CHW01

Sunday, December 3, 2023 10:33 PM

Pouya Lahabi - 400109843

1. CDP Representation with CVXPY

Sunday, December 3, 2023 10:36 PM

1 DCP Representation with CVXPY

The function $f(x, y) = 1/(xy)$ with $x, y \in \mathbb{R}$, $\mathrm{dom} f = \mathbb{R}^2_{++}$, is convex. How do we represent it using disciplined convex programming (DCP), and the functions $1/u$, $\sqrt{uv}$, $\sqrt{u}$, u^2, u^2/v, addition, subtraction, and scalar multiplication? (These functions have the obvious domains, and you can assume a sign-sensitive version of DCP, e.g., u^2/v increasing in u for $u \geq 0$.)
To learn about CVXPY and how to solve optimization problems with it, you can study the examples on this page.

$$f(x,y) = \left(\frac{1}{\sqrt{xy}}\right)^2 = g_3\left(g_2\left(g_1(x,y)\right)\right)$$

where: $g_1(u,v) = \sqrt{uv}$, $g_2(u) = \frac{1}{u}$, $g_3(u) = u^2$

$g_1(u,v) \rightarrow$ Concave on R^2_{++} , Increasing
$g_2(u) \longrightarrow$ Convex and decreasing on R_{++}
$g_3(u) \rightarrow$ Convex and Increasing on R_{++}

$\rightsquigarrow f(x,y)$ is Convex

2. Optimizing a set of disks

Sunday, December 3, 2023 11:31 PM

First Problem:

$$\text{minimize } \sum \pi r_i^2 \longrightarrow \text{Eq: minimize } \sum r_i^2$$

$$\text{s.t. } \quad c_i = c_i^{fix} \quad r_i = r_i^{fix} \quad \text{for } i = 1, \dots, k$$

$$r_i \geq 0 \quad \text{for} \quad i = 1, \dots, n$$

$$\|c_i - c_j\|_2 \leq r_i + r_j \quad \text{for} \quad (i,j) \in \mathcal{L}$$

Second Problem:

$$\text{minimize } \sum 2\pi r_i \longrightarrow \text{Eq: minimize } \sum r_i$$

$$\text{s.t. } \quad c_i = c_i^{fix} \quad r_i = r_i^{fix} \quad \text{for } i = 1, \dots, k$$

$$r_i \geq 0 \quad \text{for} \quad i = 1, \dots, n$$

$$\|c_i - c_j\|_2 \leq r_i + r_j \quad \text{for} \quad (i,j) \in \mathcal{L}$$

3. Bandlimited signal recovery from zero-crossings

Sunday, December 3, 2023 10:44 PM

$$y_t = \sum_{j=1}^{B} a_j \cos\left(\frac{2\pi}{n}(f_{\min} + j - 1)t\right) + b_j \sin\left(\frac{2\pi}{n}(f_{\min} + j - 1)t\right), \quad t = 1, \ldots, n$$

main prob: minimize $\|y\|_2$

s.t. $\mathrm{sgn}(y)$

$|y|_1 = n$

$\hat{y} = Ax \xrightarrow{\text{Where}} x = (a, b) \in \mathbb{R}^{2B} \rightarrow$ Coefficients of Sin, Cos

$A = [C \quad S] \in \mathbb{R}^{n \times 2B} \rightarrow C, S \in \mathbb{R}^{n \times B}$

$\hookrightarrow C_{tj} = \cos\left(\frac{2\pi}{n}(f_{\min} + j - 1)t\right)$

$S_{tj} = \sin\left(\frac{2\pi}{n}(f_{\min} + j - 1)t\right)$

for sign: $s_t, \hat{y}_t$ Should have the Same Sign

$\hookrightarrow s_t A_t^T x \geq 0 \rightsquigarrow A_t^T \rightarrow$ rows of A

$|y|_1 = n \rightarrow s^T A x = n$

Convex Prob: $\min \|Ax\|_2$

s.t. $s_t A_t^T x \geq 0$

$s^T A x = n$

4. Fitting a periodic Poisson distribution to data

Sunday, December 3, 2023 11:33 PM

1) $P_{(k \text{ events})} = e^{-\lambda_t} \dfrac{\lambda_t^{k}}{k!} \rightsquigarrow P_{(N_t)} = e^{-\lambda_t} \dfrac{\lambda_t^{N_t}}{N_t!}$

Log-likelihood: $\log(P_{N_t}) = -\lambda_t + N_t \log(\lambda_t) - \log(N_t!)$

$-$Log-likelikhood $= \sum \left(\lambda_t - N_t \log(\lambda_t) + \log(N_t!)\right)$

$\hookrightarrow$ minimize $\sum \left(\lambda_t - N_t \log(\lambda_t) + \log(N_t!)\right)$

s.t. $\lambda \geq 0$

با توجه به اینکه در هر زمان مستقل است. این مسئله را می‌توان برای هر t جداگانه حل کرد

$N_t \neq 0 \rightarrow \min \left(\lambda_t - N_t \log(\lambda_t) + \log(N_t!)\right)$

$\hookrightarrow \dfrac{d}{d\lambda_t} \rightarrow 1 - \dfrac{N_t}{\lambda_t} = 0 \longrightarrow \lambda_t = N_t$

$N_t = 0 \rightsquigarrow \min(\lambda_t) \rightsquigarrow \lambda_t = 0$

2) max: loglikelihood $- \rho \left\{ \sum (\lambda_{t+1} - \lambda_t)^2 + (\lambda_1 - \lambda_{24})^2 \right\}$

$\hookrightarrow$ minimize: $\displaystyle\sum_{t=1}^{24} \left(\lambda_t - N_t \log \lambda_t\right) + \rho \left\{ \sum_{t=1}^{23} (\lambda_{t+1} - \lambda_t)^2 + (\lambda_1 - \lambda_{24})^2 \right\}$

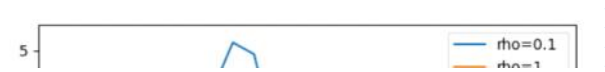
s.t. $\lambda \geq 0$

3) اگر ρ به سمت بی‌نهایت برود می‌توان گفت که λ_t ها باید با هم برابر شوند

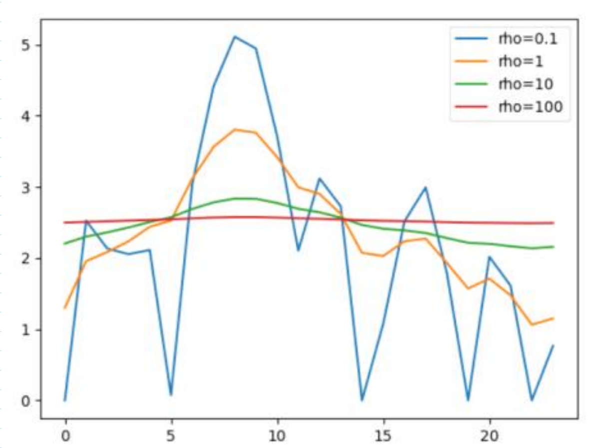
minimize $24\lambda - \displaystyle\sum_{t=1}^{24} N_t \log \lambda$

s.t. $\lambda \geq 0$

4)



4)



→ همانطور که انتظار داشتیم برای ρ زیاد تابع Smooth تر شد.

5)

rho = 0.1, log likelihood = -83.29373337277248
rho = 1, log likelihood = -37.74828664528782
rho = 10, log likelihood = -41.71490239549287
rho = 100, log likelihood = -43.76226937992665

← خروجی برنامه

→ $\rho = 1$ → کمترین log-likelihood را دارد.