

《地球物理特殊方程》实验报告



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实验一 边值问题的有限差分方法

1 实验目的

1. 利用有限差分法求解一维 Poisson 方程的近似解，并设计 MATLAB 程序实现；
2. 利用有限差分法求解二维 Poisson 方程定解问题；
3. 编写 MATLAB 程序计算 Neumann 条件下二维 Poisson 方程的有限差分近似解。

2 实验内容

任务 1 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} u''(x) = -\pi^2 \sin(\pi x), & 0 < x < 1 \\ u(0) = u(1) = 0 \end{cases}$$

其解析解为

$$u(x) = \sin(\pi x)$$

任务 2 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} u''(x) = -\pi^2 \sin(\pi x), & 0 < x < 1 \\ u'(0) - u(0) = 0.1, & u(1) = 0 \end{cases}$$

其解析解为

$$u(x) = \frac{1}{2}x^2 - 0.2x - 0.3$$

任务 3 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = (1 - \pi^2)e^x \sin(\pi y), & 0 < x < 2, 0 < y < 1 \\ u(0, y) = \sin(\pi y), & u(2, y) = e^2 \sin(\pi y), & 0 \leq y \leq 1 \\ u(x, 0) = 0, & u(x, 1) = 0, & 0 \leq x \leq 2 \end{cases}$$

其解析解为

$$u(x, y) = e^x \sin(\pi y)$$

任务 4 有限差分法求解下列问题并编写 MATLAB 程序

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -13\pi^2 \sin\left(3\pi x + \frac{\pi}{4}\right) \sin\left(2\pi y + \frac{\pi}{4}\right), & 0 < x < 1, 0 < y < 1 \\ \frac{\partial u}{\partial x}\bigg|_{x=0} = 3\pi \cos\left(\frac{\pi}{4}\right) \sin\left(2\pi y + \frac{\pi}{4}\right) \\ \frac{\partial u}{\partial x}\bigg|_{x=1} = -3\pi \cos\left(\frac{\pi}{4}\right) \sin\left(2\pi y + \frac{\pi}{4}\right) \\ u|_{y=0} = \sin\left(\frac{\pi}{4}\right) \sin\left(3\pi x + \frac{\pi}{4}\right) \\ u|_{y=1} = \sin\left(\frac{\pi}{4}\right) \sin\left(3\pi x + \frac{\pi}{4}\right) \end{cases}$$

其解析解为

$$u(x, y) = \sin\left(3\pi x + \frac{\pi}{4}\right) \sin\left(2\pi y + \frac{\pi}{4}\right)$$

3 实验过程及结果

对于任务 1，编写 MATLAB 程序如下

```
1  clc;
2  clear;
3
4  X = 1;
5  N = 100;
6  dx = X/N;
7  x = 0:dx:X;
8
9  A = sparse(N+1,N+1);
10 b = zeros(N+1,1);
11
12 A(1,1) = 1;
13 A(N+1,N+1) = 1;
14 b(1,1) = 0;
15 b(N+1,1) = 0;
16 d = 1/dx^2;
17 for i = 2:N
18     A(i,i) = -2*d;
19     A(i,i-1) = d;
20     A(i,i+1) = d;
21     b(i,1) = -pi^2*sin(pi*i*dx);
22 end
23 u = A\b;
24
25 figure;
26 plot(x,u,'ro');
27 xlabel('x');
28 ylabel('u');
29 title('Ex1');
```

对于任务 2，编写 MATLAB 程序如下

```
1  clc;
2  clear;
3
4  X = 1;
5  N = 100;
6  dx = X/N;
7  x = 0:dx:X;
8
9  A = sparse(N+1,N+1);
10 b = zeros(N+1,1);
11 A(1,1) = -1-dx;
12 A(1,2) = 1;
13 A(N+1,N+1) = 1;
14 b(1,1) = 0.1*dx;
15 b(N+1,1) = 0;
16
17 d = 1/dx^2;
18 for i = 2:N
19     A(i,i) = -2*d;
20     A(i,i-1) = d;
21     A(i,i+1) = d;
22     b(i,1) = 1;
23 end
24 u = A\b;
25
26 figure;
27 plot(x,u,'ro');
28 xlabel('x');
29 ylabel('u');
30 title('Ex2');
```

针对任务 3，编写 MATLAB 程序如下

```
1  clc;
2  clear;
3
4  X = 2; Y = 1;
5  N = 50; M = 50;
6  dx = X/N; dy = Y/M;
7  x = 0:dx:X; y = 0:dy:Y;
8
9  A = sparse((N+1)*(M+1), (N+1)*(M+1));
10 b = zeros((N+1)*(M+1), 1);
11 p = 1/dx^2; q = 1/dy^2;
12 for i = 1:M+1
13     for j = 1:N+1
14         k = (j-1)*(M+1)+i;
15         if(i==0 || i==M+1)
16             A(k,k) = 1; b(k,1) = 0;
17         elseif(j==1)
18             A(k,k) = 1; b(k,1) = sin(pi*i*dy);
19         elseif(j==N+1)
20             A(k,k) = 1; b(k,1) = exp(2)*sin(pi*i*dy);
21         else
22             A(k,k) = -2*p-2*q; A(k,k-1) = q; A(k,k+1) = q;
23             A(k,k+(M+1)) = p; A(k,k-(M+1)) = p;
24             b(k,1) = (1-pi^2)*exp(j*dx)*sin(pi*i*dy);
25         end
26     end
27 end
28 uij = A\b;
29 u = reshape(uij, M+1, N+1);
30
31 figure;
32 surf(x,y,u);
33 xlabel('x');
34 ylabel('y');
35 zlabel('u');
36 title('Ex3');
```

针对任务 4，编写 MATLAB 程序如下

```
1  clc;clear;
2
3  X = 1;Y = 1;
4  N = 50;M = 50;
5  dx = X/N;dy = Y/M;
6  x = 0:dx:X;y = 0:dy:Y;
7
8  A = sparse((N+1)*(M+1),(N+1)*(M+1));
9  b = zeros((N+1)*(M+1),1);p = 1/dx^2;q = 1/dy^2;
10 for i = 1:M+1
11     for j = 1:N+1
12         k = (j-1)*(M+1)+i;
13         if(i==1)
14             A(k,k) = 1;b(k,1) = sin(0.25*pi)*sin(3*pi*j*dx+0.25*pi);
15         elseif(i==M+1)
16             A(k,k) = 1;b(k,1) = sin(0.25*pi)*sin(3*pi*j*dx+0.25*pi);
17         elseif(j==1)
18             A(k,k) = -1;A(k,k+(M+1)) = 1;b(k,1) =
19                 ↪ 3*pi*cos(0.25*pi)*sin(2*pi*i*dy+0.25*pi)*dx;
20         elseif(j==N+1)
21             A(k,k) = -1;A(k,k-(M+1)) = 1;b(k,1) =
22                 ↪ -3*pi*cos(0.25*pi)*sin(2*pi*i*dy+0.25*pi)*dx;
23         else
24             A(k,k) = -2*p-2*q;A(k,k-1) = q;A(k,k+1) = q;
25             A(k,k+(M+1)) = p;A(k,k-(M+1)) = p;
26             b(k,1) = -13*pi^2*sin(3*pi*j*dx+0.25*pi)*sin(2*pi*i*dy+0.25*pi);
27         end
28     end
29 end
30 uij = A\b;u = reshape(uij,M+1,N+1);
31
32 figure;
33 surf(x,y,u);
34 xlabel('x');ylabel('y');zlabel('u');
35 title('Ex4');
```

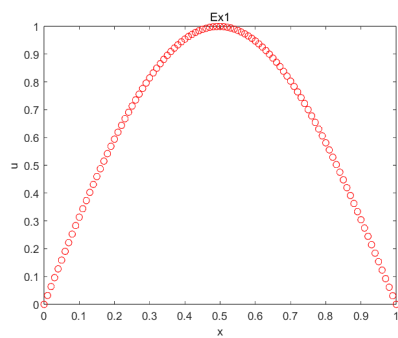


图 1: 程序 1 运行结果

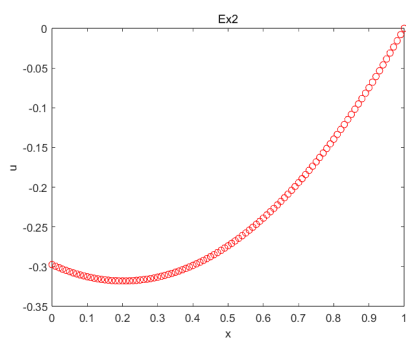


图 2: 程序 2 运行结果

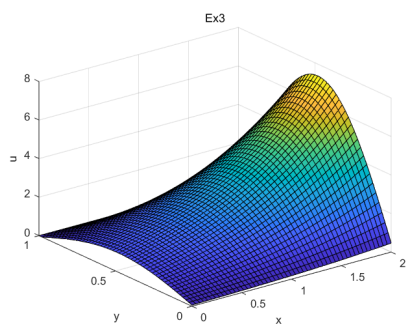


图 3: 程序 3 运行结果

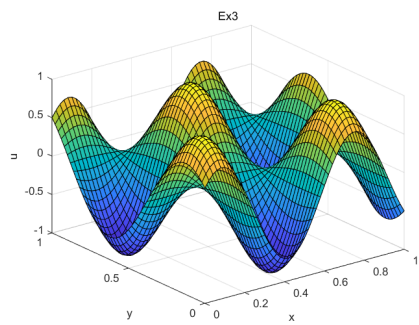


图 4: 程序 4 运行结果

4 实验总结及收获

1. 学习了有限差分法求解一维二维 Poisson 方程;
2. 学习了 Neumann 边界条件的有限差分法处理;
3. 学习了 MATLAB 程序编写。