15\ \mathrm{V}$ and $R = 1\ \mathrm{k}\,\Omega$ ;

c. $V_{SS} = 1.0\ \mathrm{V}$ and $R = 20\ \ \Omega$ .

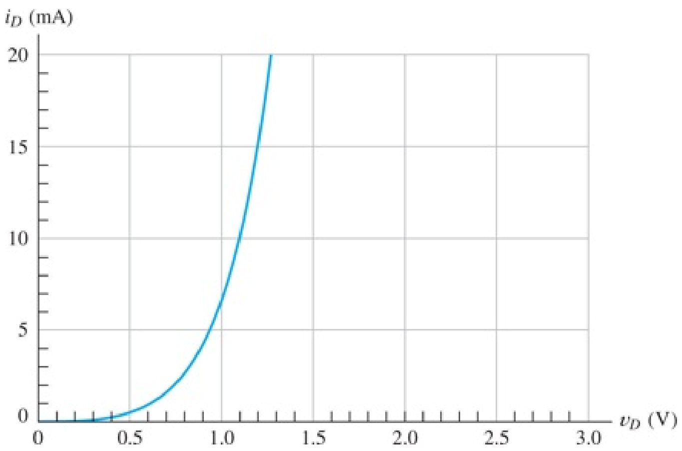



**Figure 9.8**
Diode characteristic for **Exercise 9.3**.

**Answer**

a. $V_{DQ} \cong 1.1\ \mathrm{V},\ I_{DQ} \cong 9.0\ \mathrm{mA};$

b. $V_{DQ} \cong 1.2\ \mathrm{V},\ I_{DQ} \cong 13.8\ \mathrm{mA};$

c. $V_{DQ} \cong 0.91\ \mathrm{V},\ I_{DQ} \cong 4.5\ \mathrm{mA}$ .

## 9.3 Zener-Diode Voltage-Regulator Circuits

Sometimes, a circuit that produces constant output voltage while operating from a variable supply voltage is needed. Such circuits are called **voltage regulators**. For example, if we wanted to operate computer circuits from the battery in an automobile, a voltage regulator would be needed. Automobile battery voltage typically varies between about 10 and 14 V (depending on the state of the battery and whether or not the engine is running). Many computer circuits require a nearly constant voltage of 5 V. Thus, a regulator is needed that operates from the 10 to 14 V supply and produces a nearly constant 5-V output.

A voltage regulator circuit provides a nearly constant voltage to a load from a variable source.

In this section, we use the load-line technique that we introduced in **Section 9.2** to analyze a simple regulator circuit. The regulator circuit is shown in **Figure 9.9**. (For proper operation, it is necessary for the minimum value of the variable source voltage to be somewhat larger than the desired output voltage.) The Zener diode has a breakdown voltage equal to the desired output voltage. The resistor *R* limits the diode current to a safe value so that the Zener diode does not overheat.

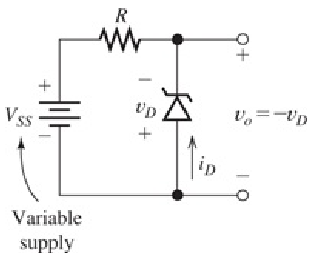



**Figure 9.9**
A simple regulator circuit that provides a nearly constant output voltage $v_o$ from a variable supply voltage.

Assuming that the characteristic for the diode is available, we can construct a load line to analyze the operation of the circuit. As before, we use Kirchhoff's voltage law to write an equation relating $v_D$ and $i_D$. (In this circuit, the diode operates in the breakdown region with negative values for $v_D$ and $i_D$.) For the circuit of **Figure 9.9**, we obtain

$$V_{SS} + Ri_D + v_D = 0 \tag{9.6}$$

Once again, this is the equation of a straight line, so location of any two points is sufficient to construct the load line. The intersection of the load line with the diode characteristic yields the operating point.

**Example 9.3 Load-Line Analysis of a Zener-Diode Voltage Regulator**

The voltage-regulator circuit of **Figure 9.9** has $R = 1\ \mathrm{k}\,\Omega$ and uses a Zener diode having the characteristic shown in **Figure 9.10**. Find the output voltage for $V_{SS} = 15\ \mathrm{V}$. Repeat for $V_{SS} = 20\ \mathrm{V}$.

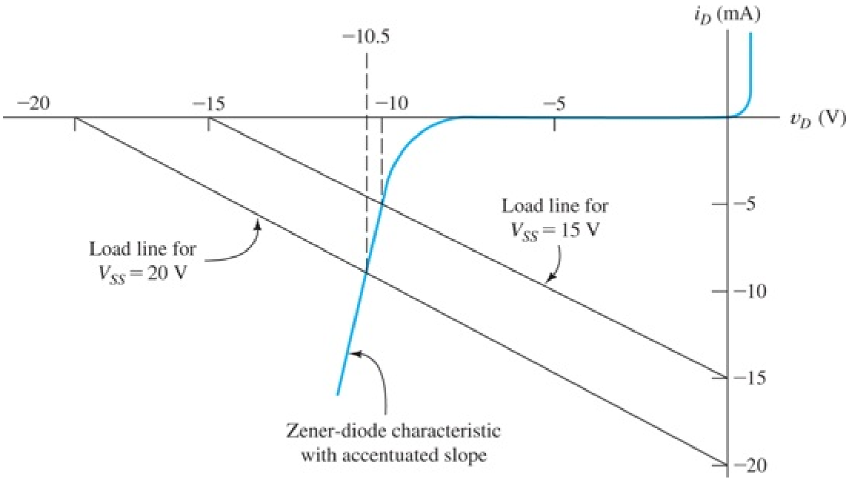



**Figure 9.10**
See **Example 9.3**.

*Solution*

The load lines for both values of $V_{SS}$ are shown in **Figure 9.10**. The output voltages are determined from the points where the load lines intersect the diode characteristic. The output voltages are found to be $v_o = 10.0\ \mathrm{V}$ for $V_{SS} = 15\ \mathrm{V}$ and $v_o = 10.5\ \mathrm{V}$ for $V_{SS} = 20\ \mathrm{V}$. Thus, a 5-V change in the supply voltage results in only a 0.5-V change in the regulated output voltage.

Actual Zener diodes are capable of much better performance than this. The slope of the characteristic has been accentuated in **Figure 9.10** for clarity–actual Zener diodes have a more nearly vertical slope in breakdown.

## Slope of the Load Line

Notice that the two load lines shown in **Figure 9.10** are parallel. Inspection of **Equation 9.5** or **Equation 9.6** shows that the slope of the load line is $-1/R$. Thus, a change of the supply voltage changes the position, but not the slope of the load line.

> Load lines for different source voltages (but the same resistance) are parallel.

## Load-Line Analysis of Complex Circuits

Any circuit that contains resistors, voltage sources, current sources, and a single two-terminal nonlinear element can be analyzed by the load-line technique. First, the Thévenin equivalent is found for the linear portion of the circuit as illustrated in **Figure 9.11**. Then, a load line is constructed to find the operating point on the characteristic of the nonlinear device. Once the operating point of the nonlinear element is known, voltages and currents can be determined in the original circuit.

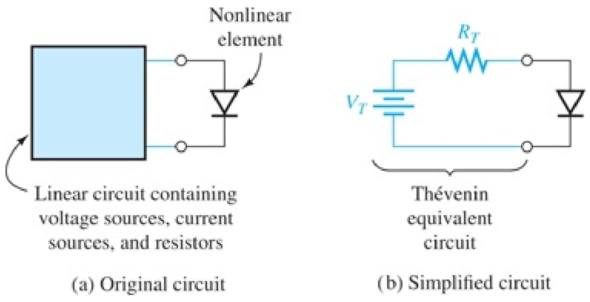



**Figure 9.11**
Analysis of a circuit containing a single nonlinear element can be accomplished by load-line analysis of a simplified circuit.

***Example 9.4 Analysis of a Zener-Diode Regulator with a Load***

Consider the Zener-diode regulator circuit shown in **Figure 9.12(a)**. The diode characteristic is shown in **Figure 9.13**. Find the load voltage $v_L$ and source current $I_S$ if $V_{SS} = 24\ \text{V},\ R = 1.2\ \text{k}\,\Omega$, and $R_L = 6\ \text{k}\,\Omega$ .

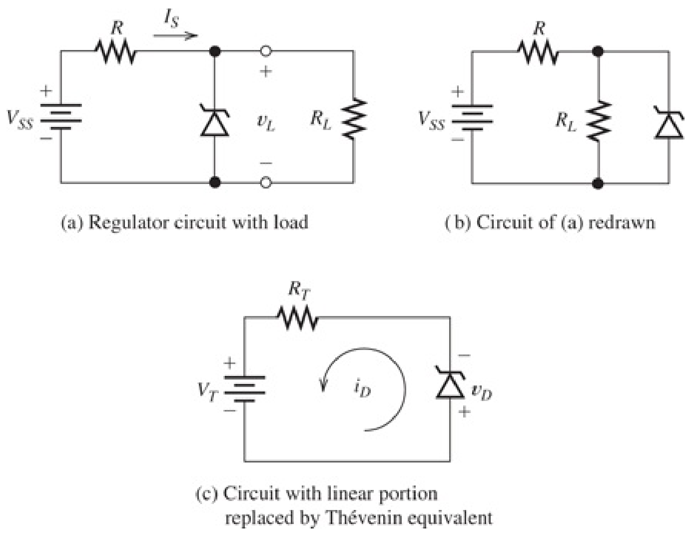



**Figure 9.12**
See **Example 9.4**.

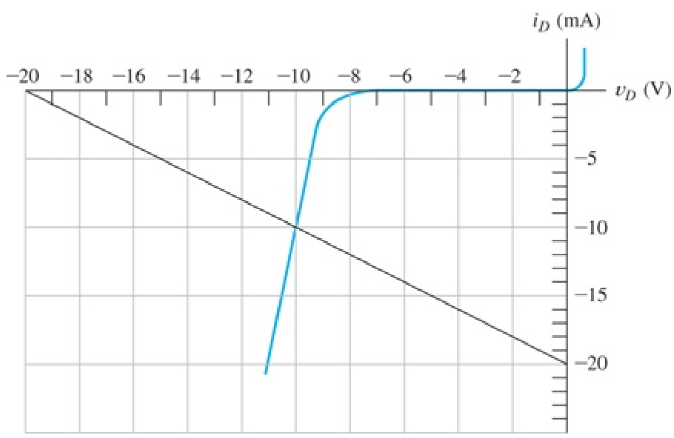



**Figure 9.13**
Zener-diode characteristic for **Example 9.4** and **Exercise 9.4**.

Solution

First, consider the circuit as redrawn in **Figure 9.12(b)**, in which we have grouped the linear elements together on the left-hand side of the diode. Next, we find the Thévenin equivalent for the linear portion of the circuit. The Thévenin voltage is the open-circuit voltage (i.e., the voltage across $R_L$ with the diode replaced by an open circuit), which is given by

$$V_T = V_{SS}\frac{R_L}{R + R_L} = 20 \text{ V}$$

The Thévenin resistance can be found by zeroing the voltage source and looking back into the circuit from the diode terminals. This is accomplished by reducing $V_{SS}$ to zero so that the voltage source becomes a short circuit. Then, we have *R* and $R_L$ in parallel, so the Thévenin resistance is

$$R_T = \frac{RR_L}{R + R_L} = 1 \text{ k}\,\Omega$$

The resulting equivalent circuit is shown in **Figure 9.12(c)**.

Now, we can use Kirchhoff's voltage law to write the load-line equation from the equivalent circuit as

$$V_T + R_T i_D + v_D = 0$$

Using the values found for $V_T$ and $R_T,$ we can construct the load line shown in **Figure 9.13** and locate the operating point. This yields $v_L = -v_D = 10.0 \text{ V}$.

Once $v_L$ is known, we can find the voltages and currents in the original circuit. For example, using the output voltage value of 10.0 V in the original circuit of **Figure 9.12(a)**, we find that $I_S = \left( V_{SS} - v_L \right) / R = 11.67 \text{ mA}$.

*Exercise 9.4*

Find the voltage across the load in **Example 9.4** if:

a. $R_L = 1.2 \text{ k}\,\Omega$;

b. b. $R_L = 400 \ \ \Omega$ .

**Answer**

a. $v_L \cong 9.4 \text{ V};$

b. $v_L \cong 6.0 \text{ V}$. (Notice that this regulator is not perfect because the load voltage varies as the load current changes.)

*Exercise 9.5*

Consider the circuit of **Figure 9.14(a)**. Assume that the breakdown characteristic is vertical, as shown in **Figure 9.14(b)**. Find the output voltage $v_o$ for:

a. $i_L = 0;$

b. $i_L = 20\ \mathrm{mA};$

c. $i_L = 100\ \mathrm{mA}$. [*Hint:* Applying Kirchhoff's voltage law to the circuit, we have

$$15 = 100\left( i_L - i_D\right) - v_D$$

Construct a different load line for each value of $i_L$.]

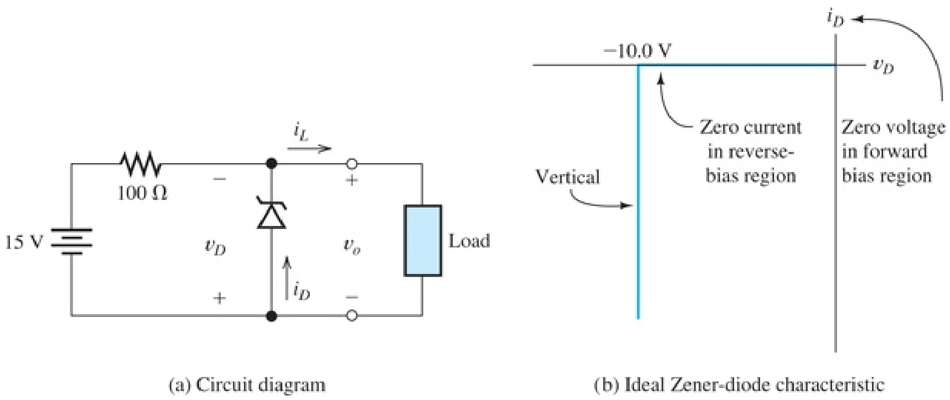



**Figure 9.14**
See **Exercise 9.5**.

**Answer**

a. $v_o = 10.0\ \mathrm{V};$

b. $v_o = 10.0\ \mathrm{V};$

c. $v_o = 5.0\ \mathrm{V}$. (Notice that the regulator is not effective for large load currents.)

# 9.4 Ideal-Diode Model

Graphical load-line analysis is useful for some circuits, such as the voltage regulator studied in **Section 9.3**. However, it is too cumbersome for more complex circuits. Instead, we often use simpler models to approximate diode behavior.

One model for a diode is the **ideal diode**, which is a perfect conductor with zero voltage drop in the forward direction. In the reverse direction, the ideal diode is an open circuit. We use the ideal-diode assumption if our judgment tells us that the forward diode voltage drop and reverse current are negligible, or if we want a basic understanding of a circuit rather than an exact analysis.

The ideal diode acts as a short circuit for forward currents and as an open circuit with reverse voltage applied.

The volt–ampere characteristic for the ideal diode is shown in **Figure 9.15**. If $i_D$ is positive, $v_D$ is zero, and we say that the diode is in the *on* state. On the other hand, if $v_D$ is negative, $i_D$ is zero, and we say that the diode is in the *off* state.

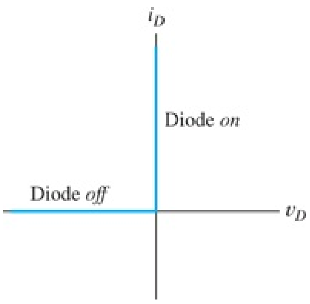



**Figure 9.15**
Ideal-diode volt–ampere characteristic.

## Assumed States for Analysis of Ideal-Diode Circuits

In analysis of a circuit containing ideal diodes, we may not know in advance which diodes are on and which are off. Thus, we are forced to make a considered guess. Then, we analyze the circuit to find the currents in the diodes assumed to be on and the voltages across the diodes assumed to be off. If $i_D$ is positive for the diodes assumed to be on and if $v_D$ is negative for the diodes assumed to be off, our assumptions are correct, and we have solved the circuit. (We are assuming that $i_D$ is referenced positive in the forward direction and that $v_D$ is referenced positive at the anode.) Otherwise, we must make another assumption about the diodes and try again. After a little practice, our first guess is usually correct, at least for simple circuits.

A step-by-step procedure for analyzing circuits that contain ideal diodes is to:

1. Assume a state for each diode, either on (i.e., a short circuit) or off (i.e., an open circuit). For *n* diodes there are $2^n$ possible combinations of diode states.
2. Analyze the circuit to determine the current through the diodes assumed to be on and the voltage across the diodes assumed to be off.
3. Check to see if the result is consistent with the assumed state for each diode. Current must flow in the forward direction for diodes assumed to be on. Furthermore, the voltage across the diodes assumed to be off must be positive at the cathode (i.e., reverse bias).

4. If the results are consistent with the assumed states, the analysis is finished. Otherwise, return to step 1 and choose a different combination of diode states.

### *Example 9.5 Analysis by Assumed Diode States*

Use the ideal-diode model to analyze the circuit shown in **Figure 9.16(a)**. Start by assuming that $D_1$ is off and $D_2$ is on.

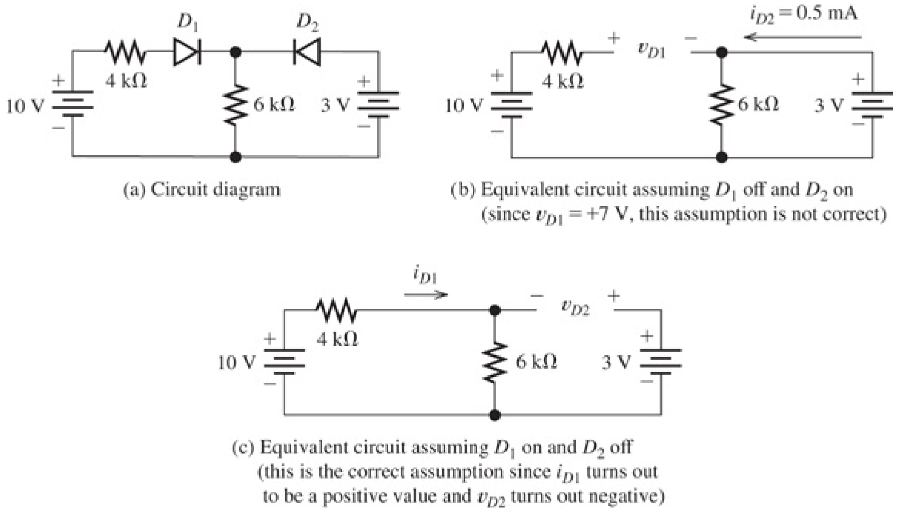



**Figure 9.16**
Analysis of a diode circuit, using the ideal-diode model. See **Example 9.5**.

Solution

With $D_1$ off and $D_2$ on, the equivalent circuit is shown in **Figure 9.16(b)**. Solving results in $i_{D2} = 0.5 \text{ mA}$. Since the current in $D_2$ is positive, our assumption that $D_2$ is on seems to be correct. However, continuing the solution of the circuit of **Figure 9.16(b)**, we find that $v_{D1} = +7 \text{ V}$. This is not consistent with the assumption that $D_1$ is off. Therefore, we must try another assumption.

This time, we assume that $D_1$ is on and $D_2$ is off. The equivalent circuit for these assumptions is shown in **Figure 9.16(c)**. We can solve this circuit to find that $i_{D1} = 1 \text{ mA}$ and $v_{D2} = -3 \text{ V}$. These values are consistent with the assumptions about the diodes ($D_1$ on and $D_2$ off) and, therefore, are correct.

Notice in **Example 9.5** that even though current flows in the forward direction of $D_2$ for our first guess about diode states ($D_1$ off and $D_2$ on), the correct solution is that $D_2$ is off. Thus, in general, we cannot decide on the state of a particular diode until we have found a combination of states that works for all the diodes in the circuit.

In general, we cannot decide on the state of a particular diode until we have found a combination of states that works for all of the diodes in the circuit.

For a circuit containing *n* diodes, there are $2^n$ possible states. Thus, an exhaustive search eventually yields the solution for each circuit.

*Exercise 9.6*

Show that the condition $D_1$ off and $D_2$ off is not valid for the circuit of **Figure 9.16(a)**.

*Exercise 9.7*

Show that the condition $D_1$ on and $D_2$ on is not valid for the circuit of **Figure 9.16(a)**.

*Exercise 9.8*

Find the diode states for the circuits shown in **Figure 9.17**. Assume ideal diodes.

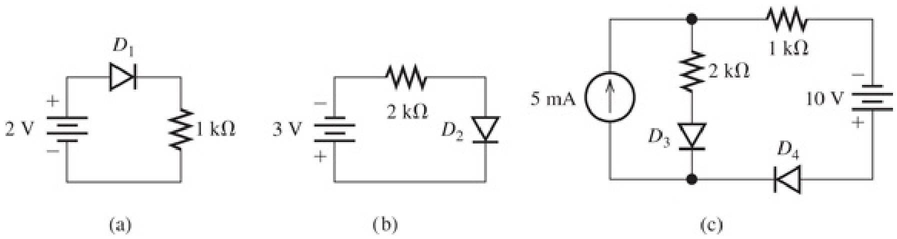



**Figure 9.17**
Circuits for **Exercise 9.8**.

**Answer**

a. $D_1$ is on;

b. $D_2$ is off;

c. $D_3$ is off; and $D_4$ is on.

## 9.5 Piecewise-Linear Diode Models

Sometimes, we want a more accurate model than the ideal-diode assumption, but do not want to resort to nonlinear equations or graphical techniques. Then, we can use **piecewise-linear models** for the diodes. First, we approximate the actual volt–ampere characteristic by straight-line segments. Then, we model each section of the diode characteristic with a resistance in series with a constant-voltage source. Different resistance and voltage values are used in the various sections of the characteristic.

Consider the resistance $R_a$ in series with a voltage source $V_a$ shown in **Figure 9.18(a)**. We can write the following equation, relating the voltage and current of the series combination:

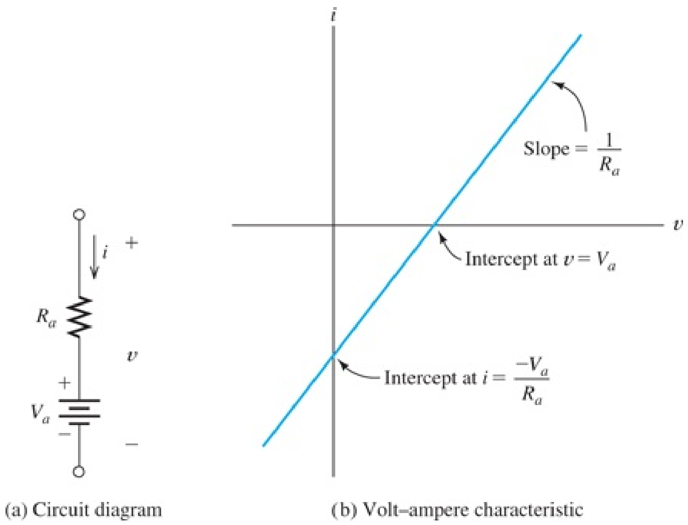



**Figure 9.18**
Circuit and volt–ampere characteristic for piecewise-linear models.

$$v = R_a i + V_a \tag{9.7}$$

The current *i* is plotted versus *v* in **Figure 9.18(b)**. Notice that the intercept on the voltage axis is at $v = V_a$ and that the slope of the line is $1/R_a$.

Given a straight-line volt–ampere characteristic, we can work backward to find the corresponding series voltage and resistance. Thus, after a nonlinear volt–ampere characteristic has been approximated by several straight-line segments, a circuit model consisting of a voltage source and series resistance can be found for each segment.

**Example 9.6 Piecewise-Linear Model for a Zener Diode**

Find circuit models for the Zener-diode volt–ampere characteristic shown in **Figure 9.19**. Use the straight-line segments shown.

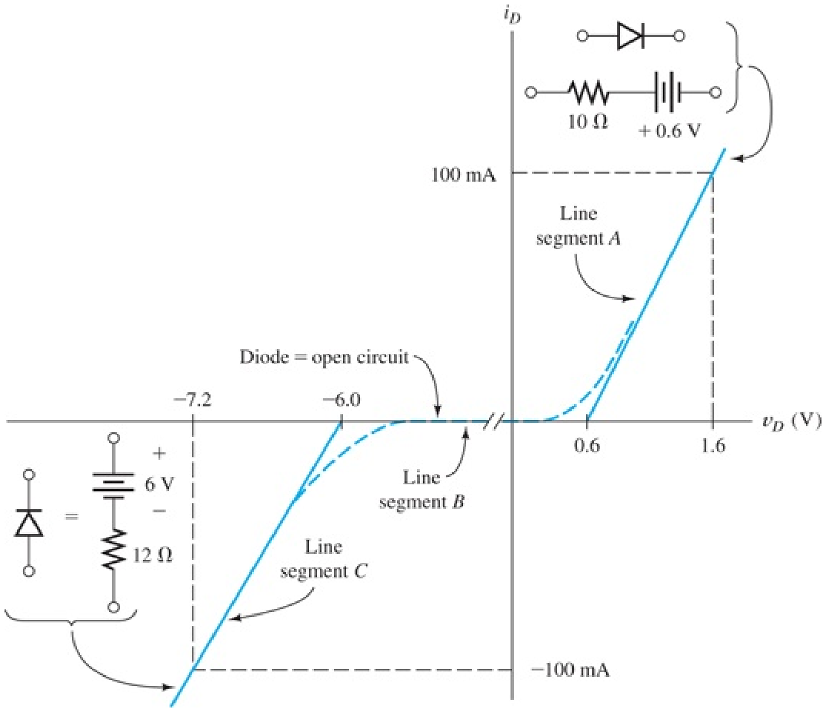


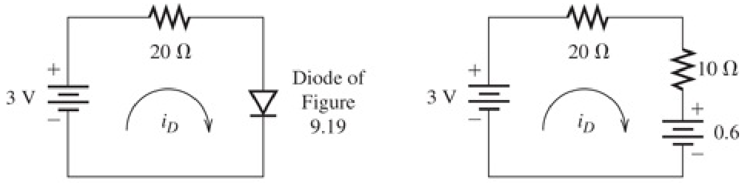

**Figure 9.19**
Piecewise-linear models for the diode of **Example 9.6**.

*Solution*

For line segment *A* of **Figure 9.19**, the intercept on the voltage axis is 0.6 V and the reciprocal of the slope is $10\ \Omega$ . Hence, the circuit model for the diode on this segment is a $10\text{-}\ \Omega$ resistance in series with a 0.6-V source, as shown in the figure. Line segment *B* has zero current, and therefore, the equivalent circuit for segment *B* is an open circuit, as illustrated in the figure. Finally, line segment *C* has an intercept of $-6\ \mathrm{V}$ and a reciprocal slope of $12\ \ \Omega$ , resulting in the equivalent circuit shown. Thus, this diode can be approximated by one of these linear circuits, depending on where the operating point is located.

**Example 9.7 Analysis Using a Piecewise-Linear Model**

Use the circuit models found in **Example 9.6** to solve for the current in the circuit of **Figure 9.20(a)**.




(a) Circuit diagram

(b) Circuit with diode modeled by the equivalent circuit for the forward-bias region

**Figure 9.20**
Circuit for **Example 9.7**.

*Solution*

Since the 3-V source has a polarity that results in forward bias of the diode, we assume that the operating point is on line segment *A* of **Figure 9.19**. Consequently, the equivalent circuit for the diode is the one for segment *A*. Using this equivalent circuit, we have the circuit of **Figure 9.20(b)**. Solving, we find that $i_D = 80 \text{ mA}$.

## Exercise 9.9

Use the appropriate circuit model from **Figure 9.19** to solve for $v_o$ in the circuit of **Figure 9.21** if:

a. $R_L = 10 \text{ k}\Omega$; and

b. $R_L = 1 \text{ k}\Omega$. (*Hint:* Be sure that your answers are consistent with your choice of equivalent circuit for the diode–the various equivalent circuits are valid only for specific ranges of diode voltage and current. The answer must fall into the valid range for the equivalent circuit used.)

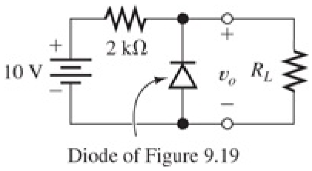



**Figure 9.21**
Circuit for **Exercise 9.9**.

**Answer**

a. $v_o = 6.017 \text{ V}$;

b. $v_o = 3.333 \text{ V}$.

## Exercise 9.10

Find a circuit model for each line segment shown in **Figure 9.22(a)**. Draw the circuit models identifying terminals *a* and *b* for each equivalent circuit.

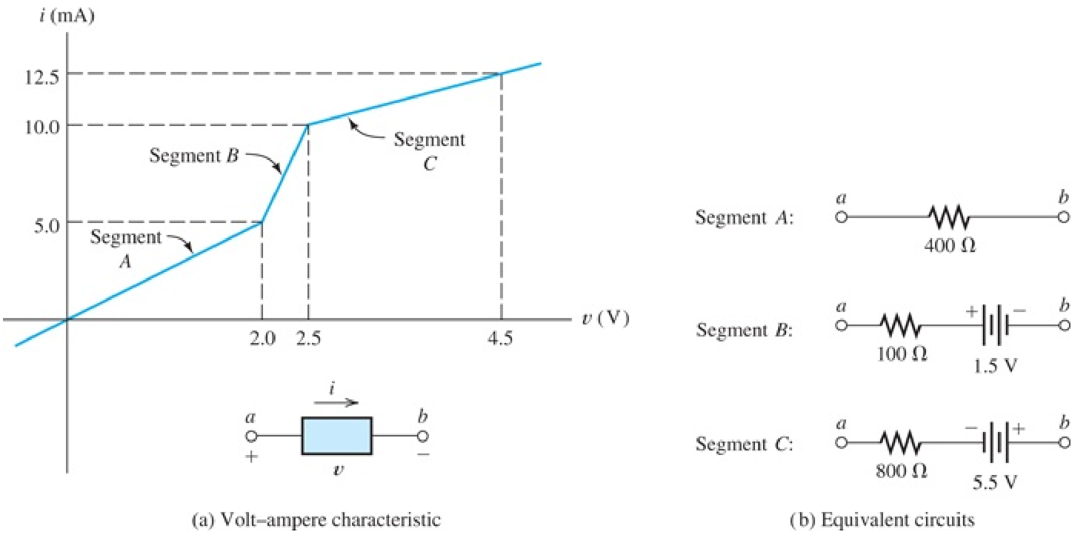



**Figure 9.22**

Hypothetical nonlinear device for **Exercise 9.10**.

**Answer**

See **Figure 9.22(b)**. Notice the polarity of the voltage sources with respect to terminals *a* and *b*.

## Simple Piecewise-Linear Diode Equivalent Circuit

**Figure 9.23** shows a simple piecewise-linear equivalent circuit for diodes that is often sufficiently accurate. It is an open circuit in the reverse-bias region and a constant voltage drop in the forward direction. This model is equivalent to a battery in series with an ideal diode.

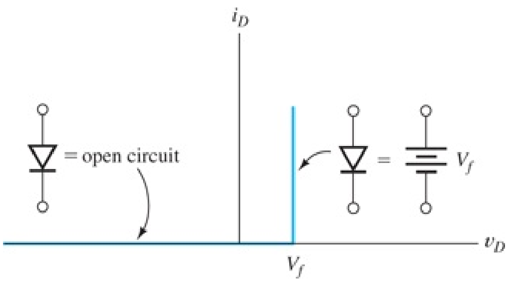



**Figure 9.23**

Simple piecewise-linear equivalent for the diode.

## 9.6 Rectifier Circuits

Now that we have introduced the diode and some methods for analysis of diode circuits, we consider some additional practical circuits. First, we consider several types of **rectifiers**, which convert ac power into dc power. These rectifiers form the basis for electronic **power supplies** and battery-charging circuits. Typically, a power supply takes power from a raw source, which is often the 60-Hz ac power line, and delivers steady dc voltages to a load such as computer circuits or television circuits. Other applications for rectifiers are in signal processing, such as demodulation of a radio signal. (*Demodulation* is the process of retrieving the message, such as a voice or video signal.) Another application is precision conversion of an ac voltage to dc in an electronic voltmeter.

### Half-Wave Rectifier Circuits

A **half-wave rectifier** with a sinusoidal source and resistive load $R_L$ is shown in **Figure 9.24**. When the source voltage $v_s(t)$ is positive, the diode is in the forward-bias region. If an ideal diode is assumed, the source voltage appears across the load. For a typical real diode, the output voltage is less than the source voltage by an amount equal to the drop across the diode, which is approximately 0.7 V for silicon diodes at room temperature. When the source voltage is negative, the diode is reverse biased and no current flows through the load. Even for typical real diodes, only a very small reverse current flows. Thus, only the positive half-cycles of the source voltage appear across the load.

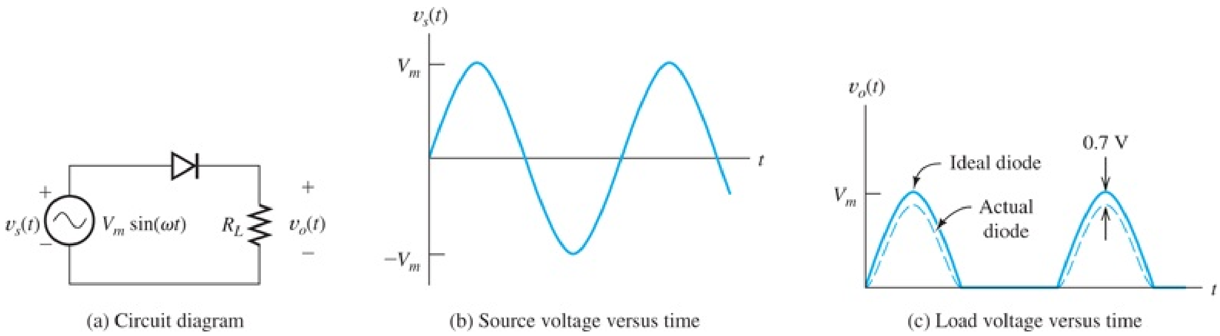



**Figure 9.24**
Half-wave rectifier with resistive load.

***Battery-Charging Circuit.***

We can use a half-wave rectifier to charge a battery as shown in **Figure 9.25**. Current flows whenever the instantaneous ac source voltage is higher than the battery voltage. As shown in the figure, it is necessary to add resistance to the circuit to limit the magnitude of the current. When the ac source voltage is less than the battery voltage, the diode is reverse biased and the current is zero. Hence, the current flows only in the direction that charges the battery.

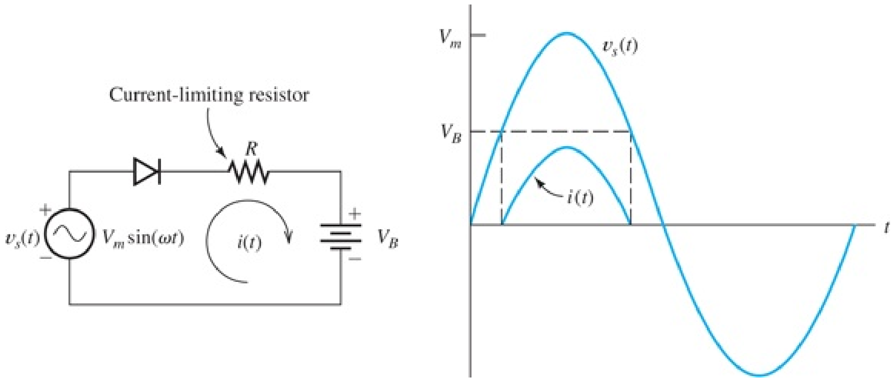



**Figure 9.25**
Half-wave rectifier used to charge a battery.

***Half-Wave Rectifier with Smoothing Capacitor.***

Often, we want to convert an ac voltage into a nearly constant dc voltage to be used as a power supply for electronic circuits. One approach to smoothing the rectifier output voltage is to place a large capacitance across the output terminals of the rectifier. The circuit and waveforms of current and voltage are shown in **Figure 9.26**. When the ac source reaches a positive peak, the capacitor is charged to the peak voltage (assuming an ideal diode). When the source voltage drops below the voltage stored on the capacitor, the diode is reverse biased and no current flows through the diode. The capacitor continues to supply current to the load, slowly discharging until the next positive peak of the ac input. As shown in the figure, current flows through the diode in pulses that recharge the capacitor.

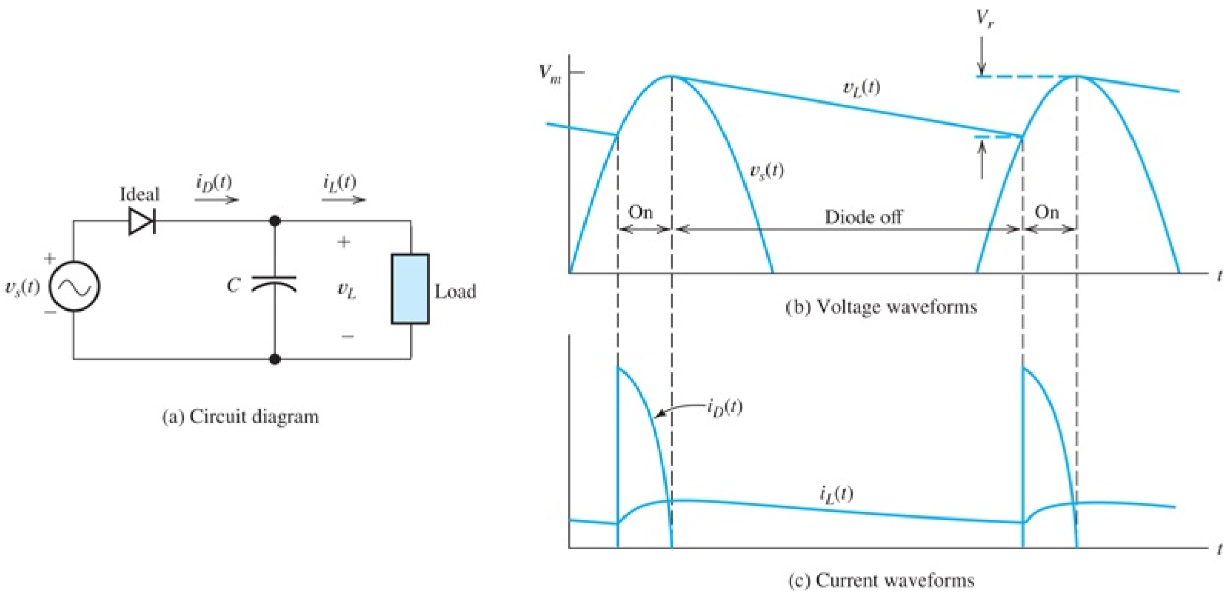



**Figure 9.26**
Half-wave rectifier with smoothing capacitor.

Because of the charge and discharge cycle, the load voltage contains a small ac component called **ripple**. Usually, it is desirable to minimize the amplitude of the ripple, so we choose the largest capacitance value that is practical. In this case, the capacitor discharges for nearly the entire cycle, and the charge removed from the capacitor during one discharge cycle is

$$Q \cong I_L T \tag{9.8}$$

where $I_L$ is the average load current and *T* is the period of the ac voltage. Since the charge removed from the capacitor is the product of the change in voltage and the capacitance, we can also write

$$Q = V_r C \tag{9.9}$$

where $V_r$ is the peak-to-peak ripple voltage and *C* is the capacitance. Equating the right-hand sides of **Equations 9.8** and **9.9** allows us to solve for *C*:

$$C = \frac{I_L T}{V_r} \tag{9.10}$$

In practice, **Equation 9.10** is approximate because the load current varies and because the capacitor does not discharge for a complete cycle. However, it gives a good starting value for calculating the capacitance required in the design of power-supply circuits.

The average voltage supplied to the load if a smoothing capacitor is used is approximately midway between the minimum and maximum voltages. Thus, referring to **Figure 9.26**, the average load voltage is

$$V_L \cong V_m - \frac{V_r}{2} \tag{9.11}$$

## Peak Inverse Voltage

An important aspect of rectifier circuits is the **peak inverse voltage** (PIV) across the diodes. Of course, the breakdown specification of the diodes should be greater in magnitude than the PIV. For example, in the half-wave circuit with a resistive load, shown in **Figure 9.24**, the PIV is $V_m$.

The addition of a smoothing capacitor in parallel with the load increases the PIV to (approximately) $2V_m$. Referring to **Figure 9.26**, for the negative peak of the ac input, we see that the reverse bias of the diode is the sum of the source voltage and the voltage stored on the capacitor.

## Full-Wave Rectifier Circuits

Several **full-wave rectifier** circuits are in common use. One approach uses two ac sources and two diodes, as shown in **Figure 9.27(a)**. One feature of this diagram is the **ground symbol**. Usually in electronic circuits, many components are connected to a common point known as *ground*. Often, the chassis containing the circuit is the electrical ground. Therefore, in **Figure 9.27(a)**, the lower end of $R_L$ and the point between the voltage sources are connected together.

**Figure 9.27**
Full-wave rectifier.

> A wire or other conductor, not shown explicitly in the diagram, connects all of the points that are connected to ground symbols.

When the upper source applies a positive voltage to the left-hand end of diode *A*, the lower source applies a negative voltage to the left-hand end of diode *B*, and vice versa. We say that the sources are **out of phase**. Thus, the circuit consists of two half-wave rectifiers with out-of-phase source voltages and a common load. The diodes conduct on alternate half-cycles.

Usually, the two out-of-phase ac voltages are provided by a **transformer**. (Transformers are discussed in **Chapter 15**.) Besides providing the out-of-phase ac voltages, the transformer also allows the designer to adjust $V_m$ by selection of the turns ratio. This is important, because the ac voltage available is often not of a suitable amplitude for direct rectification–usually either a higher or lower dc voltage is required.

A second type of full-wave rectifier uses the **diode bridge** shown in **Figure 9.28**. When the ac voltage, $V_m \sin(\omega t)$, is positive, current flows through diode *A*, then through the load, and returns through diode *B*, as shown in the figure. For the opposite polarity, current flows through diodes *C* and *D*. Notice that in either case, current flows in the same direction through the load.

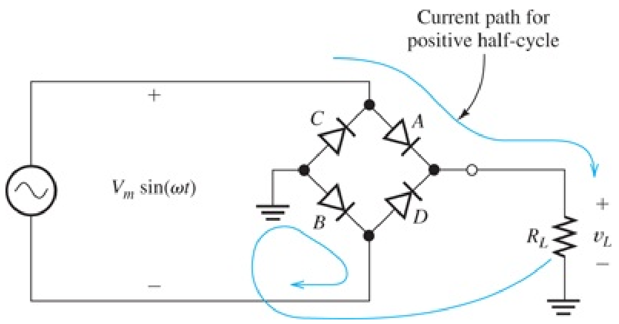



**Figure 9.28**
Diode-bridge full-wave rectifier.

Usually, neither of the ac source terminals is connected to ground. This is necessary if one side of the load is to be connected to ground, as shown in the figure. (If both the ac source and the load have a common ground connection, part of the circuit is shorted.)

If we wish to smooth the voltage across the load, a capacitor can be placed in parallel with the load, similar to the half-wave circuit discussed earlier. In the full-wave circuits, the capacitor discharges for only a half-cycle before being recharged. Hence, the capacitance required is only half as much in the full-wave circuit as for the half-wave circuit. Therefore, we modify **Equation 9.10** to obtain

$$C = \frac{I_L T}{2 V_r} \tag{9.12}$$

for the full-wave rectifier with a capacitive filter.

*Exercise 9.11*

Consider the battery-charging circuit of **Figure 9.25** with $V_m = 20 \text{ V}$, $R = 10 \ \Omega$, and $V_B = 14 \text{ V}$.

a. Find the peak current assuming an ideal diode.

b. Find the percentage of each cycle for which the diode is in the on state.

**Answer**

a. $I_{\text{peak}} = 600 \text{ mA}$;

b. the diode is on for 25.3 percent of each cycle.

*Exercise 9.12*

A power-supply circuit is needed to deliver 0.1 A and 15 V (average) to a load. The ac source has a frequency of 60 Hz. Assume that the circuit of **Figure 9.26** is to be used. The peak-to-peak ripple voltage is to be 0.4 V. Instead of assuming an ideal diode, allow 0.7 V for forward diode drop. Find the peak ac voltage $V_m$ needed and the approximate value of the smoothing capacitor. (*Hint:* To achieve an average load voltage of 15 V with a ripple of 0.4 V, design for a peak load voltage of 15.2 V.)

**Answer**

$V_m = 15.9, \ C = 4166 \ \mu\text{F}$.

*Exercise 9.13*

Repeat **Exercise 9.12** using the circuit of **Figure 9.28** with the smoothing capacitor in parallel with the load $R_L$.

**Answer**

$V_m = 16.6, \ C = 2083 \ \mu\text{F}$.

## 9.7 Wave-Shaping Circuits

A wide variety of **wave-shaping circuits** are used in electronic systems. These circuits are used to transform one waveform into another. Numerous examples of wave- shaping circuits can be found in transmitters and receivers for television or radar. In this section, we discuss a few examples of wave-shaping circuits that can be constructed with diodes.

### Clipper Circuits

> A clipper circuit "clips off" part of the input waveform to produce the output waveform.

Diodes can be used to form **clipper circuits**, in which a portion of an input signal waveform is "clipped" off. For example, the circuit of **Figure 9.29** clips off any part of the input waveform above 6 V or less than $-9\ \mathrm{V}$. (We are assuming ideal diodes.) When the input voltage is between $-9$ and $+6\ \mathrm{V}$, both diodes are off and no current flows. Then, there is no drop across *R* and the output voltage $v_o$ is equal to the input voltage $v_{\mathrm{in}}$. On the other hand, when $v_{\mathrm{in}}$ is larger than 6 V, diode *A* is on and the output voltage is 6 V, because the diode connects the 6-V battery to the output terminals. Similarly, when $v_{\mathrm{in}}$ is less than $-9\ \mathrm{V}$, diode *B* is on and the output voltage is $-9\ \mathrm{V}$. The output waveform resulting from a 15-V-peak sinusoidal input is shown in **Figure 9.29(b)**, and the transfer characteristic of the circuit is shown in **Figure 9.29(c)**.

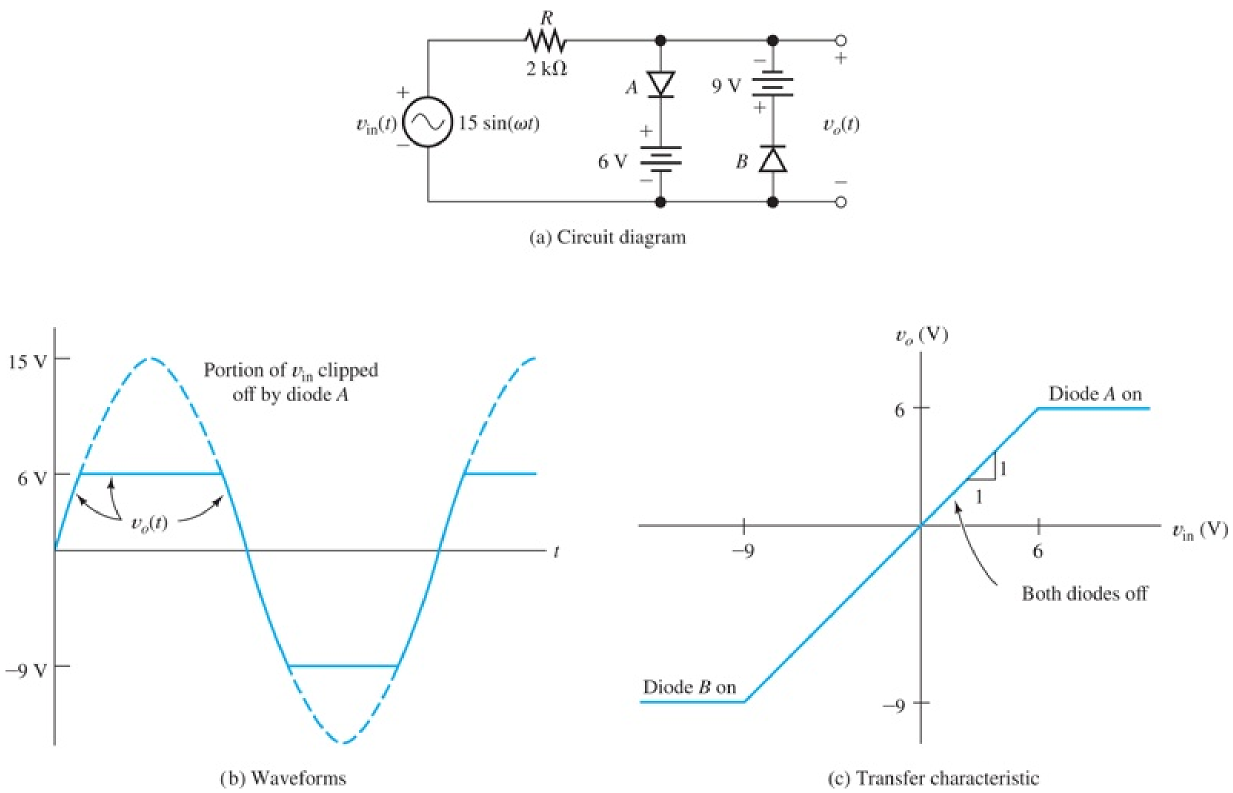
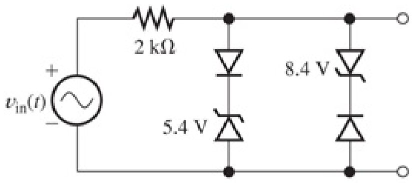


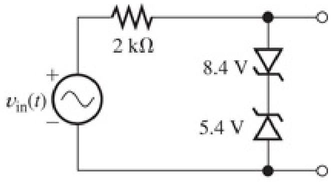

**Figure 9.29**
Clipper circuit.

The resistance *R* is selected large enough so that the forward diode current is within reasonable bounds (usually, a few milliamperes), but small enough so that the reverse diode current results in a negligible voltage drop. Often, we find that a wide range of resistance values provides satisfactory performance in a given circuit.

In **Figure 9.29**, we have assumed ideal diodes. If small-signal silicon diodes are used, we expect a forward drop of 0.6 or 0.7 V, so we should reduce the battery voltages to compensate. Furthermore, batteries are not desirable for use in circuits if they can be avoided, because they may need periodic replacement. Thus, a better design uses Zener diodes instead of batteries. Practical circuits equivalent to - **Figure 9.29** are shown in **Figure 9.30**. The Zener diodes are labeled with their breakdown voltages.




(a) Circuit of Figure 9.29 with batteries replaced by Zener diodes and allowance made for a 0.6-V forward diode drop




(b) Simpler circuit

**Figure 9.30**

Circuits with nearly the same performance as the circuit of **Figure 9.29**.

*Exercise 9.14*

a. Sketch the transfer characteristics to scale for the circuits of **Figure 9.31(a)** and **(b)**. Allow a 0.6-V forward drop for the diodes.

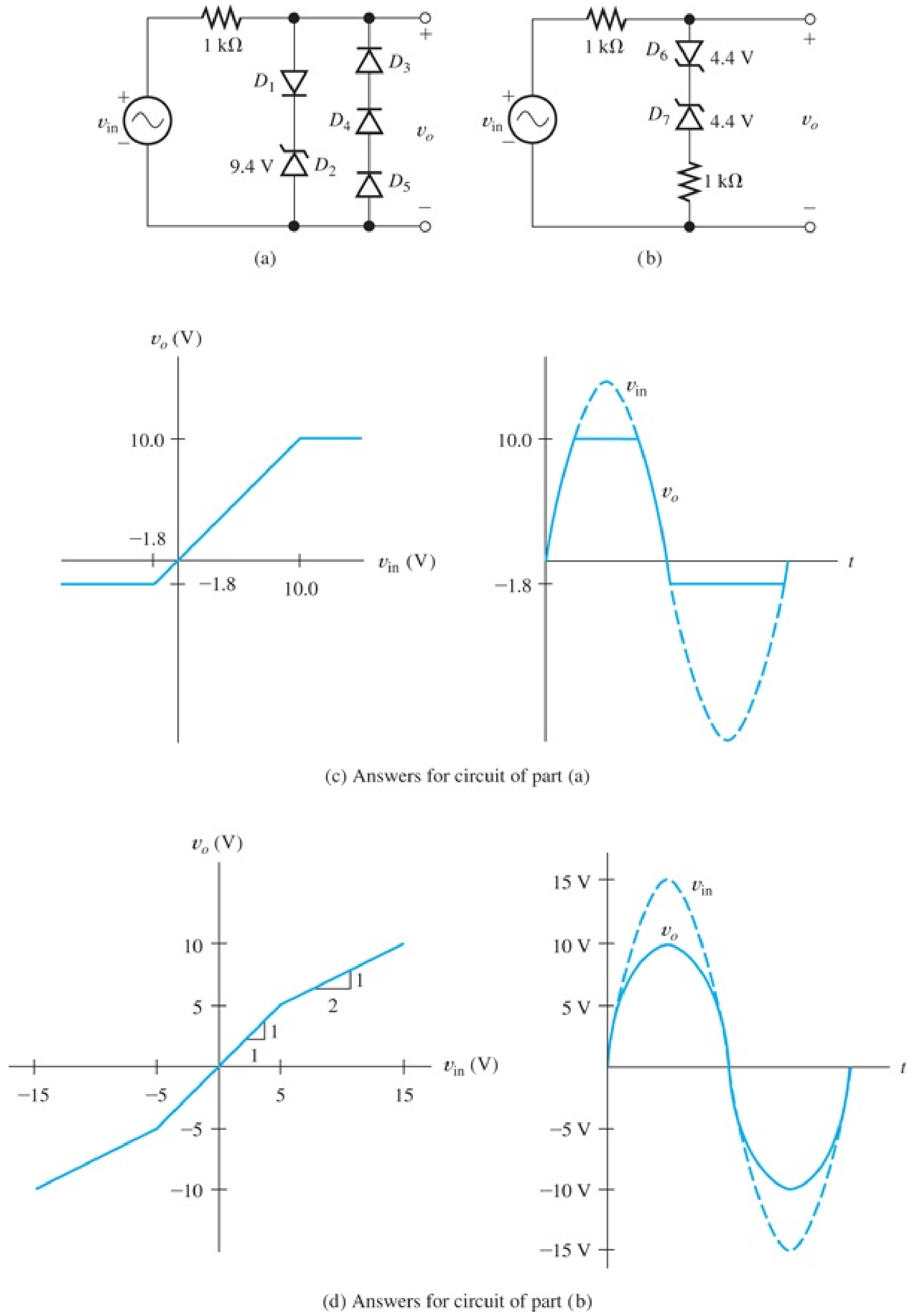



**Figure 9.31**
See **Exercise 9.14**.

b. Sketch the output waveform to scale if $v_{in}(t) = 15\sin(\omega t)$.

**Answer**

a. See **Figure 9.31(c)**;

b. see **Figure 9.31(d)**.

*Exercise 9.15*

Design clipper circuits that have the transfer characteristics shown in

a. **Figure 9.32(a)** and

b. **Figure 9.32(b)**. Allow for a 0.6-V drop in the forward direction for the diodes. [*Hint for part (b):* Include a resistor in series with the diode that begins to conduct at $v_{\text{in}} = 3\ \text{V}$ to achieve the slope required for the section between $v_{\text{in}} = 3\ \text{V}$ and 6 V.]

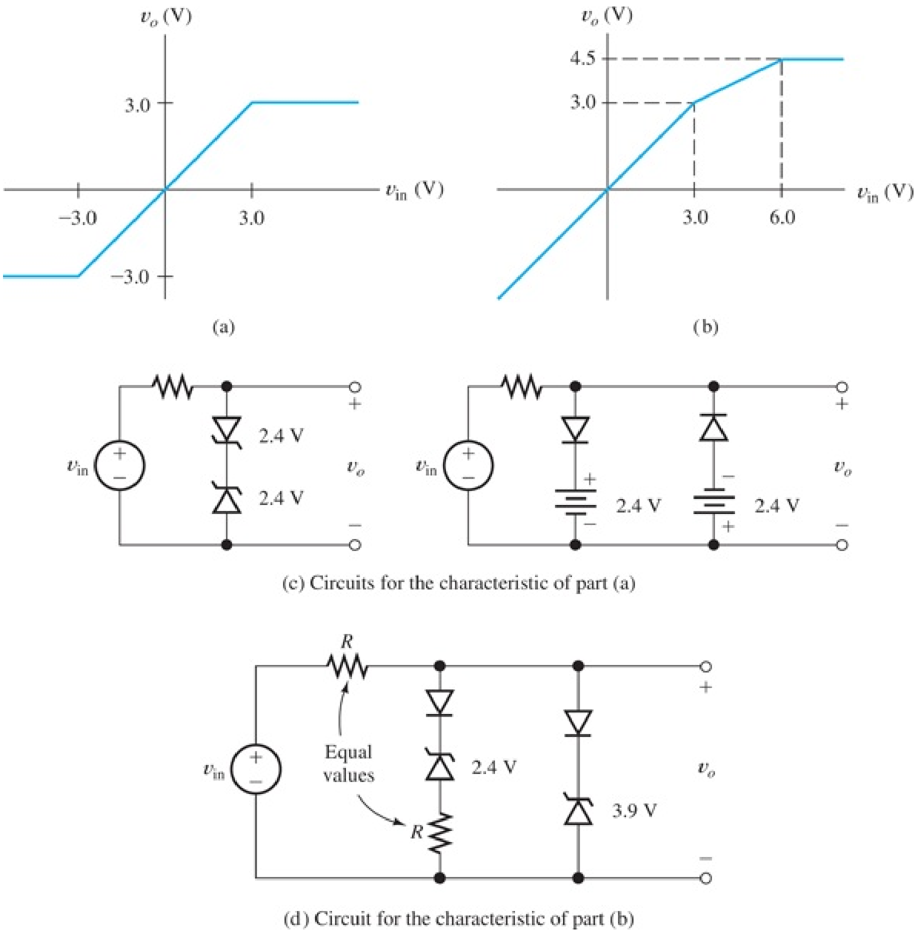



**Figure 9.32**
See **Exercise 9.15**.

**Answer**

a. See **Figure 9.32(c)**;

b. see **Figure 9.32(d)**.

## Clamp Circuits

Another diode wave-shaping circuit is the **clamp circuit**, which is used to add a dc component to an ac input waveform so that the positive (or negative) peaks are forced to take a specified value. In other words, the peaks of the waveform are “clamped” to a specified voltage value. An example circuit is shown in **Figure 9.33**. In this circuit, the positive peaks are clamped to $-5\ \text{V}$.

**Figure 9.33**
Example clamp circuit.

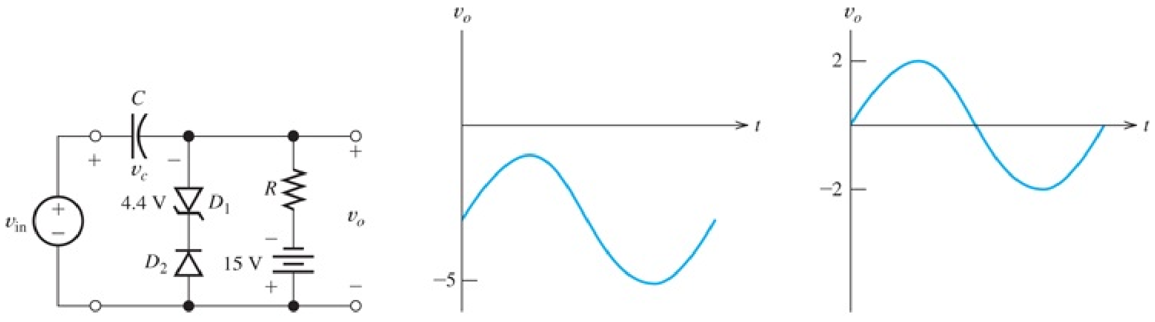
> In a clamp circuit, a variable dc voltage is added to the input waveform so that one of the peaks of the output is clamped to a specified value.

The capacitance is a large value, so it discharges only very slowly and we can consider the voltage across the capacitor to be constant. Because the capacitance is large, it has a very small impedance for the ac input signal. The output voltage of the circuit is given by

$$v_o(t) = v_{\mathrm{in}}(t) - V_C \tag{9.13}$$

If a positive swing of the input signal attempts to force the output voltage to become greater than $-5\ \mathrm{V}$, the diode conducts, increasing the value of $V_C$. Thus, the capacitor is charged to a value that adjusts the maximum value of the output voltage to $-5\ \mathrm{V}$. A large resistance *R* is provided so that the capacitor can discharge slowly. This is necessary so the circuit can adjust if the input waveform changes to a smaller peak amplitude.

Of course, we can change the voltage to which the circuit clamps by changing the battery voltage. Reversing the direction of the diode causes the negative peak to be clamped instead of the positive peak. If the desired clamp voltage requires the diode to be reverse biased, it is necessary to return the discharge resistor to a suitable dc supply voltage to ensure that the diode conducts and performs the clamping operation. Furthermore, it is often more convenient to use Zener diodes rather than batteries. A circuit including these features is shown in **Figure 9.34**.




(a) A circuit that clamps the negative peaks to −5 V (an allowance of 0.6-V forward drop is made)

(b) Output for $v_{in} = 2\sin(\omega t)$

(c) Output for $v_{in} = 2\sin(\omega t)$ and $R$ returned directly to ground (diodes do not turn on)

**Figure 9.34**
See **Exercise 9.16**.

*Exercise 9.16*

Consider the circuit of **Figure 9.34(a)**. Assume that the capacitance is large enough so that the voltage across it does not discharge through *R* appreciably during one cycle of input.

a. What is the steady-state output voltage if $v_{in}(t) = 0$?
b. Sketch the steady-state output to scale versus time if $v_{in}(t) = 2\sin(\omega t)$.
c. Suppose that the resistor is returned directly to ground instead of $-15\ \mathrm{V}$ (i.e., replace the 15-V source by a short circuit). In this case, sketch the steady-state output versus time if $v_{in}(t) = 2\sin(\omega t)$.

**Answer**

a. For $v_{in}(t) = 0,$ we have $v_o = -5\ \mathrm{V};$
b. see **Figure 9.34(b)**;
c. c. see **Figure 9.34(c)**.

*Exercise 9.17*

Design a circuit that clamps the negative peaks of an ac signal to $+6\ \mathrm{V}$. You can use batteries, resistors, and capacitors of any value desired in addition to Zener or conventional diodes. Allow 0.6 V for the forward drop.

**Answer**

A solution is shown in **Figure 9.35**. Other solutions are possible.

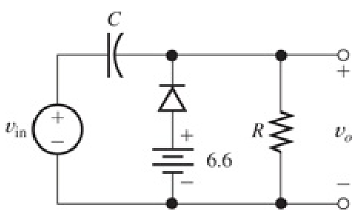



**Figure 9.35**
Answer for **Exercise 9.17**.

*Exercise 9.18*

Repeat **Exercise 9.17** for a circuit that clamps the positive peaks to $+6\ \mathrm{V}$.

**Answer**

A solution is shown in **Figure 9.36**. Other solutions are possible.

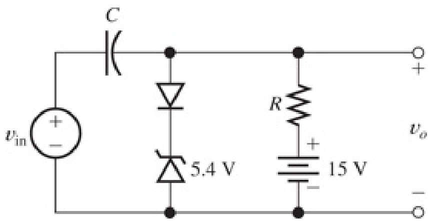



**Figure 9.36**
Answer for **Exercise 9.18**.

# 9.8 Linear Small-Signal Equivalent Circuits

We will encounter many examples of electronic circuits in which dc supply voltages are used to **bias** a nonlinear device at an operating point, and a small ac signal is injected into the circuit. We often split the analysis of such circuits into two parts. First, we analyze the dc circuit to find the operating point. In this analysis of bias conditions, we must deal with the nonlinear aspects of the device. In the second part of the analysis, we consider the small ac signal. Since virtually any nonlinear characteristic is approximately linear (straight) if we consider a sufficiently small portion, we can find a **linear small-signal equivalent circuit** for the nonlinear device to use in the ac analysis.

Often, the main concern in the design of such circuits is what happens to the ac signal. The dc supply voltages simply bias the device at a suitable operating point. For example, in a portable radio, the main interest is the signal being received, demodulated, amplified, and delivered to the speaker. The dc currents supplied by the battery are required for the devices to perform their intended function on the ac signals. However, most of our design time is spent in consideration of the small ac signals to be processed.

The small-signal linear equivalent circuit is an important analysis approach that applies to many types of electronic circuits. In this section, we demonstrate the principles with a simple diode circuit. In **Chapters 11** and **12**, we use similar techniques for transistor amplifier circuits.

> The small-signal equivalent circuit for a diode is a resistance.

Now, we show that in the case of a diode, the small-signal equivalent circuit consists simply of a resistance. Consider the diode characteristic shown in **Figure 9.37**. Assume that the dc supply voltage results in operation at the **quiescent point**, or *Q* **point**, indicated on the characteristic. Then, a small ac signal injected into the circuit swings the instantaneous point of operation slightly above and below the *Q* point.

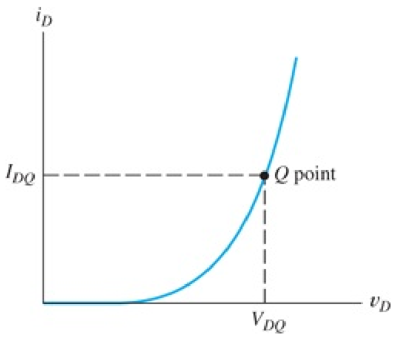



**Figure 9.37**
Diode characteristic, illustrating the *Q* point.

For a sufficiently small ac signal, the characteristic is straight. Thus, we can write

$$\Delta\, i_D \cong \left(\frac{di_D}{dv_D}\right)_Q \Delta\, v_D \tag{9.14}$$

where $\Delta\, i_D$ is the small change in diode current from the *Q*-point current caused by the ac signal, $\Delta\, v_D$ is the change in the diode voltage from the *Q*-point value, and $(di_D/dv_D)_Q$ is the slope of the diode characteristic evaluated at the *Q* point. Notice that the slope has the units of inverse resistance.

Hence, we define the **dynamic resistance** of the diode as

$$r_d = \left[\left(\frac{di_D}{dv_D}\right)_Q\right]^{-1} \tag{9.15}$$

and **Equation 9.14** becomes

$$\Delta\, i_D \cong \frac{\Delta\, v_D}{r_d} \tag{9.16}$$

We find it convenient to drop the $\Delta$ notation and denote changes of current and voltage from the *Q*-point values as $v_d$ and $i_d$. (Notice that lowercase subscripts are used for the small changes in current and voltage.) Therefore, for these small ac signals, we write

$$i_d = \frac{v_d}{r_d} \tag{9.17}$$

As shown by **Equation 9.15**, we can find the equivalent resistance of the diode for the small ac signal as the reciprocal of the slope of the characteristic curve. The current of a junction diode is given by the Shockley equation (**Equation 9.1**), repeated here for convenience:

$$i_D = I_s\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right]$$

The slope of the characteristic can be found by differentiating the Shockley equation, resulting in

$$\frac{di_D}{dv_D} = I_s\frac{1}{nV_T}\exp\left(\frac{v_D}{nV_T}\right) \tag{9.18}$$

Substituting the voltage at the *Q* point, we have

$$\left(\frac{di_D}{dv_D}\right)_Q = I_s\frac{1}{nV_T}\exp\left(\frac{V_{DQ}}{nV_T}\right) \tag{9.19}$$

For forward-bias conditions with $V_{DQ}$ at least several times as large as $V_T$, the $-1$ inside the brackets of the Shockley equation is negligible. Thus, we can write

$$I_{DQ} \cong I_s\exp\left(\frac{V_{DQ}}{nV_T}\right) \tag{9.20}$$

Substituting this into **Equation 9.19**, we have

$$\left(\frac{di_D}{dv_D}\right)_Q = \frac{I_{DQ}}{nV_T} \tag{9.21}$$

Taking the reciprocal and substituting into **Equation 9.15**, we have the dynamic small-signal resistance of the diode at the *Q* point:

$$r_d = \frac{nV_T}{I_{DQ}} \tag{9.22}$$

To summarize, for signals that cause small changes from the *Q* point, we can treat the diode simply as a linear resistance. The value of the resistance is given by **Equation 9.22** (provided that the diode is forward biased). As the *Q*-point current $I_{DQ}$ increases, the resistance becomes smaller. Thus, an ac voltage of fixed amplitude produces an ac current that has higher amplitude as the *Q* point moves higher. This is illustrated in **Figure 9.38**.

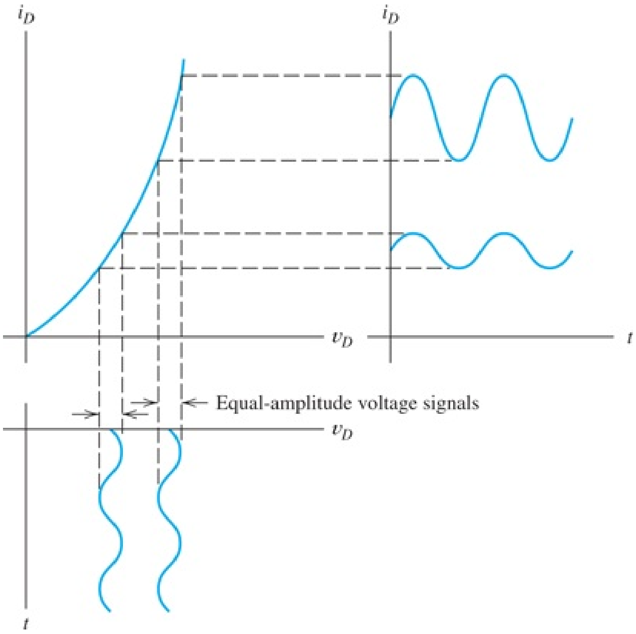



**Figure 9.38**
As the *Q* point moves higher, a fixed-amplitude ac voltage produces an ac current of larger amplitude.

## Notation for Currents and Voltages in Electronic Circuits

Perhaps we should review the notation we have used for the diode currents and voltages, because we use similar notation throughout this book:

- $v_D$ and $i_D$ represent the total instantaneous diode voltage and current. At times, we may wish to emphasize the time-varying nature of these quantities, and then we use $v_D(t)$ and $i_D(t)$.
- $V_{DQ}$ and $I_{DQ}$ represent the dc diode current and voltage at the quiescent point.
- $v_d$ and $i_d$ represent the (small) ac signals. If we wish to emphasize their time-varying nature, we use $v_d(t)$ and $i_d(t)$.

This notation is illustrated for the waveform shown in **Figure 9.39**.

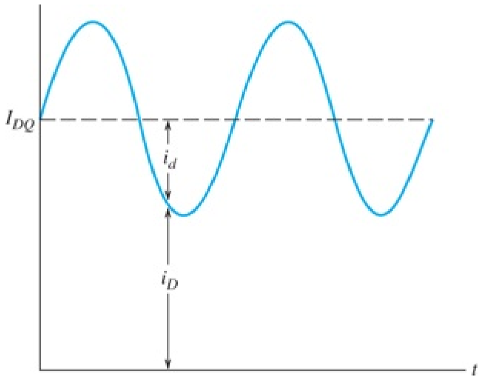



**Figure 9.39**
Illustration of diode currents.

*Exercise 9.19*

Compute the dynamic resistance of a junction diode having $n = 1$ at a temperature of 300 K for $I_{DQ} =$

a. 0.1 mA;

b. 1 mA;

c. 10 mA.

**Answer**

a. $260\ \Omega$;

b. $26\ \Omega$;

c. $2.6\ \Omega$.

## Voltage-Controlled Attenuator

Now, we consider an example of linear-equivalent-circuit analysis for the relatively simple, but useful circuit shown in **Figure 9.40**. The function of this circuit is to produce an output signal $v_o(t)$ that is a variable fraction of the ac input signal $v_{\text{in}}(t)$. It is similar to the resistive voltage divider (see **Section 2.3**), except that in this case, we want the division ratio to depend on another voltage $V_C$ called the **control signal**. We refer to the process of reduction of the amplitude of a signal as **attenuation**. Thus, the circuit to be studied is called a **voltage-controlled attenuator**. The degree of attenuation depends on the value of the dc control voltage $V_C$.

**Figure 9.40**
Variable attenuator using a diode as a controlled resistance.

Notice that the ac signal to be attenuated is connected to the circuit by a **coupling capacitor**. The output voltage is connected to the load $R_L$ by a second coupling capacitor. Recall that the impedance of a capacitance is given by

$$Z_C = \frac{1}{j\omega C}$$

in which $\omega$ is the angular frequency of the ac signal. We select the capacitance values large enough so that they are effectively short circuits for the ac signal. However, the coupling capacitors are open circuits for dc. Thus, the quiescent operating point (*Q* point) of the diode is unaffected by the signal source or the load. This can be important for a circuit that must work for various sources and loads that could affect the *Q* point. Furthermore, the coupling capacitors prevent (sometimes undesirable) dc currents from flowing in the source or the load.

Because of the coupling capacitors, we only need to consider $V_C$, $R_C$, and the diode to perform the bias analysis to find the *Q* point. Hence, the dc circuit is shown in **Figure 9.41**. We can use any of the techniques discussed earlier in this chapter to find the *Q* point. Once it is known, the *Q*-point value of the diode current $I_{DQ}$ can be substituted into **Equation 9.22** to determine the dynamic resistance of the diode.

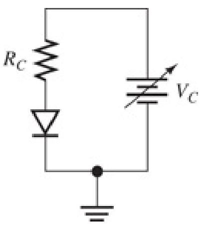



**Figure 9.41**
Dc circuit equivalent to **Figure 9.40** for *Q*-point analysis.

Now, we turn our attention to the ac signal. The dc control source should be considered as a short circuit for ac signals. The signal source causes an ac current to flow through the $V_C$ source. However, $V_C$ is a dc voltage source, and by definition, the voltage across it is constant. *Since the dc voltage source has an ac component of current, but no ac voltage, the dc voltage source is equivalent to a short circuit for ac signals.* This is an important concept that we will use many times in drawing ac equivalent circuits.

> Dc sources and coupling capacitors are replaced by short circuits in small-signal ac equivalent circuits. Diodes are replaced with their dynamic resistances.

The equivalent circuit for ac signals is shown in **Figure 9.42**. The control source and the capacitors have been replaced by short circuits, and the diode has been replaced by its dynamic resistance. This circuit is a voltage divider and can be analyzed by ordinary linear-circuit analysis. The parallel combination of $R_C$, $R_L$, and $r_d$ is denoted as $R_p$, given by

$$R_p = \frac{1}{1/R_C + 1/R_L + 1/r_d} \tag{9.23}$$

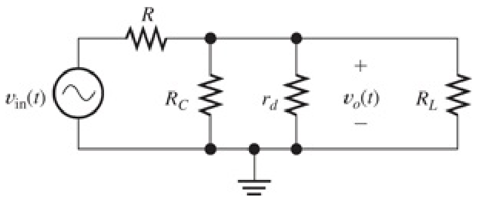



**Figure 9.42**
Small-signal ac equivalent circuit for **Figure 9.40**.

Then, the **voltage gain** of the circuit is

$$A_v = \frac{v_o}{v_{in}} = \frac{R_p}{R + R_p} \tag{9.24}$$

(Of course, $A_v$ is less than unity.)

*Exercise 9.20*

Suppose that the circuit of **Figure 9.40** has $R = 100\ \Omega$, $R_C = 2\ \mathrm{k}\Omega$, and $R_L = 2\ \mathrm{k}\Omega$. The diode has $n = 1$ and is at a temperature of 300 K. For purposes of Q-point analysis, assume a constant diode voltage of 0.6 V. Find the Q-point value of the diode current and $A_v$ for $V_C =$

a. 1.6 V;

b. 10.6 V.

**Answer**

a. $I_{DQ} = 0.5\ \mathrm{mA}$ and $A_v = 0.331;$

b. $I_{DQ} = 5\ \mathrm{mA}$ and $A_v = 0.0492.$

An application for voltage-controlled attenuators occurs in digital voice recorders in which the audio signal from a microphone is amplified to a suitable level, converted to digital form in an analog-to-digital converter (ADC), and stored in a digital memory. (Analog-to-digital conversion is discussed in **Section 6.10**, starting on page **331**.) A problem frequently encountered in recording audio is that some persons speak quietly, while others speak loudly. Furthermore, some may be far from the microphone, while others are close. If an amplifier with fixed gain is used between the microphone and the ADC, either the weak signals are small compared with the quantization error or the strong signals exceed the maximum limits of the ADC so that severe distortion occurs.

A solution is to use a voltage-controlled attenuator in a system such as the one shown in **Figure 9.43**. The attenuator is placed between the microphone and a highgain amplifier.When the signal being recorded is weak, the control voltage is small and very little attenuation occurs. On the other hand, when the signal is strong, the control voltage is large so that the signal is attenuated, preventing distortion. The control voltage is generated by rectifying the output of the amplifier. The rectified signal is filtered by a long-time constant *RC* filter so that the attenuation responds to the average signal amplitude rather than adjusting too rapidly.With proper design, this system can provide an acceptable signal at the converter for a wide range of input signal amplitudes.

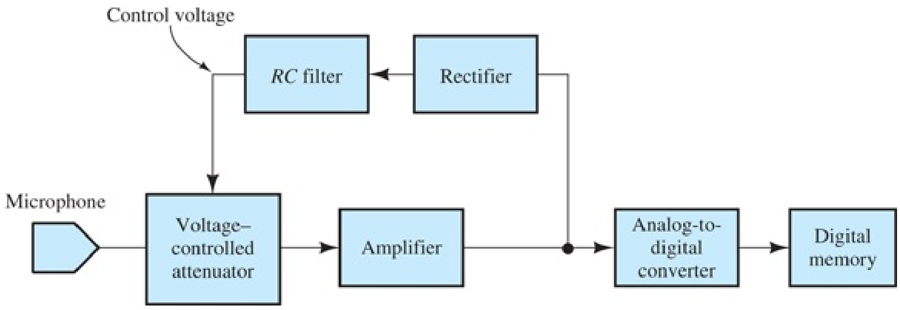



**Figure 9.43**
The voltage-controlled attenuator is useful in maintaining a suitable signal amplitude at the recording head.

While the diode circuit we have discussed is convenient for illustrating principles, integrated-circuit transistor amplifiers in which gain is controlled by changing the *Q*-points of the transistors offer better performance. Examples are the AN-934 from Analog Devices and the MAX9814 from Maxim Integrated Products.

## Summary

1. A *pn*-junction diode is a two-terminal device that conducts current easily in one direction (from anode to cathode), but not in the opposite direction. The volt–ampere characteristic has three regions: forward bias, reverse bias, and reverse breakdown.
2. The Shockley equation relates current and voltage in a *pn*-junction diode.
3. Nonlinear circuits, such as those containing a diode, can be analyzed by using the load-line technique.
4. Zener diodes are intended to be operated in the reverse-breakdown region as constant-voltage references.
5. Voltage regulators are circuits that produce a nearly constant output voltage while operating from a variable source.
6. The ideal-diode model is a short circuit (on) if current flows in the forward direction and an open circuit (off) if voltage is applied in the reverse direction.
7. In the method of assumed states, we assume a state for each diode (on or off), analyze the circuit, and check to see if the assumed states are consistent with the current directions and voltage polarities. This process is repeated until a valid set of states is found.
8. In a piecewise-linear model for a nonlinear device, the volt–ampere characteristic is approximated by straight-line segments. On each segment, the device is modeled as a voltage source in series with a resistance.
9. Rectifier circuits can be used to charge batteries and to convert ac voltages into constant dc voltages. Half-wave rectifiers conduct current only for one polarity of the ac input, whereas full-wave circuits conduct for both polarities.
10. Wave-shaping circuits change the waveform of an input signal and deliver the modified waveform to the output terminals. Clipper circuits remove that portion of the input waveform above (or below) a given level. Clamp circuits add or subtract a dc voltage, so that the positive (or negative) peaks have a specified voltage.
11. The small-signal (incremental) equivalent circuit of a diode consists of a resistance. The value of the resistance depends on the operating point (Q point).
12. Dc sources and coupling capacitors are replaced by short circuits in small-signal ac equivalent circuits. Diodes are replaced with their dynamic resistances.

## Problems

### Section 9.1: Basic Diode Concepts

**P9.1.** Draw the circuit symbol for a diode, labeling the anode and cathode.
**P9.2.** Draw the volt–ampere characteristic of a typical diode and label the various regions.
**P9.3.** Describe a fluid-flow analogy for a diode.
**P9.4.** Write the Shockley equation and define all of the terms.
**P9.5.** Compute the values of $V_T$ for temperatures of 20°C and 150°C.
***P9.6.** Sketch *i* versus *v* to scale for the circuits shown in **Figure P9.6**. The reverse-breakdown voltages of the Zener diodes are shown. Assume voltages of 0.6 V for all diodes including the Zener diodes when current flows in the forward direction.
* Denotes that answers are contained in the Student Solutions files. See **Appendix E** for more information about accessing the Student Solutions.

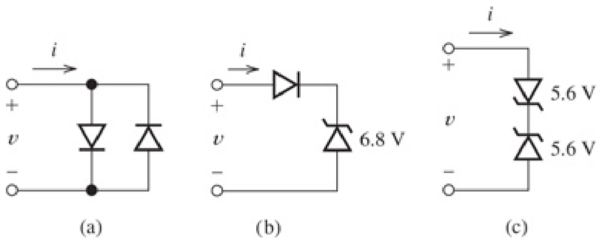



**Figure P9.6**

**P9.7.** Repeat **Problem P9.6** for the circuits shown in **Figure P9.7**.

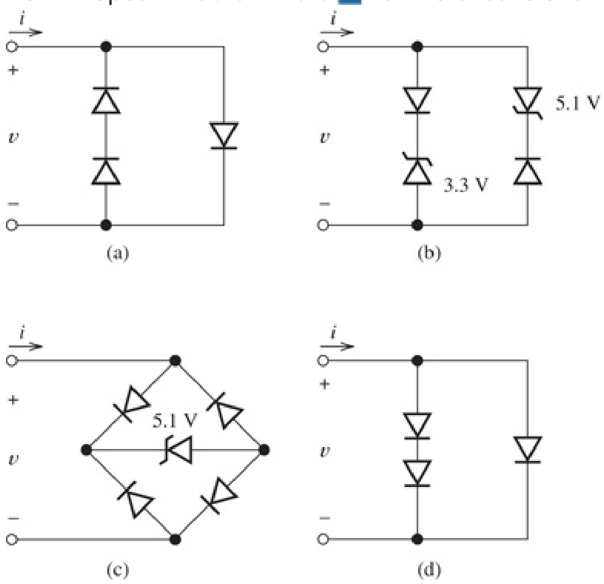



**Figure P9.7**

***P9.8.** A diode operates in forward bias and is described by **Equation 9.4**, with $V_T = 0.026\ \mathrm{V}$. For $v_{D1} = 0.600\ \mathrm{V}$, the current is $i_{D1} = 1\ \mathrm{mA}$. For $v_{D2} = 0.680\ \mathrm{V}$, the current is $i_{D2} = 10\ \mathrm{mA}$. Determine the values of $I_s$ and *n*.
**P9.9.** With constant current flowing in the forward direction in a small-signal silicon diode, the voltage across the diode decreases with temperature by about 2 mV/K. Such a diode has a voltage

of 0.650 V, with a current of 1 mA at a temperature of 25°C. Find the diode voltage at 1 mA and a temperature of 175°C.

**P9.10.** We have a junction diode that has $i_D = 0.2\ \mathrm{mA}$ for $v_D = 0.6\ \mathrm{V}$. Assume that $n = 2$ and $V_T = 0.026\ \mathrm{V}$. Use the Shockley equation to compute the diode current at $v_D = 0.65\ \mathrm{V}$ and at $v_D = 0.70\ \mathrm{V}$.

**P9.11.** We have a diode with $n = 1,\ I_s = 10^{-14}\ \mathrm{A},$ and $V_T = 26\ \mathrm{mV}$.

a. Using a computer program of your choice, obtain a plot of $i_D$ versus $v_D$ for $i_D$ ranging from $10\ \mu\mathrm{A}$ to 10 mA. Choose a logarithmic scale for $i_D$ and a linear scale for $v_D$. What type of curve results?
b. Place a $100\text{-}\ \Omega$ resistance in series with the diode, and plot current versus voltage across the series combination on the same axes used for part (a). Compare the two curves. When is the added series resistance significant?

**P9.12.** A silicon diode described by the Shockley equation has $n = 2$ and operates at 150°C with a current of 1 mA and voltage of 0.25 V. Determine the current after the voltage is increased to 0.30 V.

***P9.13.** The diodes shown in **Figure P9.13** are identical and have $n = 1$. The temperature of the diodes is constant at 300 K. Before the switch is closed, the voltage *v* is 600 mV. Find *v* after the switch is closed. Repeat for $n = 2$.

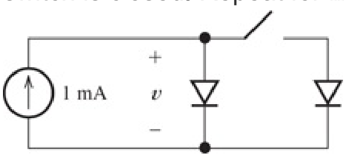



**Figure P9.13**

**P9.14.** Suppose we have a junction diode operating at a constant temperature of 300 K. With a forward current of 1 mA, the voltage is 600 mV. Furthermore, with a current of 10 mA, the voltage is 700 mV. Find the value of *n* for this diode.

***P9.15. Current hogging.** The diodes shown in **Figure P9.15** are identical and have $n = 1$. For each diode, a forward current of 100 mA results in a voltage of 700 mV at a temperature of 300 K.

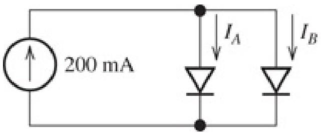



**Figure P9.15**

a. If both diodes are at 300 K, what are the values of $I_A$ and $I_B$?
b. If diode *A* is at 300 K and diode *B* is at 305 K, again find $I_A$ and $I_B,$ given that $I_s$ doubles in value for a 5-K increase in temperature. [*Hint:* Answer part (a) by use of symmetry. For part (b), a transcendental equation for the voltage across the diodes can be found. Solve by trial and error. An important observation to be made from this problem is that, starting at the same temperature, the diodes should theoretically each conduct half of the total current. However, if one diode conducts slightly more, it becomes warmer, resulting in even more current. Eventually, one of the diodes “hogs” most of the current. This is particularly noticeable for devices that are thermally isolated from one another with large currents, for which significant heating occurs.]

## Section 9.2: Load-Line Analysis of Diode Circuits

***P9.16.** The nonlinear circuit element shown in **Figure P9.16** has $i_x = [\exp(v_x) - 1]/10.$ Also, we have $V_s = 3$ V and $R_s = 1\ \Omega$. Use graphical load-line techniques to solve for $i_x$ and $v_x$. (You may prefer to use a computer program to plot the characteristic and the load line.)

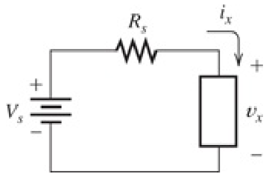
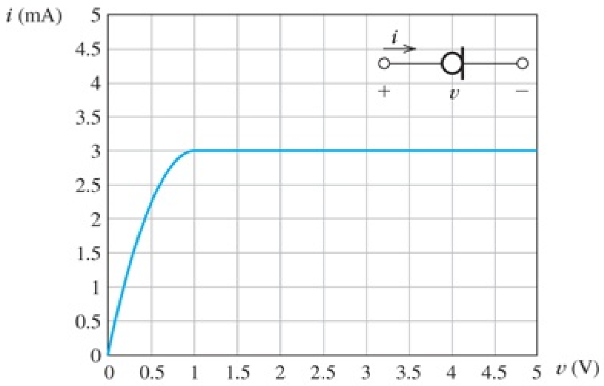


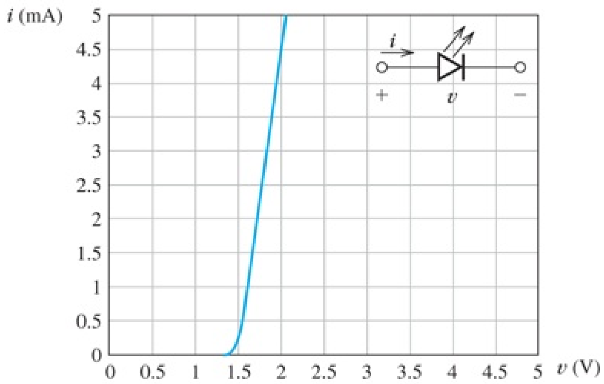

**Figure P9.16**

**P9.17.** Repeat **Problem P9.16** for $V_s = 20$ V, $R_s = 5$ k$\Omega$, and $i_x = 0.01/(1 - v_x/5)^3$ mA.
**P9.18.** Repeat **Problem P9.16** for $V_s = 6$ V, $R_s = 3\ \Omega$, and $i_x = v_x^3/8.$
**P9.19.** Repeat **Problem P9.16** for $V_s = 3$ V, $R_s = 1\ \Omega$, and $i_x = v_x + v_x^2$.
**P9.20.** Several types of special-purpose diodes exist. One is the constant-current diode for which the current is constant over a wide range of voltage. The circuit symbol and volt–ampere characteristic for a constant-current diode are shown in **Figure P9.20(a)**. Another special type is the light-emitting diode (LED) for which the circuit symbol and a typical volt–ampere characteristic are shown in **Figure P9.20(b)**. Sometimes, the series combination of these two devices is used to provide constant current to the LED from a variable voltage shown in **Figure P9.20(c)**.

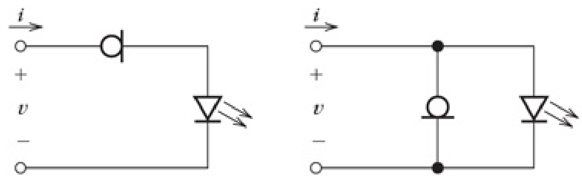
**b.** Sketch the overall volt–ampere characteristic to scale for the parallel combination shown in **Figure P9.20(d)**.




(a) Volt–ampere characteristic of a constant-current diode




(b) Volt–ampere characteristic of a light-emitting diode (LED).




(c) (d)

**Figure P9.20**

**P9.21.** Determine the values for *i* and *v* for the circuit of **Figure P9.21**. The diode is the LED having the characteristic shown in **Figure P9.20(b)**.

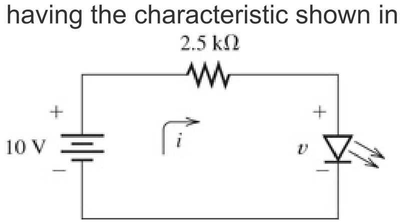



**Figure P9.21**

**P9.22.** Determine the values for $i_1$ and $i_2$ for the circuit of **Figure P9.22**. The device is the constant-current diode having the characteristic shown in **Figure P9.20(a)**.

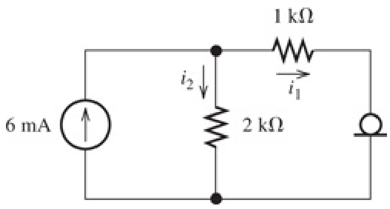



Figure P9.22

**P9.23.** Determine the values for *i* and *v* for the circuit of **Figure P9.23**. The diode is the LED having the characteristic shown in **Figure P9.20(b)**.

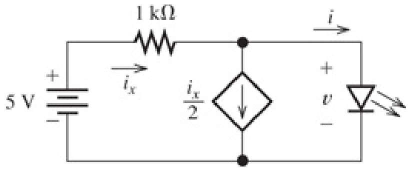



Figure P9.23

**P9.24.** Repeat **Problem P9.23** for the circuit of **Figure P9.24**.

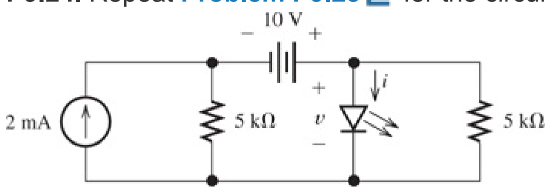



Figure P9.24

## Section 9.3: Zener-Diode Voltage-Regulator Circuits

**P9.25.** What is a Zener diode? For what is it typically used? Draw the volt–ampere characteristic of an ideal 5.8-V Zener diode.

***P9.26.** Draw the circuit diagram of a simple voltage regulator.

**P9.27.** Consider the Zener-diode regulator shown in **Figure 9.14** on page **470**. What is the minimum load resistance for which $v_o$ is 10 V?

**P9.28.** Consider the voltage regulator shown in **Figure P9.28**. The source voltage $V_s$ varies from 10 to 14 V, and the load current $i_L$ varies from 50 to 100 mA. Assume that the Zener diode is ideal. Determine the largest value allowed for the resistance $R_s$ so that the load voltage $v_L$ remains constant with variations in load current and source voltage. Determine the maximum power dissipation in $R_s$.

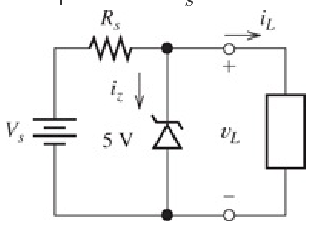



**Figure P9.28**

**P9.29.** Design a voltage-regulator circuit to provide a constant voltage of 5 V to a load from a variable supply voltage. The load current varies from 0 to 100 mA, and the source voltage varies from 8 to 10 V. You may assume that ideal Zener diodes are available. Resistors of any value may be specified. Draw the circuit diagram of your regulator, and specify the value of each component. Also, find the worst case (maximum) power dissipated in each component in your regulator. Try to use good judgment in your design.

**P9.30.** Repeat **Problem P9.29** if the supply voltage ranges from 6 to 10 V.

**P9.31.** Repeat **Problem P9.29** if the load current varies from 0 to 1 A.

**P9.32.** Outline a method for solving a circuit that contains a single nonlinear element plus resistors, dc voltage sources, and dc current sources, given the volt–ampere characteristic of the nonlinear device.

***P9.33.** A certain linear two-terminal circuit has terminals *a* and *b*. Under open-circuit conditions, we have $v_{ab} = 10\ \mathrm{V}$. A short circuit is connected across the terminals, and a current of 2 A flows from *a* to *b* through the short circuit. Determine the value of $v_{ab}$ when a nonlinear element that has $i_{ab} = \sqrt[3]{v_{ab}}$ is connected across the terminals.

## Section 9.4: Ideal-Diode Model

**P9.34.** What is an ideal diode? Draw its volt–ampere characteristic. After solving a circuit with ideal diodes, what check is necessary for diodes initially assumed to be on? Off?

**P9.35.** Two ideal diodes are placed in series, pointing in opposite directions. What is the equivalent circuit for the combination? What is the equivalent circuit if the diodes are in parallel and pointing in opposite directions?

**P9.36.** Find the values of *I* and *V* for the circuits of **Figure P9.36**, assuming that the diodes are ideal.

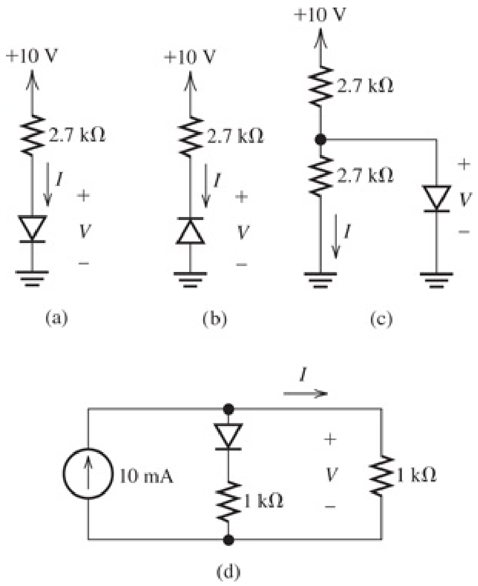



**Figure P9.36**

***P9.37.** Find the values of *I* and *V* for the circuits of **Figure P9.37**, assuming that the diodes are ideal.

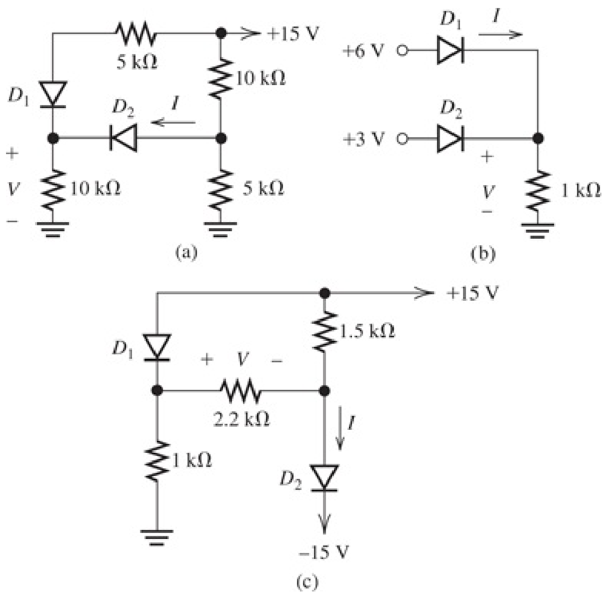



**Figure P9.37**

**P9.38.** Find the values of *I* and *V* for the circuits of **Figure P9.38**, assuming that the diodes are ideal. For part (b), consider $V_{\text{in}} = 0,\ 2,\ 6,$ and 10 V. Also, for part (b) of the figure, plot *V* versus $V_{\text{in}}$ for $V_{\text{in}}$ ranging from $-10\ \text{V}$ to 10 V.

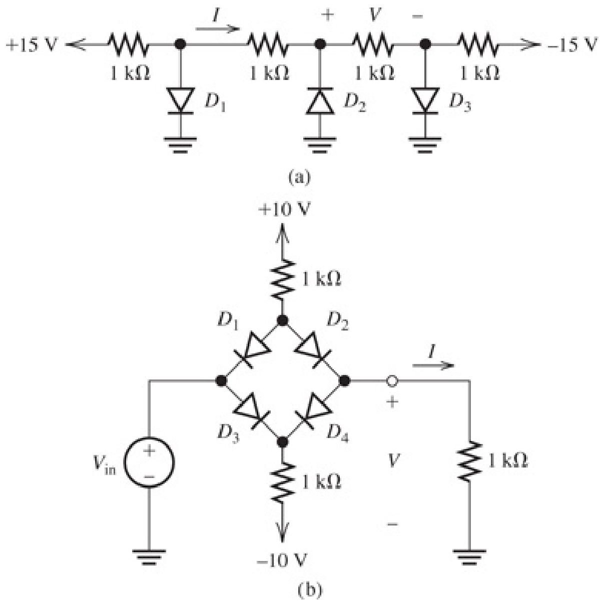



**Figure P9.38**

**P9.39.** Sketch *i* versus *v* to scale for each of the circuits shown in **Figure P9.39**. Assume that the diodes are ideal and allow *v* to range from $-10\text{ V}$ to $+10\text{ V}$.

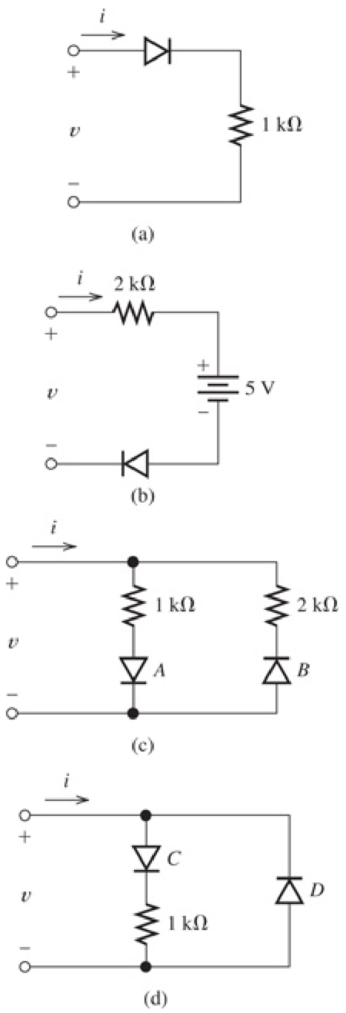



**Figure P9.39**

**P9.40.**

a. The circuit shown in **Figure P9.40(a)** is a type of logic gate. Assume that the diodes are ideal. The voltages $V_A$ and $V_B$ independently have values of either 0 V (for logic 0, or low) or 5 V (for logic 1, or high). For which of the four combinations of input voltages is the output high (i.e., $V_o = 5\ \mathrm{V}$)? What type of logic gate is this?

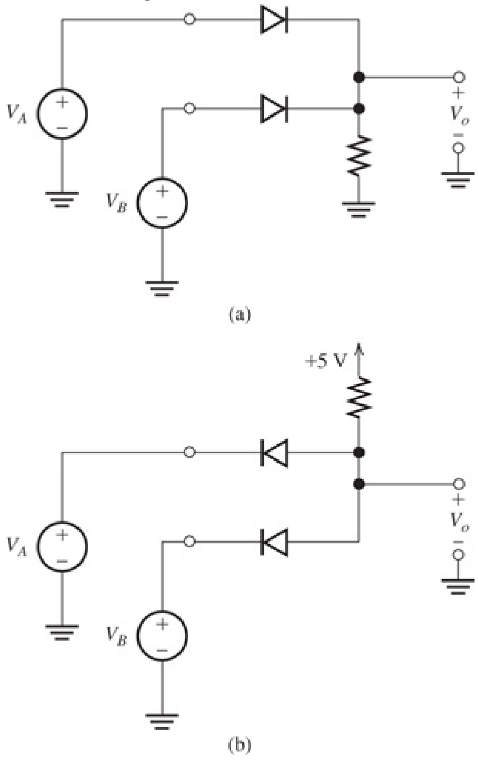



**Figure P9.40**

b. Repeat for the circuit of **Figure P9.40(b)**.

**P9.41.** Sketch $v_o(\ t)$ to scale versus time for the circuit shown in **Figure P9.41**. Assume that the diodes are ideal.

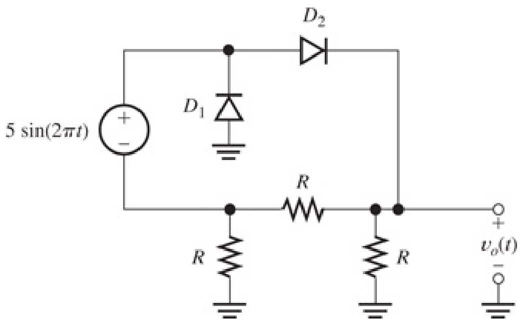



**Figure P9.41**

## Section 9.5: Piecewise-Linear Diode Models

**P9.42.** If a nonlinear two-terminal device is modeled by the piecewise-linear approach, what is the equivalent circuit of the device for each linear segment?

**P9.43.** A resistor $R_a$ is in series with a voltage source $V_a$. Draw the circuit. Label the voltage across the combination as *v* and the current as *i*. Draw and label the volt–ampere characteristic (*i* versus *v*).

**P9.44.** The volt–ampere characteristic of a certain two-terminal device is a straight line that passes through the points (2 V, 5 mA) and (3 V, 15 mA). The current reference points into the positive reference for the voltage. Determine the equivalent circuit for this device.

**P9.45.** Consider the volt–ampere characteristic of an ideal 10-V Zener diode shown in **Figure 9.14** on page **470**. Determine the piecewise-linear equivalent circuit for each segment of the characteristic.

***P9.46.** Assume that we have approximated a nonlinear volt–ampere characteristic by the straight-line segments shown in **Figure P9.46(c)**. Find the equivalent circuit for each segment. Use these equivalent circuits to find *v* in the circuits shown in **Figure P9.46(a)** and **(b)**.

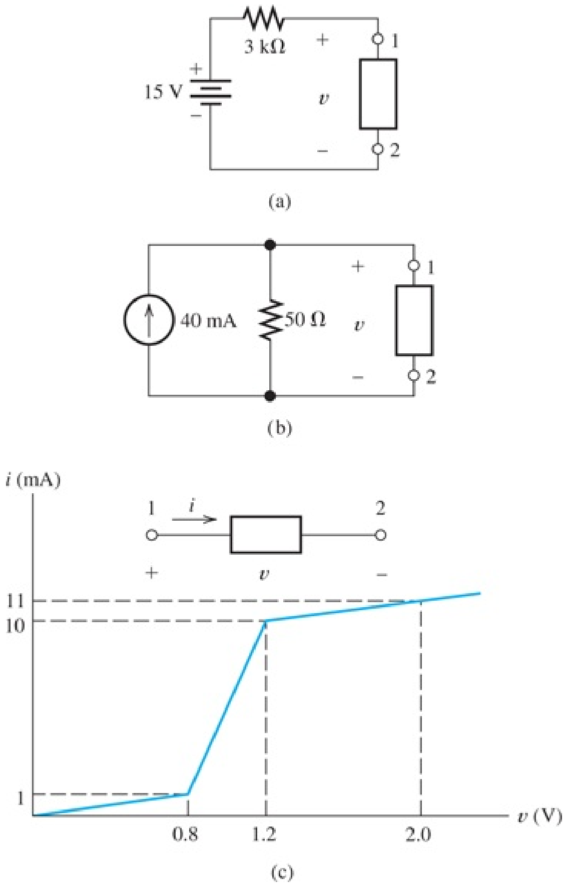



**Figure P9.46**

***P9.47.** The Zener diode shown in **Figure P9.47** has a piecewise-linear model shown in **Figure 9.19** on page **473**. Plot load voltage $v_L$ versus load current $i_L$ for $i_L$ ranging from 0 to 100 mA.

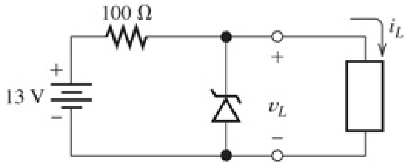



**Figure P9.47**

**P9.48.** The diode shown in **Figure P9.48** can be represented by the model of **Figure 9.23** on page **475**, with $V_f = 0.7$ V.

a. Assume that the diode operates as an open circuit and solve for the node voltages $v_1$ and $v_2$. Are the results consistent with the model? Why or why not?
b. Repeat part (a), assuming that the diode operates as a 0.7-V voltage source.

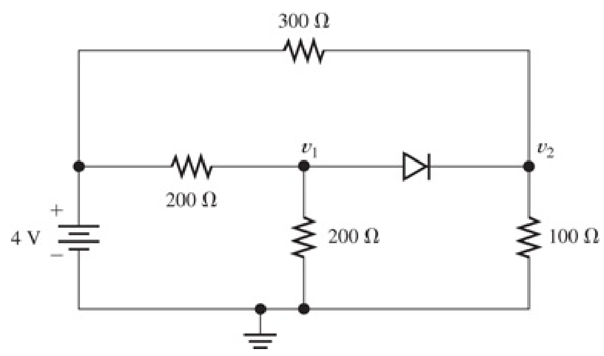



**Figure P9.48**

## Section 9.6: Rectifier Circuits

**P9.49.** Draw the circuit diagram of a half-wave rectifier for producing a nearly steady dc voltage from an ac source. Draw two different full-wave circuits.

**P9.50.** A 20-V-rms 60-Hz ac source is in series with an ideal diode and a $100\text{-}\ \Omega$ resistance. Determine the peak current and PIV for the diode.

**P9.51.** Consider the battery charging circuit shown in **Figure 9.25** on page **476**. The ac source has a peak value of 24 V and a frequency of 60 Hz. The resistance is $2\ \ \Omega$, the diode is ideal, and $V_B = 12\ \text{V}$. Determine the average current (i.e., the value of the charge that passes through the battery in 1 second). Suppose that the battery starts from a totally discharged state and has a capacity of 100 ampere hours. How long does it take to fully charge the battery?

**P9.52.** Consider the half-wave rectifier shown in **Figure 9.26** on page **477**. The ac source has an rms value of 20 V and a frequency of 60 Hz. The diodes are ideal, and the capacitance is very large, so the ripple voltage $V_r$ is very small. The load is a $100\text{-}\ \Omega$ resistance. Determine the PIV across the diode and the charge that passes through the diode per cycle.

**P9.53.** Most dc voltmeters produce a reading equal to the average value of the voltage measured. The mathematical definition of the average value of a periodic waveform is

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)dt$$

in which *T* is the period of the voltage *v*(*t*) applied to the meter.

a. What does a dc voltmeter read if the applied voltage is $v(\ t) \ = V_m \sin(\ \omega t)$ ?
b. What does the meter read if the applied voltage is a half-wave rectified version of the sinewave?
c. What does the meter read if the applied voltage is a full-wave rectified version of the sinewave?

***P9.54.** Design a half-wave rectifier power supply to deliver an average voltage of 9 V with a peak-to-peak ripple of 2 V to a load. The average load current is 100 mA. Assume that ideal diodes and 60-Hz ac voltage sources of any amplitudes needed are available. Draw the circuit diagram for your design. Specify the values of all components used.

**P9.55.** Repeat **Problem P9.54** with a full-wave bridge rectifier.

**P9.56.** Repeat **Problem P9.54** with two diodes and out-of-phase voltage sources to form a full-wave rectifier.

**P9.57.** Repeat **Problem P9.54**, assuming that the diodes have forward drops of 0.8 V.

***P9.58.** A half-wave rectifier is needed to supply 15-V dc to a load that draws an average current of 250 mA. The peak-to-peak ripple is required to be 0.2 V or less. What is the minimum value allowed for the smoothing capacitance? If a full-wave rectifier is needed?

**P9.59.** Consider the battery-charging circuit shown in **Figure 9.25** on page **476**, in which $v_s(\ t) \ = 20 \sin(\ 200\pi t)\ ,\ R = 80\ \ \Omega,\ \ V_B = 12\ \text{V},$ and the diode is ideal.

a. Sketch the current $i(t)$ to scale versus time.
b. Determine the average charging current for the battery.

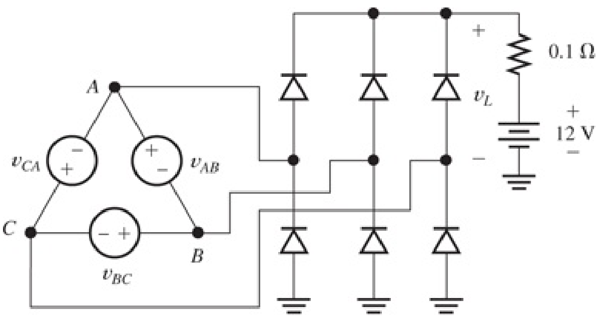
[*Hint:* The average current is the charge that flows through the battery in one cycle, divided by the period.]

**P9.60.**

a. Consider the full-wave rectifier shown in **Figure 9.27** on page **478**, with a large smoothing capacitance placed in parallel with the load $R_L$ and $V_m = 12\ \text{V}$. Assuming that the diodes are ideal, what is the approximate value of the load voltage? What PIV appears across the diodes?
b. Repeat for the full-wave bridge shown in **Figure 9.28** on page **479**.

**P9.61.** **Figure P9.61** shows the equivalent circuit for a typical automotive battery charging system. The three-phase delta-connected source represents the stator coils of the alternator. (Three-phase ac sources are discussed in **Section 5.7**. Actually, the alternator stator is usually wye connected, but the terminal voltages are the same as for the equivalent delta.) Not shown in the figure is a voltage regulator that controls the current applied to the rotor coil of the alternator and, consequently, $V_m$ and the charging current to the battery.

a. Sketch the load voltage $v_L(t)$ to scale versus time. Assume ideal diodes and that $V_m$ is large enough that current flows into the battery at all times. [*Hint:* Each source and four of the diodes form a full-wave bridge rectifier.]
b. Determine the peak-to-peak ripple and the average load voltage in terms of $V_m$.
c. Determine the value of $V_m$ needed to provide an average charging current of 30 A.
d. What additional factors would need to be considered in a realistic computation of $V_m$?




$v_{AB} = V_m \cos(\omega t)$

$v_{BC} = V_m \cos(\omega t - 120°)$

$v_{CA} = V_m \cos(\omega t + 120°)$

**Figure P9.61**

Idealized model of an automotive battery-charging system.

## Section 9.7: Wave-Shaping Circuits

**P9.62.** What is a clipper circuit? Draw an example circuit diagram, including component values, an input waveform, and the corresponding output waveform.

**P9.63.** Sketch to scale the output waveform for the circuit shown in **Figure P9.63**. Assume that the diodes are ideal.

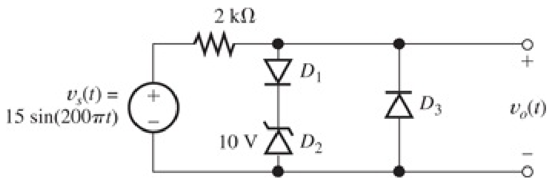



**Figure P9.63**

**P9.64.** Sketch the transfer characteristic ( $v_o$ versus $v_{\text{in}}$) to scale for the circuit shown in **Figure P9.64**. Assume that the diode is ideal.

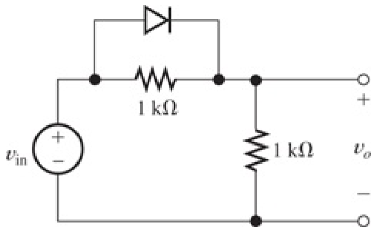



**Figure P9.64**

**P9.65.** Sketch the transfer characteristic ( $v_o$ versus $v_{\text{in}}$) to scale for the circuit shown in **Figure P9.65**. Assume that the diodes are ideal.

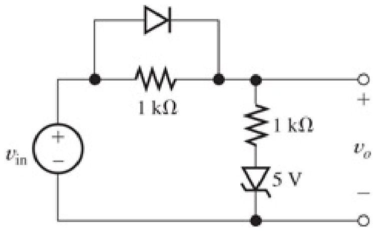



**Figure P9.65**

**P9.66.** Sketch the transfer characteristic ( $v_o$ versus $v_{\text{in}}$) to scale for the circuit shown in **Figure P9.66**. Allow $v_{\text{in}}$ to range from $-5\text{ V}$ to $+5\text{ V}$ and assume that the diodes are ideal.

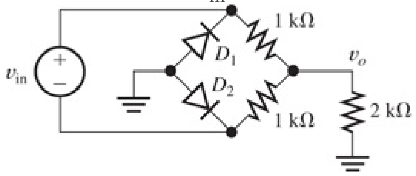



**Figure P9.66**

**P9.67.** Sketch the transfer characteristic ( $v_o$ versus $v_{\text{in}}$) for the circuit shown in **Figure P9.67**, carefully labeling the breakpoint and slopes. Allow $v_{\text{in}}$ to range from $-5\text{ V}$ to $+5\text{ V}$ and assume that the diodes are ideal.

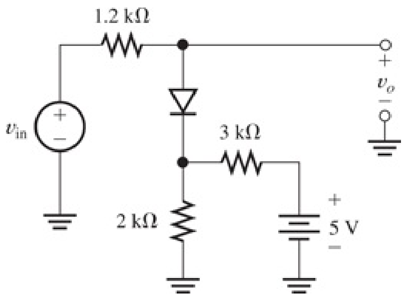



**Figure P9.67**

**P9.68.** What is a clamp circuit? Draw an example circuit diagram, including component values, an input waveform, and the corresponding output waveform.

**P9.69.** Consider the circuit shown in **Figure P9.69**, in which the *RC* time constant is very long compared with the period of the input and in which the diode is ideal. Sketch $v_o(t)$ to scale versus time.

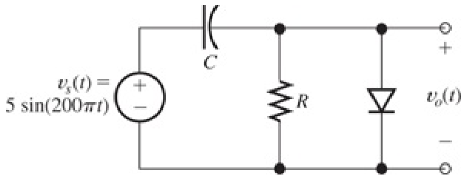



**Figure P9.69**

***P9.70.** Sketch to scale the steady-state output waveform for the circuit shown in **Figure P9.70**. Assume that *RC* is much larger than the period of the input voltage and that the diodes are ideal.

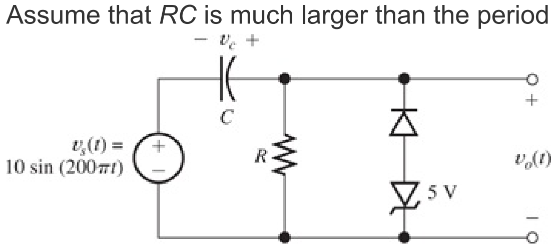



**Figure P9.70**

**P9.71. Voltage-doubler circuit.** Consider the circuit of **Figure P9.71**. The capacitors are very large, so they discharge only a very small amount per cycle. (Thus, no ac voltage appears across the capacitors, and the ac input plus the dc voltage of $C_1$ must appear at point *A*.) Sketch the voltage at point *A* versus time. Find the voltage across the load. Why is this called a voltage doubler? What is the PIV across each diode?

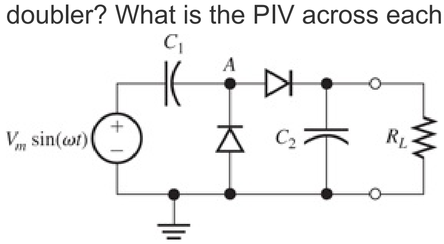



**Figure P9.71**

***P9.72.** Design a clipper circuit to clip off the portions of an input voltage that fall above 3 V or below $-5$ V. Assume that diodes having a constant forward drop of 0.7 V are available. Ideal Zener diodes of any breakdown voltage required are available. Dc voltage sources of any value needed are available.

**P9.73.** Repeat **Problem P9.72**, with clipping levels of $+2\ \mathrm{V}$ and $+5\ \mathrm{V}$ (i.e., every part of the input waveform below $+2$ or above $+5$ is clipped off).

**P9.74.** Design circuits that have the transfer characteristics shown in **Figure P9.74**. Assume that $v_{\mathrm{in}}$ ranges from $-10$ to $+10\ \mathrm{V}$. Use diodes, Zener diodes, and resistors of any values needed. Assume a 0.6-V forward drop for all diodes and that the Zener diodes have an ideal characteristic in the breakdown region. Power-supply voltages of $\pm 15\ \mathrm{V}$ are available.

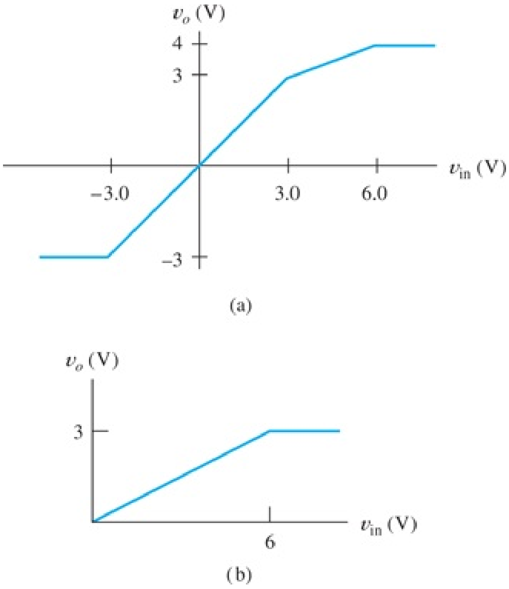



**Figure P9.74**

***P9.75.** Design a clamp circuit to clamp the negative extreme of a periodic input waveform to $-5\ \mathrm{V}$. Use diodes, Zener diodes, and resistors of any values required. Assume a 0.6-V forward drop for all diodes and that the Zener diodes have an ideal characteristic in the breakdown region. Power-supply voltages of $\pm 15\ \mathrm{V}$ are available.

**P9.76.** Repeat **Problem P9.75** for a clamp voltage of $+5\ \mathrm{V}$.

## Section 9.8: Linear Small-Signal Equivalent Circuits

**P9.77.** A certain diode has $I_{DQ} = 4\ \mathrm{mA}$ and $i_d(t) = 0.5\cos(200\pi t)\ \mathrm{mA}$. Find an expression for $i_D(t)$, and sketch it to scale versus time.

**P9.78.** Of what does the small-signal equivalent circuit of a diode consist? How is the dynamic resistance of a nonlinear circuit element determined at a given operating point?

**P9.79.** With what are dc voltage sources replaced in a small-signal ac equivalent circuit? Why?

**P9.80.** With what should we replace a dc current source in a small-signal ac equivalent circuit? Justify your answer.

***P9.81.** A certain nonlinear device has $i_D = v_D^3/8$. Sketch $i_D$ versus $v_D$ to scale for $v_D$ ranging from $-2\ \mathrm{V}$ to $+2\ \mathrm{V}$. Is this device a diode? Determine the dynamic resistance of the device and sketch it versus $v_D$ to scale for $v_D$ ranging from $-2\ \mathrm{V}$ to $+2\ \mathrm{V}$.

**P9.82.** A breakdown diode has

$$i_D = \frac{-10^{-6}}{(1 + v_D/5)^3} \qquad \text{for } -5\ \mathrm{V} < v_D < 0$$

where $i_D$ is in amperes. Plot $i_D$ versus $v_D$ in the reverse-bias region. Find the dynamic resistance of this diode at $I_{DQ} = -1\ \mathrm{mA}$ and at $I_{DQ} = -10\ \mathrm{mA}$.

**P9.83.** A certain nonlinear device is operating with an applied voltage given by

$$v_D(t) = 5 + 0.01\cos(\omega t)\ \mathrm{V}$$

The current is given by

$$i_D(t) = 3 + 0.2\cos(\omega t)\ \mathrm{mA}$$

Determine the dynamic resistance and *Q* point of the device under the conditions given.

**P9.84.** Ideally, we want the voltage for a Zener diode to be constant in the breakdown region. What does this imply about the dynamic resistance in the breakdown region for an ideal Zener diode?

***P9.85.** Consider the voltage-regulator circuit shown in **Figure P9.85**. The ac ripple voltage is 1 V peak to peak. The dc (average) load voltage is 5 V. What is the *Q*-point current in the Zener diode? What is the maximum dynamic resistance allowed for the Zener diode if the output ripple is to be less than 10 mV peak to peak?

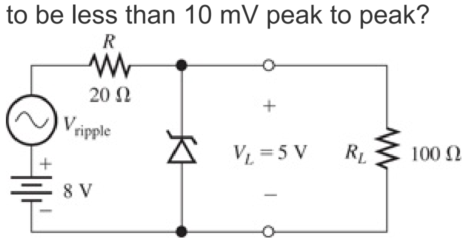



**Figure P9.85**

## Practice Test

Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in **Appendix D** and complete solutions are included in the Student Solutions files. See **Appendix E** for more information about the Student Solutions.

**T9.1.** Determine the value of $i_D$ for each of the circuits shown in **Figure T9.1**. The characteristic for the diode is shown in **Figure 9.8** on page **465**.

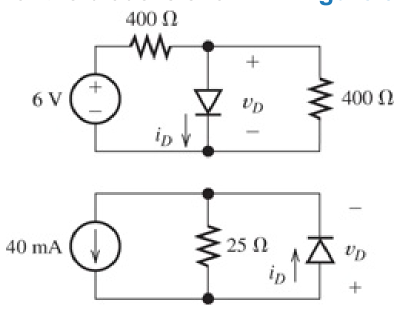



**Figure T9.1**

**T9.2.** The diode shown in **Figure T9.2** is ideal. Determine the state of the diode and the values of $v_x$ and $i_x$.

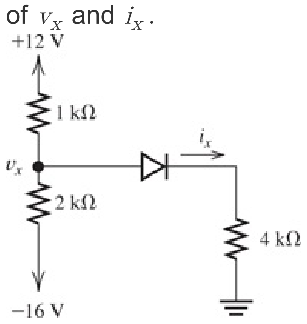



**Figure T9.2**

**T9.3.** The current versus voltage characteristic of a certain two-terminal device passes through the points (5 V, 2 mA) and (10 V, 7 mA). The reference for the current points into the positive reference for the voltage. Determine the values for the resistance and voltage source for the piecewise linear equivalent circuit for this device between the two points given.

**T9.4.** Draw the circuit diagram for a full-wave bridge rectifier with a resistance as the load.

**T9.5.** Suppose we have a 10-V-peak sinusoidal voltage source. Draw the diagram of a circuit that clips off the part of the sinusoid above 5 V and below $-4\ \mathrm{V}$. The circuit should be composed of ideal diodes, dc voltage sources, and other components as needed. Be sure to label the terminals across which the clipped output waveform $v_o(t)$ appears.

**T9.6.** Suppose we have a 10-Hz sinusoidal voltage source, $v_{\text{in}}(t)$. Draw the diagram of a circuit that clamps the positive peaks to $-4\ \mathrm{V}$. The circuit should be composed of ideal diodes, dc voltage sources, and other components as needed. List any constraints that should be observed in selecting component values. Be sure to label the terminals across which the clamped output waveform $v_o(t)$ appears.

**T9.7.** Suppose we have a silicon diode operating with a bias current of 5 mA at a temperature of 300 K. The diode current is given by the Shockley equation with $n = 2$. Draw the small-signal equivalent circuit for the diode including numerical values for the components.