

# Electrical Properties of Body

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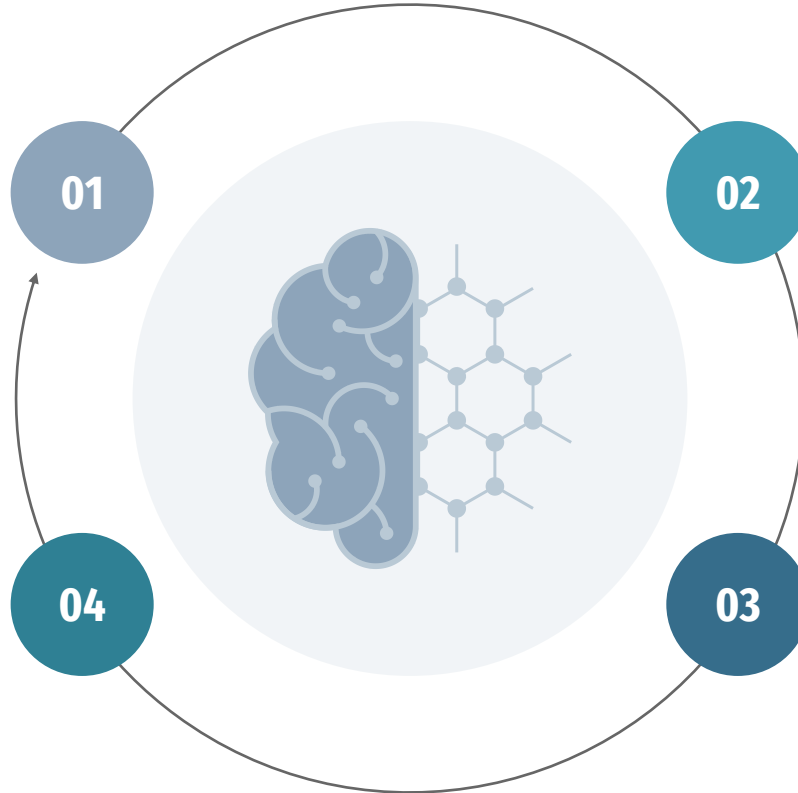
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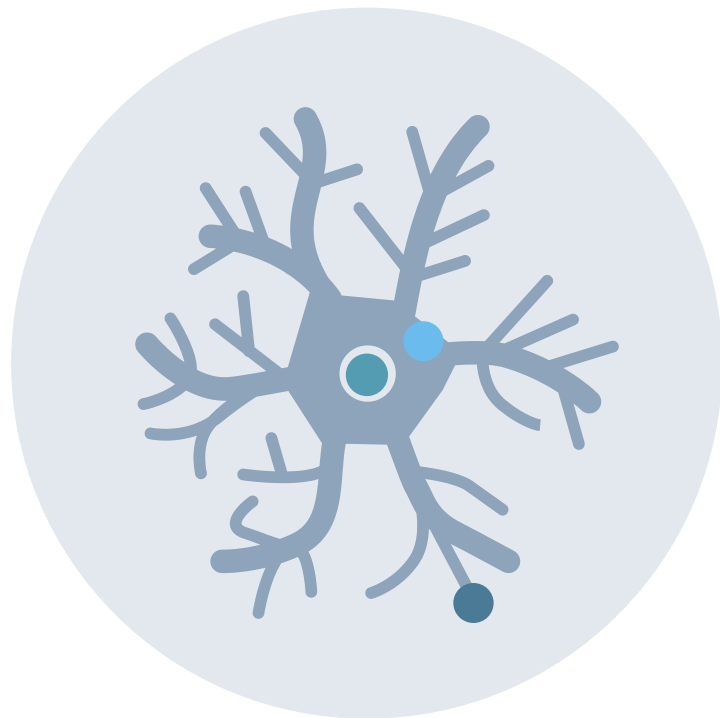
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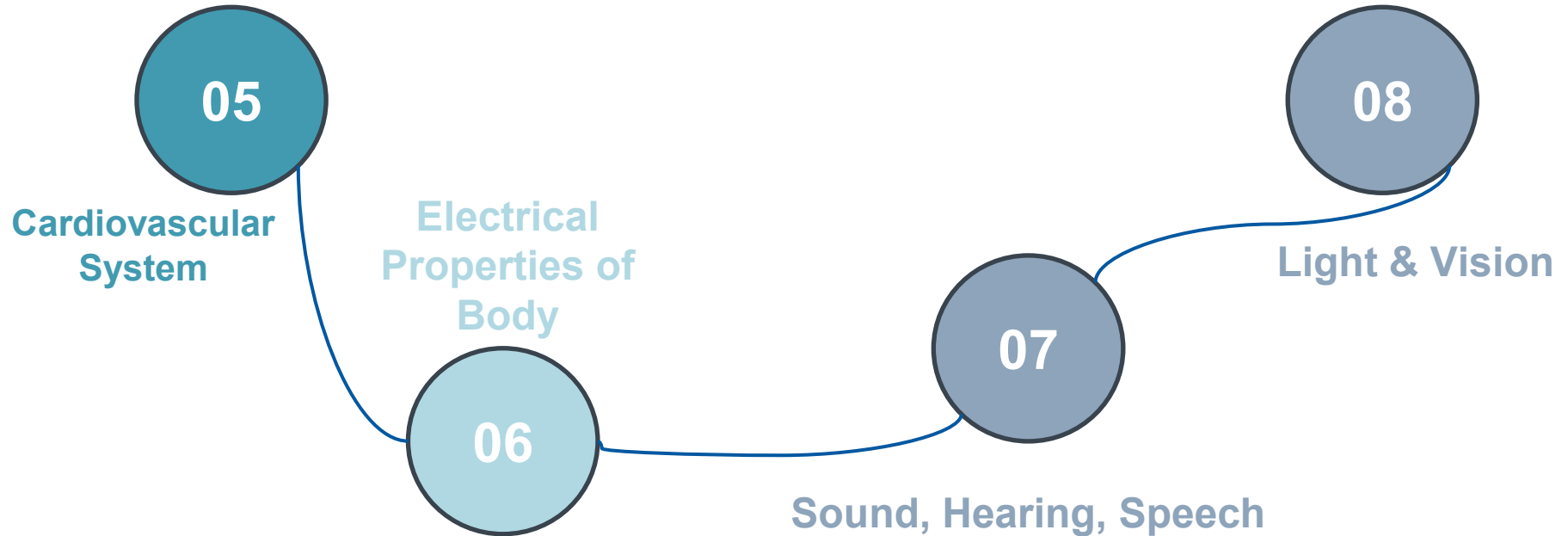


01

# Overview

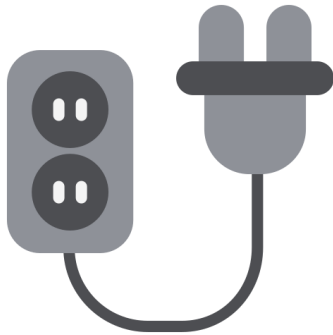


# Final Exam



# Introduction

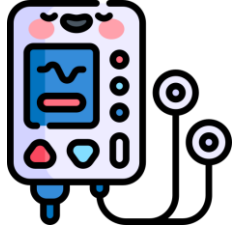
## Chapter 12



Charge movement, electric fields, and voltages play essential roles in the body. The driving forces that induce such charge motion are complicated chemical and biological processes that are only partially understood. The interplay of the resulting charges and fields is physical in nature and is well understood. We have addressed the importance of electricity in the body only briefly in previous chapters. In Chap. 3 we examined the electromyograms (EMGs) of muscle activity, in Chap. 5 we saw that muscles are activated by electrical stimuli and the release of  $\text{Ca}^{2+}$  ions, and in Chap. 8 we learned that the polarization and depolarization of cell membranes in the heart provide the signals for electrocardiograms (EKGs, ECGs). We now discuss such electrical interactions in more depth as we focus on the electrical properties of the body, the propagation of electrical signals in the axons of nerves, and electrical potentials in the body.

# The most important signals in human body

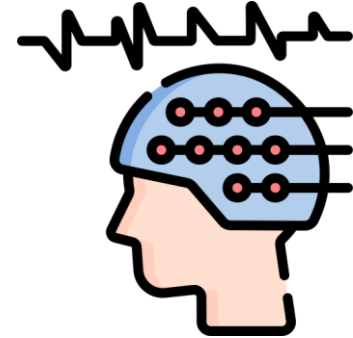
**EMG**



**ECG**



**EEG**



“Let the future tell the truth and evaluate each one according to his work and accomplishments. The present is theirs; the future, for which I have really worked, is mine.”

## **Nikola Tesla**



02

# Generals





# General rules and laws

Coulomb Law:

$$E = \frac{kq}{r^2} \hat{r}$$

The potential of that charge is:

$$V = \frac{kq}{r}$$

The potential difference between two points caused by a field is:

$$\Delta V = V_b - V_a = - \int_{r_a}^{r_b} E \cdot dr$$

We can conclude:

$$E = -\nabla V \text{ or in one dimension } E = -\frac{dV}{dx}$$

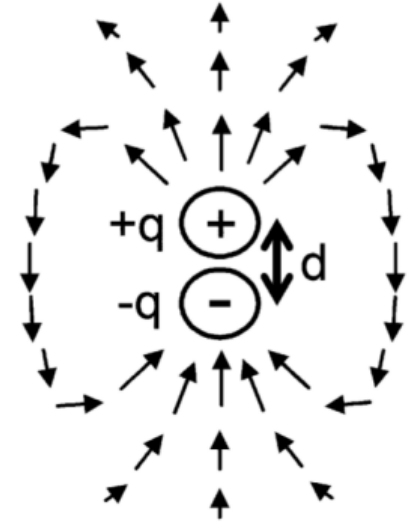
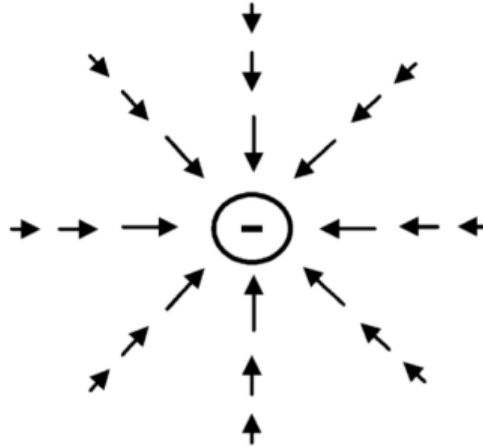
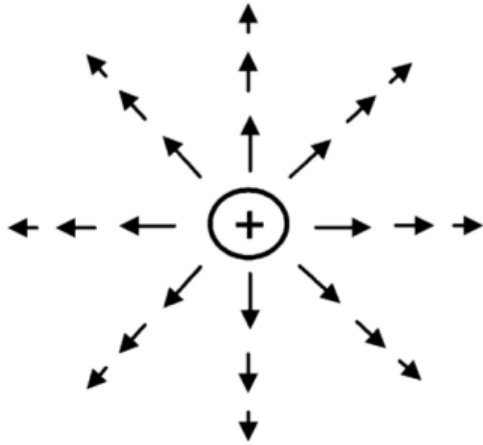
If there are two charges  $q$  and  $-q$  in vacuum in distance  $d$ :

$$V = \frac{kP \cdot r}{r^3}$$

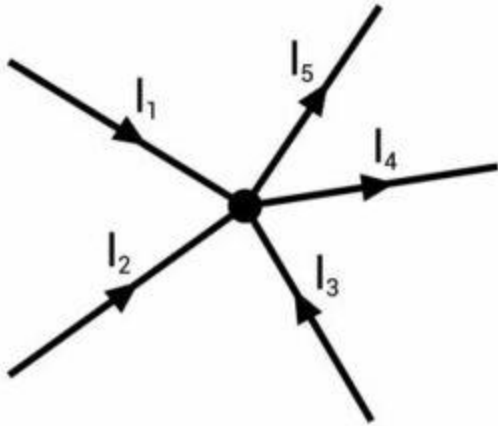
Considering the angle between  $P$  and distance vector  $r$  is  $\theta$ :

$$V = \frac{kP \cos \theta}{r^2}$$

# Electric Field

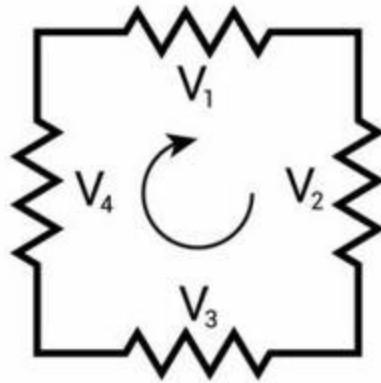


kirchhoff's current law



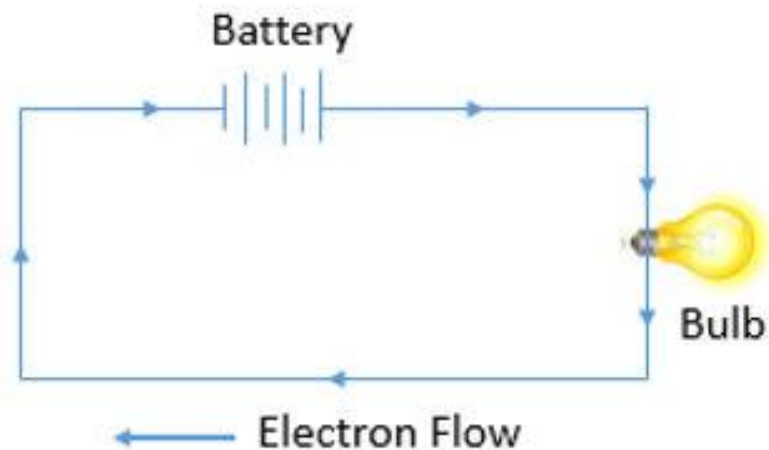
$$I_1 + I_2 + I_3 = I_4 + I_5$$

kirchhoff's voltage law



$$V_1 + V_2 + V_3 + V_4 = 0$$

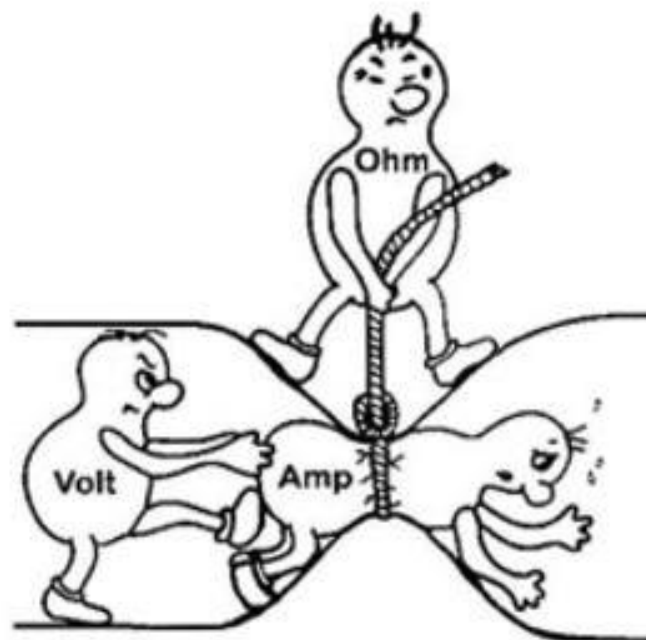
# Ohm's Law

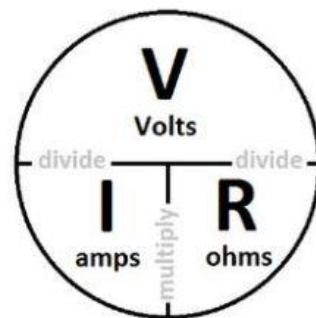
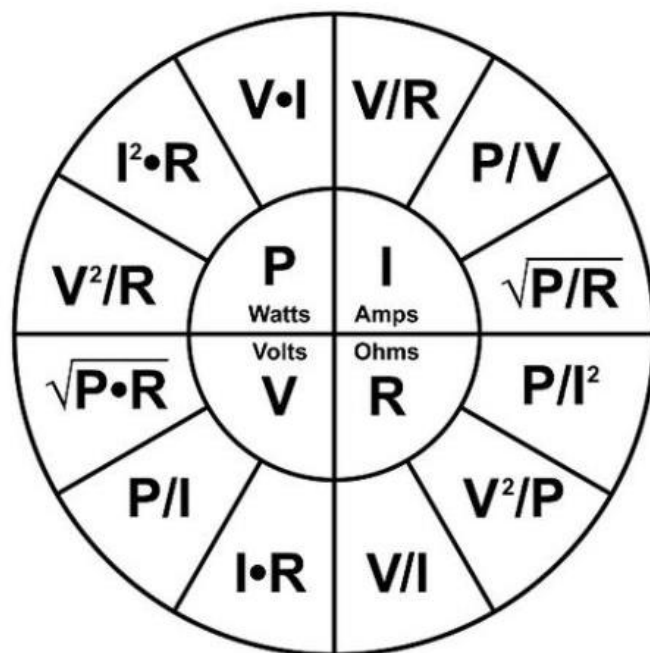
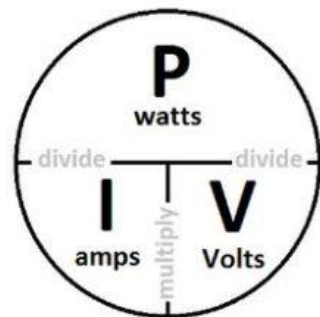


**Resistance (R)** = Bulb

**Current (I)** = Flow of Electron

**Voltage (V)** = Battery

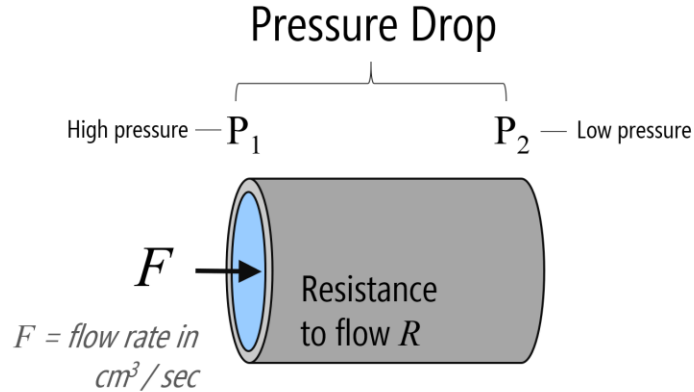




# POISEUILLE'S LAW AND OHM'S LAW



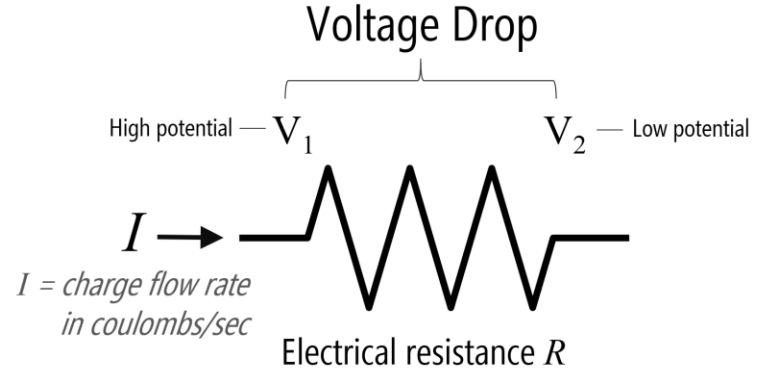
Poiseuille's Law for smooth flow (laminar flow) of fluids  
circa ~1838



$$\text{Flow } F = \frac{P_1 - P_2}{R}$$

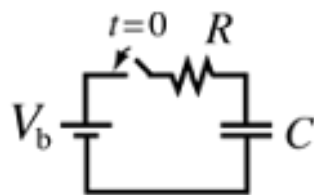


Ohm's Law for electric circuits  
circa ~1825



$$\text{Current } I = \frac{V_1 - V_2}{R}$$





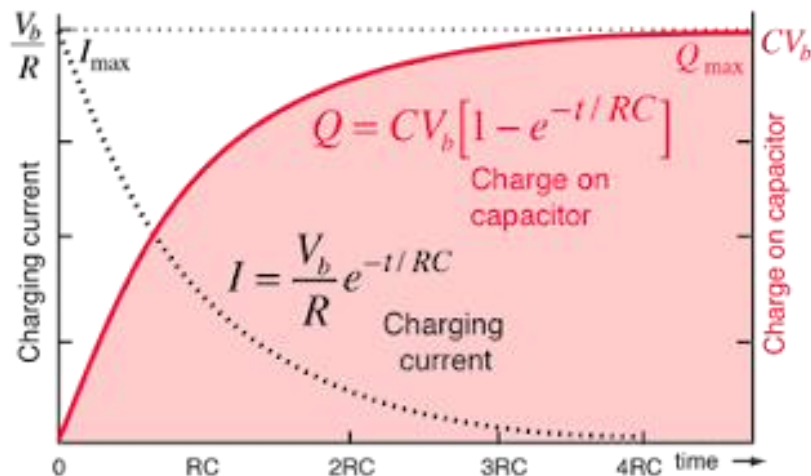
$$V_b = V_R + V_C$$

$$V_b = IR + \frac{Q}{C}$$

As charging progresses,

$$V_b = IR + \frac{Q}{C}$$

current decreases and  
charge increases.



At  $t = 0$

$$Q = 0$$

$$V_C = 0$$

$$I = \frac{V_b}{R}$$

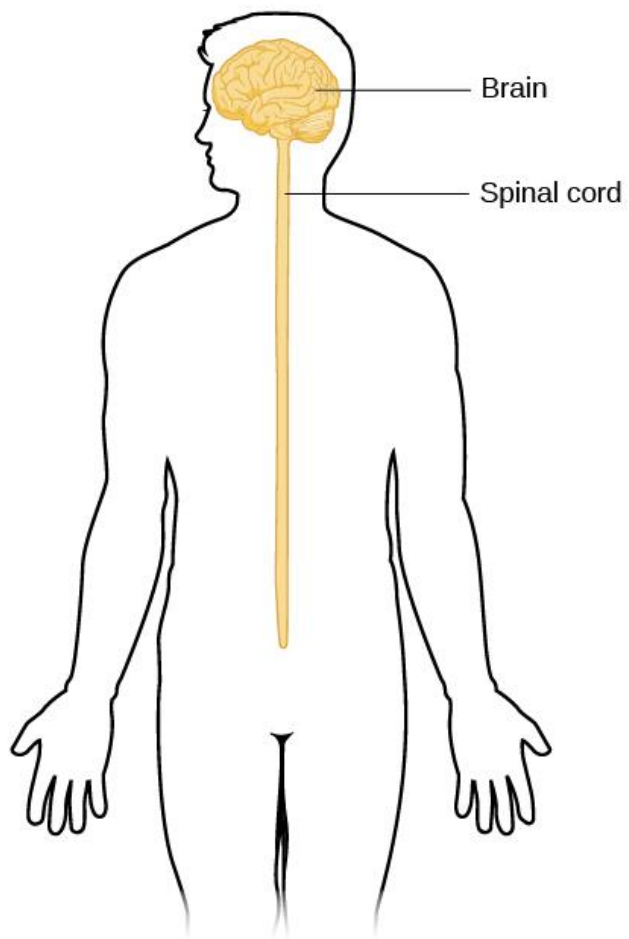
As  $t \rightarrow \infty$

$$Q \rightarrow CV_b$$

$$V_C \rightarrow V_b$$

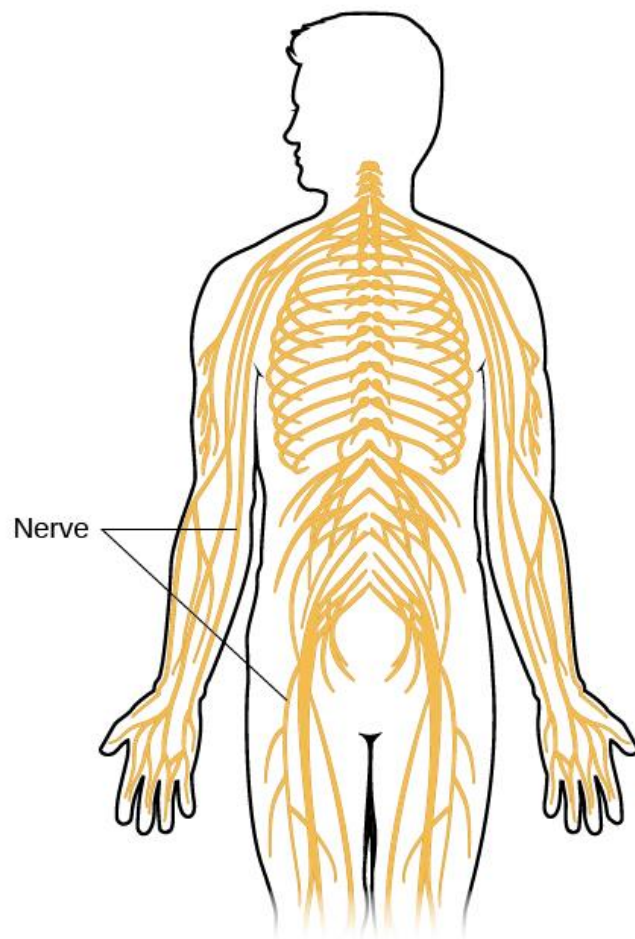
$$I \rightarrow 0$$

## Central Nervous System



(a)

## Peripheral Nervous System



(b)



# NERVOUS SYSTEM

## CENTRAL NERVOUS SYSTEM



BRAIN

SPINAL CORD

## PERIPHERAL NERVOUS SYSTEM



### SOMATIC

Voluntary movement of skeletal muscle

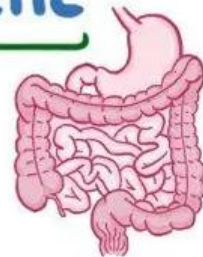


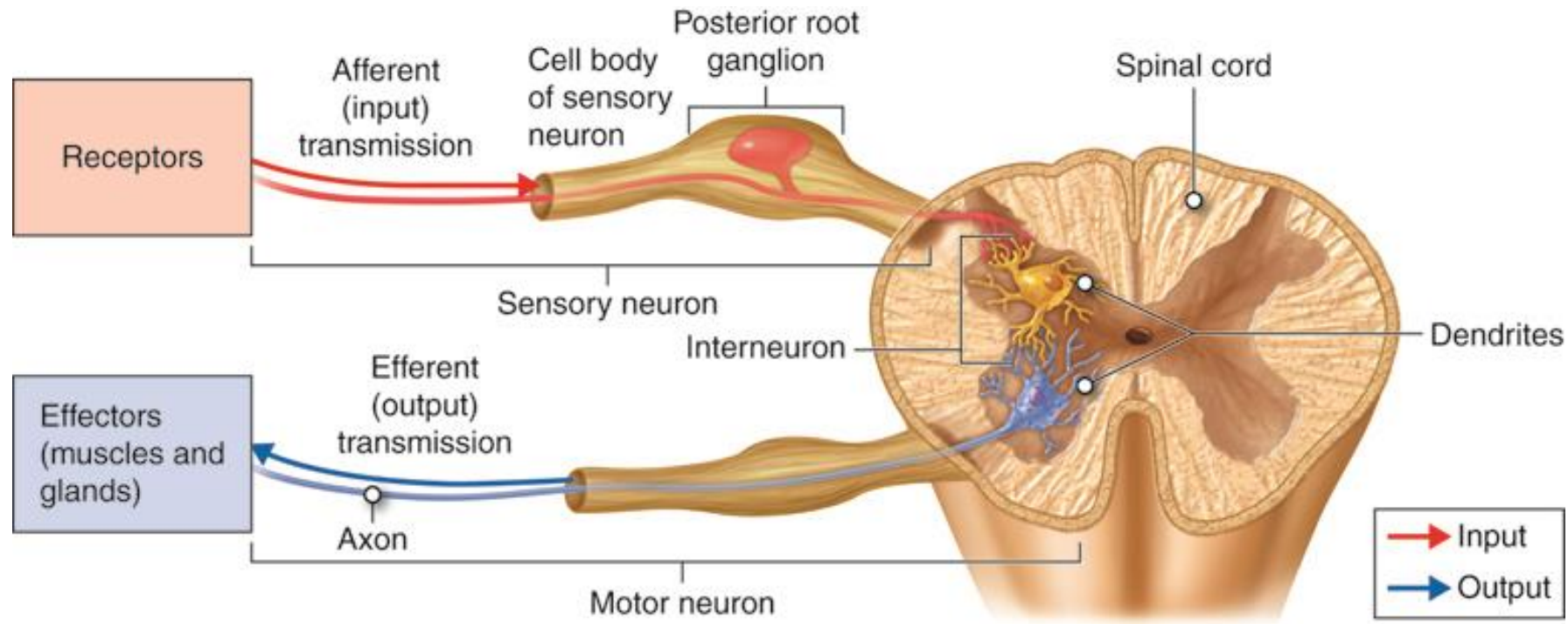
### AUTONOMIC

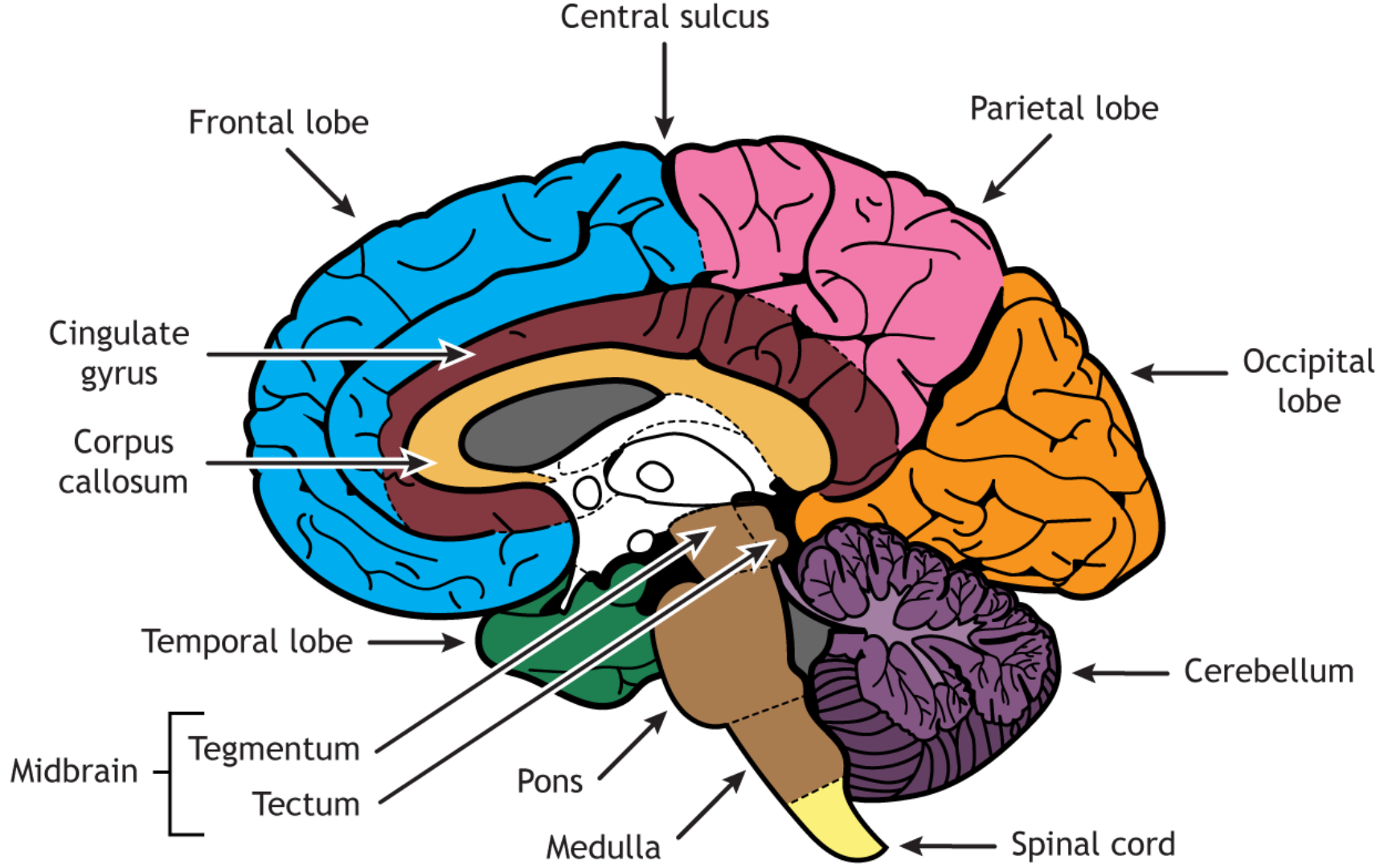
#### SYMPATHETIC PARASYMPATHETIC

Involuntary movement of

- \* Smooth muscles
- \* Glands of organs

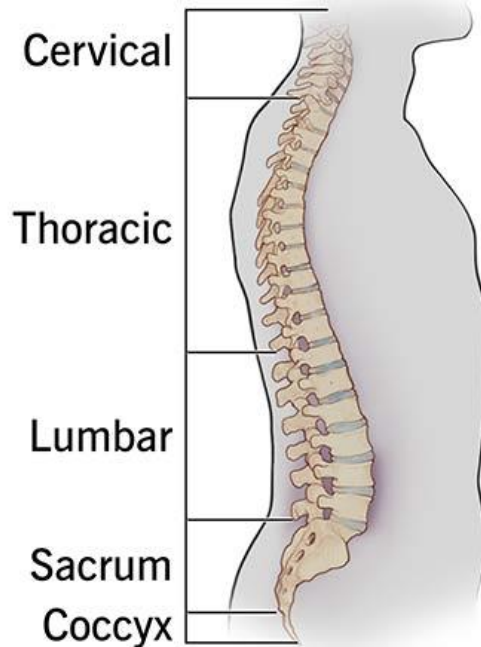




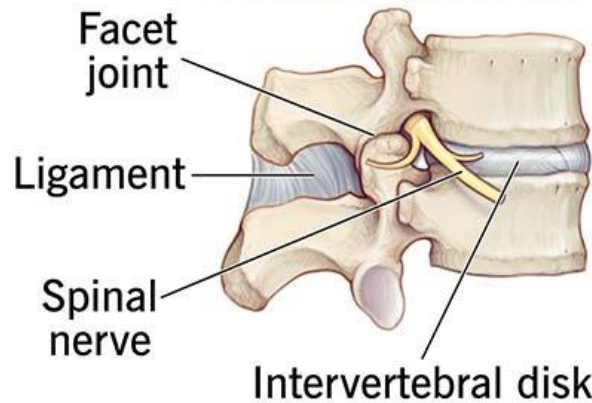


# Spine (backbone)

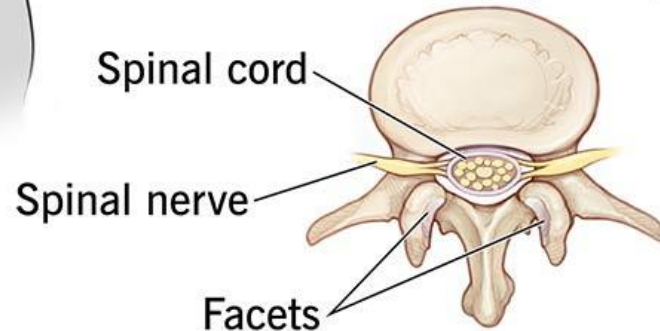
## Parts of the spine



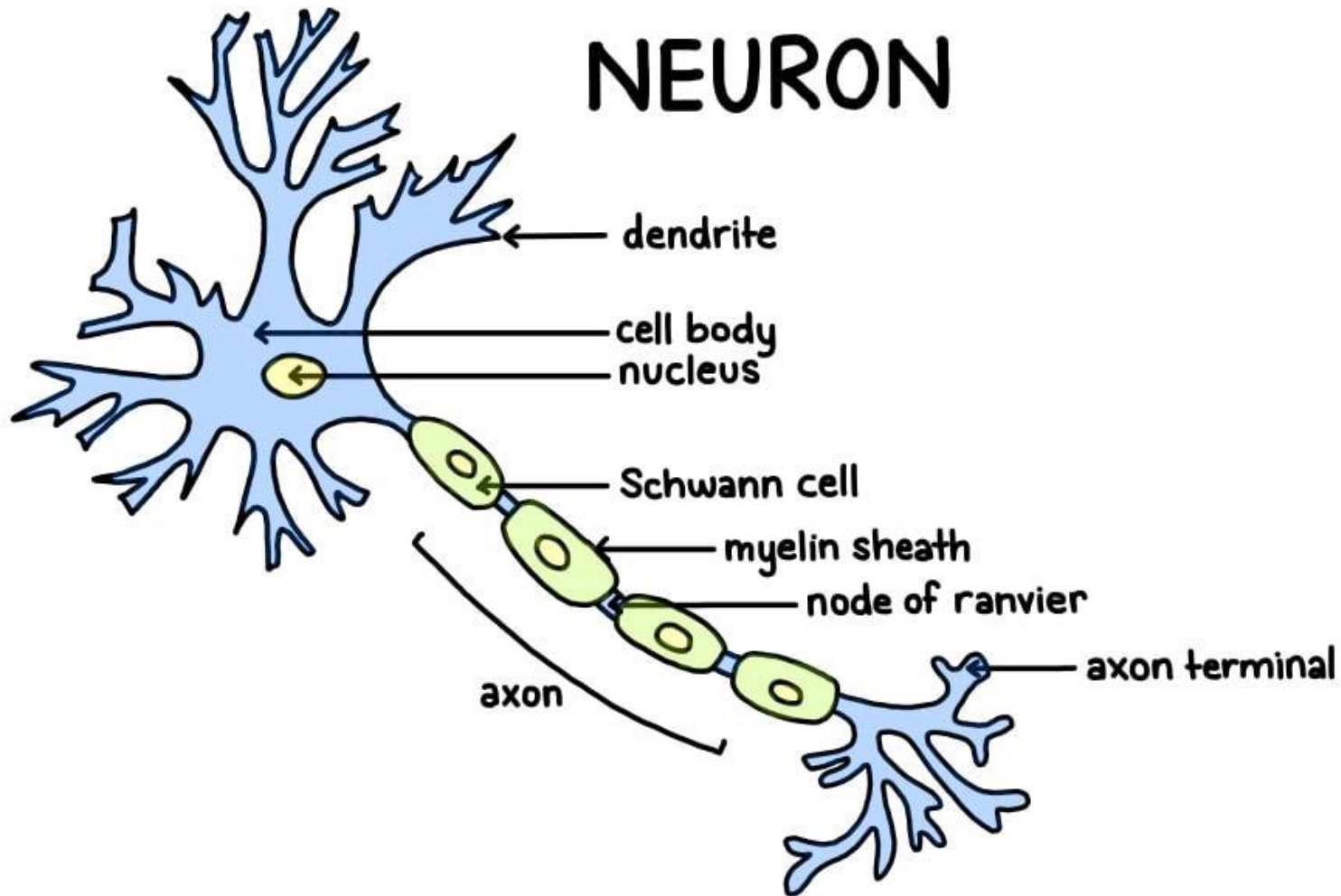
## Vertebrae (side view)



## Vertebra (top view)



# NEURON



# Neuron infographics

**Dendrite**

**Nucleus**

**Cell body**

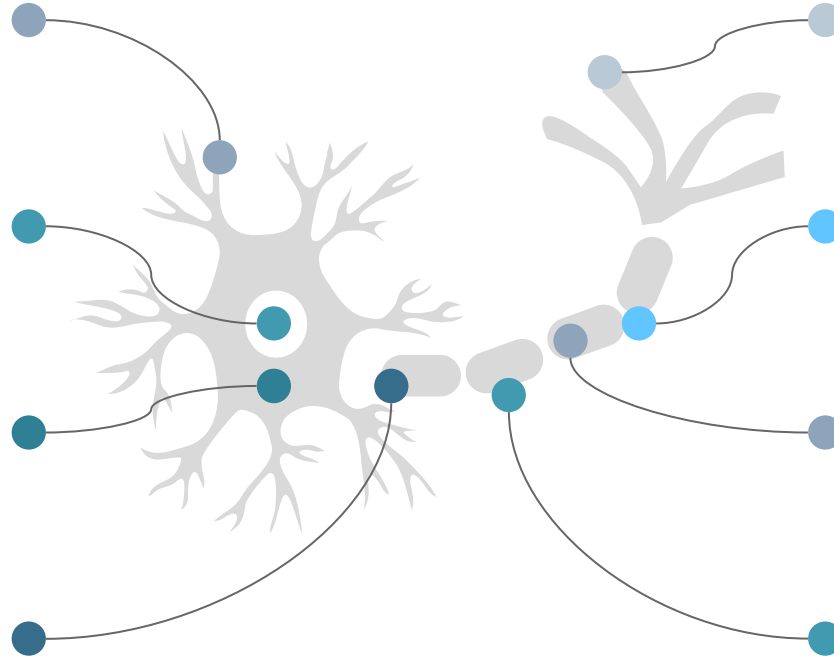
**Axon**

**Axon terminal**

**Node of Ranvier**

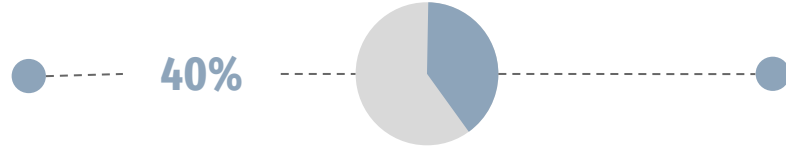
**Schwann cell**

**Myelin sheath**

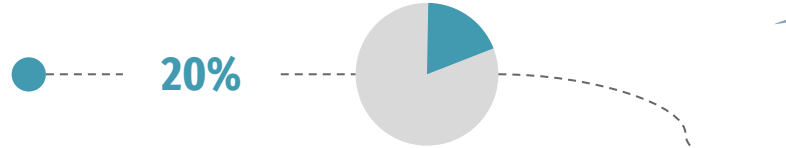


# Neuron infographics

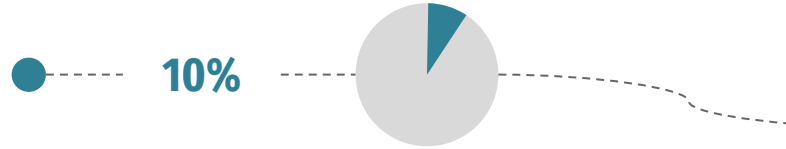
**Cell body**



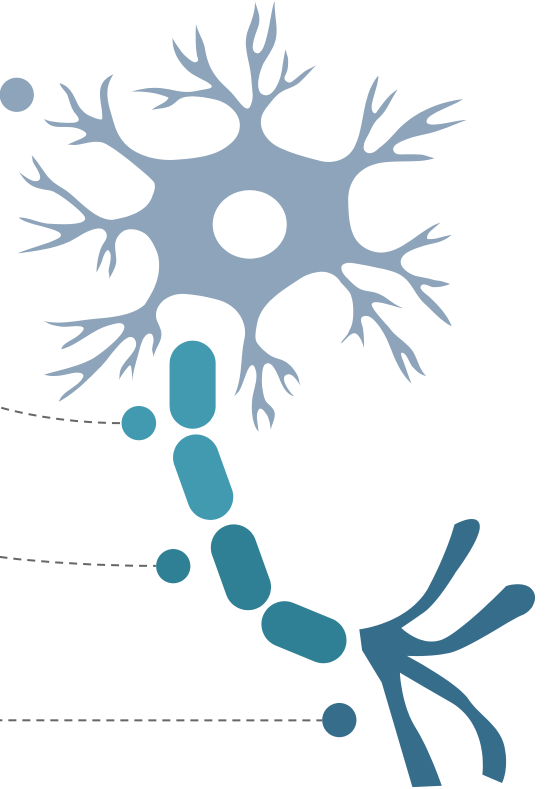
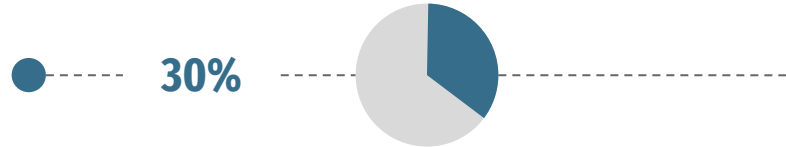
**Myelin sheath**



**Schwann cell**



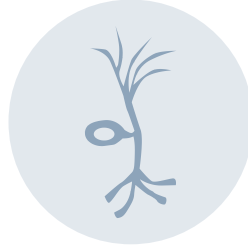
**Axon terminal**





# Neuron infographics

**Sensory**



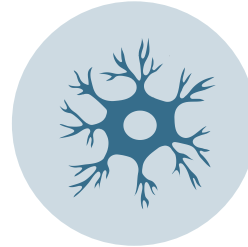
**Motor**



**Pyramidal**



**Astrocyte**



**Betz cell**



**Microglia**





# Neuron infographics

01



**Sensory**

02



**Pyramidal**

03



**Betz cell**

# Neuron infographics

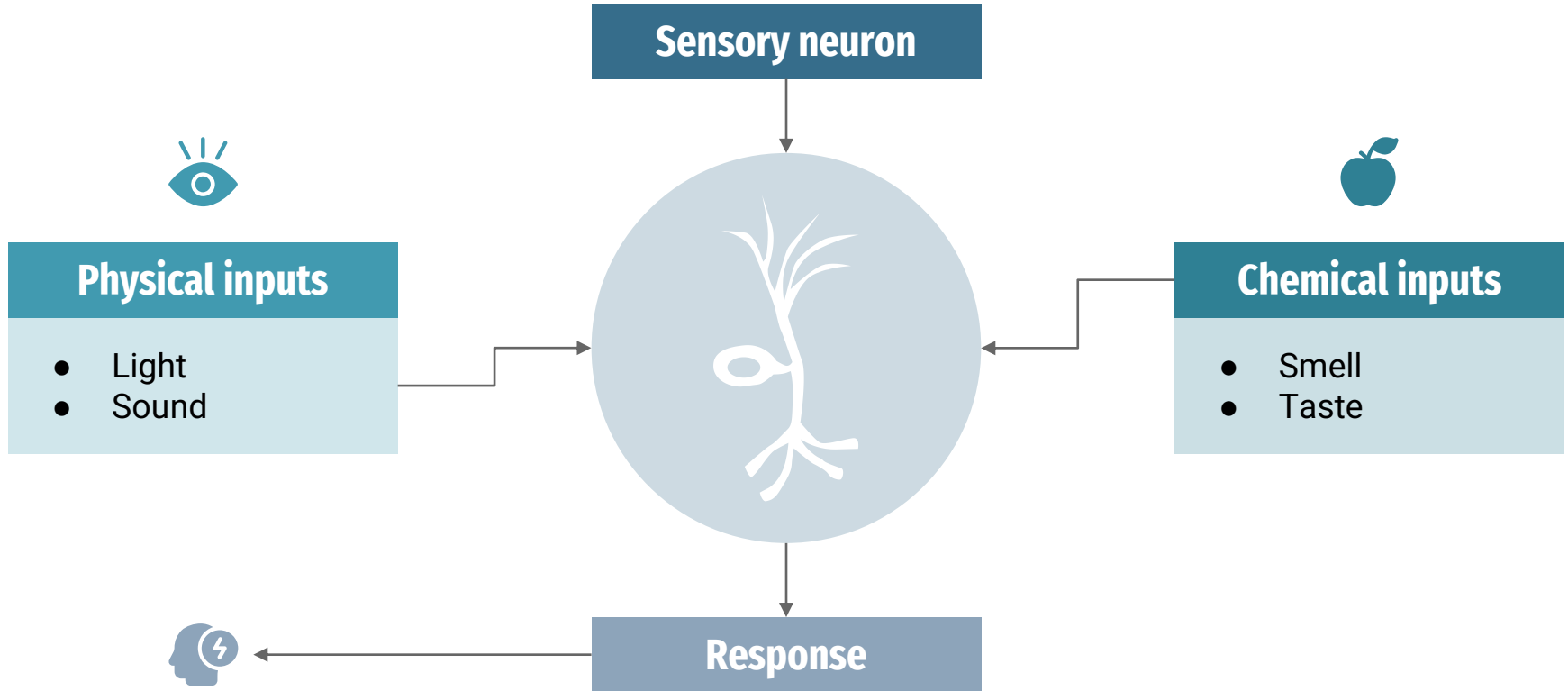


**Motor neurons**

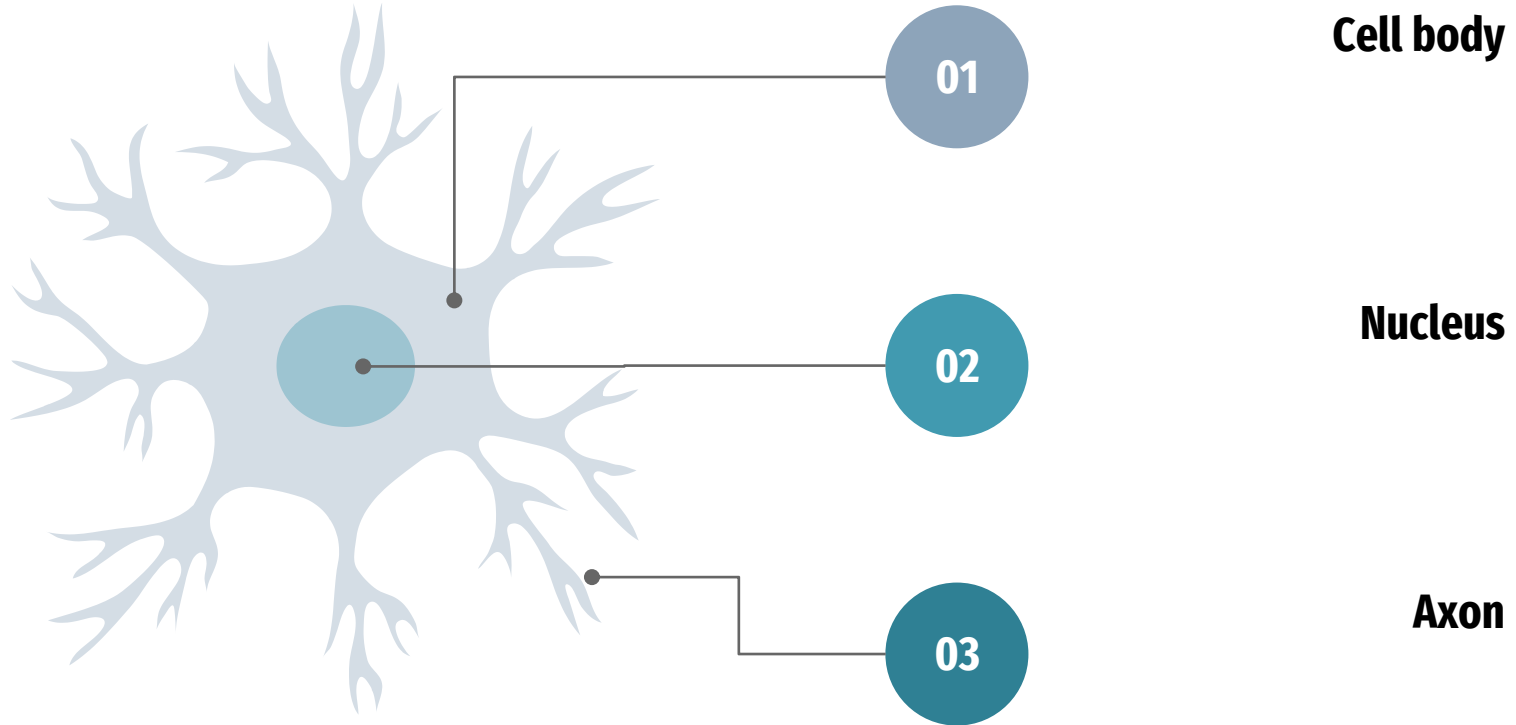
# Neuron infographics



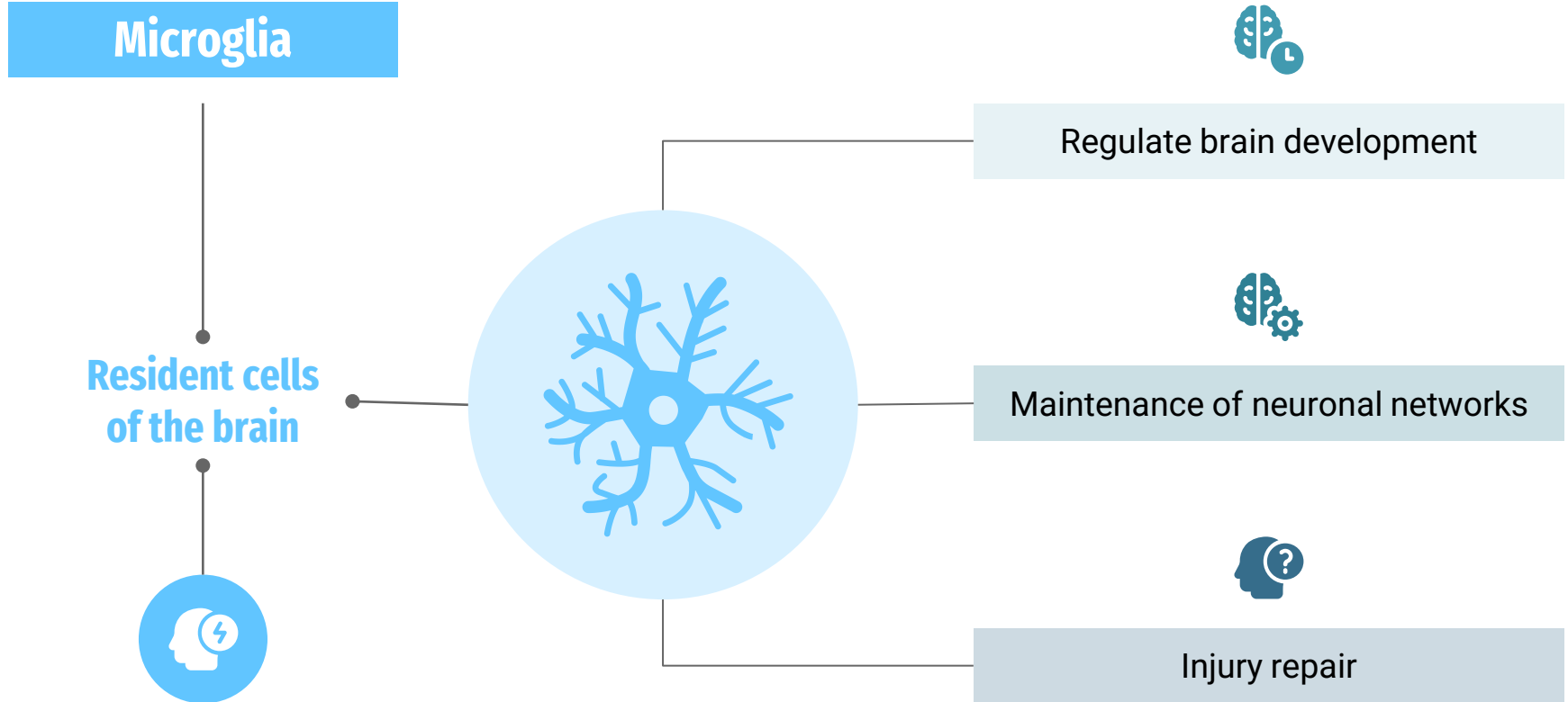
# Neuron infographics



# Neuron infographics

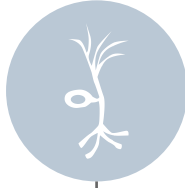


# Neuron infographics

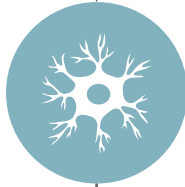


# Neuron infographics

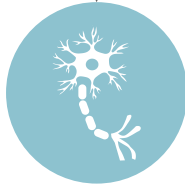
**Sensory neurons**



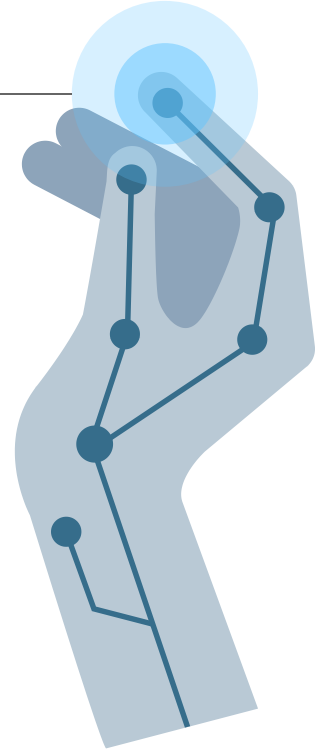
**Interneurons**



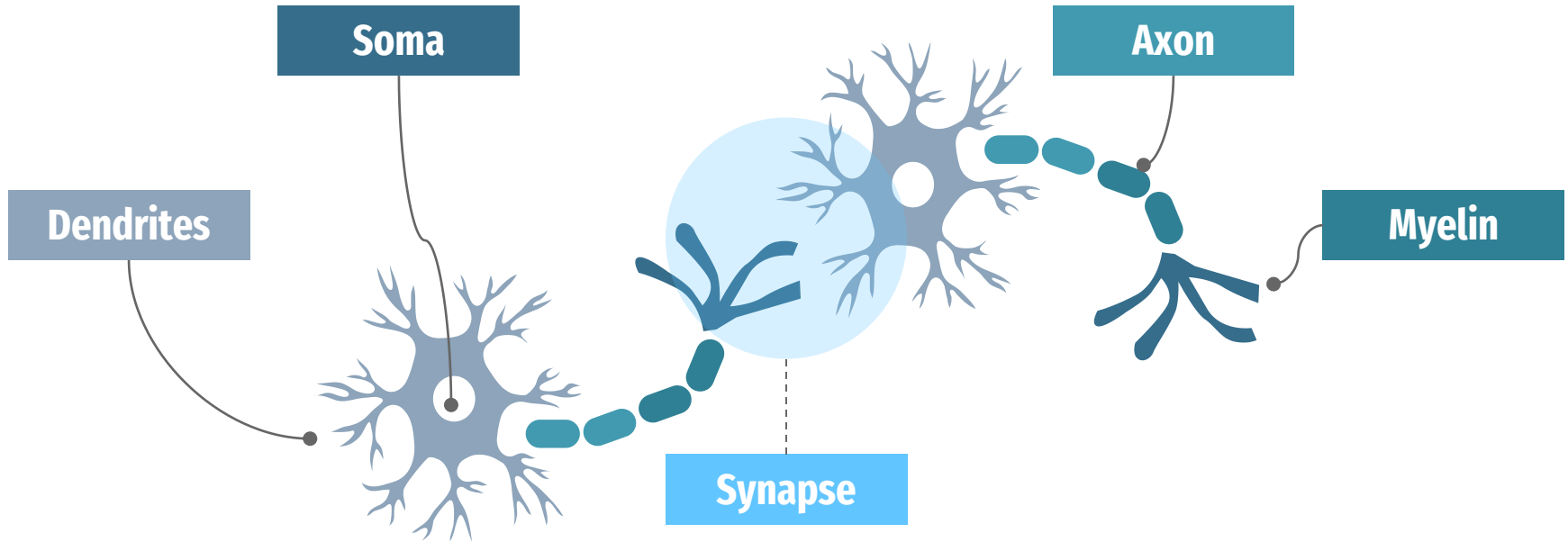
**Motor neurons**



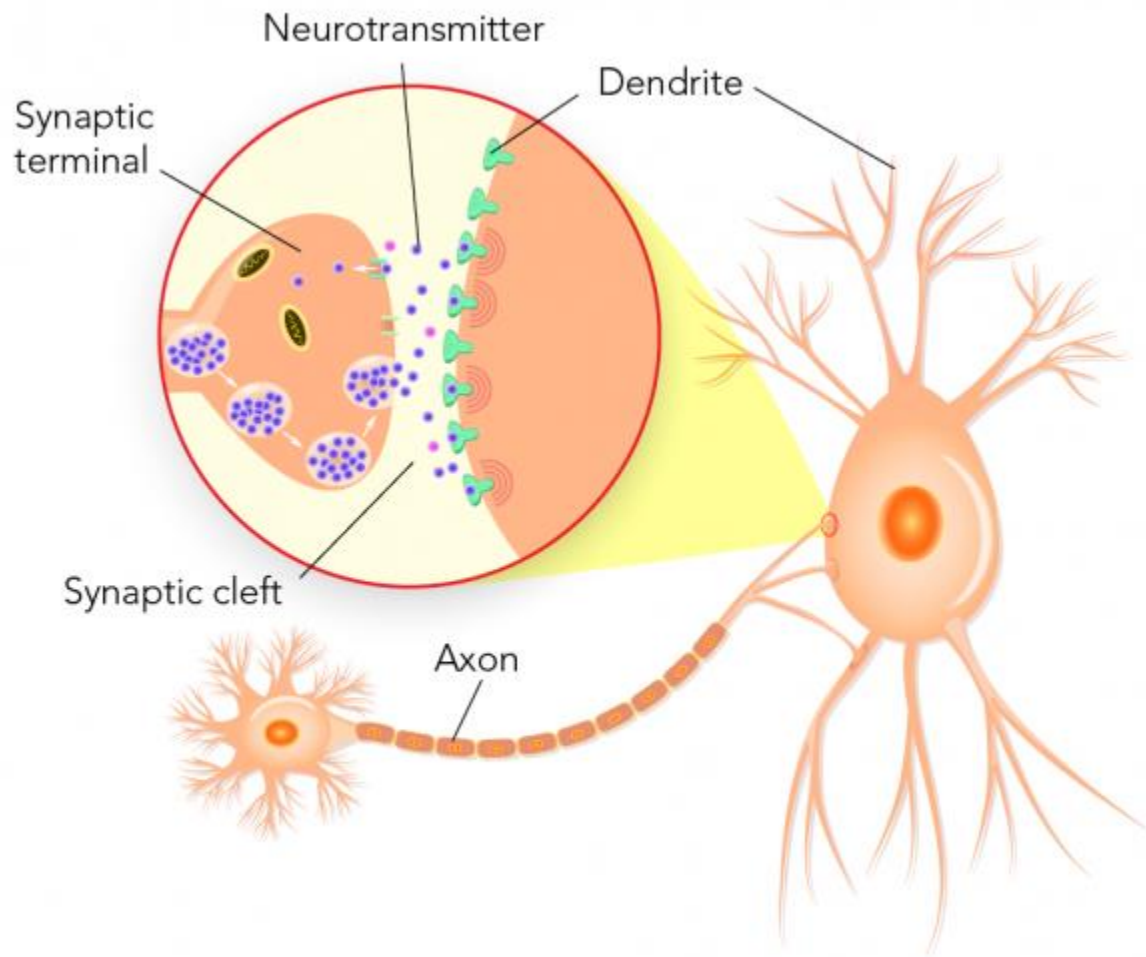
**Response**



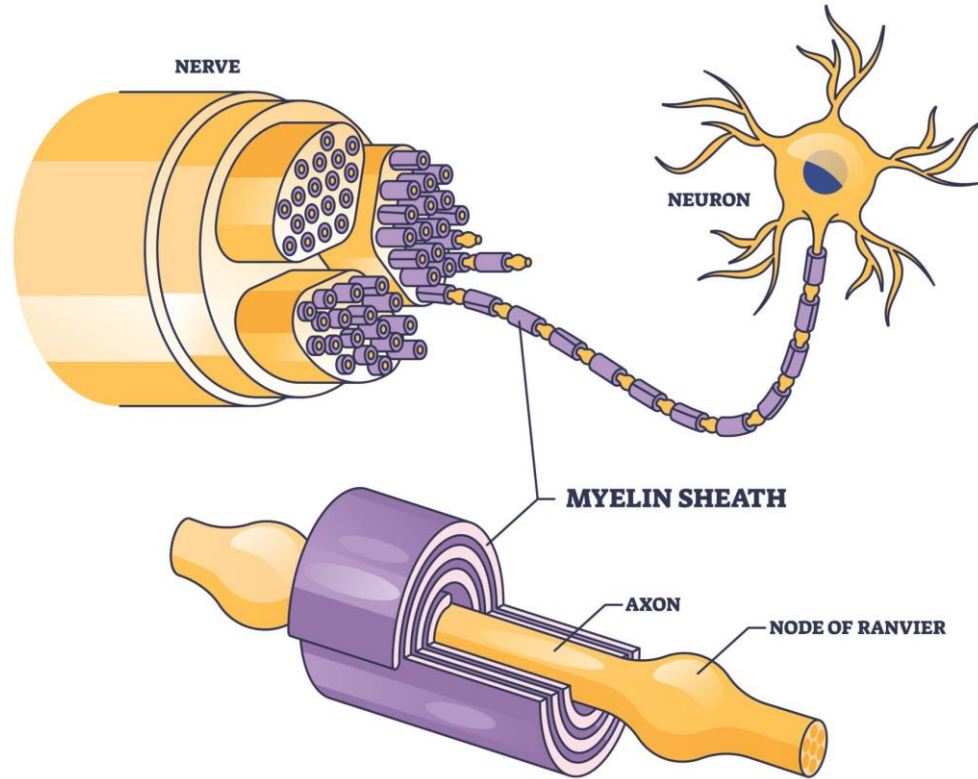
# Neuron infographics





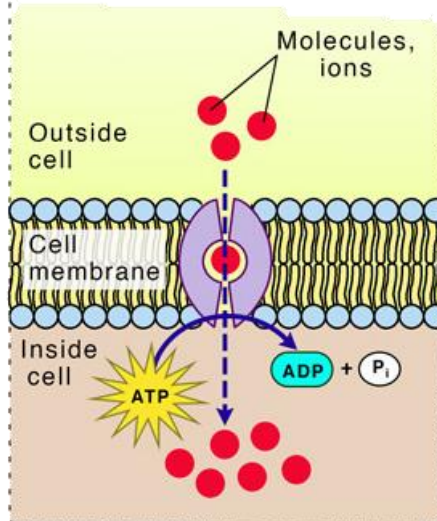


# MYELIN SHEATH



## Active Transport

Movement against the concentration gradient using energy (ATP)

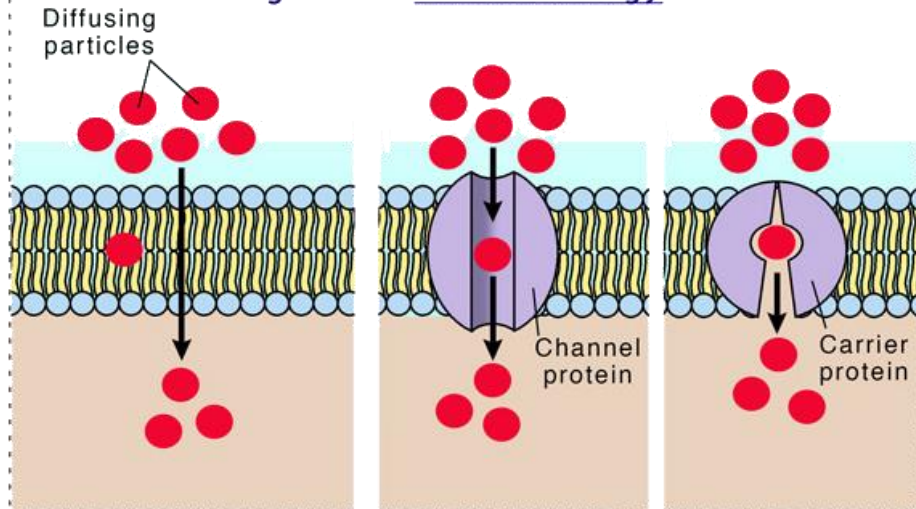


## Passive Transport

Simple Diffusion

Facilitated Diffusion

Movement along the concentration gradient without energy



# Fick's First Law

Movement of particles (diffusion flux) from high to low concentration is directly proportional to the particle's concentration gradient

$$J \propto \frac{d\phi}{dx} \quad \text{or} \quad J = -D \frac{d\phi}{dx}$$

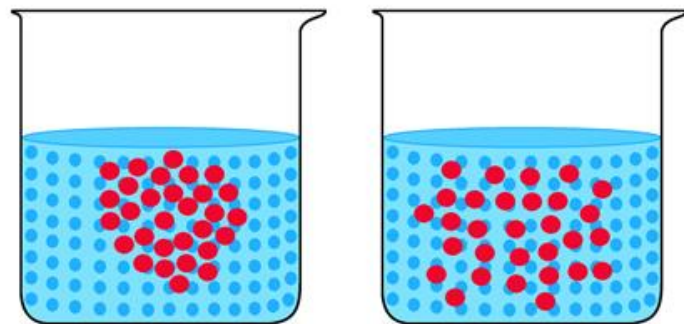
$J$  = diffusion flux

$D$  = diffusion coefficient or diffusivity

$d\phi$  = change in concentration of the particle

$dx$  = change in position

$\frac{d\phi}{dx}$  = concentration gradient of the particle



Particles diffusing from  
high to low concentration

$$E = -\frac{dV}{dx}$$

$$J_{elect} = nv_{drift} = n\mu E$$

$$\mu = \frac{qD_{diff}}{k_B T}$$

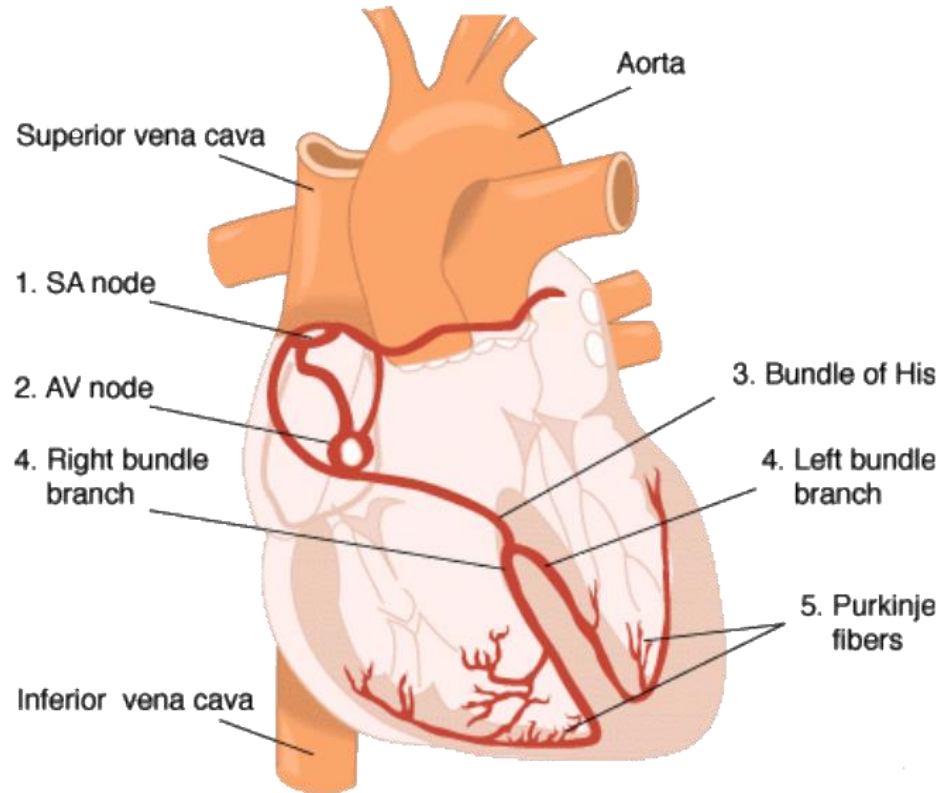
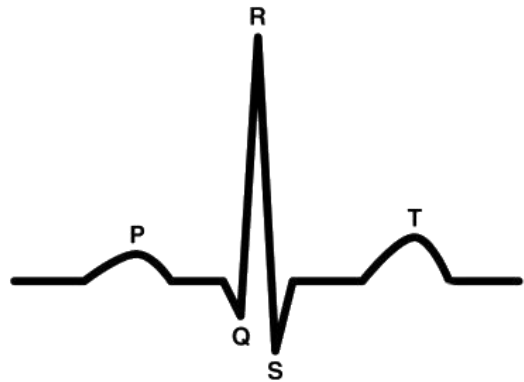
$$J_{diff} + J_{elect} = 0$$

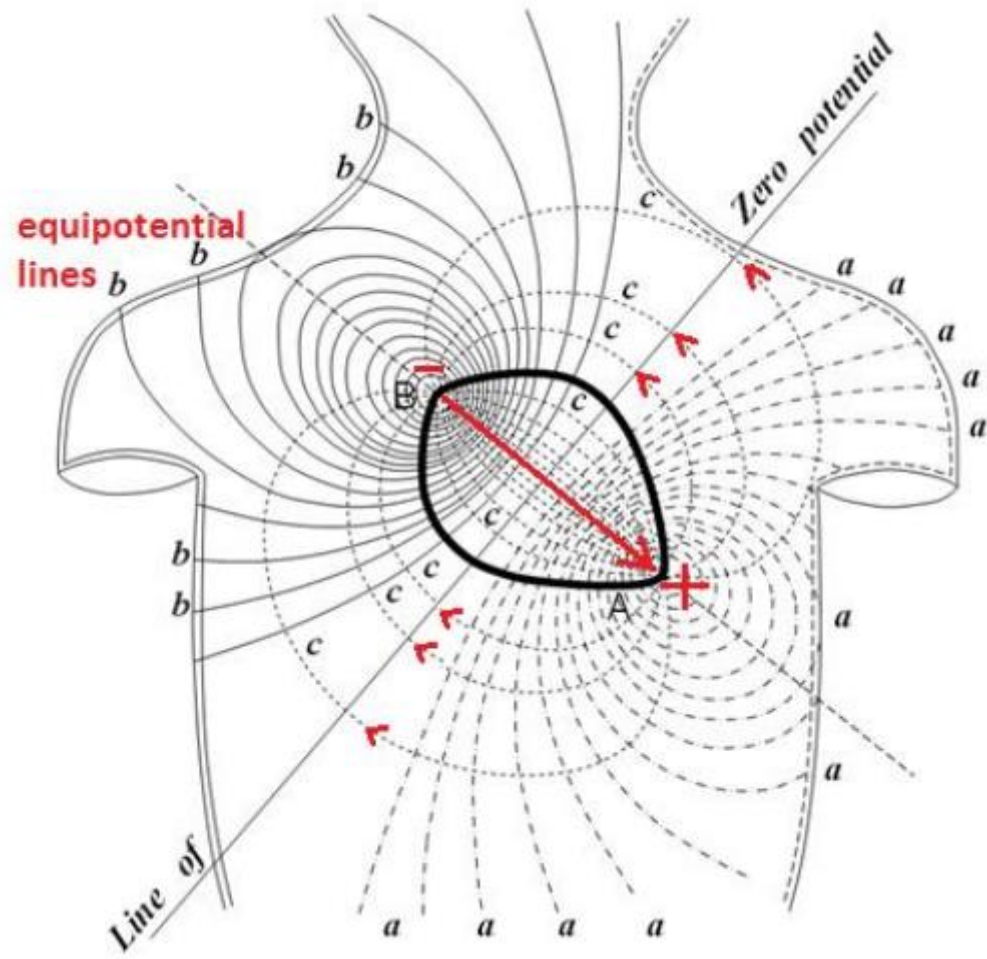
$$D_{diff} \frac{dn}{dx} = n\mu E$$

$$\Delta V = -\frac{k_B T}{q} \ln \frac{n_i}{n_o} \times \left( \frac{1}{z} \right)$$

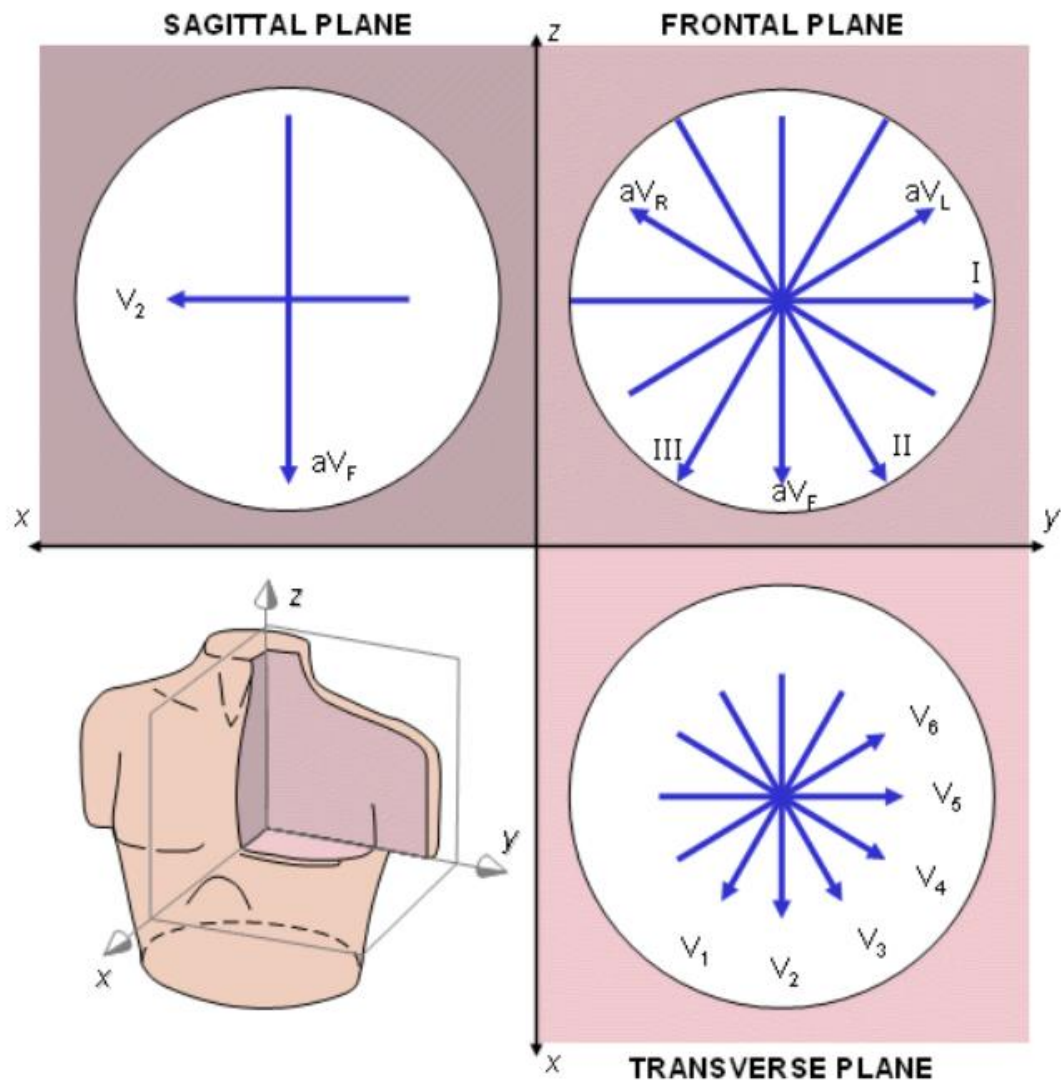
$$\Delta V = -\frac{k_B T}{q} \ln \frac{p_{Na} n_{Na, i} + p_k n_{k, i} + p_{Cl} n_{Cl, o}}{p_{Na} n_{Na, o} + p_k n_{k, o} + p_{Cl} n_{Cl, i}}$$

$$\rightarrow \frac{kT}{q} \ln \frac{[Na^+]_o + [k^+]_o + [Cl^-]_i}{[Na^+]_i + [k^+]_i + [Cl^-]_o}$$

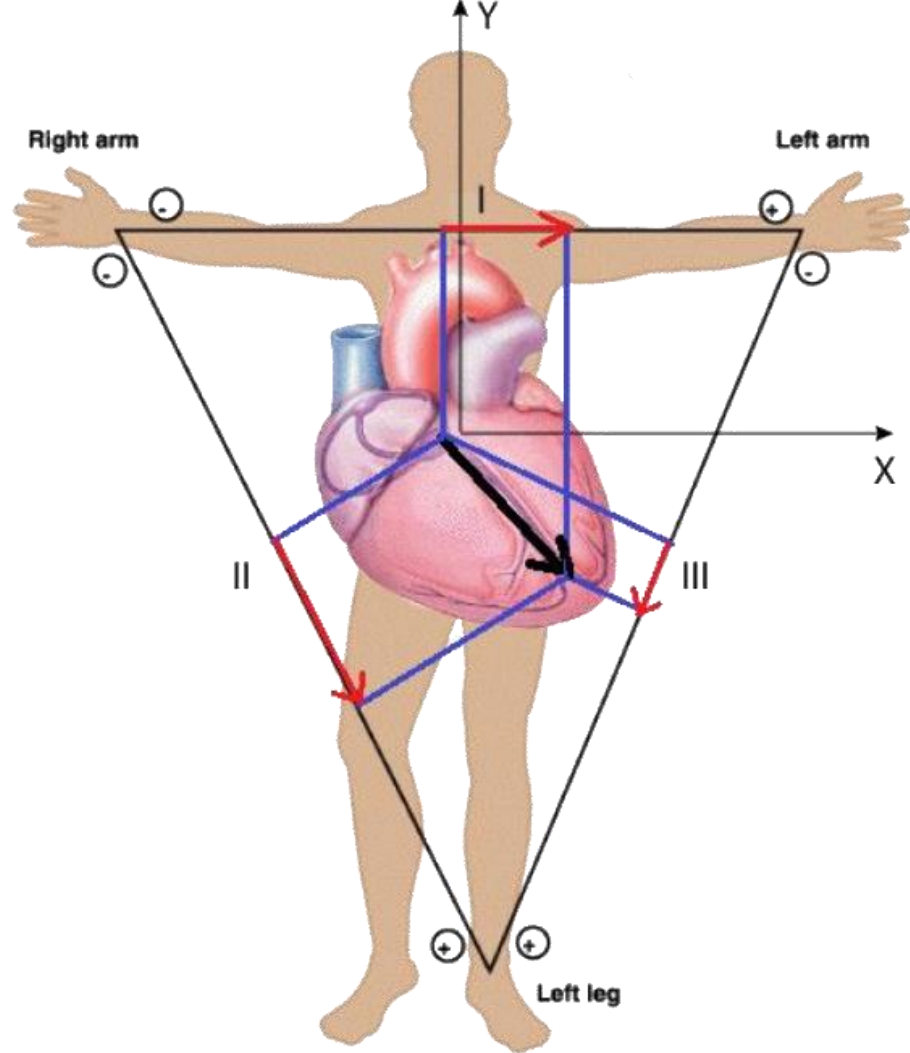


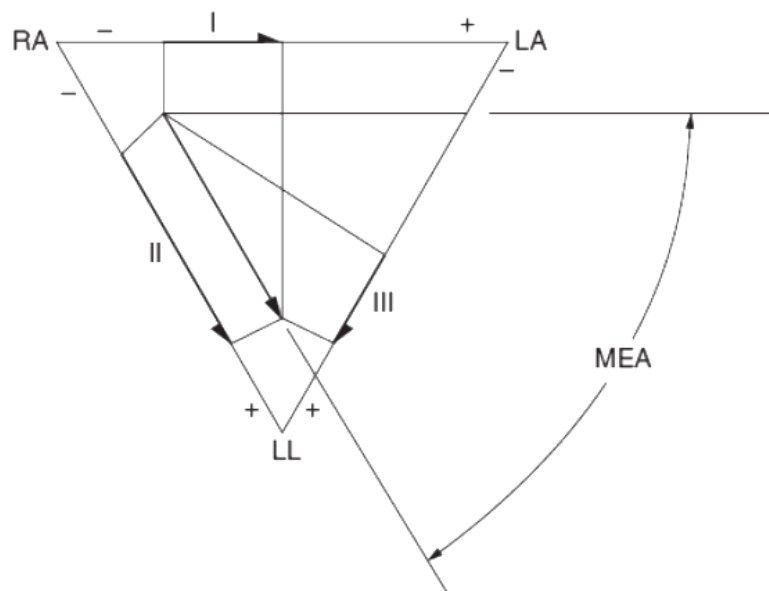
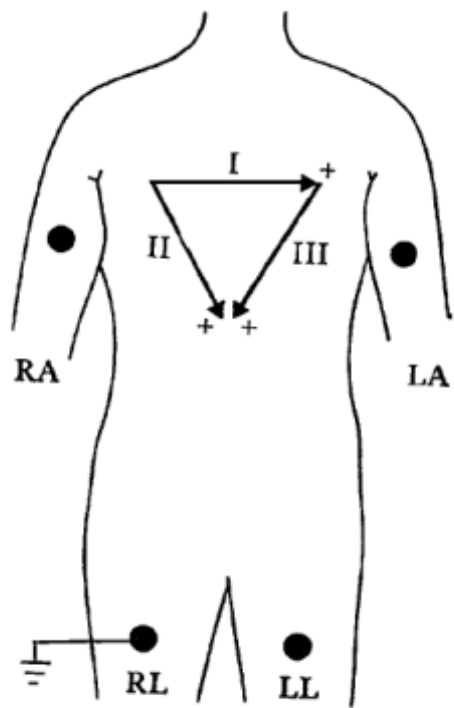


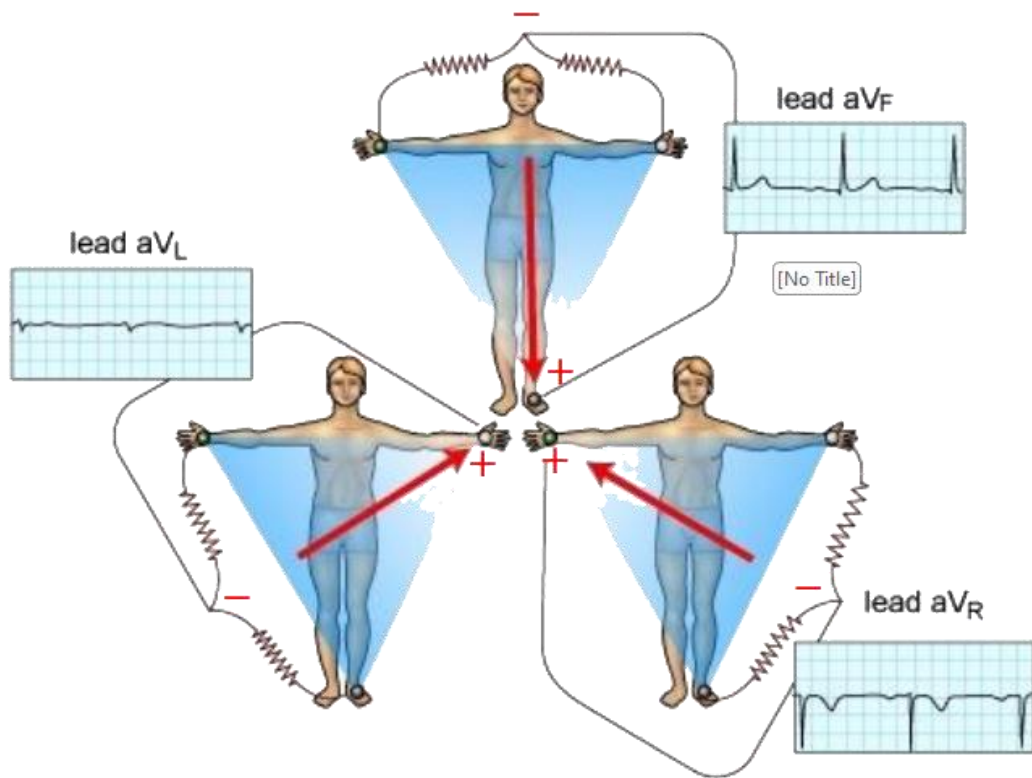


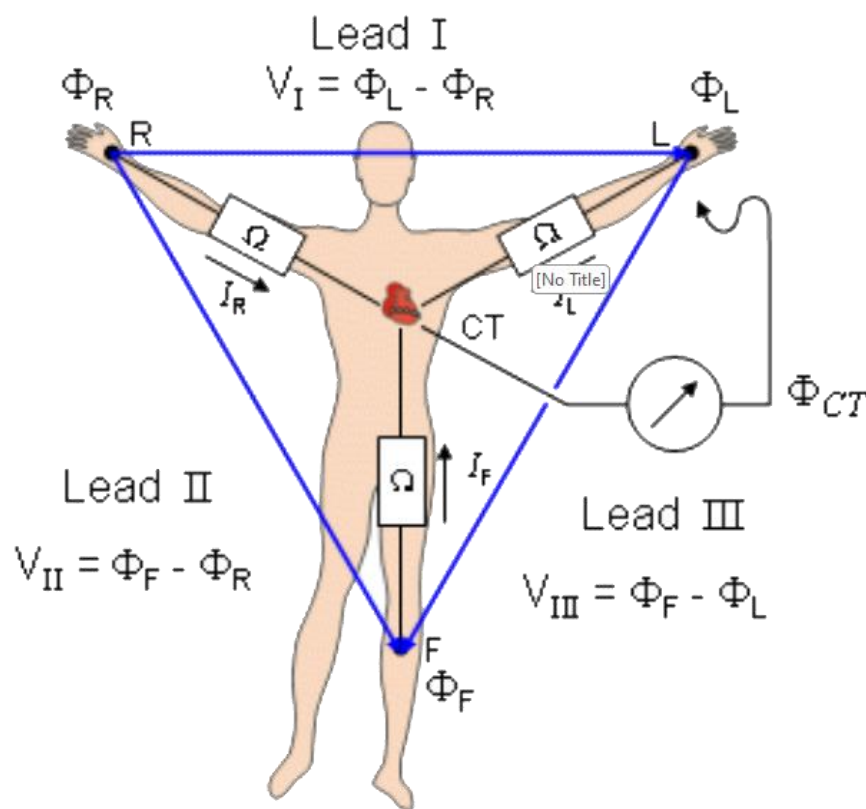












$$V_I = aV_L - aV_R$$

$$V_{II} = aV_F - aV_R$$

$$V_{III} = aV_F - aV_L$$

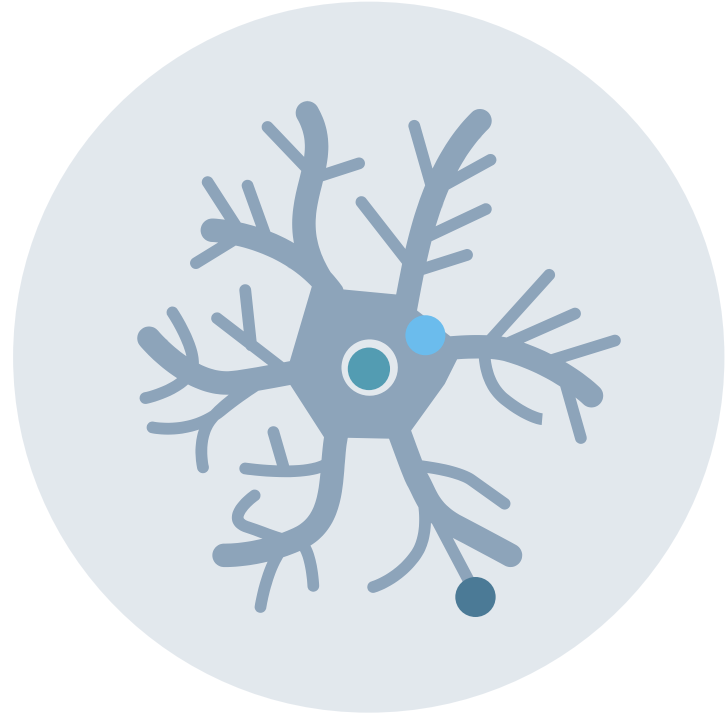
**Wilson Central Terminal  
(WCT)**

$$aV_L + aV_F + aV_R = 0$$

$$V_{II} - V_{III} = V_I$$

03

# Examples



## Example 6-1

- A. Calculate the Nernst potential for each ion as well as the membrane potential assuming the following permeability coefficients.
- B. If the active sodium-potassium pump in this cell fails due to a chemical drug, assuming that the membrane potential does not change much, calculate the new sodium ion concentration outside and inside the cell. (Hint: consider the volume of intracellular and extracellular fluid to be almost equal and use the law of conservation of mass.)

<i>Ion</i>	<b>Intracellular concentration</b>	<b>Extracellular concentration</b>	<b>Permeability coefficient</b>
$Na^+$	50	440	0.04
$K^+$	400	20	1
$Cl^-$	52	560	0.45

## Example 6-1

**Answer**

A)

$$V_{Na} = \frac{kT}{q} \ln \frac{[Na^+]_o}{[Na^+]_i} = 25 \times \ln \frac{440}{50} = 54.4 \text{ mV}$$

$$V_k = \frac{kT}{q} \ln \frac{[k^+]_o}{[k^+]_i} = 25 \times \ln \frac{20}{400} = -74.8 \text{ mV}$$

$$V_{Na} = \frac{kT}{q} \ln \frac{[Cl^-]_i}{[Cl^-]_o} = 25 \times \ln \frac{52}{560} = -59.4 \text{ mV}$$

$$V_{total} = \frac{kT}{q} \ln \frac{[Na^+]_o + [k^+]_o + [Cl^-]_i}{[Na^+]_i + [k^+]_i + [Cl^-]_o} = 25 \times \ln \frac{20 + 0.04 \times 440 + 0.45 \times 52}{400 + 0.04 \times 50 + 0.45 \times 560} = -59.1 \text{ mV}$$

## Example 6-1

### Answer

B)

When the pump is broken, Nernst potential and membrane potential will be equal:

$$\left. \begin{array}{l} \frac{[k^+]_o}{[k^+]_i} = \frac{61}{654} \\ 420 - [k^+]_o = [k^+]_i \end{array} \right\} \frac{x}{420 - x} = \frac{61}{654} \rightarrow x = 35.9 = [k^+]_o \rightarrow [k^+]_i = 420 - 35.9 = 384.1 \text{ mM}$$



## Example 6-2

The cell membrane is permeable to k and cl ions and not permeable to R ions. Find the concentration of ions at equilibrium. The concentration of ions before equilibrium is as follows.

$$[Kcl]_o = 400 \text{ mM}$$

$$[Kcl]_i = 100 \text{ mM}$$

$$[Rcl]_i = 500 \text{ mM}$$

## Example 6-2

**Answer**

$$\left. \begin{array}{l} [k^+]_o + [k^+]_i = 500 \\ [Cl^-]_i + [Cl^-]_o = 1000 \\ [Cl^-]_i = [k^+]_i + 500 \\ [k^+]_o = [Cl^-]_o \\ E_{cl} = E_K \end{array} \right\} 26 \ln \frac{[Cl^-]_i}{[Cl^-]_o} = 26 \ln \frac{[k^+]_o}{[k^+]_i} \rightarrow \frac{[k^+]_o}{[k^+]_i} = \frac{[Cl^-]_i}{[Cl^-]_o}$$

$$\left. \begin{array}{l} [k^+]_i = 500 - [k^+]_o \\ [Cl^-]_i = 1000 - [Cl^-]_o \\ [k^+]_o = [Cl^-]_o \end{array} \right\} \rightarrow x^2 = (500 - x) \times (1000 - x)$$

$$x = 333$$

$$[k^+]_i = 167 \text{ mM}$$

$$[Cl^-]_i = 667 \text{ mM}$$

$$[k^+]_o = [Cl^-]_o = 333 \text{ mM}$$

## Example 6-3

The potential recorded from two lead II, aVF is as follows. Calculate the MEA and electrical Doppler vector of the heart. Is the MEA in the normal range? What kind of deviation does it have?

## Example 6-3

### Answer

$$aVF = 5.5 \text{ mV}, V_{II} = 3.5 \text{ mV}, V_{II} = aVF - aVR \rightarrow 3.5 = 5.5 - aVR \rightarrow aVR = 2 \text{ mV}$$

$$\frac{(aVF + aVR + aVL)}{3} = 0 \rightarrow (5.5 + 2 + aVL) = 0 \rightarrow aVL = -7.5 \text{ mV}$$

$$V_I = aVL - aVR = -7.5 - 2 = -9.5 \text{ mV} : M_x \vec{i}$$

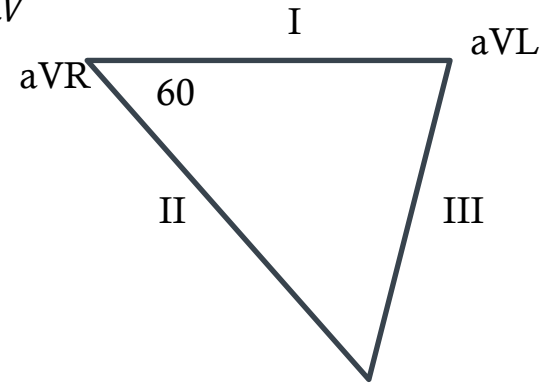
$$\vec{M} = (-9.5 \vec{i} + M_y \vec{j})$$

$$V_{II} = \vec{M} \cdot (\cos 60^\circ \vec{i} - \sin 60^\circ \vec{j})$$

$$3.5 = (-9.5 \vec{i} + M_y \vec{j}) \cdot \left( \frac{1}{2} \vec{i} - 0.86 \vec{j} \right) = M_y = -9.6 \text{ mV}$$

$$\vec{M} = (-9.5 \vec{i} + -9.6 \vec{j}), \theta = \tan^{-1} \frac{9.6}{9.5} = 45.2^\circ \rightarrow MEA = 134.8$$

$$120 < MEA < 180 \rightarrow RAD$$



## Example 6-4

- A. If the resting potential is -52 mV at room temperature, find the concentration of potassium in the cytoplasm.
- B. Find the Nernst potential of each ion.
- C. Is the membrane potential equal to the ionic Nernst potential? What factor can cause equality, and in this case, what is the flow of each of the ions? Explain

<i>Ion</i>	Cytoplasm	outside the cell	Permeability ratio
$Na^+$	41	276	0.017
$K^+$	?	4	1
$Cl^-$	52	340	0.412

## Example 6-4

### Answer

A)

$$V_{total} = \frac{kT}{q} \ln \frac{[Na^+]_o + [k^+]_o + [Cl^-]_i}{[Na^+]_i + [k^+]_i + [Cl^-]_o} = -52 = 26 \ln \left( \frac{4 + 17 \times 276 + 412 \times 10^{-3} \times 52}{x + 17 \times 10^{-3} \times 41 + 412 \times 10^{-3} \times 340} \right)$$
$$\rightarrow x = 81.76 \text{ mM}$$

B)

$$V_{Na} = \frac{kT}{q} \ln \frac{[Na^+]_o}{[Na^+]_i} = 26 \times \ln \frac{276}{41} = 49.577 \text{ mV}$$

$$V_k = \frac{kT}{q} \ln \frac{[k^+]_o}{[k^+]_i} = -78.95 \text{ mV}$$

$$V_{Na} = \frac{kT}{q} \ln \frac{[Cl^-]_i}{[Cl^-]_o} = 26 \times \ln \frac{52}{340} = -46.82 \text{ mV}$$

C)

What do you think about this one? This one will be left as your **HWc05**

## Example 6-5

Consider a membrane that has a passive channel for chlorine ions and an active pump for potassium ions separate from the potassium channel. If the concentration of  $[KCl]$  is not in equilibrium on both sides of the membrane of this cell, find an expression for the current of the potassium pump.

(**Hint:** Use the proof of existing relationships for the flow of ions.)

## Example 6-5

Answer

$$\begin{aligned} J_k &= J_p - \mu_k Z_k [k^+] \frac{dV}{dx} - \frac{kT}{q} \mu_k \frac{d[k^+]}{dx} \\ J_{Cl} &= -\mu_{Cl} Z_{Cl} [Cl^-] \frac{dV}{dx} - \frac{kT}{q} \mu_{Cl} \frac{d[Cl^-]}{dx} \\ \rightarrow \frac{dV}{dx} &= \frac{kT}{q[Cl^-]} \frac{d[Cl^-]}{dx} \xrightarrow{[Cl^-]=[k^+]} \frac{dV}{dx} = \frac{kT}{q[k^+]} \frac{d[k^+]}{dx} \end{aligned}$$

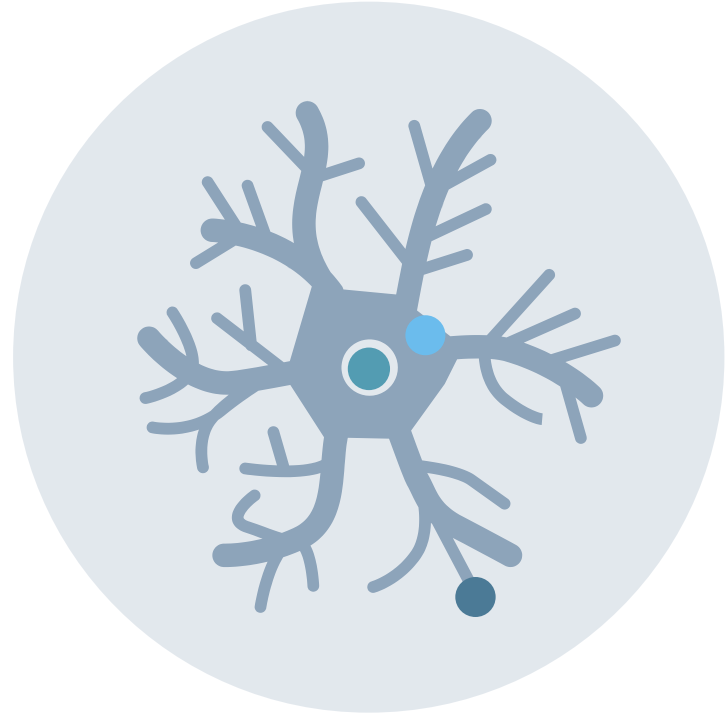
$$\begin{aligned} \text{Steady State: } J_k &= 0 = J_p - \mu_k Z_k [k^+] \frac{dV}{dx} - \frac{kT}{q} \mu_k \frac{d[k^+]}{dx} \\ &= J_p - \mu_k [k^+] \frac{kT}{q[k^+]} \frac{d[k^+]}{dx} - \frac{kT}{q} \mu_k \frac{d[k^+]}{dx} \rightarrow J_p - \frac{2kT\mu_k}{q} \frac{d[k^+]}{dx} \end{aligned}$$

$$-\int_0^\delta J_p dx = -\frac{2kT\mu_k}{q} \int_{[k^+]_i}^{[k^+]_o} d[k^+] \rightarrow J_p = \frac{2kT\mu_k}{q} ([k^+]_o - [k^+]_i)$$



04

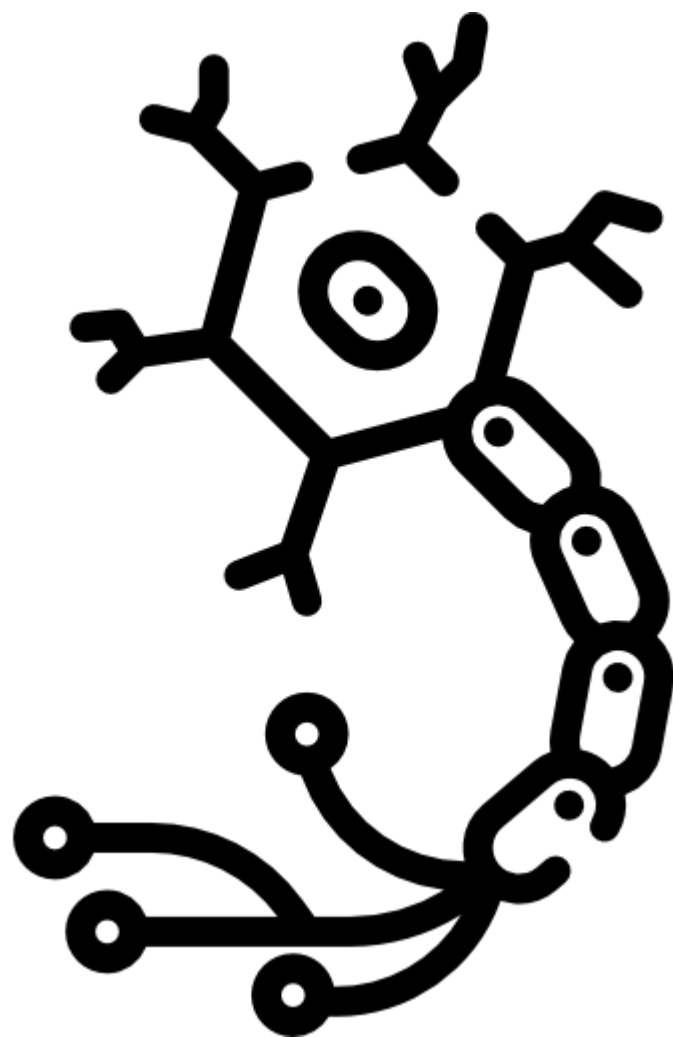
# Assignment





**HWh05**

**HWc05**



# Resources

## Dr. Malikeh Nabaei:

- Slides
- Classes

## Faezeh Jahani:

- Slides

## biological and medical physics, biomedical engineering

- The reference book



# Thanks!

Does anyone have any questions?

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**Have a good afternoon**