《地球物理特殊方程》实验报告

班　　级：地物 **2201** 班

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实验一　边值问题的有限差分方法

1　实验目的

1. 利用有限差分法求解一维 Poisson 方程的近似解，并设计 MATLAB 程序实现；
2. 利用有限差分法求解二维 Poisson 方程定解问题；
3. 编写 MATLAB 程序计算 Neumann 条件下二维 Poisson 方程的有限差分近似解。

2　实验内容

任务 1 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} u''(x) = -\pi^2 \sin(\pi x), \ 0 < x < 1 \\ u(0) = u(1) = 0 \end{cases}$$

其解析解为

$$u(x) = \sin(\pi x)$$

任务 2 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} u''(x) = -\pi^2 \sin(\pi x), \ 0 < x < 1 \\ u'(0) - u(0) = 0.1, \ u(1) = 0 \end{cases}$$

其解析解为

$$u(x) = \frac{1}{2}x^2 - 0.2x - 0.3$$

任务 3 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = (1-\pi^2)e^x \sin(\pi y), & 0 < x < 2, \ 0 < y < 1 \\ u(0,y) = \sin(\pi y), \ u(2,y) = e^2 \sin(\pi y), & 0 \leqslant y \leqslant 1 \\ u(x,0) = 0, \ u(x,1) = 0, & 0 \leqslant x \leqslant 2 \end{cases}$$

其解析解为

$$u(x,y) = e^x \sin(\pi y)$$

任务 4 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=-13\pi^2\sin\left(3\pi x+\dfrac{\pi}{4}\right)\sin\left(2\pi y+\dfrac{\pi}{4}\right), \quad 0<x<1,\ 0<y<1 \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0}=3\pi\cos\left(\dfrac{\pi}{4}\right)\sin\left(2\pi y+\dfrac{\pi}{4}\right) \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=1}=-3\pi\cos\left(\dfrac{\pi}{4}\right)\sin\left(2\pi y+\dfrac{\pi}{4}\right) \\ u|_{y=0}=\sin\left(\dfrac{\pi}{4}\right)\sin\left(3\pi x+\dfrac{\pi}{4}\right) \\ u|_{y=1}=\sin\left(\dfrac{\pi}{4}\right)\sin\left(3\pi x+\dfrac{\pi}{4}\right) \end{cases}$$

其解析解为

$$u(x,y)=\sin\left(3\pi x+\frac{\pi}{4}\right)\sin\left(2\pi y+\frac{\pi}{4}\right)$$

3 实验过程及结果

对于任务 1，编写 MATLAB 程序如下

```
clc;
clear;

X = 1;
N = 100;
dx = X/N;
x = 0:dx:X;

A = sparse(N+1,N+1);
b = zeros(N+1,1);

A(1,1) = 1;
A(N+1,N+1) = 1;
b(1,1) = 0;
b(N+1,1) = 0;
d = 1/dx^2;
for i = 2:N
    A(i,i) = -2*d;
    A(i,i-1) = d;
    A(i,i+1) = d;
    b(i,1) = -pi^2*sin(pi*i*dx);
end
u = A\b;

figure;
plot(x,u,'ro');
xlabel('x');
ylabel('u');
title('Ex1');
```

对于任务 2，编写 MATLAB 程序如下

```
clc;
clear;

X = 1;
N = 100;
dx = X/N;
x = 0:dx:X;

A = sparse(N+1,N+1);
b = zeros(N+1,1);
A(1,1) = -1-dx;
A(1,2) = 1;
A(N+1,N+1) = 1;
b(1,1) = 0.1*dx;
b(N+1,1) = 0;

d = 1/dx^2;
for i = 2:N
    A(i,i) = -2*d;
    A(i,i-1) = d;
    A(i,i+1) = d;
    b(i,1) = 1;
 end
u = A\b;

figure;
plot(x,u,'ro');
xlabel('x');
ylabel('u');
title('Ex2');
```

针对任务 3，编写 MATLAB 程序如下

```
clc;
clear;

X = 2;Y = 1;
N = 50;M = 50;
dx = X/N;dy = Y/M;
x = 0:dx:X;y = 0:dy:Y;

A = sparse((N+1)*(M+1),(N+1)*(M+1));
b = zeros((N+1)*(M+1),1);
p = 1/dx^2;q = 1/dy^2;
for i = 1:M+1
    for j = 1:N+1
        k = (j-1)*(M+1)+i;
        if(i==0||i==M+1)
            A(k,k) = 1;b(k,1) = 0;
        elseif(j==1)
            A(k,k) = 1;b(k,1) = sin(pi*i*dy);
        elseif(j==N+1)
            A(k,k) = 1;b(k,1) = exp(2)*sin(pi*i*dy);
        else
            A(k,k) = -2*p-2*q;A(k,k-1) = q;A(k,k+1) = q;
            A(k,k+(M+1)) = p;A(k,k-(M+1)) = p;
            b(k,1) = (1-pi^2)*exp(j*dx)*sin(pi*i*dy);
        end
    end
end
uij = A\b;
u = reshape(uij,M+1,N+1);

figure;
surf(x,y,u);
xlabel('x');
ylabel('y');
zlabel('u');
title('Ex3');
```

针对任务 4，编写 MATLAB 程序如下

```
clc;clear;

X = 1;Y = 1;
N = 50;M = 50;
dx = X/N;dy = Y/M;
x = 0:dx:X;y = 0:dy:Y;

A = sparse((N+1)*(M+1),(N+1)*(M+1));
b = zeros((N+1)*(M+1),1);p = 1/dx^2;q = 1/dy^2;
for i = 1:M+1
    for j = 1:N+1
    k = (j-1)*(M+1)+i;
        if(i==1)
            A(k,k) = 1;b(k,1) = sin(0.25*pi)*sin(3*pi*j*dx+0.25*pi);
        elseif(i==M+1)
            A(k,k) = 1;b(k,1) = sin(0.25*pi)*sin(3*pi*j*dx+0.25*pi);
        elseif(j==1)
            A(k,k) = -1;A(k,k+(M+1)) = 1;b(k,1) =
            ↪  3*pi*cos(0.25*pi)*sin(2*pi*i*dy+0.25*pi)*dx;
        elseif(j==N+1)
            A(k,k) = -1;A(k,k-(M+1)) = 1;b(k,1) =
            ↪  -3*pi*cos(0.25*pi)*sin(2*pi*i*dy+0.25*pi)*dx;
        else
            A(k,k) = -2*p-2*q;A(k,k-1) = q;A(k,k+1) = q;
            A(k,k+(M+1)) = p;A(k,k-(M+1)) = p;
            b(k,1) = -13*pi^2*sin(3*pi*j*dx+0.25*pi)*sin(2*pi*i*dy+0.25*pi);
        end
    end
end
uij = A\b;u = reshape(uij,M+1,N+1);

figure;
surf(x,y,u);
xlabel('x');ylabel('y');zlabel('u');
title('Ex4');
```

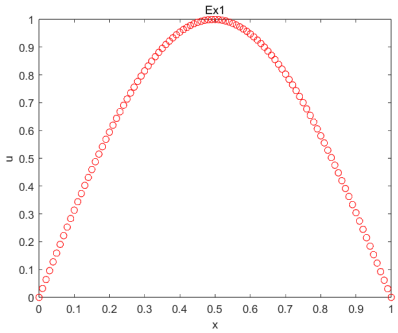

图 1: 程序 1 运行结果

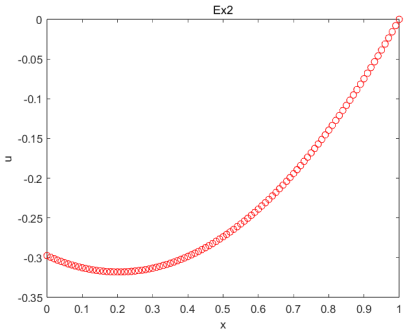

图 2: 程序 2 运行结果

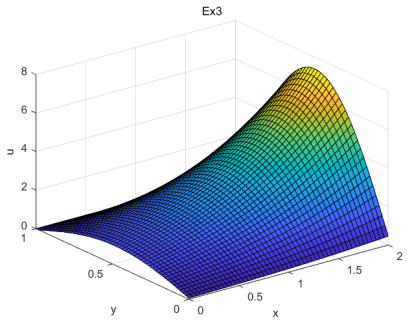

图 3: 程序 3 运行结果

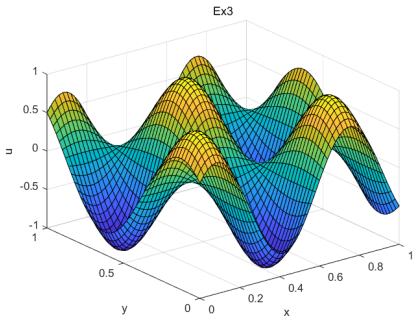

图 4: 程序 4 运行结果

4 实验总结及收获

1. 学习了有限差分法求解一维二维 Poisson 方程；
2. 学习了 Neumann 边界条件的有限差分法处理；
3. 学习了 MATLAB 程序编写。