Computer Engineering Department

Inverse Reinforcement Learning

Mohammad Hossein Rohban, Ph.D.

Spring 2024

Slides are adopted from CS 285, UC Berkeley.

Lecture Outline

1. So far: manually design reward function to define a task
2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
3. Apply approximate optimality model from last time, but now learn the reward!

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2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
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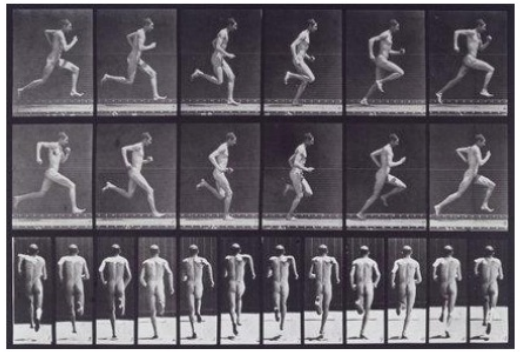
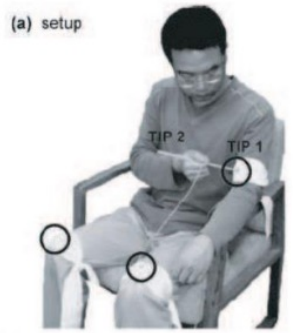
- Goals:
  - Understand the inverse reinforcement learning problem definition
  - Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
  - Understand a few practical inverse reinforcement learning algorithms we can use

Modeling Human Behavior with Optimal Control



Muybridge (c. 1870)

Mombaur et al. '09



Li & Todorov '06

Ziebart '08

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$$

$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

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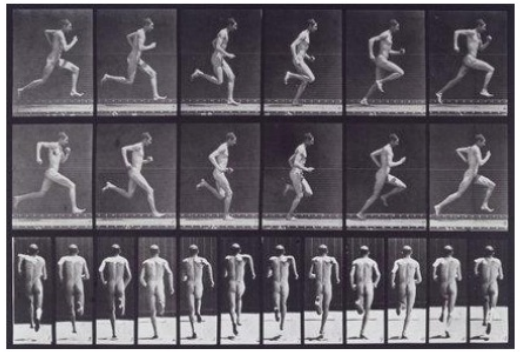

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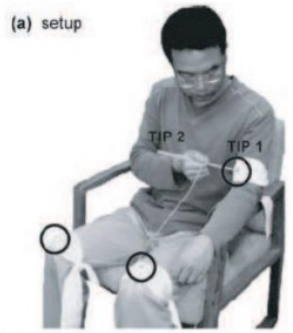

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$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$$

$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{a}_t)$$

optimize this to explain the data

$$\pi = \arg \max_{\pi} E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t), \mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)}[r(\mathbf{s}_t, \mathbf{a}_t)]$$

$$\mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)$$

Imitation learning vs RL perspective

The imitation learning perspective

Standard imitation learning:

- copy the *actions* performed by the expert
- no reasoning about outcomes of actions

Human imitation learning:

- copy the *intent* of the expert
- might take very different actions!

Imitation learning vs RL perspective

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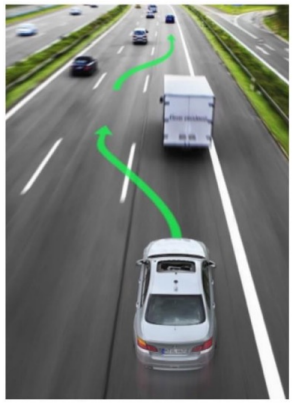
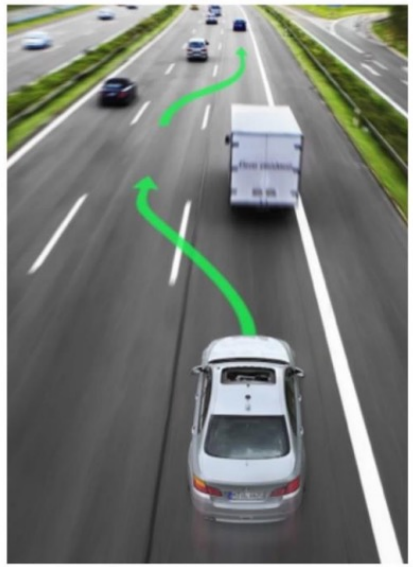


what is the reward?

Inverse Reinforcement Learning

Infer **reward functions** from **demonstrations**



$r(\mathbf{s}, \mathbf{a})$

Inverse Reinforcement Learning

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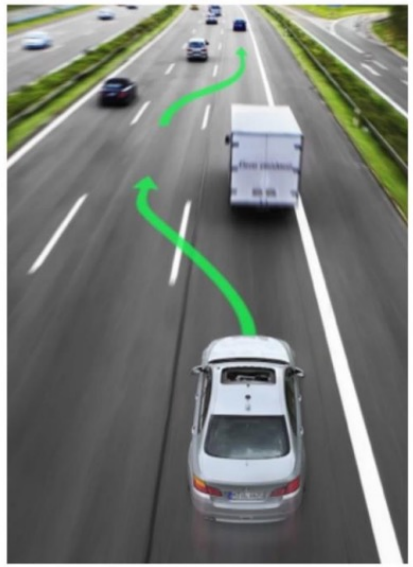
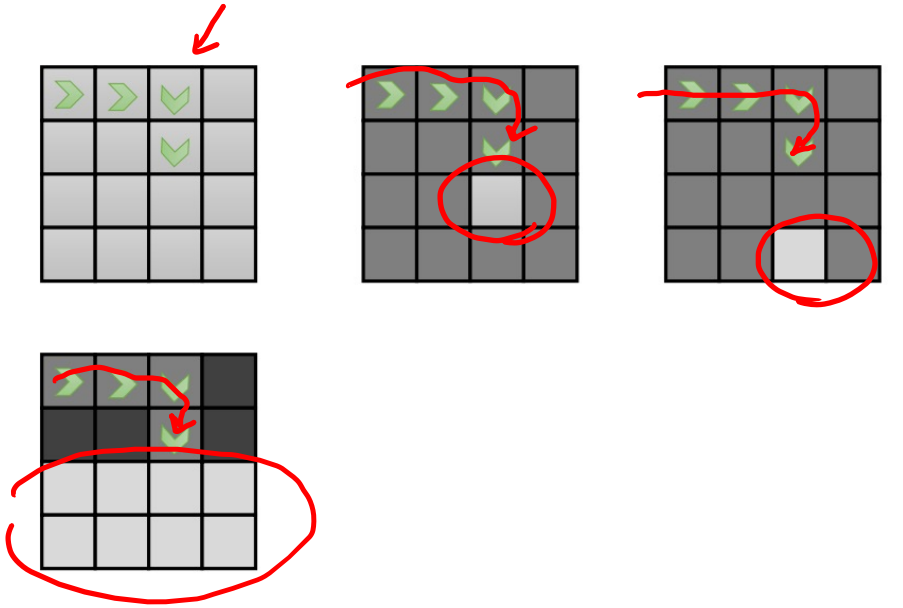


$r(\mathbf{s}, \mathbf{a})$

by itself, this is an **underspecified** problem

many reward functions can explain the **same** behavior



Inverse Reinforcement Learning Formulation

"forward" reinforcement learning

inverse reinforcement learning

Inverse Reinforcement Learning Formulation

"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$

(sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

inverse reinforcement learning

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reward parameters

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linear reward function:

$r_\psi(\mathbf{s}, \mathbf{a}) = \sum_i \psi_i f_i(\mathbf{s}, \mathbf{a}) = \psi^T \mathbf{f}(\mathbf{s}, \mathbf{a})$

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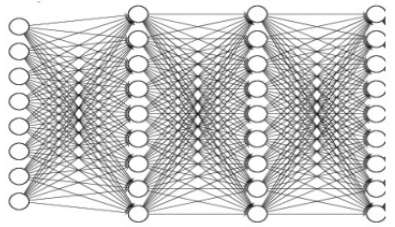
neural net reward function:



$r_\psi(\mathbf{s}, \mathbf{a})$

parameters ψ

Inverse Reinforcement Learning Formulation

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learn $r_\psi(\mathbf{s}, \mathbf{a})$

reward parameters

...and then use it to learn $\pi^\star(\mathbf{a}|\mathbf{s})$

linear reward function:

$$r_\psi(\mathbf{s}, \mathbf{a}) = \sum_i \psi_i f_i(\mathbf{s}, \mathbf{a}) = \psi^T \mathbf{f}(\mathbf{s}, \mathbf{a})$$

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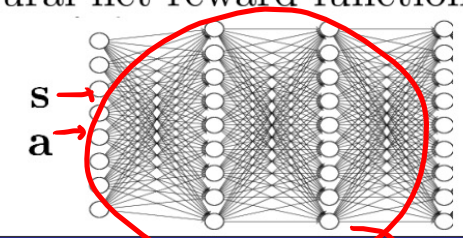

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$f(s, a)$

Feature Matching Inverse RL

linear reward function:

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linear reward function:

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Unknown

$D = \{\tau_i\}$

$\mathbb{E}_{\pi^*}[f(s,a)] = \mathbb{E}_{\pi_{r_\psi}}[f(s,a)]$

if features $\mathbf{f}$ are important, what if we match their expectations?

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let π^{r_ψ} be the optimal policy for r_ψ

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pick ψ such that $E_{\pi^{r_\psi}}[\mathbf{f}(\mathbf{s}, \mathbf{a})] = E_{\pi^\star}[\mathbf{f}(\mathbf{s}, \mathbf{a})]$

$\psi \in \mathbb{R}^k$

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state-action marginal under π^{r_ψ}

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unknown optimal policy

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still ambiguous!

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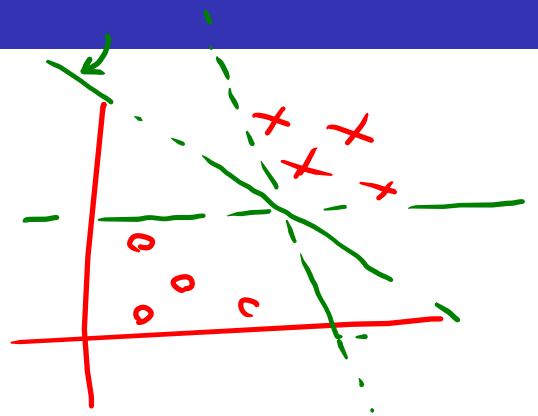

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state-action marginal under π^{r_ψ}

unknown optimal policy
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still ambiguous!

$r_\psi = \psi^T f$

$\mathbb{E}(r_\psi) = \mathbb{E}(\psi^T f)$

$= \psi^T \mathbb{E}(f(s,a))$

maximum margin principle:

$\max_{\psi, m} m$ such that $\psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \geq \max_{\pi \in \Pi} \psi^T E_\pi[\mathbf{f}(\mathbf{s},\mathbf{a})] + m$

$\pi^* \in \Pi$

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need to somehow "weight" by similarity between $\pi^\star$ and π

Feature Matching IRL & Maximum Margin

remember the "SVM trick":

$$\max_{\psi,m} m \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \geq \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s},\mathbf{a})] + m$$

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$$\Updownarrow$$

$$\min_{\psi} \frac{1}{2}\|\psi\|^2 \quad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s}, \mathbf{a})] \geq \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s}, \mathbf{a})] + 1$$

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Adaptive Margin

$\pi \simeq \pi^*$

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e.g., difference in feature expectations!

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$$\min_{\psi} \frac{1}{2}\|\psi\|^2 + \sum \xi_i \quad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s}, \mathbf{a})] \geq \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s}, \mathbf{a})] + D(\pi, \pi^\star) - \xi_i$$

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Further reading:

- Abbeel & Ng: Apprenticeship learning via inverse reinforcement learning
- Ratliff et al: Maximum margin planning

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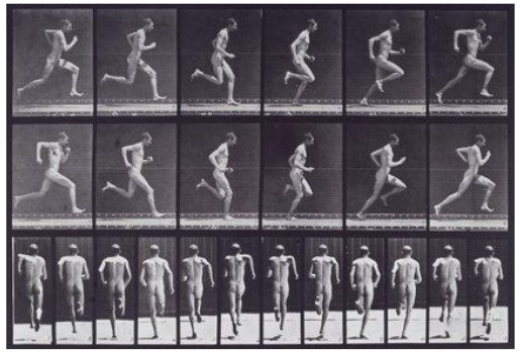

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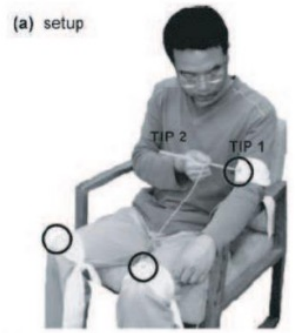
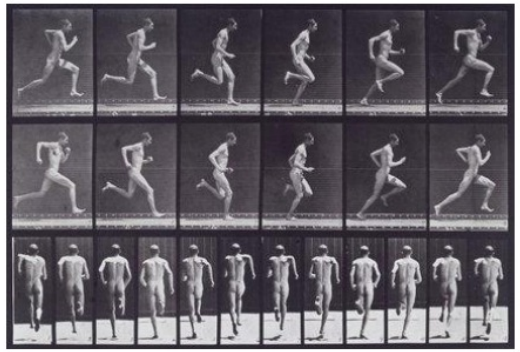


Li & Todorov '06

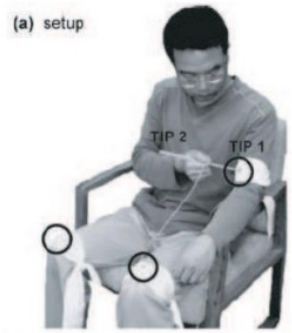
Ziebart '08

Modeling Human Behavior with Optimal Control



Muybridge (c. 1870)

Mombaur et al. '09

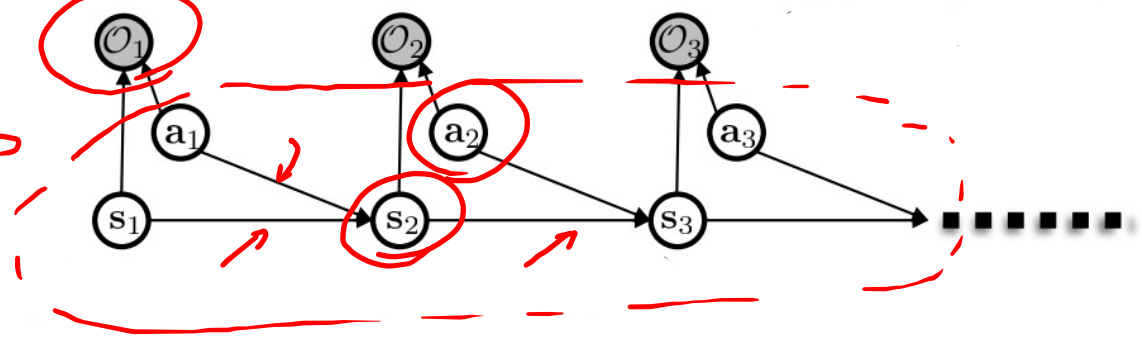


Li & Todorov '06

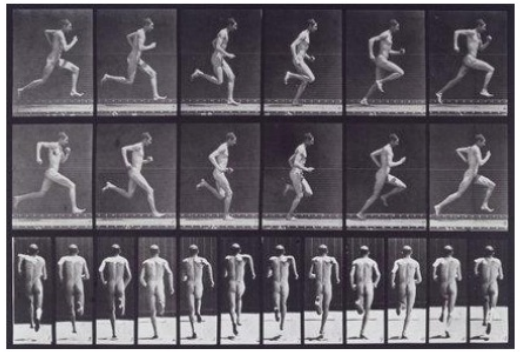
Ziebart '08

optimality = Variable $O_i \in \{0, 1\}$

MDP



Modeling Human Behavior with Optimal Control



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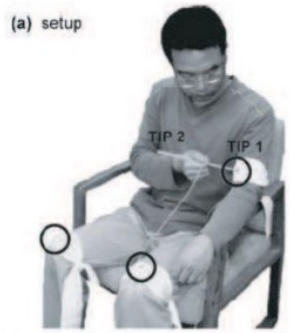

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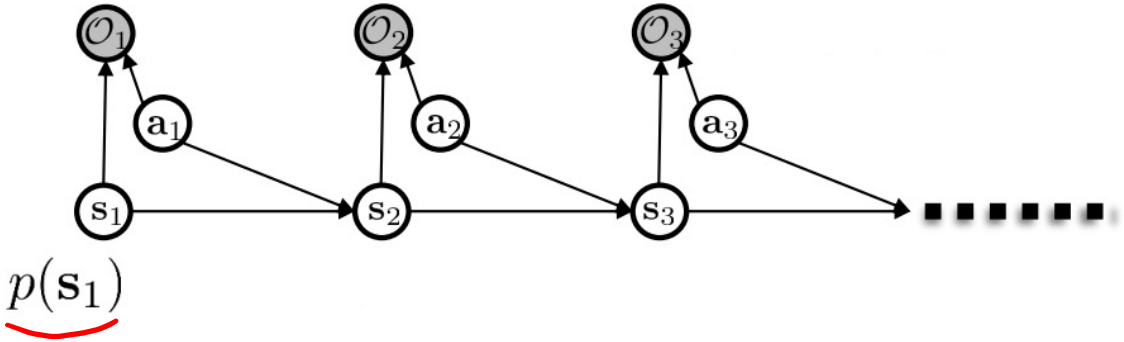
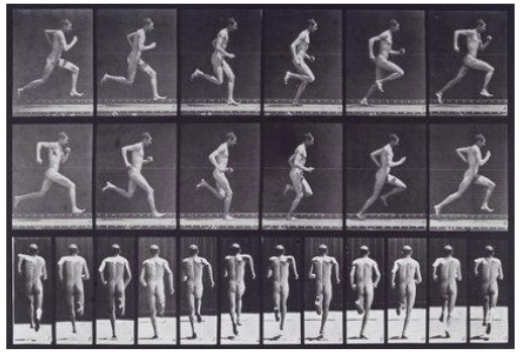


Modeling Human Behavior with Optimal Control



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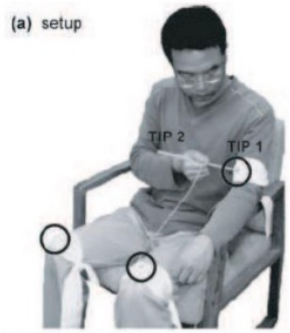

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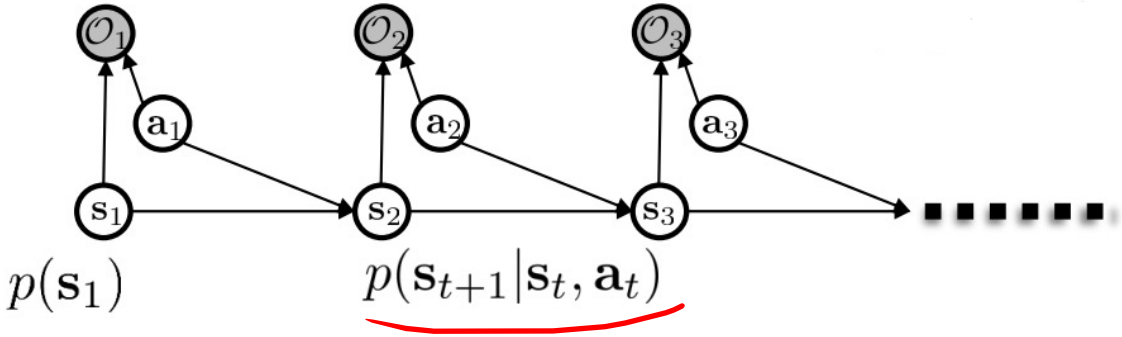
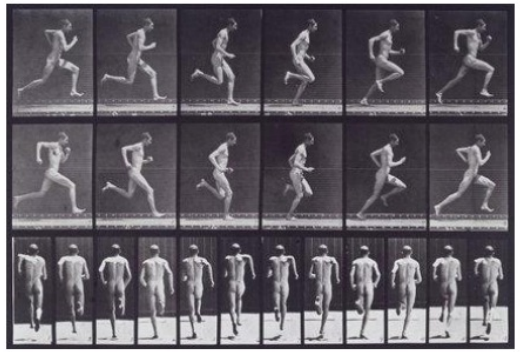


Modeling Human Behavior with Optimal Control



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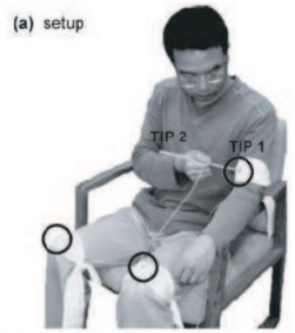

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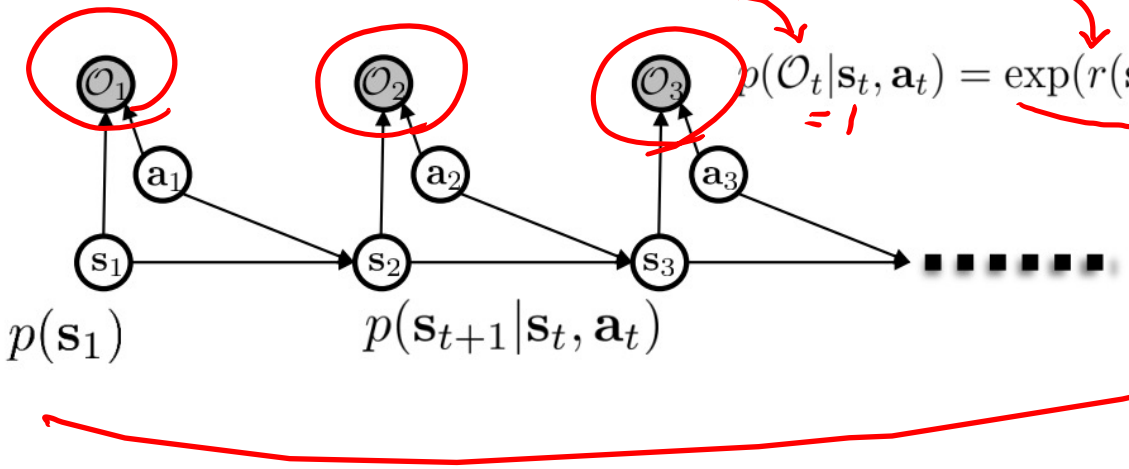

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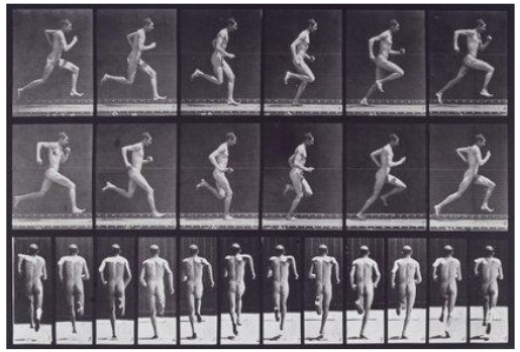

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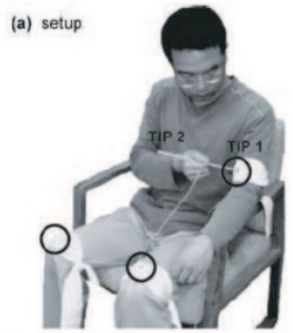

Li & Todorov '06

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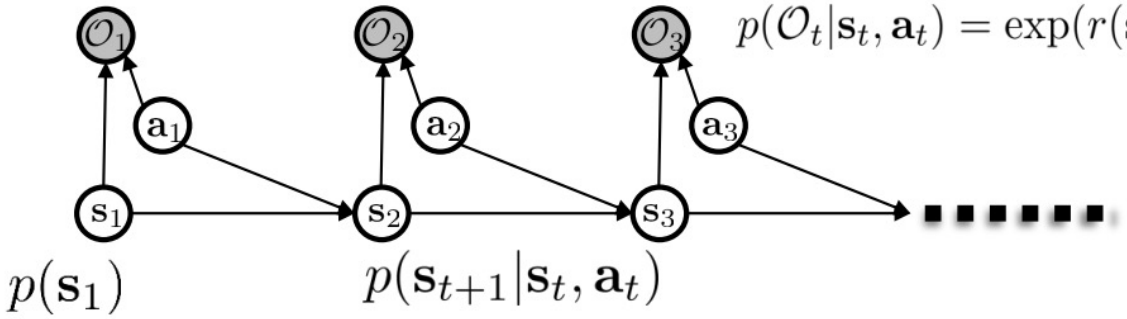
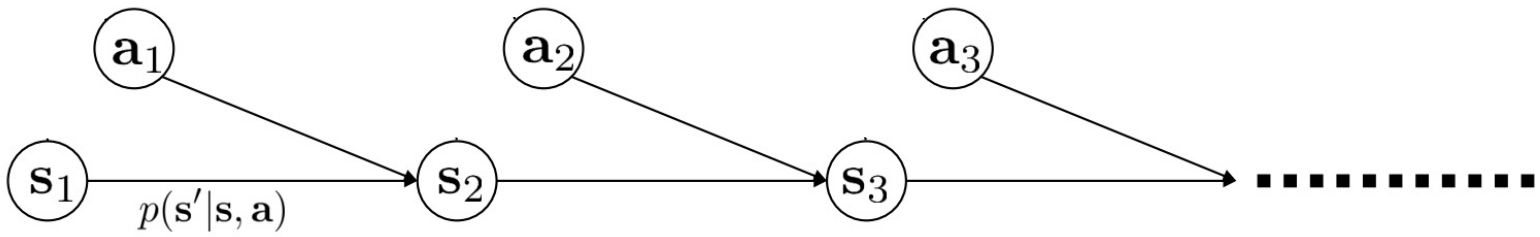
$$p(\mathbf{s}_{1:T}, \mathbf{a}_{1:T}|\mathcal{O}_{1:T})$$



$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

A probabilistic graphical model of decision making

$$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$$

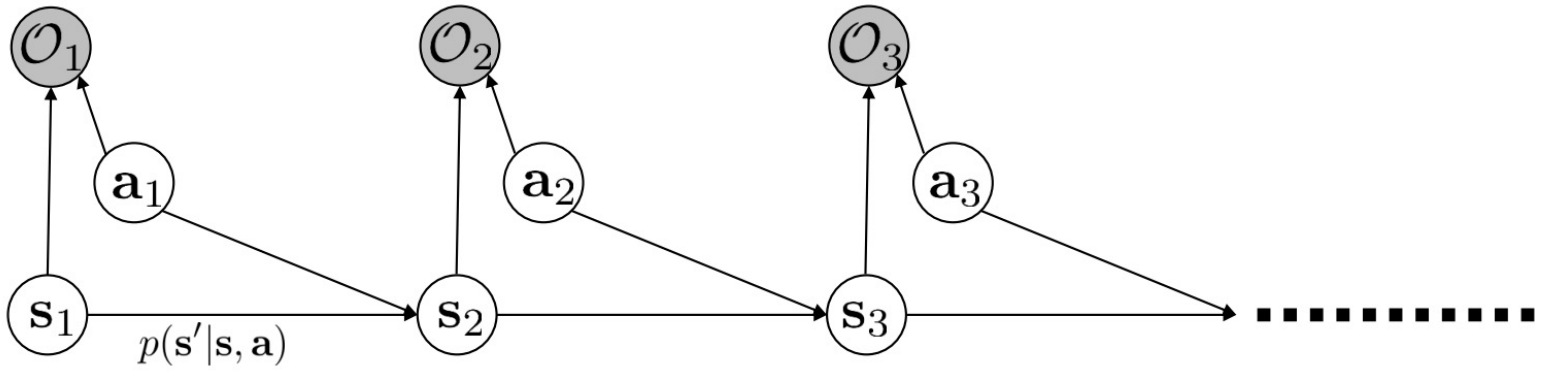


A probabilistic graphical model of decision making

$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$ no assumption of optimal behavior!

$p(\tau|\mathcal{O}_{1:T})$

$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$



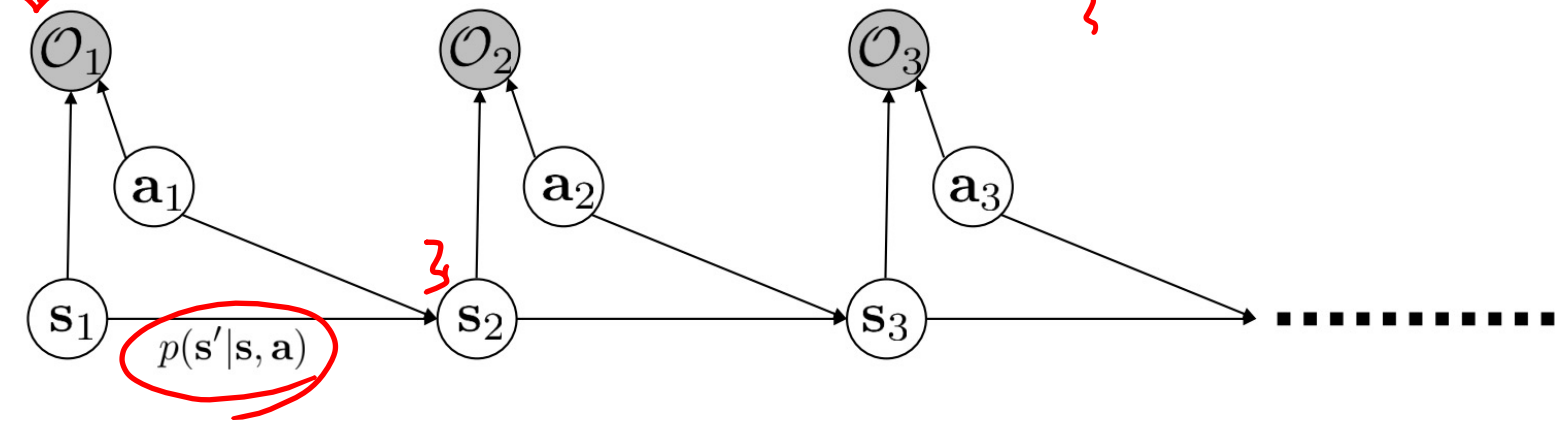
A probabilistic graphical model of decision making

$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$ no assumption of optimal behavior!

$p(\tau|\mathcal{O}_{1:T})$ $\quad p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

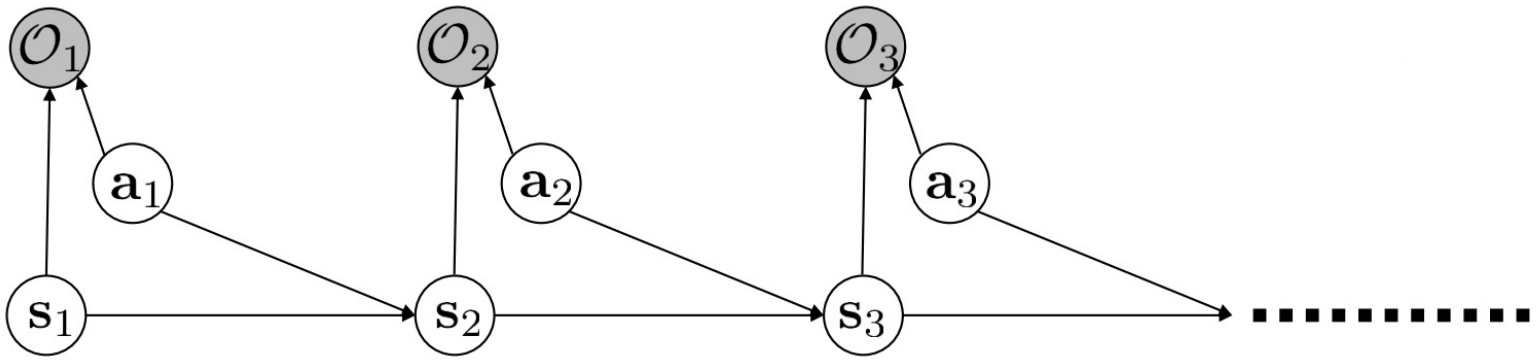
$$p(\tau|\mathcal{O}_{1:T}) = \frac{p(\tau, \mathcal{O}_{1:T})}{p(\mathcal{O}_{1:T})}$$

$$\propto p(\tau) \prod_t \exp(r(\mathbf{s}_t, \mathbf{a}_t)) = p(\tau) \exp\left(\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right)$$



Learning the optimality variable

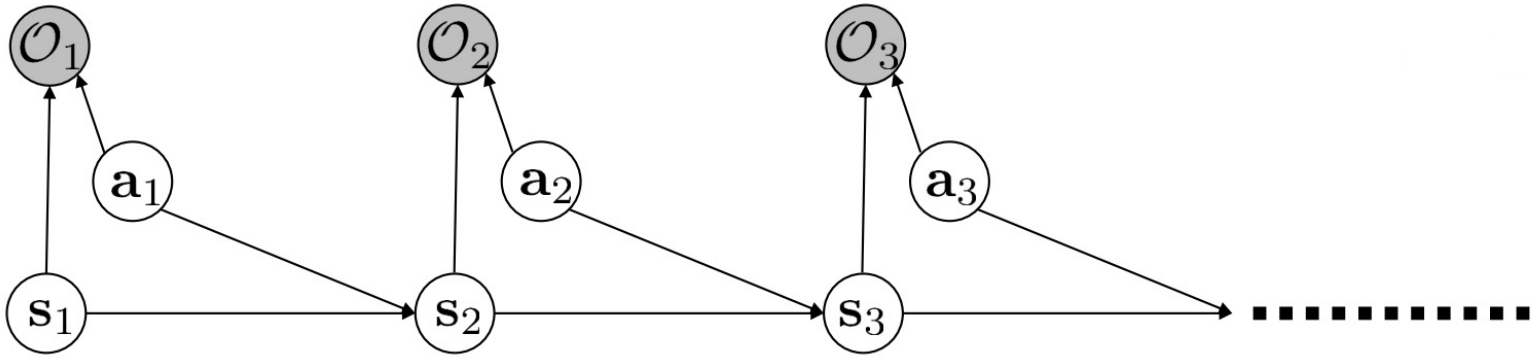
$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$



Learning the optimality variable

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

reward parameters



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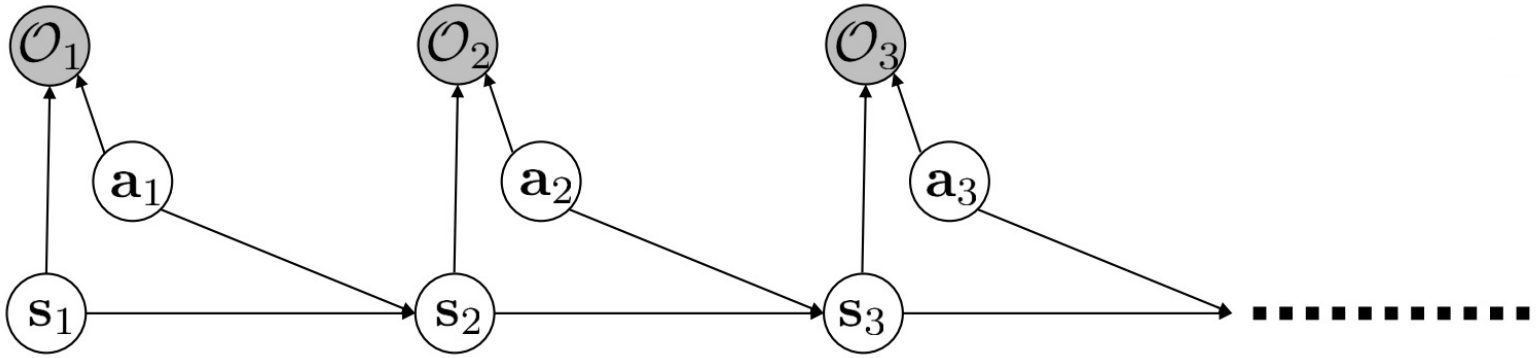
$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t, \psi) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

$$p(\tau|\mathcal{O}_{1:T}, \psi)$$



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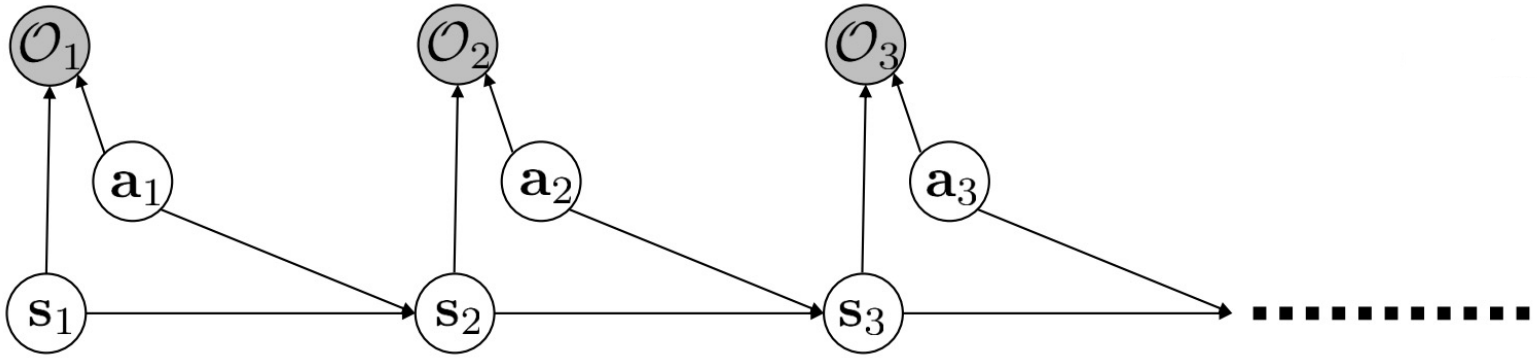

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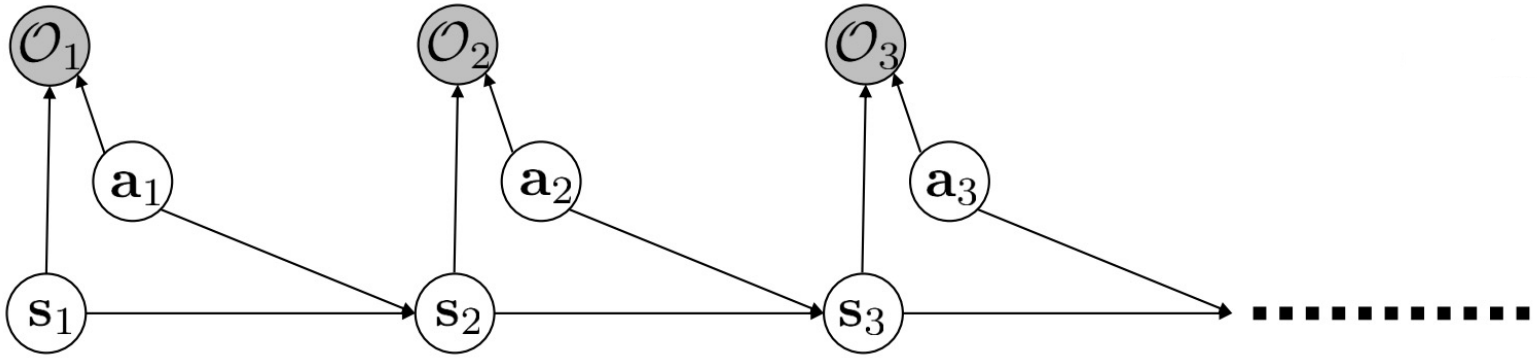
given:

samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$



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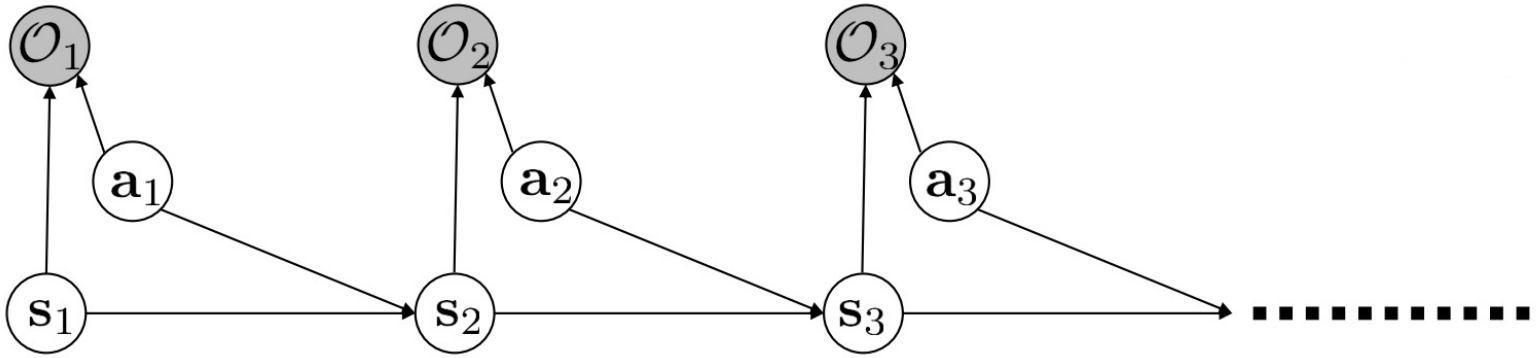
maximum likelihood learning

given:

samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$



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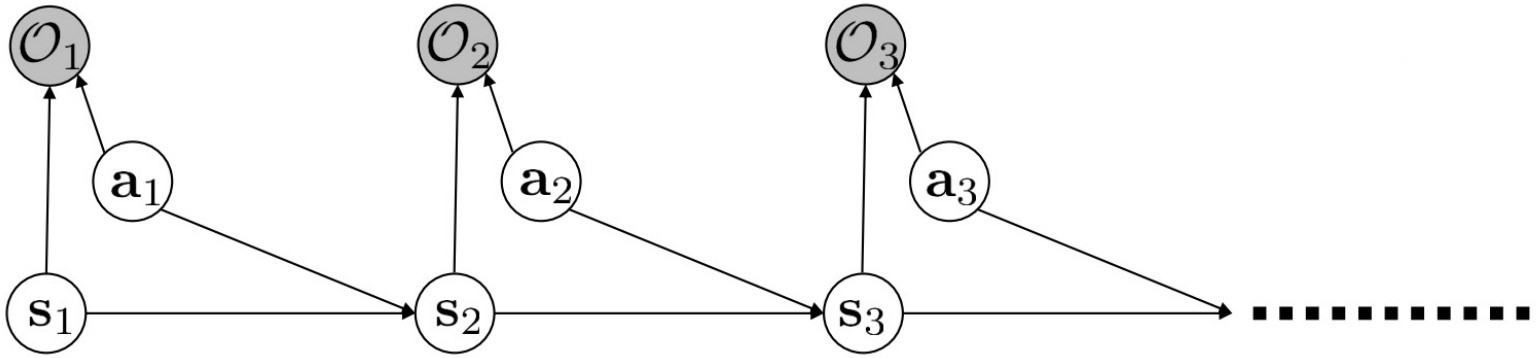
maximum likelihood learning: $\max_\psi \frac{1}{N} \sum_{i=1}^{N} \log p(\tau_i|\mathcal{O}_{1:T}, \psi)$



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can ignore (independent of ψ)

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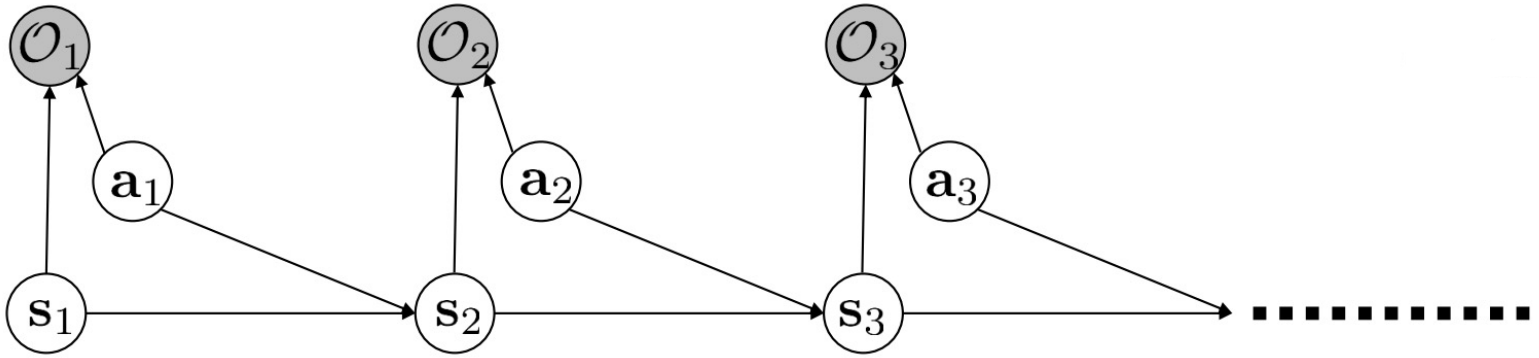


Learning the optimality variable

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t, \psi) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

$Z = \text{const}(\tau)$

given:

samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$

$$p(\tau|\mathcal{O}_{1:T}, \psi) \propto p(\tau) \exp\left(\sum_t r_\psi(\mathbf{s}_t, \mathbf{a}_t)\right)$$

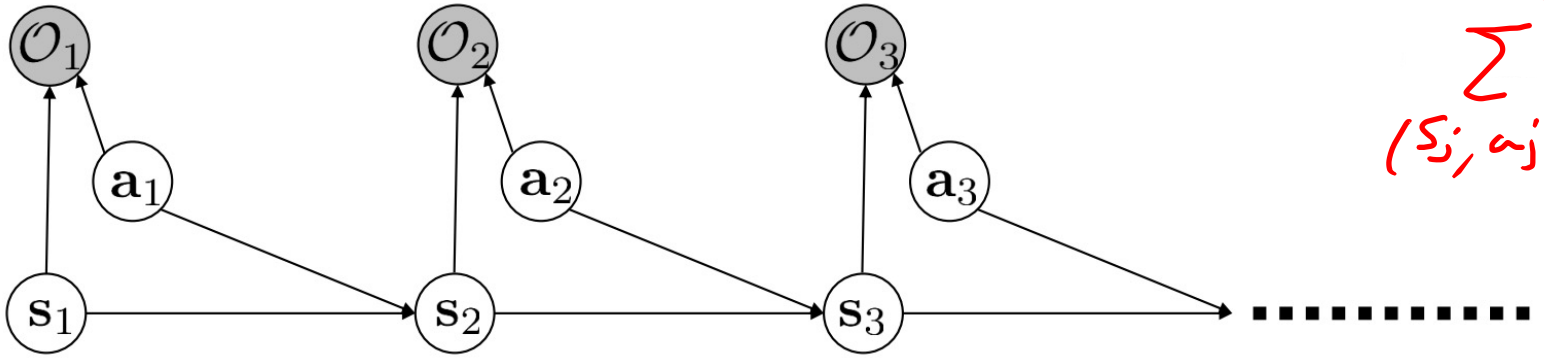
can ignore (independent of ψ)

Z

$\psi^T f(s,a)$

maximum likelihood learning: $\max_\psi \frac{1}{N} \sum_{i=1}^{N} \log p(\tau_i|\mathcal{O}_{1:T}, \psi) = \max_\psi \frac{1}{N} \sum_{i=1}^{N} r_\psi(\tau_i) - \log Z$

$r_\psi(\tau_i) = \sum_{(s_j, a_j) \in \tau_i} r_\psi(s_j, a_j)$



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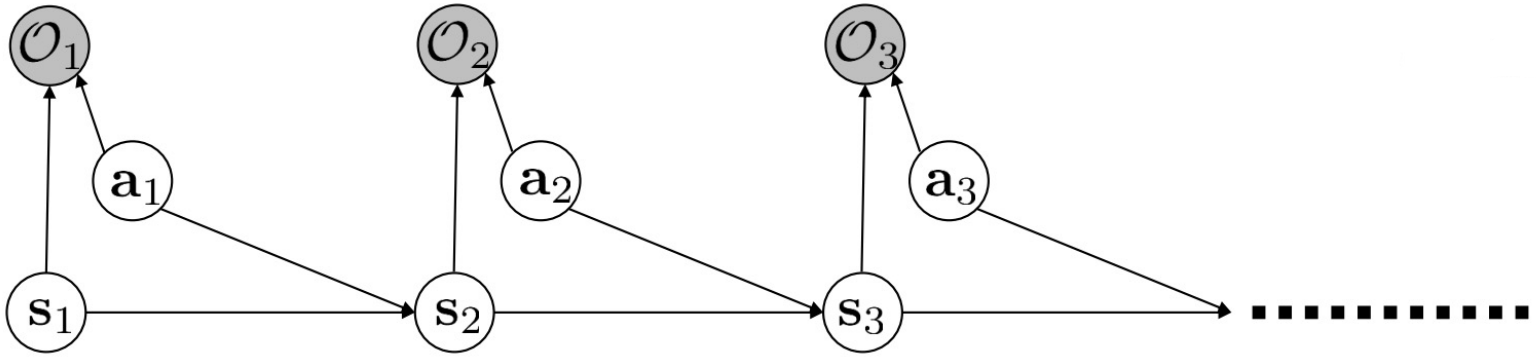
partition function
(the hard part)



The IRL Partiotion Function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z$$

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$$Z = \int p(\tau) \exp(r_\psi(\tau)) d\tau$$

$$p(\tau | o_{1..T}) = \int \frac{p(\tau) \exp(r_\psi(\tau))}{Z} d\tau = 1$$

$$\Rightarrow Z = \int p(\tau) e^{r(\tau)} d\tau$$

The IRL Partiotion Function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_\psi(\tau_i) - \log Z$$

$$Z = \int p(\tau) \exp(r_\psi(\tau)) d\tau$$

$$\nabla_\psi \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{Z} \int p(\tau) \exp(r_\psi(\tau)) \nabla_\psi r_\psi(\tau) d\tau$$

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$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T}, \psi)}[\nabla_\psi r_\psi(\tau)]$$

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$f(\tau_i)$

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T}, \psi)}[\nabla_\psi r_\psi(\tau)]$$

estimate with expert samples

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estimate with expert samples

soft optimal policy under current reward

Unknown Dynamics & Large State/Action Spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

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idea: learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

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$$J(\theta) = \sum_t E_{\pi(\mathbf{s}_t,\mathbf{a}_t)}[r_\psi(\mathbf{s}_t, \mathbf{a}_t)] + E_{\pi(\mathbf{s}_t)}[\mathcal{H}(\pi(\mathbf{a}|\mathbf{s}_t))]$$

Unknown Dynamics & Large State/Action Spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]$$

estimate with expert samples — soft optimal policy under current reward

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sum over expert samples — sum over policy samples

More Efficient Sample-Based Updates

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$\pi_{r_\psi^{old}}$ —1-step→ r_ψ^{new}

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expert — π — 1 step improvement of $\pi_{\psi_{old}}$

Importance Sampling

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j} \sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j) \qquad w_j = \frac{p(\tau) \exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

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$$\frac{p(\mathbf{s}_1) \prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t) \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))}{p(\mathbf{s}_1) \prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t) \pi(\mathbf{a}_t|\mathbf{s}_t)}$$

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each policy update w.r.t. r_ψ brings us closer to the target distribution!

Guided Cost Learning Algorithm (Finn et al. ICML '16)

$p(\tau | O_{1:T}, \psi)$

initial policy π

human demonstrations

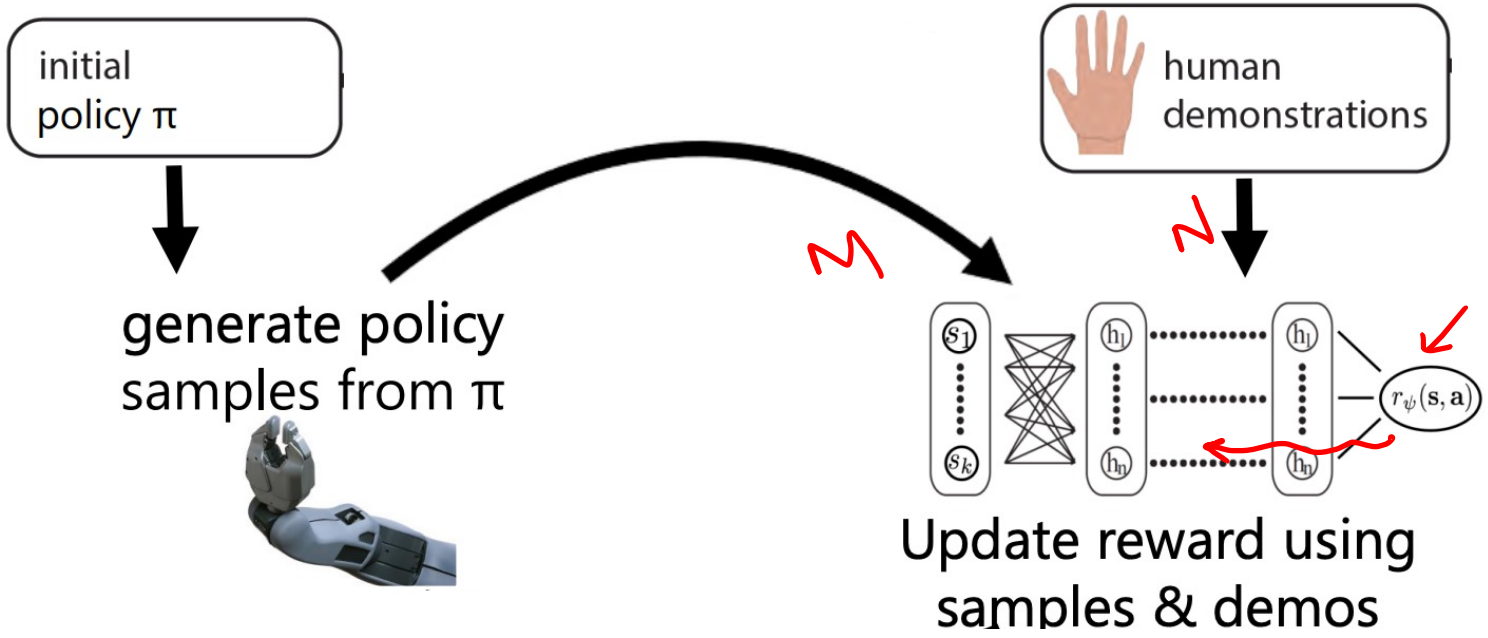
Guided Cost Learning Algorithm (Finn et al. ICML '16)

initial
policy π

generate policy
samples from π

human
demonstrations

Guided Cost Learning Algorithm (Finn et al. ICML '16)



obj

$$\nabla_\psi \mathcal{L} = \sum_{i=1}^{N} r_\psi(\tau_i) - \frac{1}{\sum w_j} \sum_{i=1}^{M} w_i r_\psi(\tau_i)$$

expert

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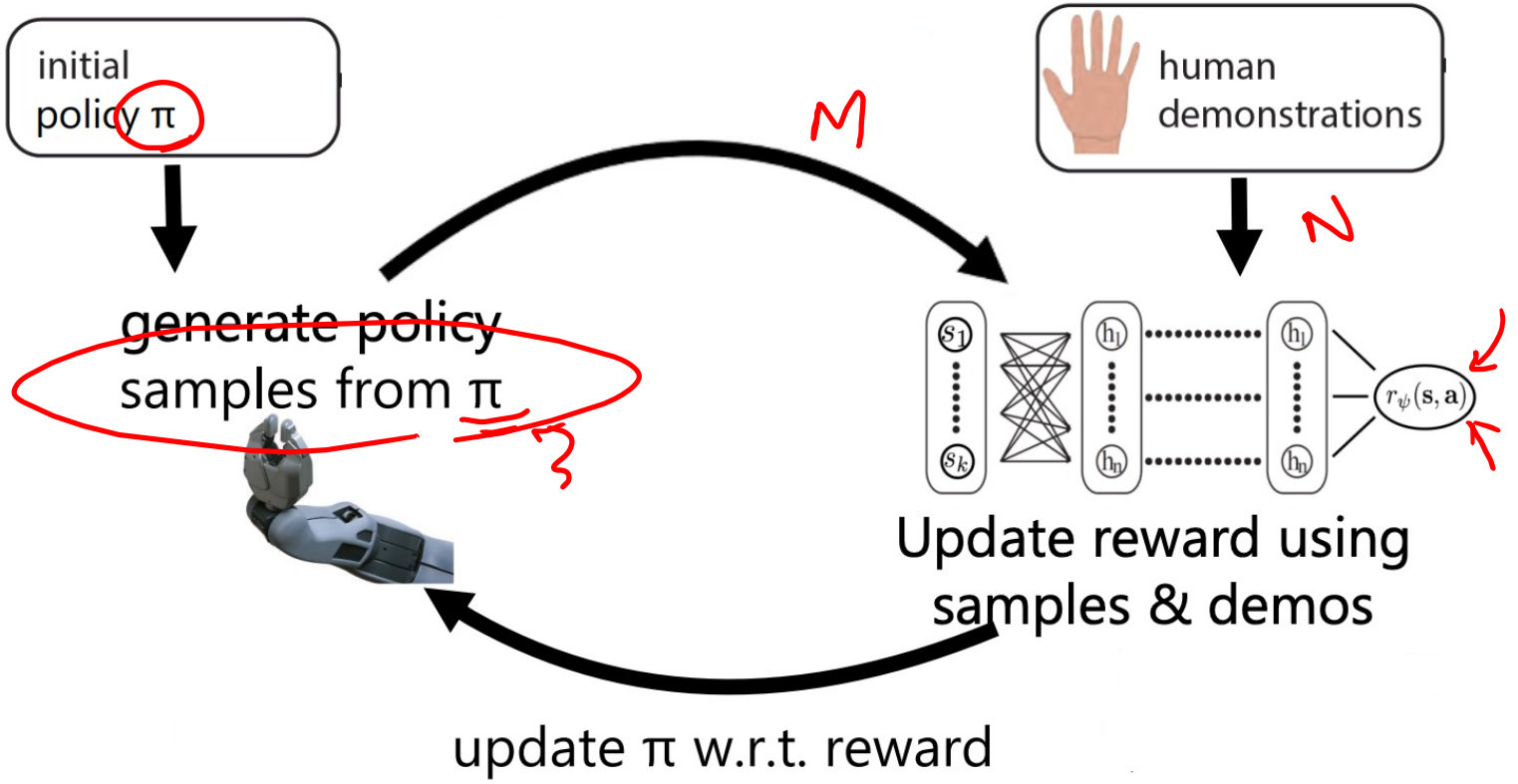

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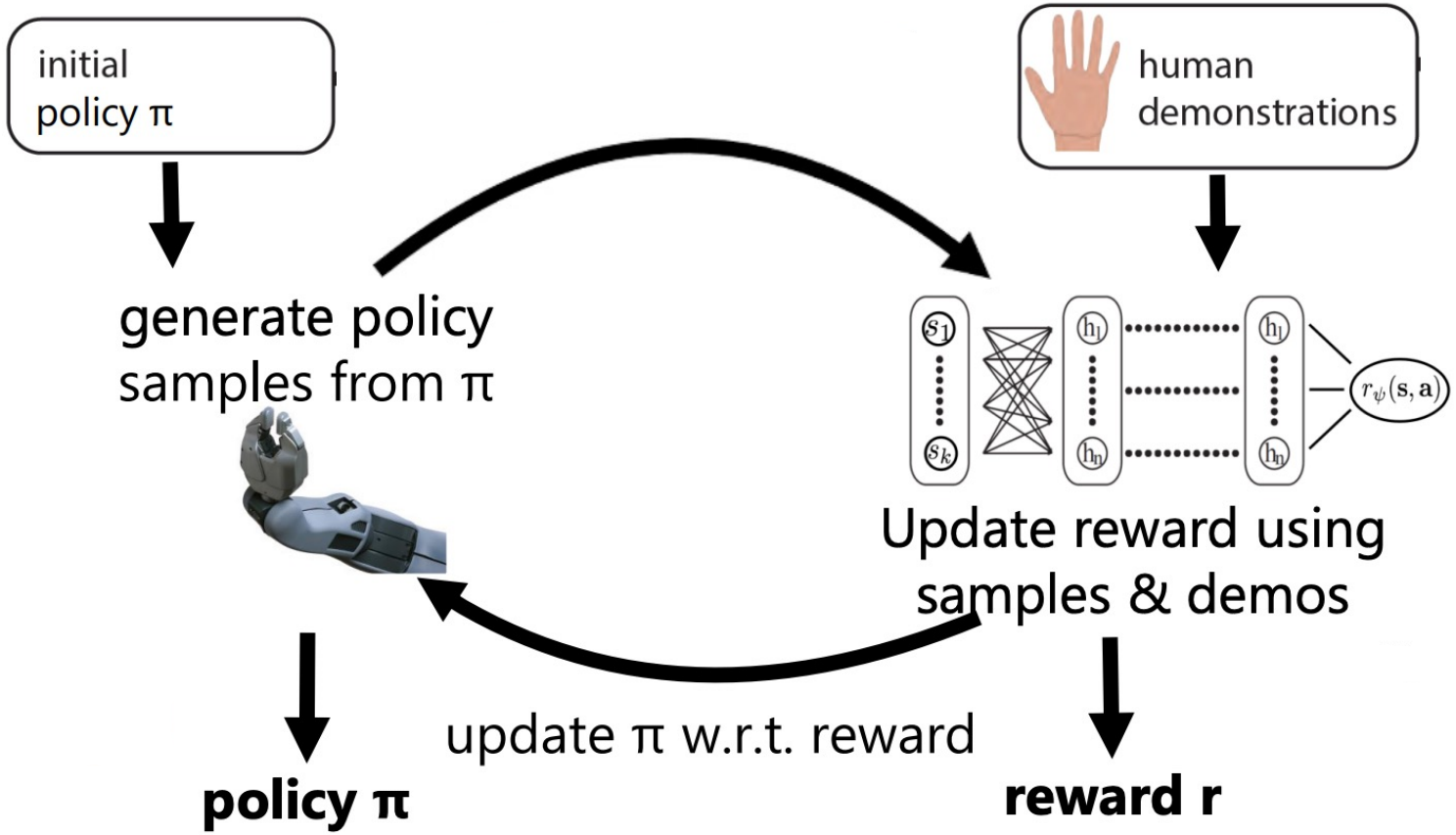

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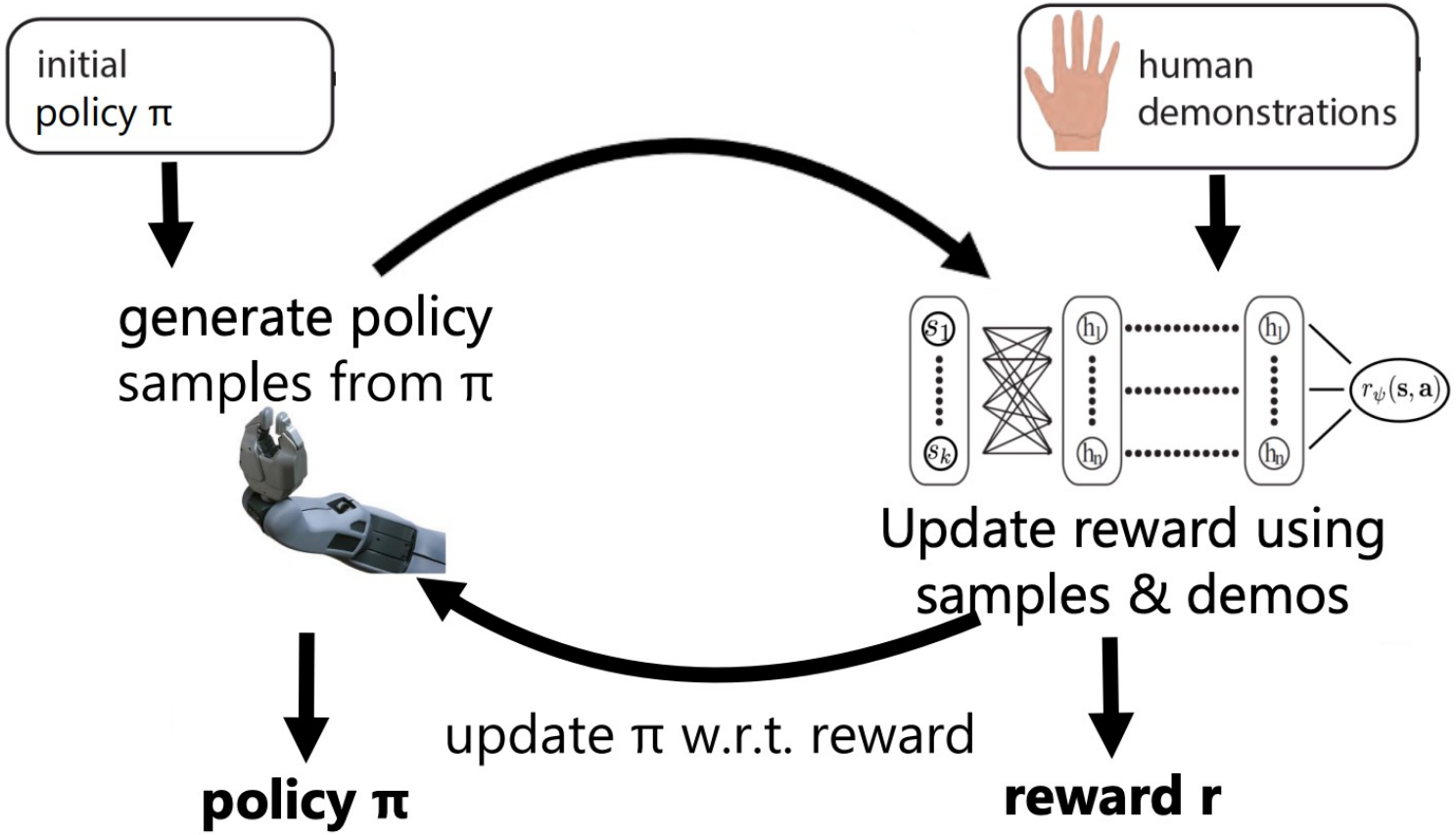

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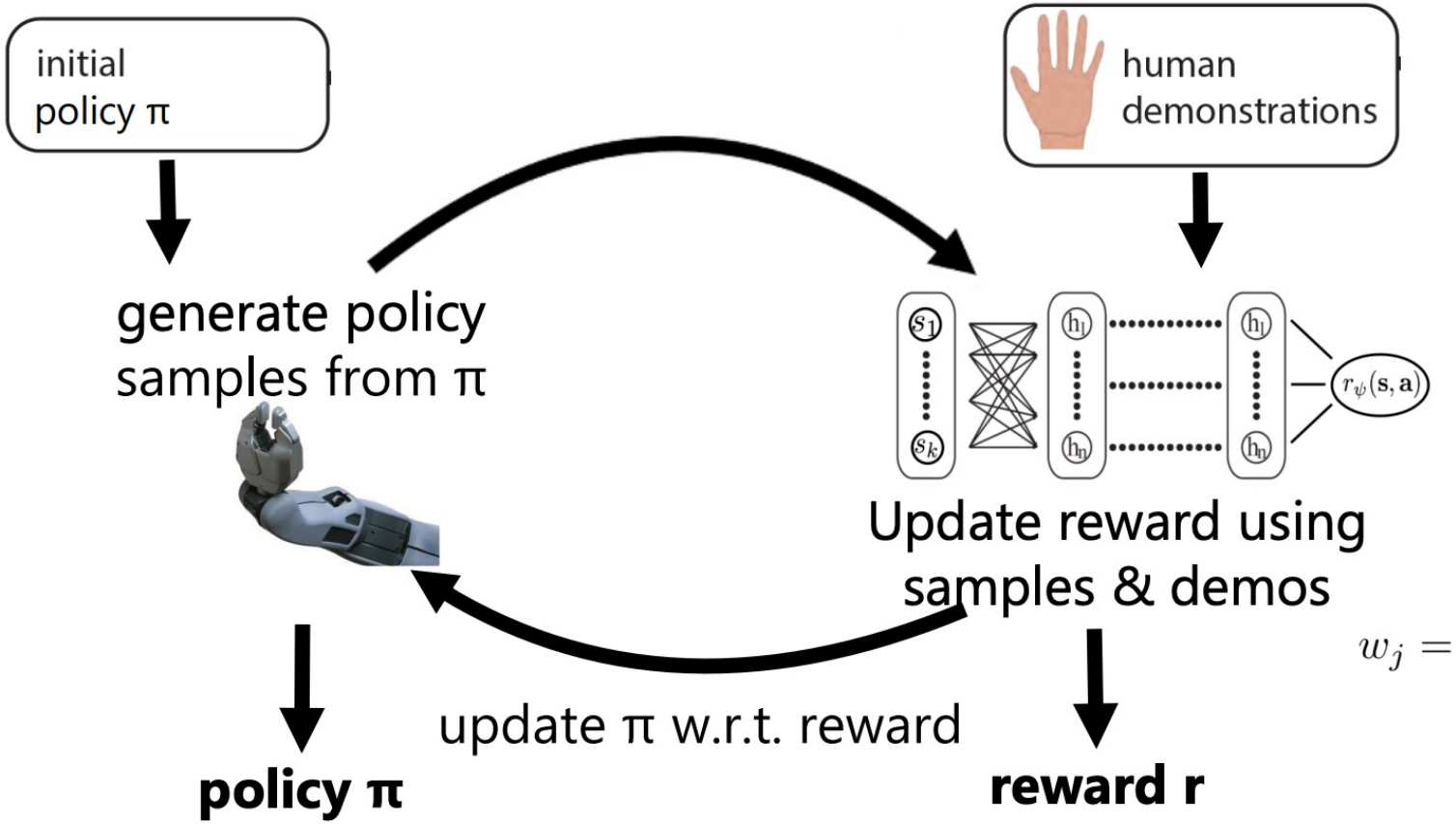

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Suggested Reading on Inverse RL

Classic Papers:
Abbeel & Ng ICML '04. *Apprenticeship Learning via Inverse Reinforcement Learning*. Good introduction to inverse reinforcement learning
Ziebart et al. AAAI '08. *Maximum Entropy Inverse Reinforcement Learning*. Introduction to probabilistic method for inverse reinforcement learning

Modern Papers:
Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions
Wulfmeier et al. arXiv '16. *Deep Maximum Entropy Inverse Reinforcement Learning*. MaxEnt inverse RL using deep reward functions
Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks
Fu, Luo, Levine ICLR '18. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning