$$\dfrac{\dfrac{}{\vdash \varphi_1, \varphi_1}}{\vdash \varphi_1 \vee \varphi_1}\ (\vdash \vee)$$
Valid ? false

$$\dfrac{\dfrac{}{\varphi_1 \vdash \varphi_2}\quad \dfrac{\varphi_1 \vdash \varphi_2, \varphi_2}{\vdash \varphi_1 \to \varphi_2, \varphi_2}\ (\vdash\to)}{\dfrac{(\varphi_1 \to \varphi_2) \to \varphi_1 \vdash \varphi_2}{\vdash ((\varphi_1 \to \varphi_2) \to \varphi_1) \to \varphi_2}\ (\vdash\to)}\ (\to\vdash)$$
Valid ? false

$$\dfrac{\dfrac{\dfrac{}{\varphi_1 \vdash \varphi_1}\ (Ax)}{\vdash \neg\varphi_1, \varphi_1}\ (\vdash \neg)}{\vdash \varphi_1 \vee (\neg\varphi_1)}\ (\vdash \vee)$$
Valid ? true

$$\dfrac{\dfrac{}{\varphi_1 \vdash \varphi_1}\ (Ax)\quad \dfrac{\dfrac{}{\varphi_1 \vdash \varphi_2, \varphi_1}\ (Ax)}{\vdash \varphi_1 \to \varphi_2, \varphi_1}\ (\vdash\to)}{\dfrac{(\varphi_1 \to \varphi_2) \to \varphi_1 \vdash \varphi_1}{\vdash ((\varphi_1 \to \varphi_2) \to \varphi_1) \to \varphi_1}\ (\vdash\to)}\ (\to\vdash)$$
Valid ? true

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_3}\ (Ax)\quad \dfrac{}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_2}\ (Ax)}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_2 \wedge \varphi_3}\ (\vdash \wedge)\quad \dfrac{}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_1}\ (Ax)}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_1 \wedge \varphi_2 \wedge \varphi_3}\ (\vdash \wedge)}{\dfrac{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_1 \wedge \varphi_2 \wedge \varphi_3}{\varphi_1 \wedge \varphi_2, \varphi_3 \vdash \varphi_1 \wedge \varphi_2 \wedge \varphi_3}\ (\wedge\vdash)}\ (\wedge\vdash)}{\varphi_1 \wedge \varphi_2 \wedge \varphi_3 \vdash \varphi_1 \wedge \varphi_2 \wedge \varphi_3}$$
Valid ? true

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\varphi_3 \vdash \varphi_2, \varphi_3, \varphi_1}\ (Ax)}{\varphi_3 \vdash \varphi_2 \vee \varphi_3, \varphi_1}\ (\vdash \vee)}{\varphi_3 \vdash \varphi_1 \vee \varphi_2 \vee \varphi_3}\ (\vdash \vee)\quad \dfrac{\dfrac{\dfrac{}{\varphi_2 \vdash \varphi_2, \varphi_3, \varphi_1}\ (Ax)}{\varphi_2 \vdash \varphi_2 \vee \varphi_3, \varphi_1}\ (\vdash \vee)}{\varphi_2 \vdash \varphi_1 \vee \varphi_2 \vee \varphi_3}\ (\vdash \vee)\quad \dfrac{\dfrac{}{\varphi_1 \vdash \varphi_1, \varphi_2 \vee \varphi_3}\ (Ax)}{\varphi_1 \vdash \varphi_1 \vee \varphi_2 \vee \varphi_3}\ (\vdash \vee)}{\varphi_1 \vee \varphi_2 \vdash \varphi_1 \vee \varphi_2 \vee \varphi_3}\ (\vee\vdash)}{\varphi_1 \vee \varphi_2 \vee \varphi_3 \vdash \varphi_1 \vee \varphi_2 \vee \varphi_3}\ (\vee\vdash)$$
Valid ? true

$$\dfrac{\dfrac{\dfrac{}{\varphi_2 \vdash \varphi_2, \neg\varphi_1}\ (Ax)}{\neg\varphi_2, \varphi_2 \vdash \neg\varphi_1}\ (\neg\vdash)}{\varphi_2 \vdash (\neg\varphi_2) \to (\neg\varphi_1)}\ (\vdash\to)\quad \dfrac{\dfrac{\dfrac{}{\varphi_1 \vdash \varphi_2, \varphi_1}\ (Ax)}{\vdash \neg\varphi_1, \varphi_2, \varphi_1}\ (\vdash \neg)}{\neg\varphi_2 \vdash \neg\varphi_1, \varphi_1}\ (\neg\vdash)}{\dfrac{\varphi_1 \to \varphi_2 \vdash (\neg\varphi_2) \to (\neg\varphi_1)}{\vdash (\neg\varphi_2) \to (\neg\varphi_1), \varphi_1}\ (\vdash\to)}\ (\to\vdash)$$
Valid ? true

$$\dfrac{\dfrac{\dfrac{}{\varphi_1 \vdash \varphi_2, \varphi_1}\ (Ax)}{\vdash \varphi_1 \to \varphi_2, \varphi_1}\ (\vdash\to)}{\neg\varphi_1 \vdash \varphi_1 \to \varphi_2}\ (\neg\vdash)\quad \dfrac{\dfrac{}{\varphi_1, \varphi_2 \vdash \varphi_2}\ (Ax)}{\varphi_2 \vdash \varphi_1 \to \varphi_2}\ (\vdash\to)}{\dfrac{}{\vdash \neg\varphi_2, \varphi_1 \to \varphi_2}\ (\vdash \neg)}\ (\to\vdash)$$
$$(\neg\varphi_2) \to (\neg\varphi_1) \vdash \varphi_1 \to \varphi_2$$
Valid ? true

$$\dfrac{\dfrac{\dfrac{}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_3}\ (Ax)\quad \dfrac{}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_1}\ (Ax)}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_1 \wedge \varphi_3}\ (\vdash \wedge)}{\dfrac{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_1 \wedge \varphi_3}{\varphi_2, \varphi_1 \vdash \varphi_3 \to (\varphi_1 \wedge \varphi_3)}\ (\vdash\to)}$$
Valid ? true

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_3, \varphi_3, \varphi_3}\ (Ax)\quad \dfrac{}{\varphi_2, \varphi_1 \vdash \varphi_2, \varphi_3, \varphi_3, \varphi_3}\ (Ax)}{\varphi_2 \to \varphi_3, \varphi_2, \varphi_1 \vdash \varphi_3, \varphi_3, \varphi_3}\ (\to\vdash)}{\varphi_2, \varphi_1 \vdash (\varphi_2 \to \varphi_3) \to \varphi_3, \varphi_3, \varphi_3}\ (\to\vdash)}{((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3, \varphi_2, \varphi_1 \vdash \varphi_3, \varphi_3}\ (\to\vdash)\quad \dfrac{\dfrac{\dfrac{\dfrac{}{\varphi_3, \varphi_2, \varphi_1 \vdash \varphi_3, \varphi_3, \varphi_3}\ (Ax)\quad \dfrac{}{\varphi_2, \varphi_1 \vdash \varphi_1, \varphi_3, \varphi_3, \varphi_3}\ (Ax)}{\varphi_1 \to \varphi_3, \varphi_2, \varphi_1 \vdash \varphi_3, \varphi_3, \varphi_3}\ (\to\vdash)}{\varphi_2, \varphi_1 \vdash (\varphi_1 \to \varphi_3) \to \varphi_3, \varphi_3, \varphi_3}\ (\to\vdash)}{((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3), \varphi_2, \varphi_1 \vdash \varphi_3, \varphi_3}\ (\to\vdash)}{\varphi_2, \varphi_1 \vdash (((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3)) \to \varphi_3, \varphi_3}\ (\to\vdash)}{\varphi_1 \vdash \varphi_2 \to \varphi_3, (((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3)) \to \varphi_3}\ (\to\vdash)}{\vdash \varphi_1 \to (\varphi_2 \to \varphi_3), (((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3)) \to \varphi_3}\ (\to\vdash)$$

$$\dfrac{\dfrac{\dfrac{}{((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3), \varphi_3 \vdash \varphi_3}\ (Ax)}{\varphi_3 \vdash (((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3)) \to \varphi_3}\ (\vdash\to)}{(\varphi_1 \to (\varphi_2 \to \varphi_3)) \to \varphi_3 \vdash (((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3)) \to \varphi_3}\ (\vdash\to)$$
$$\vdash ((\varphi_1 \to (\varphi_2 \to \varphi_3)) \to \varphi_3) \to ((((\varphi_1 \to \varphi_3) \to \varphi_3) \to (((\varphi_2 \to \varphi_3) \to \varphi_3) \to \varphi_3)) \to \varphi_3)$$
Valid ? true