Journal of Optimization in Electrical Engineering

2018, 1(1), 1-12

A Novel Method on Nonlinear and Adaptive Control of Buck DC to DC Converter

Mahdi Salimi

Department of Eng., Ardabil Branch, Islamic Azad University, Ardabil, Iran

Received 26 February 2018; Received in revised form 25 May 2018; Accepted 27 July 2018

Abstract
In this paper a novel method for nonlinear and adaptive control of buck DC to DC converter is presented. Regardless of wide load variation, proposed controller is capable of regulating output voltage in Continuous Conduction Mode (CCM) and Discontinuous Conduction Mode (DCM). Output Voltage of the converter during DCM depends on ΔT_s (interval where the inductor current is zero). On the other hand, value of Δ largely depends on circuit parameters especially load resistance. Thus the idea of assuming Δ as an uncertain parameter is proposed that will be estimated according to a suitable Lyapunov function. Also it is possible to estimate load resistance and output capacitor of the converter based on the designed controller. In order to design controller in CCM, it is enough to set $\Delta = \mathbf{0}$. For this reason application of similar equations in CCM and DCM results in stable response of the controller in a wide range of operation and soft transition between these two modes. Simulation and experimental results are presented to describe and verify the proposed controller.

Keywords: Buck; Adaptive Control; Converter

Introduction

DC-DC converters are widely used in industrial applications such as computer systems, communication equipment and DC motors drives. These converters are nonlinear with uncertain parameters. Also large disturbances may be imposed to these systems [1]. For example, load resistance may vary in a wide range or input DC voltage include ripple. Variation of load resistance is more important because it could change converter's operation mode. According to switching frequency and relative sizes of load and inductor, DC-DC converter can operate in either Continuous Conduction Mode (CCM) or Discontinuous Conduction Mode. If the operation mode of the converter is changed, model and performance will be completely different. For this reason, controller design that can regulate output voltage of the converter in both CCM and DCM operating modes is challenging and most of the presented controllers are applicable just in one region.

In order to use linear controllers in power electronic circuits, model of the converter must be linearized first. The linearized model could describe behavior of the system satisfactorily around of the operating point. In spite of linear controller's design simplicity, existence of large disturbances or variation of operating mode might cause instability in DC-DC converters [2]. Thus, in a wide operating range, designing of a linear controller and compensating network might be complicated. Basically using a fixed compensating network in DCM is not

Correspondent Author: m.salimi@iauardabil.ac.ir

Doi:

accurate because load resistance should be considered in computations [3]. Compared with CCM, load resistance has dominant influence in DCM and therefore it may significantly affect the output voltage.

In DCM, one of the state variables (inductor current) become zero for a portion of switching period ($\Delta \boldsymbol{T_s}$) that is illustrated in Fig.1 and during this region output voltage is dependent to Δ(that it might vary in a wide range) [4]. Thus in controller design of DC-DC converters, usually control engineers assume that the converter operates in CCM and Most of the presented controllers for DC-DC converters are applicable just in CCM (for example, fuzzy method [5], $\boldsymbol{H}_\infty$[6] and other nonlinear controllers [7]-[10]). Regardless of large disturbances and also presence of uncertain parameters in the model, using nonlinear controllers can significantly improve system's performance. It is possible to use nonlinear sensors and measuring circuits in this case. Most important nonlinear controllers which are applied to DC-DC converters in DCM are: *feedback linearization* [7], *passivity based controller* [8], *sliding mode control* [9] and *adaptive backstepping* [10] and these methods are compared in [11].

In spite of higher switch stress, in DCM DC-DC converters could operate fast and have better dynamic response [3] because smaller inductor could be used. In some applications such as Power Factor Correction (PFC) and efficiency improvement in light loads [12], application of DC-DC converters in DCM has been developed more. Usually a boost DC-DC converter is used for PFC. This converter might enter in DCM for light loads.

Recently, several papers presented on control of DC-DC converters in DCM. For example in [12] a controller is presented for boost converters in DCM. This method is based on sampling converter's state variables and calculation of switch *ON* time. Similar method is applied to the flyback converter in [13] that is based on *peak current control*. In [2] a new modulation method is presented to solve instability problem in DCM operation of a boost converter with Constant Power Loads (CPL). This digital controller – *pulse adjustment*– uses either high or low power pulses instead of conventional PWM. Control of a boost DC-DC converter for PFC is reviewed in [15]. These papers and other similar works are based on small signal modeling of a converter and could not be used when load resistance varies widely (even in DCM). In [3] *Sliding Mode Controller* (SMC) is used to control DC-DC converters in DCM. SMC is well known for simplicity of implementation, Robustness and good dynamic response but switching frequency might vary widely and output voltage ripple might increase [16]. Using a fixed frequency SMC [17] might spoil dynamic response.

Controller design for DC-DC converters is complicated since output voltage depends on Δ and load variation has large effect on controller compared with CCM. Moreover in some applications such as photovoltaic systems, operation mode of converter may vary quickly [18]. Also transition between CCM and DCM may be done in light loads for power conversion efficiency. Therefore design and implementation of a controller which can satisfactorily control DC-DC converter in both modes is completely important. It is possible to use two separate controller for each mode: one controller for CCM and another for DCM [19] and select them according to operating mode. But switching between two different controllers might impose another dynamic to system and certainly in this case dynamic response of the controller during transition between CCM and DCM cannot be satisfactory.

To operate in CCM and DCM simultaneously, a *boundary control* is proposed in [20]. This method could control buck DC-DC converter in both regions but similar to SMC, the main problem of this controller is variation of switching frequency which results in larger filters in switch mode power supplies[21]. In [22] an adaptive method is proposed to tune parameters of the PID controller. This controller has good response during transition from CCM to DCM (or *vice versa*). In this method adaptive tuner is modeled and it adjusts parameters of the controller such that crossover frequency and phase margin match the desired values. In this method, it is possible to vary load resistance widely since parameters of the PID controller are adjusted continuously but it is not possible to regulate output voltage in a wide range due to consideration of linearized small signal models. In this reference, output voltage reference assumed to be +5 volts always.

In this paper a novel adaptive and nonlinear controller is proposed to control buck DC-DC converter in both CCM and DCM. The proposed controller is based on *Adaptive Backstepping* [14], [23]. Adaptive backstepping is based on systematic and recursive method in nonlinear system's feedback design. Despite of input disturbance rejection, ability to estimate uncertain parameters of the system is the most important advantage of this controller. Application of this controller to buck DC-DC converter has been reported recently [24]-[25] but main disadvantages of these papers could be summarized as following:

1- Design process is accomplished in CCM and it is not possible to regulate output voltage of the system in DCM.
2- Input DC voltage of the converter is assumed to be ideal and robustness of the proposed controllers to presence of ripple is not investigated.
3- Response of the controller to wide variation of load resistance in CCM is not studied and the change is just 25 percent.
4- Dynamic response of the controller to load and reference variations has not been investigated.

Averaged State-Space Modeling of Buck DC-DC Converter in DCM (and CCM):

State-space averaging is a general and powerful method for modeling of various systems such as power electronic converters. Application of this technique in DC-DC converters is quite simple. Suppose that a DC-DC convert operates in DCM and inductor's current is shown in Fig.1.

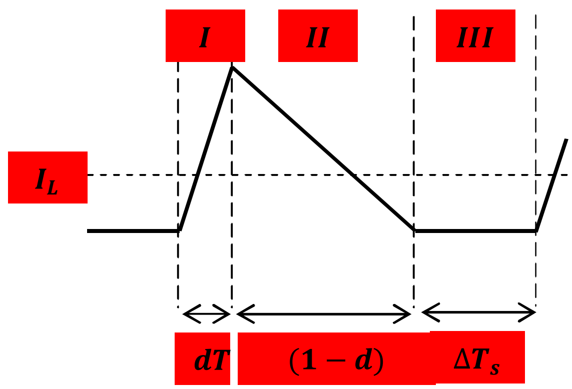


Fig.1 inductor current of a DC-DC converter in DCM

According to three different regions that is shown in Fig.1, in general it is possible to model a DC-DC converter as:

$\dot{X} = A_1X + B_1v_{in}\ [0 \leq t \leq dT_s]$ $\quad$ $(region\ I)$ $\quad$ (1-a)

$\dot{X} = A_2X + B_2v_{in}[dT_s \leq t \leq (1-\Delta)T_s]$ $\quad$ $(region\ II)$ $\quad$ (1-b)

$\dot{X} = A_3X + B_3v_{in}[(1-\Delta)T_s \leq t \leq T_s]$ $\quad$ $(region\ III)$ $\quad$ (1-c)

In buck DC-DC converter – which is shown in Fig.2 -, if the power switch be *ON*, the converter will operate in area I. Similarly in order to transfer system into area II, power switch must be off. Operation of the converter in area III (discontinuous region of inductor current) and Δ can not be controlled directly with system input (usually duty cycle of a power switch is assumed as control input.). Therefore area III shouldn't be considered during state-space modeling of the converter. It is shown that if $\lambda_{max}T_s \ll 1$, DC-DC converters can be modeled as following in DCM [26].

$\dot{\bar{X}} = [dA_1 + (1-d-\Delta)A_2]\bar{X} + [dB_1 + (1-d-\Delta)B_2]v_{in}$ or

$\dot{\bar{X}} = A\bar{X} + Bv_{in}$ $\quad$ (2)

In this equation:

d: control input of the system

$\bar{X}$: averaged state vector,

T_s : switching period

$A_1, A_2, B_1\ and\ B_2$: state equation coefficients

v_{in} : input DC voltage of converter

t : time

λ_{max} : maximum absolute Eigen values of matrix A

It should be mentioned that due to high switching frequency of DC-DC converters, usually the condition $\boldsymbol{\lambda_{max} T_s \ll 1}$ is established. Power circuit of buck DC-DC converter is shown in Fig.(2-a). When switch S is turned on, the circuit that is shown in Fig. (2-b) will be used and Fig.(2-c) during *OFF* state of the power switch.

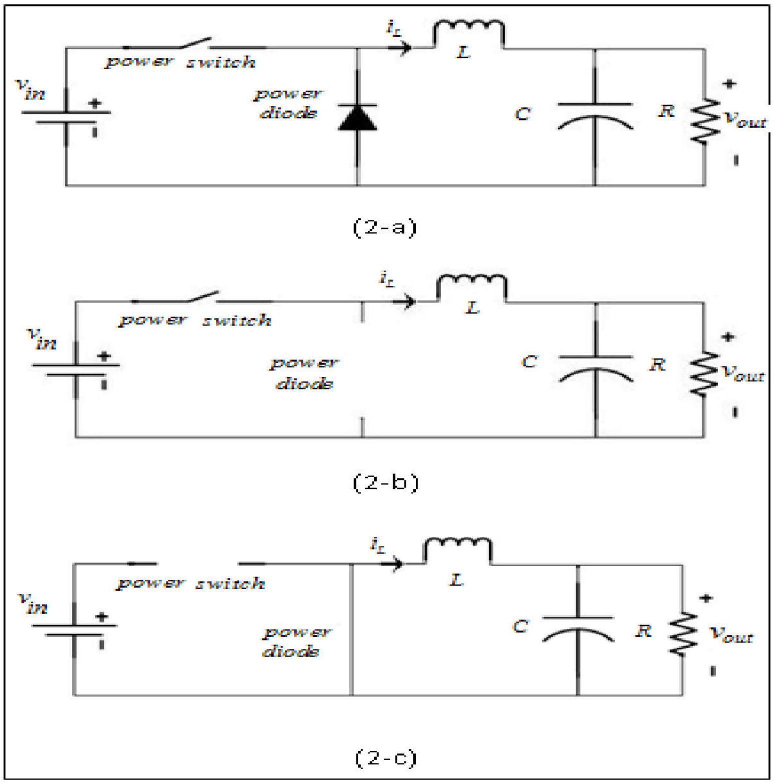


Fig.2-a: *general topology of a buck DC-DC converter*
2-b: equivalent circuit during ON state of power switch
2-c: buck converter when power diode is ON

If we assume inductor current and capacitor (output) voltage as state variables, state-space equations can be easily resulted as following:

$\boldsymbol{X = (i_L, v_C) = (i_L, v_{out}) = (x_1, x_2)}$ (3)

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} v_{in}$$

$\boldsymbol{state - space\ equation\ of\ Fig.(2-b)}$ (4)

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} v_{in}$$

$\boldsymbol{state - space\ equation\ of\ Fig.(3-b)}$(5)

According to equations (1) and (2), averaged state-space model of buck DC-DC converter in DCM could be summarized as following:

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L}(1-\Delta) \\ \frac{1}{C}(1-\Delta) & -\frac{1}{RC}(1-\Delta) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{d}{L} \\ 0 \end{pmatrix} v_{in} \quad (6)$$

These equations show average values of state-variables and for simplicity bar notation is omitted. The following points could be resulted according to equation (6):

(a) In steady-state, dynamic of the converter is zero. In this condition equation (6) results in $V_{out} = \frac{d}{1-\Delta}V_{in}$ according to$\dot{X} = (\emptyset)$ which is completely known equation in buck DC-DC converter operating in DCM [27].

(b) It is clear that Δ has dominant effect in dynamic and steady-state response of the converter. On the other hand, it's value depends on operating point of the circuit and might vary continuously. In this paper a nonlinear and adaptive controller is proposed to estimate Δ that will be completely described in next section.

(c) In equation (6), if we assume Δ to be zero, obtained model will be valid in CCM. In a similar manner, if such an assumption is done in controller's equations, it will be possible to use similar controllers in both operating modes of buck DC-DC converter. This idea will be described completely in the next section.

Proposed Controller

Since DC-DC converters are nonlinear, in this section Adaptive and nonlinear controller - Backstepping - is developed in order to control Buck DC-DC converter in DCM and CCM. The idea of nonlinear and adaptive control was suggested by P.V. Kokotovic in 1991[28]. At first this technique was applicable just to Pure Parametric Form (PPF) systems. Then it was developed to systems that their model is in Parametric Strict Form (PPF). Now this control method includes relatively wide range of systems which are reviewed completely in [29].

In this paper the idea of using adaptive and nonlinear controller is proposed in order to estimate Δ. Since dynamic and steady-state response of the controller is largely depended onΔ, estimated value of this uncertain parameter is used in final control law. Also proposed controller can estimate load resistance and output capacitance of the converter. In fact, three parameters assumed to be uncertain:$\Delta, \boldsymbol{R}$ ***and*** $\boldsymbol{C}$.

A. Rearrangement of state-space model

State-space model of a DC-DC converter in DCM is rewritten as:

$$\dot{x_1} = -\frac{\beta_1}{L}x_2 + \frac{v_{in}}{L}d \quad (7\text{-a})$$

$$\dot{x_2} = \beta_2 x_1 + \beta_3 x_2 \quad (7\text{-b})$$

In this equation, $\boldsymbol{\beta_1}, \boldsymbol{\beta_2}$ ***and*** $\boldsymbol{\beta_3}$ are related to $\Delta, \boldsymbol{R}, \boldsymbol{C}$ and assumed to be uncertain:

$$\beta_1 = 1 - \Delta \,, \beta_2 = \frac{1-\Delta}{C} \,, \beta_3 = -\frac{1-\Delta}{RC} \quad (8)$$

First, uncertain parameters are defined as following:

$$\beta_i = \beta_{in} + \Delta\beta_i \qquad i = 1, 2, 3$$

In this equation, $\boldsymbol{\beta_{in}}$ is nominal value of $\boldsymbol{\beta_i}$ and $\Delta\boldsymbol{\beta_i}$ is its possible variation. For example $\boldsymbol{\beta_2}$ and $\boldsymbol{\beta_3}$ could be written as:

$$\beta_2 = \beta_{2n} + \Delta\beta_2 = \frac{1-\Delta}{C} \Rightarrow \beta_{2n} = \frac{1}{C_n} \text{ and } \Delta\beta_2 = \frac{1-\Delta}{C} - \frac{1}{C_n} (9\text{-a})$$

$$\beta_3 = \beta_{3n} + \Delta\beta_3 = -\frac{1-\Delta}{RC} \Rightarrow \beta_{3n} = -\frac{1}{R_n C_n} \text{ and } \Delta\beta_2 = -\frac{1-\Delta}{RC} + \frac{1}{R_n C_n} \quad (9\text{-b})$$

In these equations, $\boldsymbol{R_n}$ and $\boldsymbol{C_n}$ are nominal values which are used in circuit. $\boldsymbol{R}$and$\boldsymbol{C}$are real values. In order to simplify the design process, we assume $\Delta_n = \boldsymbol{0}$ (CCM). It is clear that if converter enters to DCM region, the controller will estimate it satisfactorily:

$$\beta_1 = 1 - \Delta \Rightarrow \beta_{1n} = 1 \text{ and } \Delta\beta_1 = -\Delta \quad (9\text{-c})$$

B. Adaptive Backstepping controller design in DCM:

Controller design usually completes in several steps.

Step1) At first, new state-space variable is defined as:

$$z_1 = h(x) = x_2 = v_{out} \quad (10\text{-a})$$
$$z_2 = L_{\overline{F}}h(x) \quad (10\text{-b})$$

In this equation, $L_{\overline{F}}h(x)$ is Lie Derivative [30] of function $h(x)$along $\overline{F}$:

$$L_{\overline{F}}h(x) = \begin{pmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \end{pmatrix} \begin{pmatrix} \overline{f_1} \\ \overline{f_2} \end{pmatrix} \quad (11)$$

Matrix F is defined according to state-space model of the buck DC-DC converter:

$$\dot{X} = \overline{F(x)} + \Delta F(x) + G(x)d$$
$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} -\frac{\beta_{1n}}{L}x_2 \\ \beta_{2n}x_1 + \beta_{3n}x_2 \end{pmatrix} + \begin{pmatrix} -\frac{\Delta\beta_1}{L}x_2 \\ \Delta\beta_2 x_1 + \Delta\beta_3 x_2 \end{pmatrix} + \begin{pmatrix} \frac{v_{in}}{L} \\ 0 \end{pmatrix} d \quad (12)$$

In order to change variables according to (10), we should first calculate the following functions:

$$L_{\overline{F}}h(x) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \overline{f_1} \\ \overline{f_2} \end{pmatrix} = \beta_{2n}x_1 + \beta_{3n}x_2 \quad (13\text{-a})$$
$$L_{\overline{F}}^2 h(x) = L_{\overline{F}}(L_{\overline{F}}h(x)) = \beta_{2n}\left(-\frac{\beta_{1n}}{L}x_2\right) + \beta_{3n}(\beta_{2n}x_1 + \beta_{3n}x_2) \quad (13\text{-b})$$
$$L_G(L_{\overline{F}}h(x)) = \beta_{2n}\frac{v_{in}}{L} \quad (13\text{-c})$$

According to (10) and (13), buck DC-DC converter model can be rewritten as below:

$$\begin{pmatrix} \dot{z_1} \\ \dot{z_2} \end{pmatrix} = \begin{pmatrix} z_2 \\ L_{\overline{F}}^2 h(x) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} d_1 + \begin{pmatrix} \theta_1\varphi_1 \\ \theta_2\varphi_1 \end{pmatrix} \quad (14\text{-a})$$
$$\theta_1 = (\Delta\beta_2 \quad \Delta\beta_3) \quad (14\text{-b})$$
$$\varphi_1 = (x_1 \quad x_2)^T \quad (14\text{-c})$$
$$\theta_2 = \left(\beta_{3n}\Delta\beta_2 \quad \beta_{3n}\Delta\beta_3 - \beta_{3n}\frac{\Delta\beta_1}{L}\right) \quad (14\text{-d})$$
$$d_1 = \left(\beta_{2n}\frac{v_{in}}{L}\right)d \quad (14\text{-e})$$

Now, we can define the following reference model in a general form:

$$\begin{pmatrix} \dot{z_{m1}} \\ \dot{z_{m2}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -k_{m1} & -k_{m1} \end{pmatrix} \begin{pmatrix} z_{m1} \\ z_{m2} \end{pmatrix} + \begin{pmatrix} 0 \\ k_{m1} \end{pmatrix} x_2(ref) \quad (15)$$

In this equation, $x_2(ref)$ is the reference voltage of converter z_{m1} and z_{m2} are desired values of state-variables in new coordinate. Error vector is defined:

$$e = (z_1 - z_{m1} \quad z_2 - z_{m2}) = (e_1 \quad e_2) \quad (16\text{-a})$$
$$\dot{e_1} = \dot{z_1} - \dot{z_{m1}} = z_2 + \theta_1\varphi_1 - z_{m2} = e_2 + \theta_1\varphi_1 \quad (16\text{-b})$$
$$\dot{e_2} = \dot{z_2} - \dot{z_{m2}} = L_{\overline{F}}^2 h(x) + d_1 + \theta_2\varphi_1 + k_{m1}z_{m1} + k_{m2}z_{m2} - k_{m1}x_2(ref) \quad (16\text{-c})$$

We could rearrange equations (16-b) and (16-c):

$$\begin{pmatrix} \dot{e_1} \\ \dot{e_2} \end{pmatrix} = \begin{pmatrix} e_2 \\ L_{\overline{F}}^2 h(x) \end{pmatrix} + \begin{pmatrix} \theta_1\varphi_1 \\ \theta_2\varphi_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overline{d} \quad (16\text{-d})$$
$$\overline{d} = d_1 + k_{m1}z_{m1} + k_{m2}z_{m2} - k_{m1}x_2(ref) \quad (16\text{-e})$$

Equations (16-d) and (16-e) shows dynamic of error in this system.

Step2) if we consider equation (16-b), it is clear that we can take e_2 as a *virtual controller* for subsystemee_1.According to backstepping theory, if $\theta_1$is certain parameter and $V_1 = \frac{1}{2}e_1^2$ is considered as a Lyapunov function, virtual controller would be defined as follow:

$$\acute{\alpha} = -k_1e_1 - \theta_1\varphi_1 \quad (17\text{-a})$$

But θ_1 is not certain and e_2 is not a real controller too. Therefor the virtual controller could be defined as follow:

$$\alpha = -k_1e_1 - \widehat{\theta_1}\varphi_1 = -k_1e_1 - \widehat{\Delta\beta_2}x_1 - \widehat{\Delta\beta_3}x_2 \quad (17\text{-b})$$

In this equation, $\widehat{\theta_1}$ is the estimated value of θ_1:

$$\widetilde{\theta_1} = \theta_1 - \widehat{\theta_1} \quad (17\text{-c})$$

Since e_2 and $\alpha$are not necessarily equal, new error variables are defined:

$$\overline{e_1} = e_1 \quad (18\text{-a})$$
$$\overline{e_2} = e_2 - \alpha \quad (18\text{-b})$$

According to (16-d), (17-b) and (17-c), dynamic of new variables could be defined as following:

$\dot{\overline{e_1}} = \dot{e_1} = e_2 + \theta_1\varphi_1 = -k_1\overline{e_1} + \overline{e_2} + \widetilde{\theta_1}\varphi_1$ (19-a)

$\dot{\overline{e_2}} = L_F^2 h(x) + \theta_2\varphi_1 + \overline{d} + k_1\left(-k_1\overline{e_1} + \overline{e_2} + \widetilde{\theta_1}\varphi_1\right) + \dot{\widehat{\theta_1}}\varphi_1$

Or:

$\dot{\overline{e_2}} = L_F^2 h(x) + \widehat{\theta_2}\varphi_1 + \widetilde{\theta_2}\varphi_1 + \overline{d} - k_1^2\overline{e_1} + k_1\overline{e_2} + k_1\widetilde{\theta_1}\varphi_1 + \dot{\widehat{\theta_1}}\varphi_1$ (19-b)

Step3) According to adaptive backstepping theory, Lyapunov function may be defined as following:

$V_a = \frac{1}{2}\bar{e}_1^2 + \frac{1}{2}\bar{e}_2^1 + \frac{1}{2\gamma_1}\widetilde{\theta}_1\widetilde{\theta}_1^T + \frac{1}{2\gamma_2}\widetilde{\theta}_2\widetilde{\theta}_2^T$ (20)

In this equation, γ_1 and γ_2 are positive adaptive gains. Time derivative of Lyapunov function (20) is:

$\dot{V}_a = \overline{e_1\dot{e_1}} + \overline{e_2\dot{e_2}} + \frac{1}{\gamma_1}\widetilde{\theta}_1(-\dot{\widehat{\theta_1}})^T + \frac{1}{\gamma_2}\widetilde{\theta}_2(-\dot{\widehat{\theta_2}})^T$ (21)

In this equation, by substituting $\dot{\overline{e_1}}$ from (19-a), $\dot{\overline{e_2}}$ from (19-b) and considering equation (14), the following expression will be obtained for derivative of Lyapunov function:

$$\begin{aligned}\dot{V}_a = & -k_1\overline{e_1}^2 + \overline{e_2}\left(\overline{e_1} + L_F^2 h(x) + \widehat{\theta_2}\varphi_1 + \overline{d} - k_1^2\overline{e_1} + k_1\overline{e_2} + \dot{\widehat{\theta_1}}\varphi_1\right) \\ & + \widetilde{\Delta\beta_1}\left(\frac{\beta_{3n}\beta_{2n}}{\gamma_2 L}\dot{\widehat{\Delta\beta_3}} - \frac{\beta_{2n}^2}{\gamma_2 L^2}\dot{\widehat{\Delta\beta_1}} - \frac{\beta_{2n}x_2\overline{e_2}}{L}\right) \\ & + \widetilde{\Delta\beta_2}\left(x_1\overline{e_1} + \beta_{3n}x_1\overline{e_2} + k_1\overline{e_2}x_1 - \frac{\dot{\widehat{\Delta\beta_2}}}{\gamma_1} - \frac{\beta_{3n}^2}{\gamma_2}\dot{\widehat{\Delta\beta_2}}\right) \\ & + \widetilde{\Delta\beta_3}\left(x_2\overline{e_1} + \beta_{3n}x_2\overline{e_2} + k_1\overline{e_2}x_2 - \frac{\dot{\widehat{\Delta\beta_3}}}{\gamma_1} - \frac{\beta_{3n}^2}{\gamma_2}\dot{\widehat{\Delta\beta_3}} - \frac{\beta_{3n}\beta_{2n}}{\gamma_2 L}\dot{\widehat{\Delta\beta_1}}\right)\end{aligned} \quad (22)$$

In order to ensure stability of the system, control and adaptive laws should be selected correctly to make derivative of the Lyapunov function (equation 22) negative. In this regard, if assume coefficient of $\overline{e_2}$ is equal to $-k_2\bar{e}_2$ and coefficients of $\widetilde{\Delta\beta_i}$ is zero then equation (22) could be summarized as following:

$\dot{V}_a = -k_1\overline{e_1}^2 - k_2\overline{e_2}^2$ (23)

If k_1 and k_2 are selected positive, it is clear that the derivative of Lyapunov function will be negative.

In order to calculate control law, it is enough to equate coefficient of $\overline{e_2}$ and $-k_2\bar{e}_2$:

$\overline{d} = -k_2\overline{e_2} - \overline{e_1} - L_F^2 h(x) - \beta_{3n}\widehat{\Delta\beta}_2 x_1 - \beta_{3n}\widehat{\Delta\beta}_3 x_2 + \beta_{2n}\frac{\widehat{\Delta\beta_1}}{L}x_2 + k_1^2\overline{e_1} - k_1\overline{e_2} - \dot{\widehat{\Delta\beta}}_2 x_1 - \dot{\widehat{\Delta\beta}}_3 x_2$

(24-a)

It is possible to calculate original input of the controller (d) easily from equations (14-e) and (16-e). Equation (24-a) shows that designed controller is dependent on estimated values of uncertain parameters ($\Delta\beta_i$). Parameter estimation laws could be calculated easily by setting coefficients of $\Delta\beta_i$ in equation (22) to zero:

$\dot{\widehat{\Delta\beta}}_1 = \frac{1}{\frac{\beta_{2n}}{L}\left(\frac{1}{\gamma_1\beta_{3n}} + \frac{2\beta_{3n}}{\gamma_2}\right)}\left[\overline{e_1} + (k_1 - \frac{1}{\gamma_1})\overline{e_2}\right]x_2$ (24-b)

$\dot{\widehat{\Delta\beta}}_2 = \frac{1}{\left(\frac{1}{\gamma_1} + \frac{\beta_{3n}^2}{\gamma_2}\right)}\left[\overline{e_1} + (\beta_{3n} + k_1)\overline{e_2}\right]x_1$ (24-c)

$\dot{\widehat{\Delta\beta}}_3 = \gamma_1\left[\overline{e_1} + k_1\overline{e_2}\right]x_1$ (24-d)

Shortly control law (equation 24-a) and estimation equations of uncertain parameters (24-a,b,c) could control buck DC-DC converter in DCM. During operation of the converter, if Δ is changed, its variation will be estimated. Then estimated value will be used in equation 24-a in order to regulate output voltage of the converter. In this method, load resistance and output capacitance of the converter are also assumed to be uncertain. Variation of these parameters is estimated continuously to improve system's response.

C. is it possible to use this controller in CCM?

In order to design a controller for CCM, the converter should be modeled at first. On the other hand it is clear that during CCM, Δ becomes exactly zero. If Δ is assumed to be zero in equation (6), the same model also could be used in CCM. In fact state-space model in DCM is more general. There for all previous results and equations – which are derived in DCM – could be used in CCM if$\boldsymbol{\Delta = 0}$. Since$\boldsymbol{\Delta}$ is assumed as an uncertain parameter, $\widehat{\boldsymbol{\Delta\beta_1}}$(estimation of $\boldsymbol{\Delta}$) must be assumed zero in CCM. Briefly, the control law (equation 24-a) and estimated parameters (equations 24-b,c) could be used in CCM if $\widehat{\boldsymbol{\Delta\beta_1}}$is forced to zero.

Finally in order to control buck DC-DC converter, at first inductor current is checked to distinguish operating mode of the system. In DCM, uncertain parameters are estimated based on equations 24-b,c,d. Then equation 24-a (controller) is calculated. On the other hand, if converter operates in CCM, $\widehat{\boldsymbol{\Delta\beta_1}}$ assumed to be zero and equation 24-a isnot used. As it is completely studied in appendix, the same process could be used to calculate controller based on equations 24. it is seen that just one controller is used in both CCM and DCM and hence system dynamic during transition between CCM and DCM will be fast and completely stable.

D. stability of designed controller

It is possible to prove stability of the proposed controller based on Barbalat Lemma[30]. Suppose that $\boldsymbol{M(t)}$ is defined as follow:

$$\boldsymbol{M(t) = k_1\bar{e}_1^2 + k_2\bar{e}_2^2} \qquad (25)$$

And:

$$\boldsymbol{V_a(t) = V_a(\bar{e}(0), \tilde{\theta}(0)) + \int_0^t \dot{V}_a(\tau)d\tau = V_a(\bar{e}(0), \tilde{\theta}(0)) - \int_0^t M(\tau)\,d\tau} \qquad (26)$$

In this equation, $\boldsymbol{\bar{e} = (\bar{e}_1, \bar{e}_2)^T}$ and $\boldsymbol{\tilde{\theta} = (\tilde{\theta}_1, \tilde{\theta}_2)^T}$. According to definition of Lyapunov function in (20), it is clear that $\boldsymbol{V_a(t) \geq 0}$ and the following equation could be written according to (26):

$$\boldsymbol{\lim_{t\to\infty} \int_0^t M(\tau)\,d\tau \leq V_a(\bar{e}(0), \tilde{\theta}(0)) \leq \infty} \qquad (27)$$

As a result and according to balbalatle`mma, when $\boldsymbol{t} \to \infty$, then $\boldsymbol{M(t) \to 0}$. For this reason gradually $\boldsymbol{\bar{e}_1}$ and $\boldsymbol{\bar{e}_2}$ will be zero according to (25).

Simulation Result of Applying Designed Controller to a Buck DC-DC Converter

In this section buck DC-DC converter is simulated based on adaptive back stepping controller. Equations 24- b, c and d is used to estimate uncertain parameters ($\boldsymbol{\Delta}$, R and C). Equation 24-s is used to calculate the control input of the system.

A. Converter's elements selection

Converter design process and selection of elements is relatively straightforward. Since output voltage could be regulated in both CCM and DCM regions, L and C could be selected more easily. Parameters – which are used in design process – are listed in table (1). In voltage controlled PWM converters, inductor's size determines current ripple. In CCM, L could be selected according to the following equation:

$$\boldsymbol{L = \frac{V_{out}(1-D_{min})}{\delta I_r f_s I_{out}}} \qquad (28)$$

During converter design, the input voltage ripple is not considered:

$$\boldsymbol{D_{min} = \frac{10}{20} = 0.5 \Rightarrow L = \frac{10(1-0.5)}{0.15 * 18000 * 4} \cong 463\mu H}$$

Also capacitor selection is based on output voltage ripple:

$$\boldsymbol{\frac{\Delta V_{out}}{V_{out}} = \frac{\pi^2}{2}(1-D)\left(\frac{f_c}{f_s}\right)^2 \Rightarrow f_c \cong 444\ Hz} \qquad (29)$$

$\boldsymbol{f_c}$is corner frequency of low-pass LC filter which is used in buck DC-DC converter:

$$\boldsymbol{f_c} = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \boldsymbol{C \cong 274\mu F}$$

It should be mentioned that during practical implementation, we have assumed $\boldsymbol{C = 330\mu F}$.

In boundary of CCM and DCM, averaged inductor current maybe calculated as following:

$$\boldsymbol{I_{LB} = \frac{D}{2Lf_s}(V_{in} - V_{out}) = 0.3A} \qquad (31)$$

Since average currents of inductor and load are equals, load resistance in boundary condition is:

$$R_B = \frac{V_{out}}{I_{out}} = \frac{10}{0.3} = 33.3\Omega$$

It is clear that if load resistance is more than R_B, the converter will operate in DCM and *vice versa*. Of course the designed controller is capable of regulating output voltage in both modes.

The design process that is described in this section is based on operation of converter in CCM, constant input voltage and fixed output voltage. But in order to test the controller completely, these situations will be changed which can lead to changes in current and voltage ripples.

Table (1): parameters which is used in converter design step

1-	*nominal input voltage*(V_{in})	*20 V*
2-	*output voltage* (V_{out})	*10 V*
3-	*switching frequency*(f_s)	*18kHz*
4-	*output current*(I_{out})	*4 A*
5-	*output voltage ripple*(ΔV_o)	*10 mV*
6-	relative current ripple($\delta I_r = \frac{\Delta I}{I_L}$)	0.15

B. Control Block Diagram and Test Criteria in Closed Loop DC-DC Converters:

Proposed adaptive back stepping control is simulated in MATLAB/Simulink based on the following block diagram.

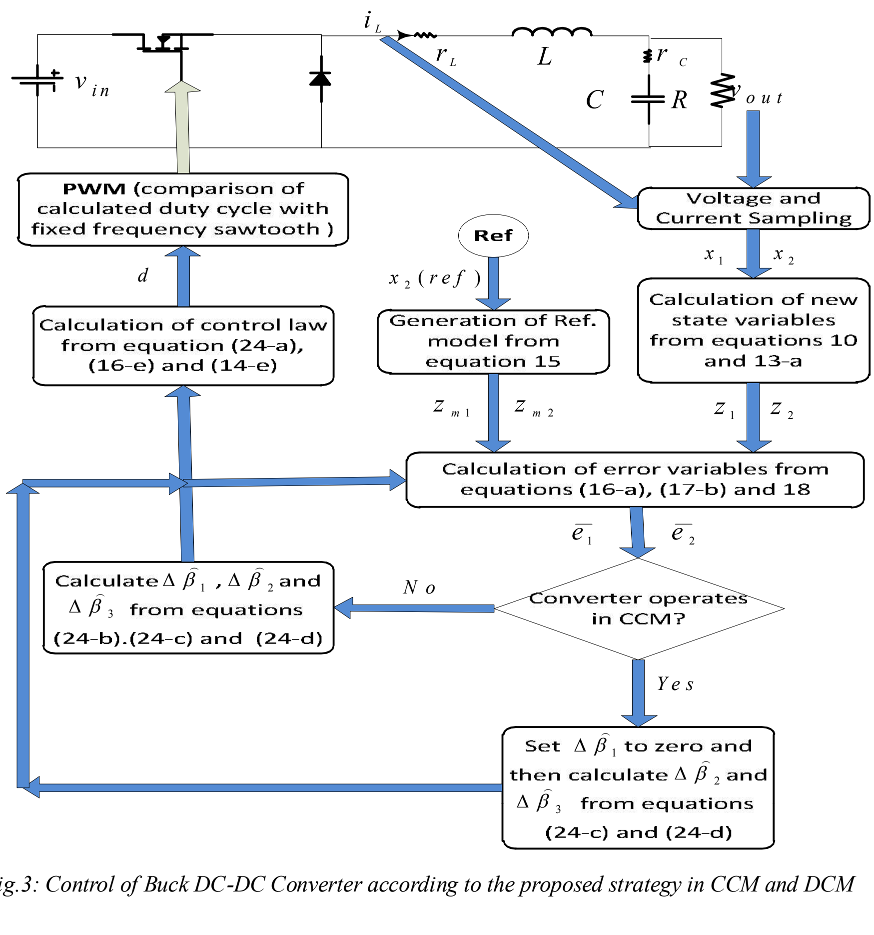


Fig.3: Control of Buck DC-DC Converter according to the proposed strategy in CCM and DCM

Simulation parameters are listed in table (2)

Table (2): simulation parameters

1-	*Input voltage* (v_{in}):	*20 V*
2-	*Converter inductor (L):*	*470μH*
3-	*Capacitor* (C):	*330μF*
4-	*Switching frequency* (f_s):	*18kHz*
5-	*Adaptive gains* (γ_1, γ_2):	*2e-4*
6-	*Inductor series resistance*(r_L)	*0.1Ω*
7-	*Capacitor series resistance*(r_C)	*0.02 Ω*
8-	k_1 *(controller gain):*	*16e3*
9-	k_2*(controller gain):*	*2e3*
10-	k_{m1}	*10e6*

Since the proposed controller is able to regulate the output voltage of the converter in a wide range of input voltage disturbances, load resistance and output reference variations, proper test criteria must be done to ensure overall capability of the system. The following criteria will be used to test the proposed controller:

1- Steady-state error and output voltage ripple.
2- Step response of the controller to reference variation and analysis of rise time, fall time, overshoot, and settling time.
3- Controller robustness to load and line variation. Quality of response during transition between CCM and DCM could be considered here.
4- Switching frequency variation. However in converters which are based on PWM and fixed frequency triangular waveform is used, this item is not important. Sometimes – for example sliding mode control and hysteresis current control- switching frequency may vary. In these cases an additional circuit should be employed to limit variation rang of the frequency.
5- Effect of controller's gains variation on steady state and dynamic response of the system.

C. Steady-state response of the controller

Generally response of the controller (and especially steady-state error in the proposed controller) largely depends on controller's gain. Because there is no integral action in the structure of the controller, we have to adjust controller's gains in order to eliminate errors. For example, steady-state relative error of the controller for different values of k_1 is plotted in Fig.4. According to steady-state and dynamic (which will be presented in the next section) responses, k_1 is assumed to be 16000.

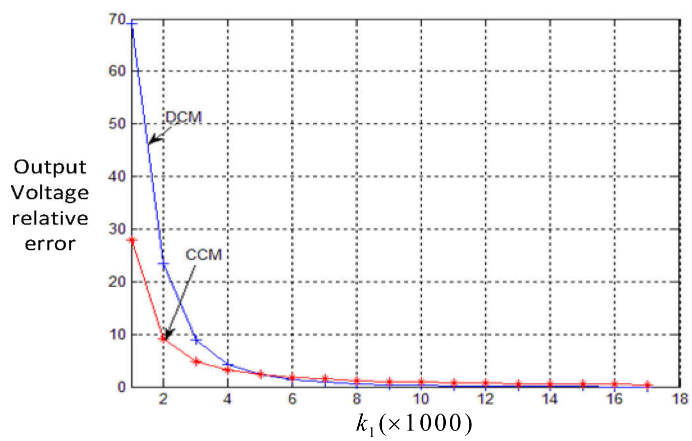
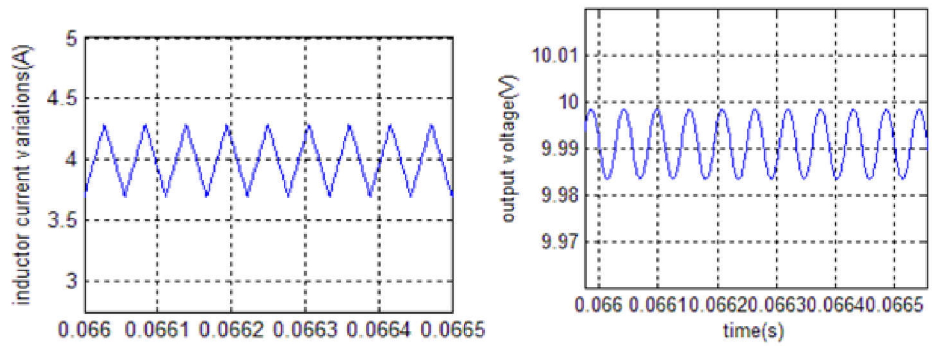


Fig.4: output voltage relative error ($\frac{reference-output\ voltage}{reference} \times 100\%$) *versus* k_1.
Simulation parameters are listed in table 2. Reference voltage is assumed to be 10 V. In CCM, load resistance is 14 ohms and during DCM 50ohms. It is clear that for $k_1 \geq 12000$ *, steady-state error is approximately zero.*

In Fig.5 inductor current variation and output voltage ripple (b) are illustrated in (a) and (b). The ripple values satisfy equations (28)-(30) which were used in choosing L and C.

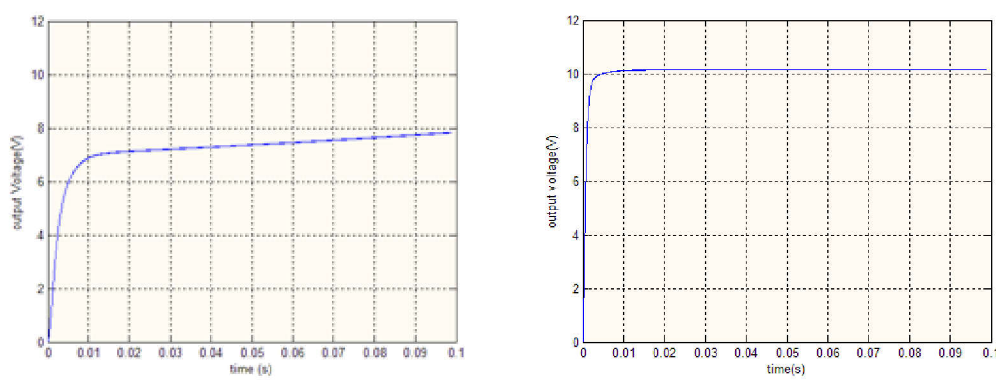


(a) *(b)*

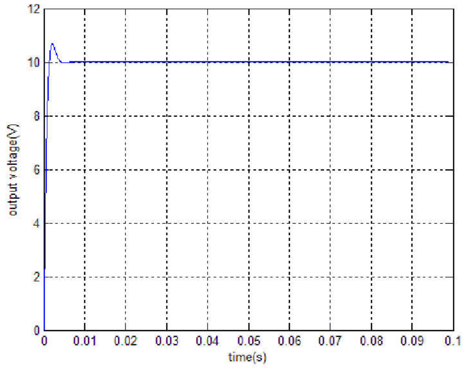
Fig (5): inductor current variation (a) and output voltage ripple (b) during steady-state. Load resistance is assumed 2.5 ohms and output reference voltage is 10 V. other simulation parameters are based on table (2).

D. *Dynamic Response of the Converter*

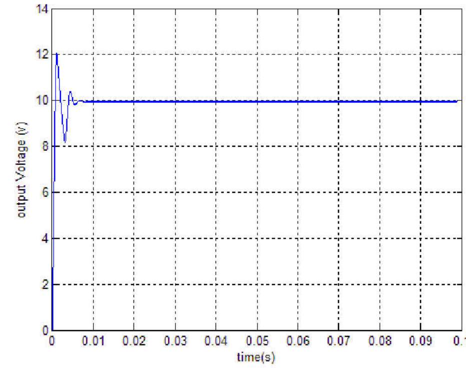
During start-up, Output voltage of the converter is plotted in Fig. 6 for different values of k_1 in CCM and DCM. It is clear that increment of k_1 may improve rise time, but on the other hand settling time and overshoot of the response will be deteriorated. For this reason k_1 is assumed to be 16000. Output voltage of the system can be regulated in a wide range and it is not limited to 10 V. For example in Fig.7 reference of the system is stepped up from 5 to 10. Load resistance is selected 30 ohms. When output voltage is 5 v, the converter operates in DCM. Increment in load voltage will result in more load and inductor current. Thus after an increment in reference, the converter will operate in CCM.



(a) $k_1 = 2000$ (b) $k_1 = 4000$

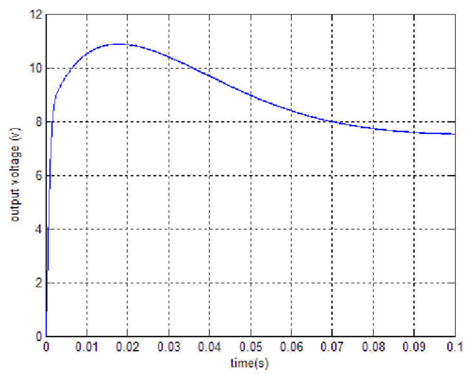


(c) $\boldsymbol{k_1 = 16000}$

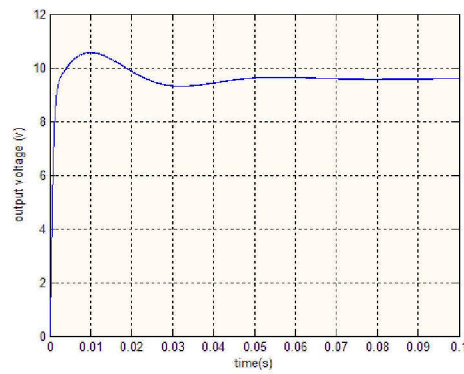


(d) $\boldsymbol{k_1 = 18000}$

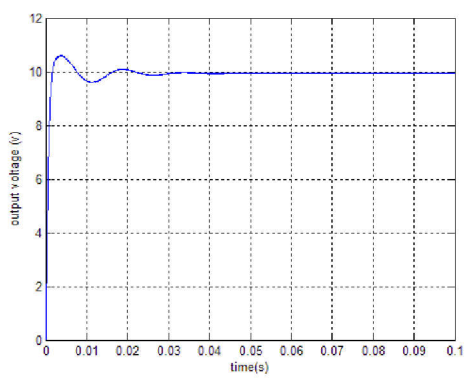
Fig.6_1: dynamic response of the controller for different values of $k_1$in CCM. Output reference voltage is 10 V and load resistance is assumed to be 14 ohms.



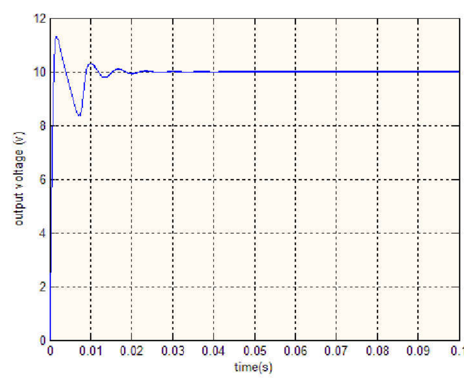
(a) $\boldsymbol{k_1 = 2000}$



(b) $\boldsymbol{k_1 = 4000}$

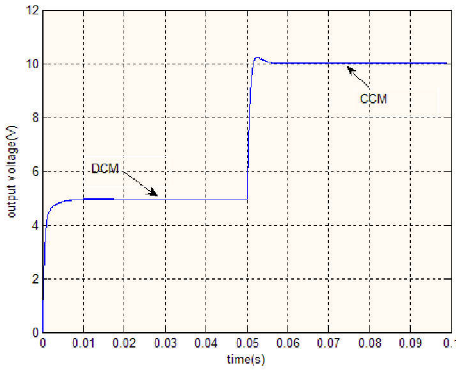


(c) $\boldsymbol{k_1 = 16000}$

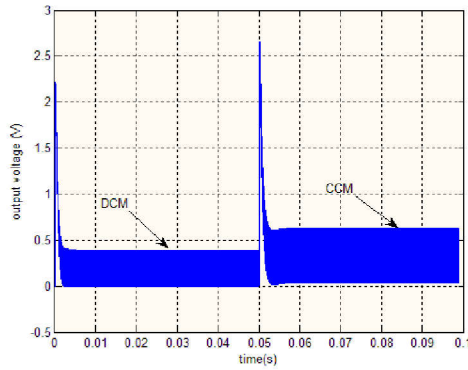


(d) $\boldsymbol{k_1 = 18000}$

Fig.6_2 (continuance): dynamic response of the controller for different values of $k_1$in DCM. Outputreference voltage is 10 V and load resistance is assumed to be 50 ohms.



(a)



(b)

Fig.7: step response of the controller and transition from DCM to CCM. (a) output voltage (b)inductor current Load resistance is assumed to be 30 ohms. At t=0.05s voltage reference is changed from 5 V to 10 V. It is clear that controller has good dynamic and steady-state response.

E. Robustness Analysis

Load variation: During operation of converter, load resistance may vary several times. This is very important from control point of view, since it may change operating mode of the converter. In Fig.8 output voltage and inductor current are plotted. In this fig. load resistance in increased from 25 ohms (CCM) to 50 ohms (DCM) at t=0.05s.It is clear that the proposed controller responses satisfactorily for different values of load resistor. Such results will be presented in next the section.

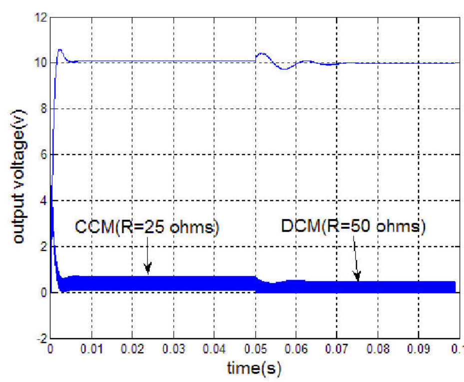
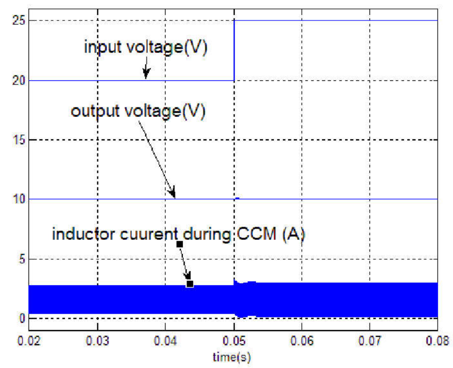
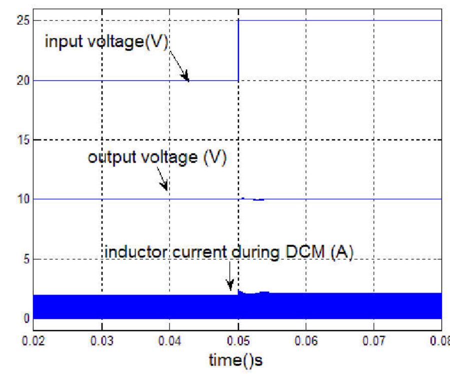


Fig.8: controller response to load variation. At t=0.05 load resistance is changed from 25 ohms to 50 ohms. Dynamic and transient response is satisfactorily during transition.

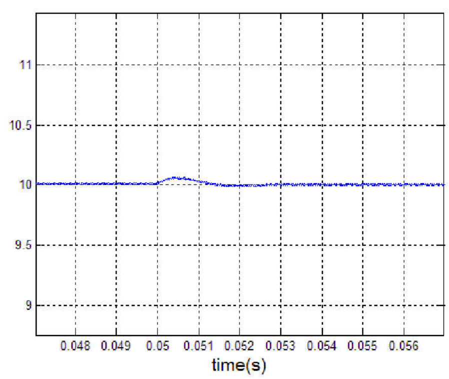
Line variation: usually input voltage of DC-DC converters is provided through a bridge rectifier and filter. For this reason variation of input voltage is completely possible and robustness of the controller is highly important. The response of the controller is plotted in Fig.9 for both operating modes. Controller robustness to presence of voltage ripple will be presented in the next section. Also effect of output capacitor variation will be illustrated in experimental results.



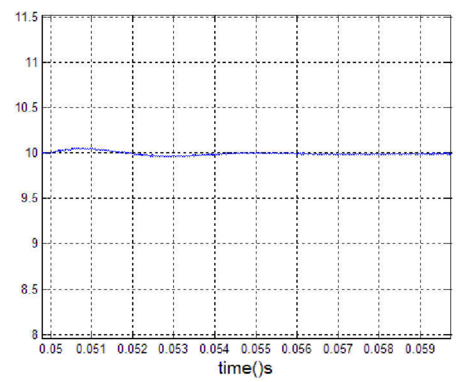
(a) CCM



(b) DCM



(c) output voltage ripple(CCM)



(d)output voltage ripple(DCM)

Fig 9: robustness of controller to line variations.
Input voltage of converter is increased from 20 V to 25 V at t=0.05s and simulation results are presented in CCM (a,c) and DCM(b,d). During CCM load resistance is assumed to be 25 ohms and during DCM is 50. Output voltage ripple is plotted in (c) ,(d).

Experimental Results

Buck DC-DC converter based on the adaptive backstepping controller is practically implemented to test response of the proposed method. In this section, simulation parameters (table 2) are used again to implement converter and controller. Laboratory set-up is shown in Fig.10. Texas DSP (TMS320F2810) is used to calculate equations of the controller. State variables of the converter are measured whit isolated sensors. For example IL300 is used for voltage measurement and Hall Effect current sensors for the next state variable. Then these analog signals are converted to 12-bit digital samples by internal ADC of processor. Since switching frequency of the converter is 18 kHz, sampling frequency of analog signals assumed to be 260 kHz. In this case, it is possible to sample about 15 point in each switching period. The processor is fast enough to update controller and estimated variables after each sampling (based on equation 24). Flowchart of developed software is completely similar to Fig.3. In order to investigate controller's response to parameters variations, power circuit of Fig.11 is used. Switch Q3 is used to change equivalent load of the converter and test robustness of the controller. Also switch Q2 is used to change output capacitor. In order to test robustness of the controller in presence of input voltage ripple, v_{in}could be implemented though diode rectifier.

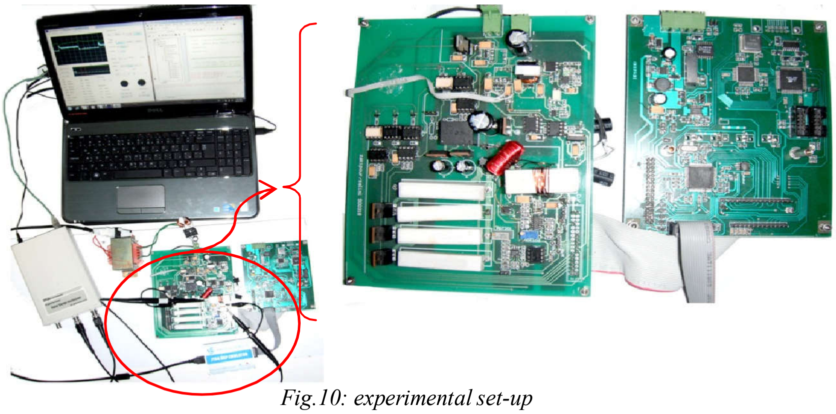


Fig.10: experimental set-up

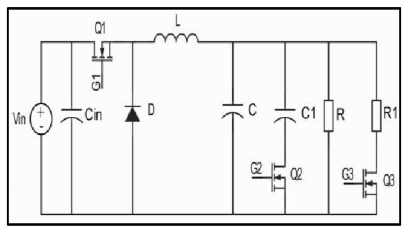
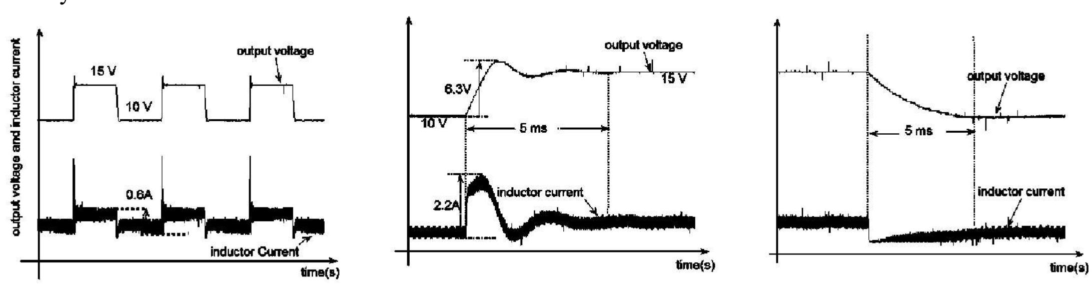
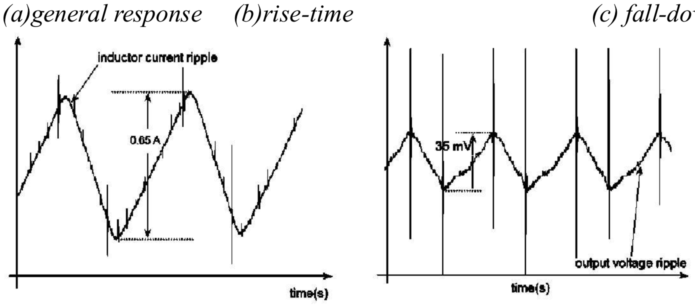


Fig. 11 : topology of implemented power circuit.
Load variation is done with switch Q3.

In order to investigate response of the proposed controller, in Fig.12 a step reference which varies periodically between 10 V and 15 V, is applied to the system and general response is given in Fig. 12.a. Such a study will be useful for dynamic analysis (and measurement of response rise-time, fall time, settling time and …). But we should consider that due to the large measuring time used in Fig.12-a, sampling frequency will be very low and it is not possible to study transient response exactly in this case. In Fig.12-b and 12-c, rise and fall time of the response is illustrated. Also output voltage ripple and inductor current variation is plotted in Fig.12-d and Fig. 12-e respectively.



(a)general response *(b)rise-time* *(c) fall-down*



(d) inductor current ripple *(e)output voltage ripple*

Fig. 12: Step response of the controller
Reference voltage of the systems between 10 and 15 continuously in order to study dynamic response of the controller

Response of the controller to load variation is illustrated if Fig.3. In this case output voltage of the converter assumed to be 10 V and it must be constant in spite of load variations. In Fig.13-a initial value of load resistance is 5.83 which is changed to 14 ohms. Both of selected load resistor result in CCM. It is clear that converter has good robustness to load variation in CCM.

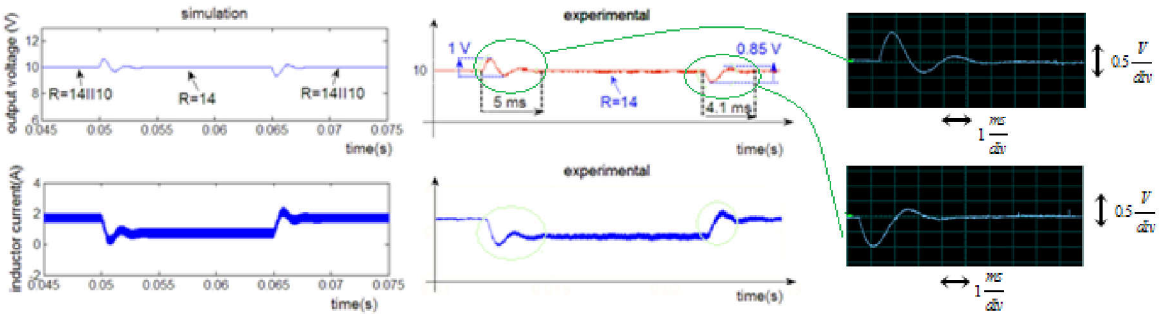


Fig. 13-a: transient response of the system during load variation in CCM

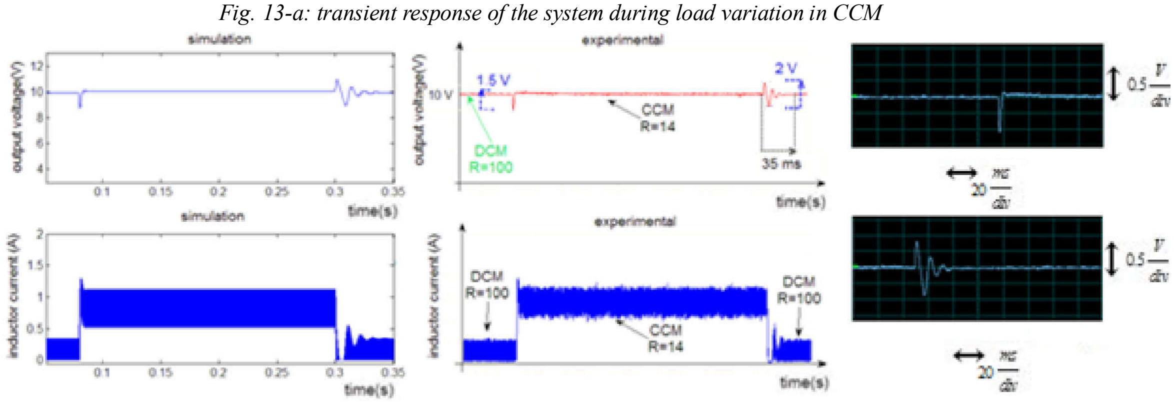


Fig 13-b: transient response of the system during transition between CCM and DCM

In fig 13-b response of the controller during variation of converter's operating mode is illustrated. In DCM load resistance is 100 ohms. In order to shift the system into CCM, load resistance is changed to 12.28 ohm. In spite of large load variation and transition between DCM and CCM, controller is completely stable. Finally, in fig.13-c load resistance is changed between 50 and 100 ohms. In this case controller will operate in DCM.

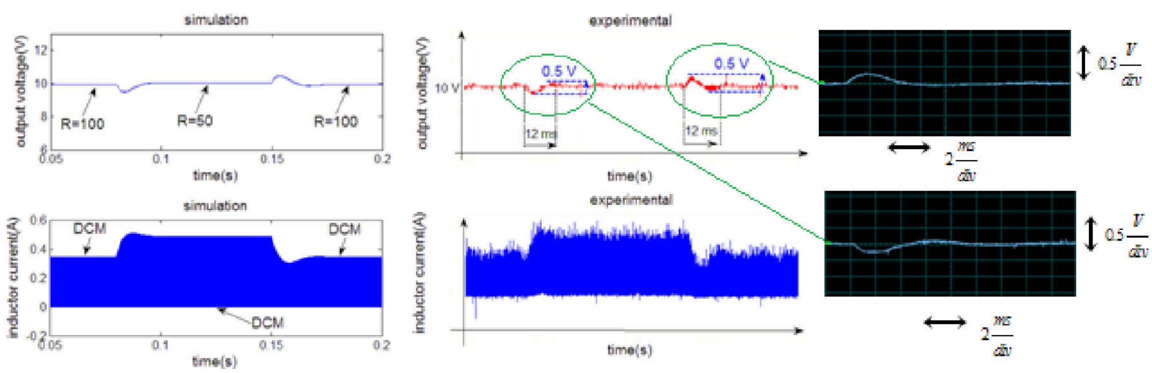


Fig.13-c stability of the proposed controller to load variation in DCM

Robustness of the proposed controller to input voltage variations is plotted in Fig.14. A diode rectifier and Capacitor filter (1mF) is used to implement input voltage. In Fig.14-a voltage reference of the Buck DC-DC converter is assumed to be 5 V and load resistance is 3 ohms. It is clear that in spite of relatively large voltage ripple, output of the system is acceptable. In order to complete the robustness test of the controller, during variation of input voltage, load resistance and operating mode of the converter is changed in Fig.14-b,c. Again output voltage reference is 5 volt. Load resistance in DCM is 100 ohms and in CCM assumed to be 3 ohms.

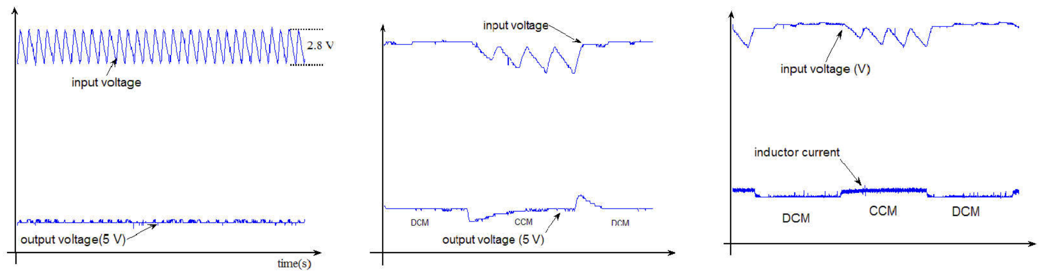


Fig. 14: Robustness analysis of the proposed controller. In a,b and reference voltage of the system is 5 V. Load resistance is assumed to be 3 ohms. Although relatively large ripple is present in the input voltage of the converter, output of the controller is completely stable.
(b),(c) Load resistance is changed between DCM(R=100 ohms) and CCM(R=3 ohms). It is clear that, in spite of large variation of load resistance (which results in transition between DCM and CCM) and relatively large voltage ripple of the input, output of the system is acceptable.

The response of the controller to simultaneous changes in the input voltage, load resistance and output capacitance of the converter is presented in Fig.15. In DCM load resistor is 100 ohms. Periodically a 3.3ohms resistor and 1mF capacitor is switched to the converter. Again input voltage ripple is present especially in CCM. During DCM power consumption of the converter is relatively small and for this reason voltage ripple of the converter could be neglected (similar to Fig14-b, c).

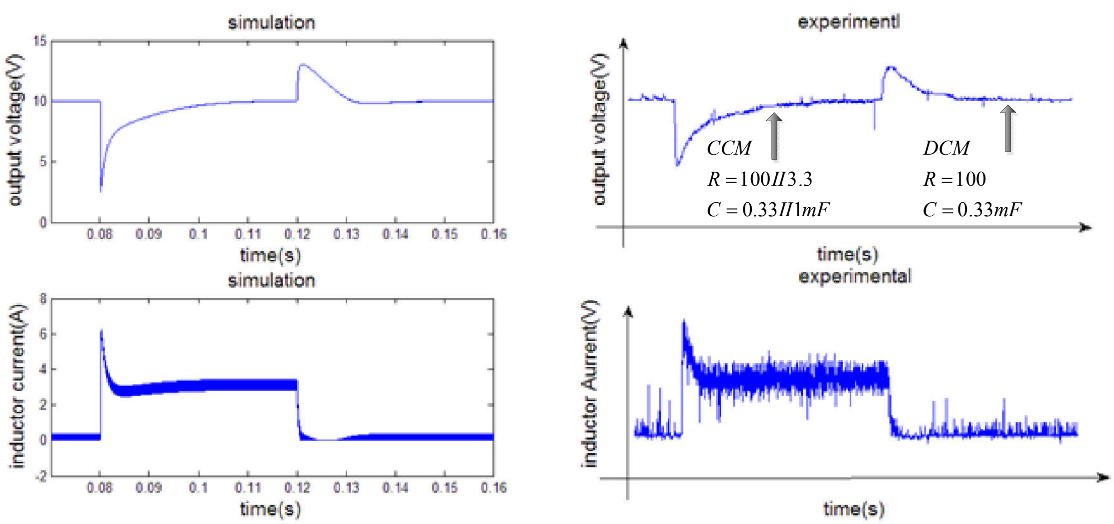


Fig.15: output voltage and inductor current of the proposed controller during simultaneous variation of load resistance, input voltage and output capacitor

Further Discussion

In this section advantages and disadvantages of proposed control strategy is summarized.

A. Advantages

The proposed controller is applicable to buck DC-DC converter in both operating mode: CCM and DCM. This is the main advantage of the proposed controller. Simulation and experimental results clearly show than response of the system during transition from CCM to DCM (or vice versa) is completely satisfactorily. **The** second advantage of this method is the possibility of regulating output voltage with a wide range of load resistance. If during variation of load resistance, operating mode is changed consequently, again stability of system will be maintained. Also robustness of this switching converter to line voltage ripple and output capacitor is acceptable. Finally, due to the development of nonlinear controller which is based on non-linearized model of the converter, it is possible to regulate output voltage in a wide range.

B. Disadvantages

Presence of steady-state error is the main problem of the proposed controller but as simulation and experimental results clearly illustrated, an acceptable response could be achieved with an adjustment of controller's parameters. In order to solve this problem, an integrator action should be added to the controller.

C. Future Work

To improve the response of the controller to the input voltage ripple, it is possible to assume $\boldsymbol{v_{in}}$ as an uncertain parameter and estimateit's variation based on the Adaptive backstepping strategy. Due to the core magnetic saturation, inductor value may vary widely. It is possible to estimate variation of this uncertain parameter similarly. It is possible to consider parasitic element during modeling of the converter. For example Equivalent Series Resistance (ESR) of inductor and output capacitor could be modeled to improve performance of the controller. Also more accurate models of power switches may be used. Application of integrator based backstepping may solve steady-state error of the converter.

Conclusion

Design process of Adaptive Backstepping controller in buck DC to DC converter is analyzed and described in details. Control law and estimation rules of uncertain parameters are obtained. Simulation and experimental results clearly show that proposed controller could be employed in both operating modes and in spite of large variation of load resistance, output capacitor and input voltage system is completely stable. Digital signal processor is used to implement and verify accuracy of proposed nonlinear and adaptive controller.

References:

[1] Sudip K. Mazumder, Ali H. Nayfeh, and DushanBorojevic´, "Robust Control of Parallel DC–DC Buck Converters by Combining Integral-Variable-Structure and Multiple-Sliding-Surface Control Schemes" IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 17, NO. 3, MAY 2002, pp.428-437

[2] AlirezaKhaligh, Amir M. Rahimi, and Ali Emadi,, "Negative Impedance Stabilizing Pulse Adjustment Control Technique for DC/DC Converters Operating in Discontinuous Conduction Mode and Driving Constant Power Loads" IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, VOL. 56, NO. 4, JULY 2007 2005, pp.2005 -2016

[3] S.-C. Tan, Y.M. Lai, C.K. Tse and L. Martinez -Salamero "Special family of PWM-based sliding-mode voltage controllers for basic DC–DC converters in discontinuous conduction mode" IET Electr. Power Appl., Vol. 1, No. 1, January 2007, pp.64-74

[4] C. Sreekumar and V. Agarwal "Hybrid control approach for the output voltage regulation in buck type DC–DC converter" IET Electr. Power Appl., 2007, 1, (6), pp. 897–906

[5] Paolo Mattavelli, Leopoldo Rossetto, Giorgio Spiazzi, and Paolo Tenti, "General-Purpose Fuzzy Controller for DC–DC Converters" IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 12, NO. 1, JANUARY 1997, pp.79-86

[6] Andreas Kugi, and Kurt Schlacher, "Nonlinear -$\mathbf{H}_\infty$Controller Design for a DC-to-DC Power Converter" IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, VOL. 7, NO. 2, MARCH 1999, pp.230-237

[7] Jinbo Liu, Wenlong Ming, Fanghong Gao "A New Control Strategy for Improving Performance of Boost DC/DC Converter Based on Input-Output Feedback Linearization" Proceedings of the 8th world

Congress on Intelligent Control and Automation, Jinan, China, July 6-9 2010

[8] R. Leyva, A. Cid-Pastor, C. Alonso, I. Queinnec, S. Tarbouriech and L. Martinez-Salamero "Passivity-based integral control of a boost converter for large-signal stability" IEE Proc.-Control Theory Appl., Vol. 153, No. 2, March 2006, pp.139-146

[9] Siew-Chong Tan, Y. M. Lai, Martin K. H. Cheung, and Chi K. Tse,"On the Practica l Design of a Sliding Mode Voltage Controlled Buck Converter" IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 20, NO. 2, MARCH 2005, pp.425-437

[10] Li-kui YI, Jun ZHAO, Dan MA "Adaptive Backstepping Sliding Mode Nonlinear Control for Buck DC/DC Switched Power Converter" 2007 IEEE International Conference on Control and Automation, Guangzhou, CHINA - May 30 to June 1, 2007

[11] G. Escobar, R. Ortega, H. Sira-Ramirez, J-P.Vilain, and I. Zein" An Experimental Comparison of Several Nonlinear Controllers for Power Converters" IEEE Control Systems , 02 72- 1708/99/$10.00 1999IEEE , pp.66-8

[12] M. Ferdowsi, and A. Emadi, "Estimative Current Mode Control Technique for DC–DC Converters Operating in Discontinuous Conduction Mode" IEEE POWER ELECTRONICS LETTERS, VOL. 2, NO. 1, MARCH 2004, pp.20-23

[13] Keng C. Wu "A Comprehensive Analysis of Current-Mode Control for DCM Buck-Boost Converters" IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, VOL. 51, NO. 3, JUNE 2004, pp.733-735

[14] Khalil, H.K.: in 'Nonlinear systems' (Prentice-Hall, 1996), 2nd Edn

[15] H.S. Athab "Control Strategy for Discontinuous Conduction Mode Boost Rectifier with Low Total Harmonic Distortion and Improved Dynamic Response" American J. of Engineering and Applied Sciences 1 (4): 329-337, 2008, pp.329-337

[16] Ming Qin and Jianping Xu "Multiduty Ratio Modulation Technique for Switching DC–DC Converters Operating in Discontinuous Conduction Mode" IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, VOL. 57, NO. 10, OCTOBER 2010, pp.3497-3507

[17] Siew-Chong Tan, Member, Y. M. Lai, Member,Chi K. Tse, and Martin K. H. Cheung, "Adaptive Feedforward and Feedback Control Schemes for Sliding Mode Controlled Power Converters" IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 21, NO. 1, JANUARY 2006, pp.182 -192

[18] Sun, J., Mitchel, D. M., Greuel, M. F., Krein, P. T., Bass, R. M., "Averaged Modeling of PWM Converters Operating in Discontinuous Conduction Mode", IEEE Transactions on Power Electronics, vol. 16, 2001, pp. 482 -492

[19] J.I.Castillejo, M.Garcia-Sanz, "Robust Control of an Ideal DCM/CCM Flyback Switching Converter" Proceeding of the 10th Mediterranean Conference Control and Automation, MED2002, Lisbon, Portugal, July 9-12, 2002

[20] Kelvin Ka-Sing Leung, and Henry Shu-hung Chung, "A Comparative Study of Boundary Control With First- and Second-Order Switching Surfaces for Buck Converters Operating in DCM" IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 22, NO. 4, JULY 2007, pp.1196-1209

[21] Y. He and F.L. Luo "Design and analysis of adaptive sliding-mode-like controller for DC–DC converters" IEE Proc.-Electr. Power Appl., Vol. 153, No. 3, May 2006

[22] Jeffrey Morroni, Luca Corradini, Regan Zane, and Dragan Maksimovi´c, "Adaptive Tuning of Switched-Mode Power Supplies Operating in Discontinuous and Continuous Conduction Modes" IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL. 24, NO. 11, NOVEMBER 2009, pp.2603 -2611

[23] A. JafariKoshkouei and A. S. I. Zinober "Adaptive Sliding Backstepping Control of Nonlinear Semi-Strict Feedback Form Systems" Proceedings of the 7th Mediterranean Conference on Control and Automation (MED99) Haifa, Israel - June 28-30, 1999, pp. 2376-2383

[24] H.ElFadil and F.Giri" Backstepping Based Control of PWM DC-DC Boost Power Converters" International Journal of Electrical and Power Engineering 1(5): Medwell Journals, 2007, pp.479-485

[25] Shui-Chun Lin and Ching-Chih Tsai "ADAPTIVE BACKSTEPPING CONTROL WITH INTEGRAL ACTION FOR PWM BUCK DC-DC CONVERTERS" Journal of the Chinese Institute of Engineers, Vol. 28, No. 6, 2005, pp. 977-984

[26] S. Cuk and R. D. Middlebrook, "A general unified approach to modeling switching DC-to-DC converters in discontinuous conduction mode," in Proc. IEEE PESC'77, 1977, pp. 36–56.

[27] Mohan, N., Undeland, T.M., and Robbins, W.P.: 'Power electronics -converters, applications and design' (John Wiley and Sons, Inc., Singapore, 2003, 3rd edn.)

[28] Petar V. Kokotovic "The Joy of Feedback: Nonlinear and Adaptive" IEEE Control Systems, Bode Prize lecture, june 1992, pp.7-17

[29] M. Krstic, I. Kanellakopoulos and P. V. Kokotovic, Nonlinear and adaptive control design. New York: John Willy & Sons, 1995.

[30] Hsin-Jang Shieh and Kuo-Kai Shyu "Nonlinear Sliding-Mode Torque Control with Adaptive Backstepping Approach for Induction Motor Drive" IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, VOL. 46, NO. 2, APRIL 1999, pp.380 - 389

Appendix: Adaptive Backstepping controller Design in CCM

During CCM,Δ is zero and according to equation 7 and 8, model could be simplified as following:

$$\dot{x_1} = -\frac{1}{L}x_2 + \frac{v_{in}}{L}d \qquad \text{(A-1)}$$

$$\dot{x_2} = \beta_2 x_1 + \beta_3 x_2 \qquad \text{(A-1)}$$

In this equation, β_2 *and* β_3 are related to R, C and assumed to be uncertain:

$$\beta_2 = \frac{1}{C}, \beta_3 = -\frac{1}{RC} \qquad \text{(A-3)}$$

$$\beta_2 = \beta_{2n} + \Delta\beta_2 = \frac{1}{C} \Rightarrow \beta_{2n} = \frac{1}{C_n} \ \ and\ \Delta\beta_2 = \frac{1}{C} - \frac{1}{C_n} \qquad \text{(A-4)}$$

$$\beta_3 = \beta_{3n} + \Delta\beta_3 = -\frac{1}{RC} \Rightarrow \beta_{3n} = -\frac{1}{R_n C_n} \ \ and\ \Delta\beta_2 = -\frac{1}{RC} + \frac{1}{R_n C_n} \qquad \text{(A-5)}$$

$$\begin{pmatrix}\dot{x_1}\\ \dot{x_2}\end{pmatrix} = \begin{pmatrix}-\frac{1}{L}x_2\\ \beta_{2n}x_1 + \beta_{3n}x_2\end{pmatrix} + \begin{pmatrix}0\\ \Delta\beta_2 x_1 + \Delta\beta_3 x_2\end{pmatrix} + \begin{pmatrix}\frac{v_{in}}{L}\\ 0\end{pmatrix}d \qquad \text{(A-6)}$$

Equation 14 will be the same except 14-d which will be summarized as following:

$$\theta_2 = (\beta_{3n}\Delta\beta_2 \quad \beta_{3n}\Delta\beta_3) \qquad \text{(A-7)}$$

Equation 14 to 21 could be used similarly. Derivative of Lyapunov function (equation 22) will be:

$$\dot{V}_a = -k_1\overline{e_1}^2 + \overline{e_2}\left(\overline{e_1} + L_F^2 h(x) + \widehat{\theta_2}\varphi_1 + \overline{d} - k_1^2\overline{e_1} + k_1\overline{e_2} + \dot{\widehat{\theta_1}}\varphi_1\right) + + \widetilde{\Delta\beta}_2\left(x_1\overline{e_1} + \beta_{3n}x_1\overline{e_2} + k_1\overline{e_2}x_1 - -\frac{\beta_{3n}^2}{\gamma_2}\dot{\widehat{\Delta\beta}}_2\right) + \widetilde{\Delta\beta}_3\left(x_2\overline{e_1} + \beta_{3n}x_2\overline{e_2} + k_1\overline{e_2}x_2 - \frac{\dot{\widehat{\Delta\beta_3}}}{\gamma_1} - \frac{\beta_{3n}^2}{\gamma_2}\dot{\widehat{\Delta\beta}}_3\right) \text{(A-8)}$$

control law and estimation rules of Buck DC-DC converter during CCM is:

$$\overline{d} = -k_2\overline{e_2} - \overline{e_1} - L_F^2 h(x) - \beta_{3n}\widehat{\Delta\beta}_2 x_1 - \beta_{3n}\widehat{\Delta\beta}_3 x_2 + k_1^2\overline{e_1} - k_1\overline{e_2} - \dot{\widehat{\Delta\beta}}_2 x_1 - \dot{\widehat{\Delta\beta}}_3 x_2$$

(A-9)

$$\dot{\widehat{\Delta\beta}}_2 = \frac{1}{\left(\frac{1}{\gamma_1} + \frac{\beta_{3n}^2}{\gamma_2}\right)}[\overline{e_1} + (\beta_{3n} + k_1)\overline{e_2}]x_1 \qquad \text{(A-10)}$$

$$\dot{\widehat{\Delta\beta}}_3 = \gamma_1[\overline{e_1} + k_1\overline{e_2}]x_1 \qquad \text{(A-11)}$$

In order to control buck DC-DC converter in CCM, equations (A-9), (A-10) and (A-11) should be used. However it will be possible to useequations (24-b), (24-c) and (24-d), if sett$\widehat{\Delta\beta}_1$.=0.