Optimal PMU-Communication Link Placement for Smart Grid Wide-Area Measurement Systems

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***Abstract*—Due to the relatively high costs of phasor measurement units (PMUs), the optimal PMU placement problem of minimizing the number of PMUs for full system observability has long been a popular research topic. However, PMUs alone will not make a system observable. An appropriate communication network that transmits all PMU data to the phasor data concentrator plays a key role. This paper proposes the optimal PMU-communication link placement (OPLP) problem that investigates the placement of PMU and communication links (CLs) for full observability in a power system. In addition to the location of PMUs and CLs, to ensure the reliable and timely transmission of PMU data, the communication capacity needed on every CL is captured by the OPLP problem. We carry out numerical studies on the IEEE 30-bus, 57-bus, 118-bus, and 300-bus systems for the proposed models. The results reveal that, when compared to the traditional optimal PMU placement model, OPLP can reduce the total installation cost significantly.**

***Index Terms*—Optimal PMU placement, communication link, bandwidth assignment.**

I. INTRODUCTION

WIDE area measurement system (WAMS) involves the use of system-wide information and the communication of selected local information to a remote data center [1]. Synchronized measurement technology is an important enabler of WAMS. By providing synchronized real-time information, including voltage and current phasors, the phasor measurement units (PMUs) are important parts of WAMS [2]. The optimal PMU placement (OPP) problem, which investigates optimal installation strategies that make the entire system observable with the minimum number of PMUs, has been extensively studied in [3]–[10].

However, PMUs alone will not make a power system observable. An appropriate communication infrastructure must also be established to support timely data transmission between PMUs and the phasor data concentrator (PDC). The topological design of the smart grid communication networks is becoming important. Not only do the applications in smart grid require communication to be real-time, reliable, scalable and manageable, but they also require communications to be interoperable, secure, future-proof, and cost-effective [11]. Although there are no industrial standards, the communication infrastructure in smart power system is envisioned as a collection of interconnected networks that will be structured into a hierarchy of at least three main tiers: local area networks (LAN), field area networks (FAN), and wide area networks (WAN) [13]. In order to fulfill the complicated transmission requirements of different applications, a variety of communication technologies has been adopted in the design of the communication network in smart grid. Early communication technologies, like power line carrier and microwave communication, have limitations in reliability, scalability and robustness [14]. However, the adoption of optical fiber communication in power systems renders the end-to-end data transmission latency low enough to meet the communication needs [13]. Thus, in this work, we investigate the use of optical fibers in smart grid.

In many existing works, such as [15] and [16], communication network availability is considered as a constraint in choosing the optimal places to install PMUs. In other words, these proposals consider a bus as a candidate to install a PMU only if it has a communication link (CL) connecting to the PDC. The OPP problem assumes the existence of a communication network with relatively good coverage and sufficient bandwidth. If this assumption does not hold, new CLs must be installed. Relatively few studies, such as [17]–[20], considered the placement of CLs jointly with the PMUs. The co-optimal placement problem of PMU and their communication infrastructure was formulated and solved by Imperialistic Competition Algorithm (ICA) and Genetic Algorithm (GA) in [17] and [20], respectively. Rather *et al.* [18] took the hidden cost into consideration, and, jointly solved the placement problem of PMU and ancillary facilities, including communication facilities by Particle Swarm Optimization (PSO). Huang *et al.* [19] proposed a solution to place a minimum number of PMUs and CLs in the power system so that the steady-state availability of synchrophasor data at each bus meets a prescribed level. Pal *et al.* [21] presented an integer linear programming methodology for the PMU placement problem while considering the communication infrastructure upgrading cost. However, these studies did not take the PMU data transmission requirements into consideration. They all assumed that the network capacity is sufficient. However, network capacity can be a very expensive resource in large-scale power systems. Appropriate allocation of the

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transmission capacity of every CL is very important in the design of WAMS.

The main objective of this paper is to extend the traditional OPP problem to the optimal PMU-CL placement (OPLP) problem to investigate the optimal placement of PMUs and CLs jointly. Both the effect of zero-injection buses (ZIBs) and PMU measurement capacity limitation are considered in the OPLP problem. Furthermore, this paper investigates the assignment of link capacity of every CL to fulfill the communication requirements. A linear CL cost model and the shortest path routing scheme are used. We fit the OPLP problem into the mixed integer quadratically constrained programming (MIQCP) framework, which can be solved efficiently using commercial numerical solvers. We also provide numerical analysis for OPLP on the IEEE 30-bus, 57-bus, 118-bus, and 300-bus systems. From the numerical results, we have the following particular findings:

- The optimal placement of CL relies on the installation of PMUs and the selection of data routing paths. The different choices on transmission paths for PMU data can affect the CL cost. Specifically, the cost is additionally reduced by the optimization of data routing path.
- Since the bandwidth cost constitutes a significant proportion of the total system installation cost, the shortest CL does not necessarily indicate minimum total cost. In some cases, the CL will be located at the branch which is not the shortest one for reducing the overall bandwidth cost.
- Since the existence of ZIBs reduces the number of buses that are needed to be measured, the number of PMUs for full observability and the total data traffic in the WAMS reduce. Furthermore, ZIBs would lessen the optimal number of PMUs as well as the cost on CLs.

The rest of the paper is organized as follows. We first model WAMS in Section II. Then, we show the OPLP model and give some brief extensions of the proposed model in Section III, followed by the MIQCP formulation of OPLP. In Section IV, we carry out numerical study and provide numerical results before we conclude in Section V.

II. Wide Area Monitoring System (WAMS) Model

In this section, the basic model of WAMS consisting of three major components, PMUs, a PDC and a communication network, is introduced. All PMUs can be placed on the buses, and CLs can be placed on the branches that connect the buses. In this way, the communication network topology overlaps with the power network topology. As summarized in [11], the overlapping of networks can reduce the maintenance cost as well as the construction fees of WAMS. Besides, the coincidence of power network and the communication network would allow the smart grid to support multiple applications [12], e.g., wide area measurement, substation automation, etc. Also, the implementations of some communication technologies, such as power-line communications that carry data on power lines, require the overlapping of power lines and communication lines. Therefore, the assumption that only communication lines can be built between two adjacent buses is widely accepted in the optimal PMU and CL placement problem [19]. In the overlapped system, the PMU data may not be transmitted to the PDC via a direct CL. Thus, as a key component of the multi-hop data transmission, the routing scheme of the PMU data will be introduced in this section.

We consider an N-bus power system and denote $\mathcal{N} = \{1, 2, \ldots, N\}$ as the set of buses. We use $\boldsymbol{M}$ to represent the connectivity matrix. Its (i, j) entry, $m_{i,j}$, satisfies:

$$m_{i,j} = \begin{cases} 1 & \text{if bus } i \text{ and bus } j \text{ are connected} \\ 1 & \text{if } \; i = j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_N]$ is defined as the PMU installation vector, where

$$\mu_i = \begin{cases} 1 & \text{if PMU is installed on bus } i \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

In this work, we assume that there is one PDC in the WAMS and the location of the PDC is fixed. In order to transmit the PMU data reliably and in a timely fashion, sufficient link capacities should be assigned to these CLs. In this paper, we use the term bandwidth inter-changeably with channel capacity. We use matrix $\underline{\boldsymbol{B}}$ to denote the power system bandwidth requirement, with its element $\underline{b}_{i,j}$ indicating the bandwidth requirement on the CL placed at branch i-j. In general, $\underline{b}_{i,j} \geq 0$. Moreover, $\underline{b}_{i,j} = 0$ indicates that there is no data transmitted via branch i-j, which means that there is no CL placed at branch i-j. Furthermore, the requirements on the CL capacity depend on the PMU data traffic and the data routing scheme. In the following, we introduce the WAMS traffic model and data routing scheme in detail.

A. WAMS Traffic Model

In WAMS, the PMUs generate phasor data to make the power system observable. However, the amount of data traffic generated by different PMUs varies. In [23], the bandwidth requirements of different smart grid applications are studied. In most cases, 100 Kbps per node are required. For the wide area situational awareness, the bandwidth requirement ranges from 600 Kbps to 1500 Kbps. According to [24], the required bandwidth for synchrophasor data transfer is dedicated based on the reporting rate and the message size. In this section, we introduce the WMAS traffic model based on the linear traffic model proposed in [25]. The amount of data generated by PMU i per second can be formulated as a linear function of the number of buses that PMU i measured as follows,

$$d_i = (W_i p + \alpha) \cdot F_s \tag{3}$$

where W_i is the number of buses that PMU i measures (including bus i), p is the size of the data portion in a single phasor data frame, α is the size of the frame overhead generated by this PMU, and F_s is the configured phasor data frame reporting frequency. In fact, α is very small compared with the size of the phasor data, and α can be dropped from D_i, resulting in,

$$d_i = W_i \cdot d \tag{4}$$

where $d = pF_s$. We assume that the data size and measurement frequency for all PMUs are the same. Thus, d is a positive constant in this paper.

B. Data Routing Scheme

For a certain data routing scheme, one or several paths connecting a PMU and the PDC will be adopted. Furthermore, the bandwidth requirement on every CL in the adopted path is determined based on the data traffic of all PMUs in the system. Thus, when the data routing paths are determined, the bandwidth requirement matrix $\underline{\boldsymbol{B}}$ is fixed for a certain placement of PMU. Similarly, the choice of data routing paths influences the bandwidth requirements even for a fixed placement of PMUs.

Let $\mathcal{P}_i$, $i \in \mathcal{N}$, be the set of all simple paths between bus i and the PDC. We denote the k_{th} element of $\mathcal{P}_i$ by an $N \times N$ path matrix, $\boldsymbol{P}_i^k$. Its (j,t) entry, where $j, t \in \mathcal{N}$, is given by:

$$p_{j,t}^{i,k} = \begin{cases} 1 & \text{if } \boldsymbol{P}_i^k \text{ has branch } j\text{-}t \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The number of hops of the routing path $\boldsymbol{P}_i^k$ is denoted by $|\boldsymbol{P}_i^k|$. Given that there may be multiple paths with the same number of hops, we constrain the elements of $\mathcal{P}_i$ by the following rule,

$$|\boldsymbol{P}_i^k| \leq |\boldsymbol{P}_i^{k+1}|, \quad \forall k \in \mathcal{N} \tag{6}$$

For simplicity, we assume the shortest path routing scheme is adopted in this work. The shortest path means the path with the minimum number of hops instead of the smallest link length. We assume that the PMU data is routed over the shortest path without traffic splitting. We use $\tilde{\mathcal{P}}_i$ to represent the set of all shortest paths from bus i to the PDC, and only the shortest path can be chosen as the data routing path. Denote the number of shortest paths connecting bus i to the PDC as C_i. Thus, $\tilde{\mathcal{P}}_i = \{\boldsymbol{P}_i^1, \ldots, \boldsymbol{P}_i^{C_i}\}$.

The shortest paths, P_i^k, $\forall k \leq C_i$, can be calculated by algorithms, such as Dijkstra Algorithm. We assume that the matrix $\tilde{\mathcal{P}}_i$ and the value of C_i are known for all i. We define a set of path selection variables, $\boldsymbol{\Gamma}_i = (\Gamma_i^1, \Gamma_i^2 \ldots, \Gamma_i^{C_i})$, where $i \in \mathcal{N}$, to indicate which path(s) will be adopted by the PMU installed on bus i. $\Gamma_i^k = 1$ if the k^{th} path connecting bus i and the PDC is used, and $\Gamma_i^k = 0$ otherwise. Assume that a PMU installed on bus i requires h_i paths to reliably communicate with the PDC, $h_i \geq 1$. $h_i \in \mathbb{Z}$, we must have:

$$\sum_{k=1}^{C_i} \Gamma_i^k = h_i \mu_i \tag{7}$$

For simplicity, we set $h_i = 1$ in this paper.

For the PMU located on bus i, it generates d_i amount of data per second. In order to ensure timely transmission, the chosen simple paths from bus i to the PDC should satisfy the communication bandwidth requirement, d_i. In other words, the bandwidth of all CLs belonging to the chosen paths should be at least d_i.

For the transmission of all PMU data, a CL may belong to multiple data transmission paths. We assume that the time-division multiple access (TDMA) scheme is adopted to avoid the collision of the data transmission of numerous PMUs over a CL. In order to fulfill the transmission requirements of all PMUs in the system, the system bandwidth matrix $\underline{\boldsymbol{B}}$ should be the sum of all PMU requirements. Therefore, $\underline{\boldsymbol{B}}$ can be calculated as:

$$\underline{\boldsymbol{B}} = \sum_{i=1}^{N} d_i \mu_i \cdot \left(\sum_{k=1}^{C_i} \Gamma_i^k \boldsymbol{P}_i^k \right) \tag{8}$$

which is a quadratic function of $\boldsymbol{\mu}$ and $\boldsymbol{\Gamma_i}$.

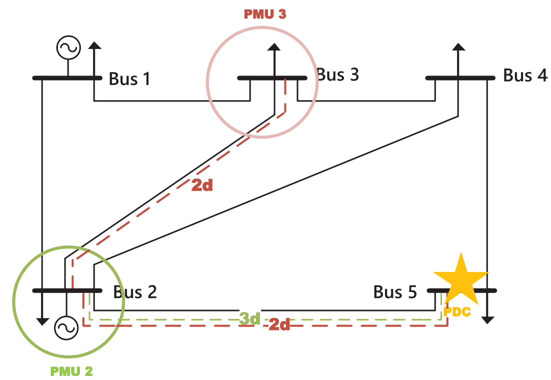

Fig. 1. Standard IEEE 5-Bus System [26].

We take the standard IEEE 5-bus system [26], as shown in Fig. 1, as an example to explain the concepts mentioned above. Suppose that two PMUs are located at bus 2 and bus 3 to make the system observable. The PDC is located at bus 5. The PMU placement here may not be the optimal one, and we just take it as an example to explain the relationship between the system bandwidth requirement and the routing path selection. Then, $\mathcal{P}_2 = \{$bus 2 $\rightarrow$ bus 5, bus 2 $\rightarrow$ bus 4 $\rightarrow$ bus 5, bus 2 $\rightarrow$ bus 3 $\rightarrow$ bus4 $\rightarrow$ bus 5, bus 2 $\rightarrow$ bus 1 $\rightarrow$ bus 3 $\rightarrow$ bus4 $\rightarrow$ bus 5$\}$ represents all the simple paths from bus 2 to the PDC. As defined in (5), the four elements in $\mathcal{P}_2$ can be represented by four 5×5 path matrices correspondingly. Here, we take the third element of $\mathcal{P}_2$, i.e., P_2^3, as an example. The corresponding path matrix is shown as below:

$$\boldsymbol{P}_2^3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{9}$$

Suppose that there are three buses, including bus 1, 2 and 5, assigned to PMU 2 to measure. According to (4), $d_2 = 3d$ holds. Since we adopt the shortest path routing scheme in this work, P_2^1 is selected as the data path and it should fulfill the bandwidth requirement. As the green slash line shown in Fig. 1, Γ_2^1 is set as one and the bandwidth of the CL at branch 2-5 should be at least $3d$.

The use of PMU 2 alone cannot yield the full observability. Another PMU is thus placed at bus 3 for observing buses 3 and 4. The set of shortest paths from bus 3 to the PDC is represented by $\tilde{\mathcal{P}}_3 = \{$bus 3 $\rightarrow$ bus 2 $\rightarrow$ bus 5, bus 3 $\rightarrow$ bus 4 $\rightarrow$ bus 5$\}$. The selections between two shortest path result in two different bandwidth required by PMU 3. If $\Gamma_3^1 =$ 1 and PMU 3 is assigned to measure the phasor data at buses 3 and 4, the bandwidth requirements of PMU 3 on CLs located on branches 3-2 and 2-5 are $2d$. Moreover, both PMUs 2 and 3 transmit data via the CL on branch 2-5. The bandwidth requirement on CL 2-5 should be $5d$. Hence, by summing the bandwidth requirements of PMU 2 and PMU 3, we can obtain the system bandwidth matrix $\underline{\boldsymbol{B}}$ as follows:

$$\underline{\boldsymbol{B}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2d & 0 & 5d \\ 0 & 2d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 5d & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Similarly, if $\Gamma_3^2 = 1$, we can calculate $\underline{\boldsymbol{B}}$ in a similar way. We thus omit the calculation here. As shown above, we can see how the data routing selection influences the system bandwidth requirement which affects the installation cost of CL directly. Thus, the data routing selection can have a significant impact on the optimal PMU-CL placement.

C. CL Construction Cost Model

In this section, we introduce a general construction cost model of CLs. The cost of communication infrastructure is calculated as the sum of two components, passive cost and active cost [20]. Active devices of a wired communication network include switches and routers, whose costs depend mainly on the communication capacity provided [17]. Here we formulate the cost of active devices as a linear function of bandwidth. On the other hand, the passive devices include optical fiber cables and fiber path panel [17]. Besides the devices cost, the passive cost includes the labour cost, e.g., installation and procurement cost of the fiber, as well. Thus, the passive cost mainly depends on the length of CL. In summary, the construction cost of a CL is calculated based on its bandwidth and length as shown in (11).

$$Cost_{comm} = E_b \cdot Bandwidth + E_l \cdot Length \tag{11}$$

where E_b is the bandwidth price and E_l is passive CL price per kilometer.

More generally, there may exist some CLs in the power system before we design the placement of CLs. For branch *i-j*, if there is an existing CL, $\alpha_{i,j} = 1$ and its preassigned bandwidth is denoted as $q_{i,j}$, otherwise, $\alpha_{i,j} = 0$. Denote the cost of building a CL placed on branch *i-j* with designed bandwidth $b_{i,j}$ and link length $d_{i,j}$ as the function $f(b_{i,j}, d_{i,j})$. Then, we have $f(b_{i,j}, d_{i,j})$ as shown in (12).

$$f\big(b_{i,j}, d_{i,j}\big) = \begin{cases} E_b b_{i,j} + E_l d_{i,j} & \text{if } b_{i,j} > 0 \text{ and } \alpha_{i,j} = 0 \\ E_b\big(b_{i,j} - q_{i,j}\big) & \text{if } b_{i,j} > q_{i,j} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

III. Optimal Joint PMU-CL Placement (OPLP) Problem

In this section, we present the OPLP problem that models how to optimally install PMUs jointly with the CLs in the proposed power system. We will first give the basic formulation of the OPLP problem. Then, we will extend the OPLP problem to make it more general.

OPLP can be modeled as:

$$\text{Minimize } \ K\sum_{j=1}^{N}\mu_j + \sum_{i=1}^{N}\sum_{j>i}^{N} f\big(b_{i,j}, d_{i,j}\big) \tag{13}$$

$$\text{Subject to } \ \boldsymbol{\mu M} \geq \boldsymbol{k} \tag{14}$$

$$b_{i,j} \geq \underline{b}_{i,j} \ \ \forall i, j \in \mathcal{N}, \tag{15}$$

where K is the installation cost of a single PMU, including the PMU device cost and other labour costs, $\boldsymbol{k} = [k, k, \ldots, k]$. Equation (14) ensures that the system is fully observable. When $k = 1$, (14) models the case without consideration of any equipment outage. Here we consider a more general formulation ($k \geq 1$) which takes PMU outage into consideration. For example, when $k = 2$, the constraint in (14) means that each bus should be measured by at least two PMUs, that is $(N - 1)$ redundancy [16]. For simplicity, we do not consider the outage of branch or CL in this work. When an equipment outage occurs, we assume here that the loss of observability can be resolved by referring the relevant historical data about the captioned equipment [27].

The following discussions in Sections III-A and III-B will expand the above formulation by incorporating specific characteristics of power networks, including ZIBs and PMU measurement limitations. These extensions can be taken into consideration simultaneously or separately. In Section III-C, we give some instructions on how to solve OPLP collaborating with the effect of ZIBs and PMU measurement limitations.

A. Effect of ZIBs

ZIBs are those buses without current injection. Therefore, ZIBs correspond to the transshipment nodes in the system, which brings the following new rule for assessing the observability. Among a ZIB and its adjacent buses, a single bus can be made observable by making the others observable [15]. Thus, for a ZIB, the only one "credit of observability" can be assigned to itself or its adjacent buses. We use $y_{i,j} \in \{0, 1\}$ to indicate the assignment of the "credit" from bus j to bus i. When $y_{i,j} = 1$, the "credit" provided by bus j is assigned to bus i, and $y_{i,j} = 0$, otherwise.

To account for ZIB in the OPLP model, we replace (14) by the following constraint:

$$g_i \geq k, \quad \forall j \in \mathcal{N} \tag{16}$$

where

$$g_i = \sum_{j\in\mathcal{N}} m_{i,j}\mu_j + \sum_{j\in\mathcal{N}} m_{i,j}z_j y_{i,j}, \quad \forall i \in \mathcal{N} \tag{17}$$

$$\sum_{i\in\mathcal{N}} m_{i,j}y_{i,j} = z_j, \quad \forall j \in \mathcal{N} \tag{18}$$

where z_j equals one when bus j is ZIB, and zero, otherwise. As shown in (18), if z_j equals one, bus j has a total number of one "credit", which can be assigned to itself or its adjacent buses. For bus i, its observability can be calculated as shown in (17), the sum of the measurements provided by PMUs and the "credit".

B. PMU Measurement Limitations

Usually, a PMU with multiple measurement channels is installed at the bus with several neighboring buses. There exist an upper bound on how many buses one PMU can measure, and this upper bound is called the measurement capacity limitation of this PMU. We consider the effect of PMU measurement capacity limitations on the PMU placement problem by substituting $\sum_{j\in\mathcal{N}} m_{i,j}\mu_j$ with $\sum_{j\in\mathcal{N}} m_{i,j}\omega_{i,j}\mu_j$ in (16). Thus,

$$g_i = \sum_{j\in\mathcal{N}} m_{i,j}\omega_{i,j}\mu_j + \sum_{j\in\mathcal{N}} m_{i,j}z_j y_{i,j}, \quad \forall i \in \mathcal{N} \tag{19}$$

In addition, we introduce the following constraints:

$$\sum_{i\in\mathcal{N}} m_{i,j}\omega_{i,j} \le \omega_j^{max}, \quad \forall j \in \mathcal{N} \tag{20}$$

$$\omega_{i,j} \le \mu_j, \quad \forall i, j \in \mathcal{N} \tag{21}$$

where $\omega_{i,j} \in \{0, 1\}$ represents the measurement at bus i with the PMU located at bus j. If $\omega_{i,j} = 1$, bus i is measured by the PMU located at bus j, and $\omega_{i,j} = 0$, otherwise. The two constraints mean that the total number of buses a PMU measures should be less than its measurement capacity limitation, ω_j^{max}.

C. Solving OPLP

In this section, we will introduce a general way to solve OPLP. All through this section, we take both the ZIB effect and PMU measurement limitation into consideration.

Constraint (15) specifies the bandwidth requirement on each CL. Since the CL cost function, $f(\cdot)$, as defined in (12), is monotonic increasing with $b_{i,j}$, the optimal solution must lie on the boundary of constraint (15). In other words, OPLP is equivalent to:

$$\begin{aligned} \text{Minimize} \quad & K\sum_{j=1}^{N}\mu_j + \sum_{i=1}^{N}\sum_{j>i}^{N} f\left(\underline{b}_{i,j}, d_{i,j}\right) \qquad (22)\\ \text{Subject to} \quad & \boldsymbol{\mu M} \ge \boldsymbol{k} \\ & \sum_{k=1}^{C_i}\Gamma_i^k = h_i\mu_i \\ & \underline{\boldsymbol{B}} = \sum_{i=1}^{N} d_i\mu_i\left(\sum_{k=1}^{C_i}\Gamma_i^k\boldsymbol{P}_i^k\right) \end{aligned}$$

where $\mathbf{X} = \{\mu_i \in \{0, 1\}, \Gamma_i^k \in \{0, 1\}, \underline{b}_{i,j} \ge 0 | \forall i \in \mathcal{N}, \forall k \le C_i\}$ denotes the decision space of (22).

In (22), the objective function has some piecewise linear functions, $f(\underline{b}_{i,j}, d_{i,j})$. Thus, the optimization problem cannot be solved by any commercial solver. To tackle the issue, we introduce two sets of binary variables, $l_{i,j} \in \{0, 1\}$ and $\eta_{i,j} \in \{0, 1\}$, where $i, j \in \mathcal{N}$, to model the piecewise function and transform the problem into a mixed integer programming problem. Then, we can transform $f(\underline{b}_{i,j}, d_{i,j})$ into $g(\underline{b}_{i,j}, d_{i,j})$ by adding constraints (24) and (25).

$$g\left(\underline{b}_{i,j}, d_{i,j}\right) = \left(1-\alpha_{i,j}\right)\left(E_l d_{i,j}\right)l_{i,j} + E_b\left(\underline{b}_{i,j} - q_{i,j}\right)\eta_{i,j} \tag{23}$$

$$d\ l_{i,j} \le \underline{b}_{i,j} \le l_{i,j}\underline{b}_{i,j} \tag{24}$$

$$\left(1-\eta_{i,j}\right)\left(\underline{b}_{i,j} - q_{i,j}\right) \le \underline{b}_{i,j} - q_{i,j} \le \eta_{i,j}\left(\underline{b}_{i,j} - q_{i,j}\right) \tag{25}$$

To realize OPLP, there may exist the installation of new CLs as well as provision more capacities to some existing CLs. Based on this fact, we can understand the physical meaning of $l_{i,j}$ and $\eta_{i,j}$ in (23) easily. The binary variable $l_{i,j}$ indicates if there is a CL placed on branch *i-j*. When there is a CL installed on branch *i-j*, $l_{i,j} = 1$ holds, and otherwise, $l_{i,j} = 0$. When there is an existing CL on branch *i-j*, $\alpha_{i,j} = 1$ holds. Then, according to the first part in $g(\underline{b}_{i,j}, d_{i,j})$, even if $l_{i,j} = 1$, the passive cost on the corresponding CL equals to zero. The binary variable, $\eta_{i,j}$, indicates if the CL needs a bandwidth upgrade. If there is an existing CL on brand *i-j* and the preassigned bandwidth is needed to be upgraded, $\eta_{i,j} = 1$, and otherwise, $\eta_{i,j} = 0$. The constraint (24) ensures that the placement of CL can only happen when the bandwidth requirement on the CL is larger than d. Here we formulate the lower bound on the bandwidth requirement as d instead of zero. Based on the linear PMU traffic model, d is the minimum unit of data traffic that a PMU can generate. The constraint (25) ensures that there will be an upgrade on the bandwidth, if and only if the existing CL bandwidth, $q_{i,j}$ cannot meet the bandwidth requirement, i.e., $\underline{b}_{i,j}$.

With the transformation, the OPLP problem can be modeled as (26) shown at the top of the next page. Since (24) and (25) are quadratic constraints, OPLP is a Mixed Integer Quadratic Constraint programming (MIQCP) problem. Although the MIQCP for OPLP is an *NP*-hard problem, many commercial solvers, such as, Cplex, Gurobi, and Mosek, can provide approximate solution efficiently [28]. Cplex utilizes McCormick Branch and Bound approximation to solve MIQCP problem. Branch and bound algorithms provide a systematic enumeration of candidate solutions by means of state space search. On each search point, a second order cone programming (SOCP) should be solved. CPLEX use McCormick method to provide a linear programming approximation of the SOCP problem. According to [29] and [30], Cplex maintains a provable upper and lower bounds on the (globally) optimal objective value. They can converge to the ϵ-suboptimal point in a finite number of steps.

IV. Numerical Analysis

In this section, we evaluate the proposed OPLP model on the IEEE 30-bus, 57-bus, 118-bus, and 300-bus test systems [31]. Cplex optimization package[1] is used for our study. We test all systems under two initial conditions, with and without existing CLs. We name the cases with existing CLs OPLPeL and the cases without existing CLs OPLPnL. By comparing the results of OPLPnL and OPLPeL, we try to find how the existing links affect the placement of PMU and CLs. In this section, we first compare and discuss the performance results of OPP, OPLPnL and OPLPeL. Then we examine the optimal placements of CLs and PMUs as well as the bandwidth requirements for CLs. Next, we show the results of OPLPnL under different PMU measurement limitations. In the end, we will test OPLPnL on different pricing scenarios of PMU and CLs.

To solve OPLP, we need to define the values of K, E_b, and E_l, where K is the cost of a PMU, E_b is the CL bandwidth price per kbit/s, and, E_l is the passive CL price per kilometer. These

[1]Cplex Optimizer, Inc., "IBM ILOG CPLEX Optimization Studio." [Online] http://https://www.ibm.com/support/knowledgecenter/en/SSSA5P_12.7.0/ilog.odms.studio.help/Optimization_Studio/topics/COS_home.html, Accessed, Mar. 2018.

$$
\begin{aligned}
&\text{Minize} \quad K\sum_{j=1}^{N}\mu_j + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} g\left(\underline{b}_{i,j}, d_{i,j}\right)\\
&\text{Subject to} \quad \begin{cases}
\sum\limits_{j\in\mathcal{N}} m_{i,j}\omega_{i,j}\mu_j + \sum\limits_{j\in\mathcal{N}} m_{i,j}z_j y_{i,j} \geq \boldsymbol{k}, \quad \forall i\in\mathcal{N}\\
\sum_{i\in\mathcal{N}} m_{i,j}y_{i,j} = z_j, \quad \forall j\in\mathcal{N}\\
\sum_{i\in\mathcal{N}} m_{i,j}\omega_{i,j} \leq \omega_j^{max}, \forall j\in\mathcal{N}\\
\omega_{i,j} \leq \mu_j, \forall i,j\in\mathcal{N}\\
g\left(\underline{b}_{i,j}, d_{i,j}\right) = \left(1-\alpha_{i,j}\right)\left(E_l d_{i,j}\right) l_{i,j} + E_b\left(\underline{b}_{i,j} - q_{i,j}\right)\eta_{i,j}\\
d\ l_{i,j} \leq \underline{b}_{i,j} \leq l_{i,j}\underline{b}_{i,j}\\
\left(1-\eta_{i,j}\right)\left(\underline{b}_{i,j} - q_{i,j}\right) \leq \underline{b}_{i,j} - q_{i,j} \leq \eta_{i,j}\left(\underline{b}_{i,j} - q_{i,j}\right)\\
\sum_{k=1}^{C_i}\Gamma_i^k = h_i\mu_i\\
\underline{\boldsymbol{B}} = \sum\limits_{i=1}^{N} d_i\mu_i\left(\sum\limits_{k=1}^{C_i}\Gamma_i^k\boldsymbol{P}_i^k\right)\\
l_{i,j}\in\{0,1\} \quad \forall i,j\in\mathcal{N}, i\neq j\\
\eta_{i,j}\in\{0,1\} \quad \forall i,j\in\mathcal{N}, i\neq j\\
\mu_i\in\{0,1\} \quad \forall i\in\mathcal{N}\\
\omega_{i,j}\in\{0,1\}, \quad \forall i,j\in\mathcal{N}\\
y_{i,j}\in\{0,1\}, \quad \forall i,j\in\mathcal{N}\\
\Gamma_i^k\in\{0,1\}, \quad \forall i\in\mathcal{N}, \quad \forall k\leq C_i
\end{cases}
\end{aligned} \tag{26}
$$

three values vary significantly for different power systems. According to [20], the price from Chinese telecommunication company[2] and U.S-based vendor,[3] we take $K = 40000$, $E_b = 120$, and $E_l = 1500$ in units of USD in our numerical studies. In [20], the installation cost of optical fiber communication line is about $10000 per kilometer, and 85% of the price is digging fee. Since we consider the model where CL overlaps with the power line, the digging fee is saved. Then, we set $E_l = 1500$. Besides, these values represent the estimated prices of a standard PMU offered by a U.S.-based vendor [3] and the Ethernet lines provided by a Chinese telecommunication company[2]. To be able to approximate the distance matrix of each IEEE test network, we have assumed that all transmission lines have the same conductors with the same configurations. Thus, the relative distances between system buses can be extracted from the system admittance matrix [20]. We assume that the total length of transmission lines in IEEE 30-bus, 57-bus, 118-bus, and 300-bus test networks equal to 3000, 5712, 9884 and 25128 kilometers, respectively.

For OPLPnL, since there are no existing CLs, we set all $\alpha_{i,j}$ equals to zero. For OPLPeL, we randomly select three branches with pre-installed CLs, each having a bandwidth between d to $5d$. The detailed configuration is shown in Table I. The bandwidth associated with these links, however, are assigned at random.

In all systems we studied, we assume that PDC is located at bus 10. Before solving the MIQCP model proposed in (26), we need to pre-calculate $\mathcal{P}_i$, where $i \in \mathcal{N}$. As mentioned

[2][Online] http://www.chinatelecomglobal.com/tariffinformation/iepliplc, Accessed, Sep 2017.

[3][Online] https://www.selinc.com/synchrophasors/products. Accessed, May 2016.

TABLE I
EXISTING CLs LOCATIONS AND BANDWIDTHS

IEEE 30-bus System			
Locations of CLs	4-6	16-17	10-17
Bandwidth (Kbps)	$4d$	$3d$	$4d$
IEEE 57-bus System			
Locations of CLs	36-37	1-16	12-16
Bandwidth (Kbps)	$4d$	$5d$	$3d$
IEEE 118-bus System			
Locations of CLs	5-8	114-115	82-96
Bandwidth (Kbps)	$5d$	$5d$	$3d$
IEEE 300-bus System			
Locations of CLs	9-11	100-102	198-209
Bandwidth(Kbops)	$5d$	$1d$	$5d$

before, in this paper, we evaluate data transmissions from any potential PMU bus to the PDC by considering the shortest path only.

A. Performance Comparisons

First, the standard IEEE 30-bus, 57-bus, 118-bus, and 300-bus test systems are investigated by the proposed OPLPeL and OPLPnL models. Table II shows the PMU placement results of OPP [15], OPLPnL and OPLPeL. Three different cases are considered, namely, the case without ZIBs or PMU outage, with PMU outage or $(N-1)$ redundancy, and, with ZIBs. Columns 2-4 of Table II show the minimum PMU number achieved by OPP under these three cases. Columns 5-16 show the minimum PMU number and CLs achieved by OPLPnL and OPLPeL under three different cases. By comparing the results of Case 1 and 3, we can see that the existence of ZIBs reduces the number of PMUs. Besides, almost double the number of PMUs are needed to achieve $(N-1)$ redundancy. Besides ZIBs, the existing of conventional

TABLE II
RESULTS OF PMU PLACEMENT FOR IEEE STANDARD TEST SYSTEMS IN DIFFERENT CASES

Test System	OPP			OPLPnL						OPLPeL					
	Case 1[4]	Case 2[5]	Case 3[6]	Case 1[4]		Case 2[5]		Case 3[6]		Case 1[4]		Case 2[5]		Case 3[6]	
				PMU	Link	PMU	Link	PMU	Link	PMU	Link	PMU	Link	PMU	Link
IEEE 30-bus	10	17	7	10	13	17	18	7	9	10	15	17	19	7	11
IEEE 57-bus	17	29	11	18	33	29	38	14	27	18	33	32	59	14	28
IEEE 118-bus	32	59	28	35	60	71	85	32	59	35	59	71	85	32	59
IEEE 300-bus	87	162	68	90	167	205	303	69	141	90	165	205	303	69	141

[4] The case without zero-injection buses or device outage.
[5] $(N-1)$ redundancy case without consideration of zero-injection buses.
[6] The case with zero-injection buses and no device outage.

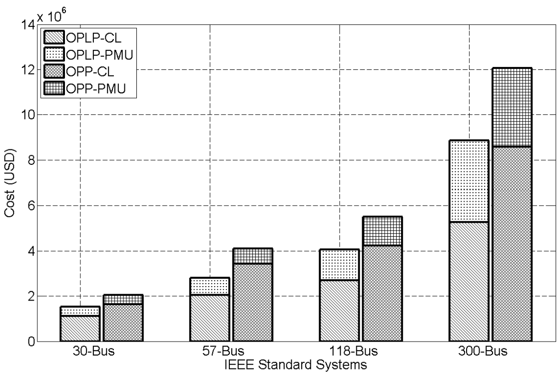

Fig. 2. Cost Comparison Between OPLP and OPP.

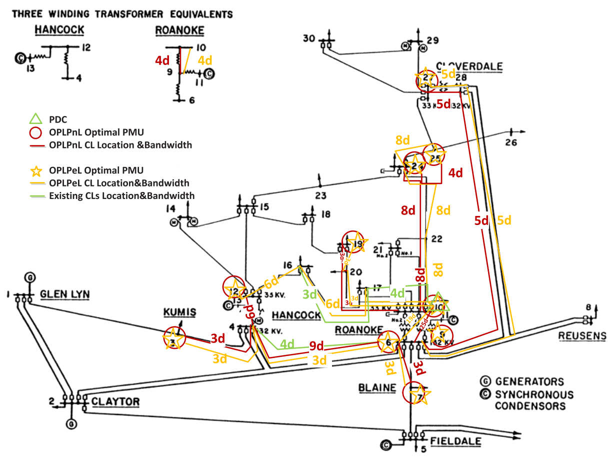

Fig. 3. OPLP Results for IEEE 30-bus system [31].

measurement (CM) sensors, e.g., power flow measurements and voltage measurement, will reduce the number of required PMUs according to [3]. Since we do not consider the effect of CMs in OPLP, there would be observability redundancies provided by the CM sensors. From another point of view, the observability redundancies enhance the resilience against potential cyberattack [32] and PMU failure [3].

Based on the results of Table II, existing CLs seem to have little influence on the number of PMUs and CLs when we compare OPLPnL and OPLPeL results. Next, we show the locations of PMUs, locations and bandwidth of CLs of OPLPnL and OPLPeL. All the results shown in Fig. 3, Table III and IV are tested under the assumption of Case 1.

By comparing the OPLP results with those of OPP, we can see that, for the case of the IEEE 30-bus system, OPLP and OPP require the same number of PMUs for system observability. However, OPLP would install more PMUs compared with OPP in 57-bus, 118-bus, and 300-bus systems.

We show the cost comparison between OPLP and OPP of IEEE 30-bus, 57-bus, 118-bus, and 300-bus systems in Fig. 2. The bars of OPLP-CL and OPLP-PMU represent the cost of CLs and PMUs achieved by OPLP. The bars of OPP-CL and OPP-PMU represent the cost of CLs and PMUs achieved by OPP, respectively. All the results are obtained from the tests under the condition that there are no existing CLs. From Fig. 2, we can see that compared with OPP, OPLP can reduce the total cost by about 25%. For small-scale systems, i.e., IEEE 30-bus system, OPLP and OPP pay the same amount of money on PMUs. OPLP would save the cost on CLs by adopting the optimal locations of PMUs and selecting the optimal data routing paths. For the large-scale systems, i.e., IEEE 57-bus, 118-bus, and 300-bus systems, although OPLP adopts more PMUs than the minimal number, it could reduce the total cost significantly by reducing the CL cost. This is because the costs of CLs will outweigh the costs of PMUs in large-scale power systems. In this case, placing more PMUs can help to reduce the cost on CLs.

B. IEEE Standard Cases

We show results of OPLPeL and OPLPnL in Fig. 3 based on the IEEE 30-bus diagram from [31]. The solution to the OPLPnL problem suggests that, to make the system fully observable, a total of ten PMUs are to be installed on buses 3, 6, 7, 9, 10, 12, 19, 24, 25, and 27. To successfully transmit all PMU data to the PDC, a total of 13 CLs with properly provisioned bandwidths is required. The symbol attached to each CL denotes its required bandwidth of the CL. OPLPnL calculates the total cost to be \$1,523,320.

If three CLs with bandwidths $4d$, $3d$, and $4d$ were previously installed in the system on branches 4-6, 16-17, and 10-17, respectively, the solution to the OPLPeL problem suggests the same installations of PMUs are adopted as shown in Fig 3. However, the installations of CLs are different for the two cases. By comparing the CL installations, we can easily find that a new CL is placed at branch $12 \rightarrow 16$ to build a new transmission path, i.e., $12 \rightarrow 16 \rightarrow 17 \rightarrow 10$. Thus, the existing CLs can be fully utilized. Furthermore, the data of the PMU located at bus 12 is routed via the new path instead of the old path, so the bandwidth requirement on CL $4 \rightarrow 6$

TABLE III
OPLP RESULTS FOR IEEE 57-BUS SYSTEM

	OPLPnL		OPLPeL	
Locations of P-MU	3 4 8 12 15 20 24 28 31 32 36 38 41 46 51 52 55 57		1 4 8 10 20 21 24 28 31 32 36 41 44 46 49 52 55 57	
Total Cost	$2,801,940.0		$2,711,520.0	
CL Placement and Bandwidth (Kbps)	Branch	Bandwidth	Branch	Bandwidth
	3-15	4d	*1-16	5d
	4-6	5d	4-6	5d
	6-8	5d	6-8	5d
	7-8	6d	7-8	7d
	7-29	6d	7-29	7d
	8-9	15d	8-9	15d
	9-10	26d	9-10	26d
	9-11	8d	9-11	8d
	9-55	3d	9-55	3d
	10-12	43d	10-12	37d
	10-51	3d	11-41	8d
	11-41	8d	12-13	32d
	12-13	37d	*12-16	4d
	13-14	3d	13-14	3d
	13-15	10d	13-49	29d
	13-49	24d	14-46	3d
	14-46	3d	20-21	3d
	20-21	3d	21-22	6d
	21-22	3d	22-23	4d
	22-23	4d	22-38	10d
	22-38	7d	23-24	4d
	23-24	4d	28-29	3d
	28-29	3d	29-52	3d
	29-52	3d	31-32	3d
	31-32	3d	32-34	7d
	32-34	7d	34-35	7d
	34-35	7c	35-36	7d
	35-36	7d	*36-37	11d
	36-37	11d	37-38	11d
	37-38	11d	38-44	3d
	38-49	24d	38-49	24d
	41-56	3d	41-56	3d
	56-57	3d	56-57	3d

* Link bandwidth upgrades required.

TABLE IV
OPLP RESULTS FOR IEEE 118-BUS SYSTEM

	OPLPnL		OPLPeL	
Locations of P-MU	2 5 10 11 12 17 20 23 25 29 34 37 40 45 49 50 51 52 59 65 66 71 75 77 80 85 87 91 94 101 105 110 114 116		2 5 10 11 12 17 20 23 25 29 34 37 40 45 49 50 51 52 59 65 66 71 75 77 80 85 87 91 94 101 105 110 114 116	
Total Cost	$4,047,520.0		$4,046,320.0	
Link Placement and Bandwidth (Kbps)	Branch	Bandwidth	Branch	Bandwidth
	2-12	3d	2-12	3d
	5-8	22d	*5-8	22d
	5-11	16d	5-11	16d
	8-9	161d	8-9	161d
	8-30	139d	8-30	139d
	9-10	161d	11-12	11d
	11-12	11d	15-17	3d
	15-17	3d	15-19	3d
	15-19	3d	17-30	16d
	17-30	16d	17-31	3d
	17-31	3d	17-113	3d
	17-113	3d	19-20	3d
	19-20	3d	23-25	5d
	23-25	5d	25-26	9d
	25-26	9d	26-30	9d
	26-30	9d	29-31	3d
	29-31	3d	30-38	114d
	30-38	114d	32-113	3d
	32-113	3d	32-114	3d
	32-114	3d	34-37	5d
	34-37	5d	37-38	17d
	37-38	17d	37-40	5d
	37-40	5d	38-65	97d
	38-65	97d	45-49	4d
	45-49	4d	49-50	3d
	49-50	3d	49-51	7d
	49-51	7d	49-66	4d
	49-66	4d	49-69	28d
	49-69	28d	51-52	3d
	51-52	3d	59-63	7d
	59-63	7d	63-64	7d
	63-64	7d	64-65	7d
	64-65	7d	65-68	86d
	65-68	86d	68-69	53d
	68-69	53d	68-81	31d
	68-81	31d	68-116	2d
	68-116	2d	69-70	4d
	69-70	4d	69-75	6d
	69-75	6d	69-77	15d
	69-77	15d	70-71	4d
	70-71	4d	77-82	8d
	77-82	8d	80-81	31d
	80-81	31d	80-96	6d
	80-96	6d	80-98	5d
	80-98	17d	80-99	12d
	82-83	8d	82-83	8d
	83-85	8d	83-85	8d
	85-86	2d	85-86	2d
	86-87	2d	86-87	2d
	91-92	3d	91-92	3d
	92-100	3d	92-100	3d
	94-96	6d	94-96	6d
	98-100	17d	*98-100	5d
	100-101	3d	99-100	12d
	100-103	11d	100-101	3d
	103-105	6d	100-103	11d
	103-110	5d	103-105	6d
			103-110	5d
			9-10	161d

* Link bandwidth upgrades required.

reduces from 9d to 3d. A reduction of bandwidth requirement on CL 6 $\rightarrow$ 10 from 25d to 19d is obtained for the same reason. Since the solution to OPLPeL utilizes the existing CLs effectively, which reduces the cost of CL bandwidth, it achieves a reduction of 39840 dollars in the total cost. In this case, OPLPeL calculates the total cost of enabling this PMU network to be $1,483,480.

Compared with IEEE 30-bus system, 57-bus and 118-bus systems have complex network topologies. Thus, showing their results in a graphic manner as Fig. 3 is not readable. Next we show the results of IEEE 57-bus and 118-bus system in Table III and Table IV. The readers can refer to the system graphics in [31] for the understanding of the results.

For the IEEE 57-bus system with no existing CLs, a total of 18 PMUs, placed on buses 3, 4, 8, 12, 15, 20, 24, 28, 31, 32, 36, 38, 41, 46, 51, 52, 55 and 57, so as to make the system fully observable is suggested. In order to support timely data delivery of these PMUs, 33 CLs with bandwidths properly provisioned are required for the system, as shown in Table III.

With three CLs in the system, OPLPeL will offer a different PMU installation strategy, as shown in Table III. On the one hand, it requires different locations for PMU installations. On the other hand, there is a dramatic reduction of $90,420 in the total installation cost as compared with OPLPnL.

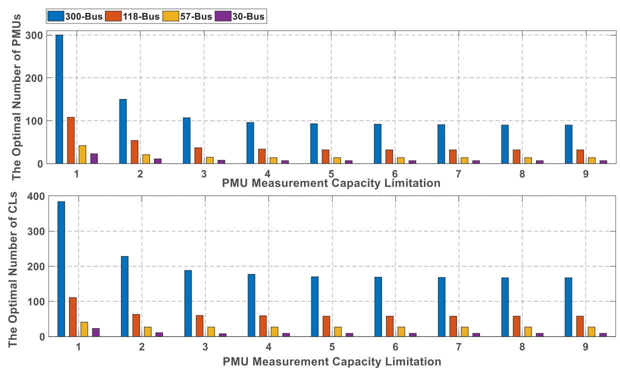

Fig. 4. Results of OPLP for IEEE standard test systems considering measurement limitation.

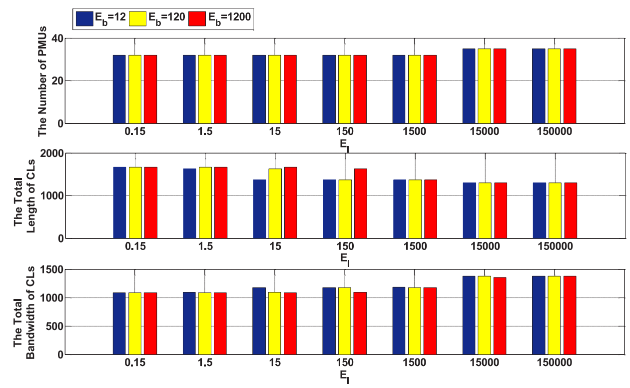

Fig. 5. Results of OPLP under different pricing scenarios.

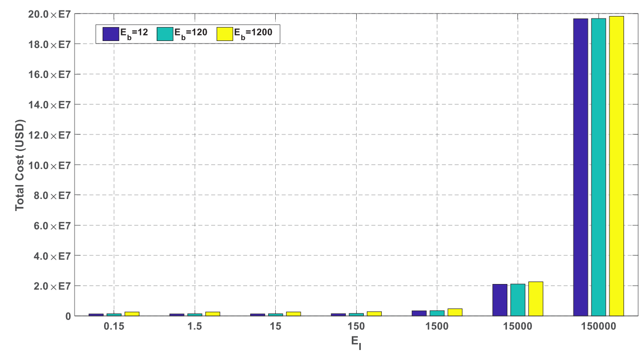

Fig. 6. Total cost of OPLP under different pricing scenarios.

For the 118-bus system, OPLPnL and OPLPeL adopt the same installation strategy of PMUs but different CL installation schemes. By changing the data routing paths, OPLPeL employs the existing CLs. Consequently, the total installation cost achieved by OPLPeL is \$1,200 cheaper than the cost of OPLPnL.

The results on these case studies suggest that existing CLs in a power system may alter the data routing paths used by the installed PMUs in order to utilize the existing CLs. Then, OPLPeL may result in a different PMU installation strategy and different CL installations. In summary, OPLPeL takes advantage of the existing CLs so that the total cost of establishing the WAMS is reduced.

C. Measurement Capacity Limitation Effects

Assuming that all PMUs come with the same measurement capacity limitation, we test nine different cases on OPLPnL model in IEEE 30-bus, 57-bus, 118-bus, and 300-bus systems. Fig. 4 shows the minimum numbers of PMUs and CLs achieved by OPLPnL. The figure shows that as PMU measurement capacity limitation increases, the numbers of PMUs and CLs decrease. However, when the PMU measurement capacity is larger than four, even though it increases, the optimal number of PMUs and CLs do not decrease. The reason is that when the PMU measurement capacity is large, the PMU measurement capacity cannot be fully utilized due to the power system topology. For bus n, since it has a limited number of adjacent buses, the maximum number of buses that need to be measured by PMU n is limited. For example, even though a PMU may have a measurement capacity of 100 buses, at least a measurement capacity of 94 buses would be unutilized when it is located at the bus with only 5 adjacent buses. Consequently, more PMUs must be placed to measure the buses that are not connected to the PMU. Therefore, as the PMU measurement capacity exceeds a certain value, the increase of PMU measurement capacity will not make the required number of PMUs and CLs reduce further.

D. OPLP Results Under Different Pricing Scenarios

In this section, we show the results of OPLP under different pricing scenarios in Fig. 5 and Fig. 6. The test is conducted in IEEE 118-bus system. The price of PMU, K, and, the price of CLs, including the passive CL price, E_l, and, the bandwidth price, E_b, may influence the placement of PMU and CL. In order to determine how the results will be influenced, we do the following tests. First, we keep the PMU price $K = 40000$ fixed. Then, we change the passive CL price E_l from 0.15 to 150000 under different bandwidth prices, including $E_b = 12$, $E_b = 120$ and, $E_b = 1200$. The results are shown in Figure 5. The blue, yellow and red bars show the results when E_b is 12, 120 and 1200, respectively. As shown in the figure, the increase of E_l results in the decrease of total CL length. As E_l increases, both the number of PMUs and total CL bandwidth increase.

Then, we have a look at each cluster of bars in Figure 5 and take $E_l = 150$ as an example. As E_b increases, although the number of PMUs remains the same, the total CL bandwidth decreases and the total CL length increases. This is because the locations of PMUs change to reduce the CL cost, although the total number of PMUs remains unchanged. However, with a large value of E_l, $E_l \geq 1500$, for example, the changes of CL placement under different bandwidth prices become tiny. We can observe similar findings in Fig. 6. When $E_l \leq 1500$, due to the relative small value of the total cost, the differences of the total cost under different bandwidth prices are relatively apparent. When $E_l > 1500$, the differences of bars within one cluster are tiny. This is because when $E_l > 1500$, the passive CL cost takes a large proportion of the total cost and the bandwidth cost becomes negligible.

In summary, the placements of PMUs and CLs change a lot under different pricing scenarios. The cost on CLs, including the passive cost and active cost, is non-negligible in the OPP problem, which shows the importance of OPLP problem.

V. Conclusion

In this paper, we extend the OPP problem by formulating an OPLP problem. The proposed model not only minimizes the total cost of PMUs and CLs, but also takes the power system observability, and data transmission bandwidth requirements into consideration. Some extensions, including ZIBs, PMU measurement capacity limitation, are considered. By carrying out numerical studies on the IEEE standard systems, we discovered that, when communication costs are considered, more PMUs may be required to minimize the total cost of building WAMS. Moreover, by comparing the cases with and without existing CLs, we have found that the presence of the existing CLs may result in the relocation of PMUs and different transmission paths for PMU data. Finally, we found that a tiny change on PMU or CL price influences optimal results significantly, which proves the importance of OPLP in saving cost.

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