Linear Regression

Data (input): $(Y_i, X_{1,i}, \ldots, X_{p-1,i})$ $(i = 1, \ldots, n)$

Model:

$$Y_i = \boldsymbol{\beta}^{\mathrm{T}} \mathbf{X}_i + \epsilon_i$$

$$\epsilon_i \sim N(0, \sigma^2)$$

$$\mathbf{X}_i = \begin{bmatrix} 1 \\ X_{1,i} \\ \vdots \\ X_{p-1,i} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{p-1} \end{bmatrix}$$

Estimation:

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{i=1}^{n} \mathbf{X}_i \mathbf{X}_i^{\mathrm{T}} \right)^{-1} \sum_{i=1}^{n} Y_i \mathbf{X}_i$$

$$\widehat{\sigma}^2 = \frac{\sum_{i=1}^{n} (Y_i - \widehat{\boldsymbol{\beta}}^{\mathrm{T}} \mathbf{X}_i)^2}{n - p}.$$

$$\mathrm{Cov}(\widehat{\boldsymbol{\beta}}) = \widehat{\sigma}^2 \left(\sum_{i=1}^{n} \mathbf{X}_i \mathbf{X}_i^{\mathrm{T}} \right)^{-1}$$

$$\mathrm{Var}(\widehat{\beta}_j) = j\text{th diagonal element of } \mathrm{Cov}(\widehat{\boldsymbol{\beta}}), \quad j = 0, 1, \ldots, p-1$$

Output:

Variable	Estimate	Variance
Constant (β_0)	xxxx.xxxx	xxxx.xxxx
Age (β_1)	xxxx.xxxx	xxxx.xxxx
Income (β_2)	xxxx.xxxx	xxxx.xxxx