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i) $x(t) = te^{-3t}\sin(4t)\,u(t) = te^{-3t} \times \left(\frac{e^{4tj} - e^{-4tj}}{2j}\right) u(t) =$

$= \frac{1}{2j} \times te^{(-3+4j)t} u(t) - \frac{1}{2j} te^{-(3+4j)t} u(t) \Rightarrow$

$\Rightarrow X(jw) = \frac{1}{2j} \times \left(\frac{1}{(3+(w-4)j)^2} - \frac{1}{(3+(w+4)j)^2}\right)$

ii) $x(t) = e^{-at}\cos(w_0 t)u(t) = e^{-at}\left(\frac{e^{jw_0 t} + e^{-jw_0 t}}{2}\right)u(t) =$

$= \frac{1}{2}\left\{e^{(jw_0 - a)t} + e^{-(jw_0 + a)t}\right\}u(t) \Rightarrow$

$\Rightarrow X(jw) = \frac{1}{2}\left\{\frac{1}{(a+j(w-w_0))} + \frac{1}{(a+j(w+w_0))}\right\}$

iii) $x(t) = e^{-3t}\left[u(t+2) - u(t-3)\right] =$

$= e^{+6} \times e^{-3(t+2)} u(t+2) - e^{-9} e^{-3(t-3)} u(t-3)$

$$\left.\begin{array}{l} f(t-t_0) \xrightarrow{F.} e^{-jwt_0} F(jw) \\ e^{-at} u(t) \xrightarrow{F.} \frac{1}{a+jw} \end{array}\right\} \Rightarrow$$

$\Rightarrow X(jw) = \frac{e^{6+2wj}}{3+jw} - \frac{e^{-(9+3wj)}}{3+jw}$

iv) $x(t) = \dfrac{e^{2jt}}{2\pi} \operatorname{Sinc}(3t)$

$$e^{2jt} h(t) \xrightarrow{F.} F(\omega-2)$$

$$\frac{1}{6} \times \frac{3}{\pi} \operatorname{Sinc}(3t) \xrightarrow{F} \frac{1}{6} \operatorname{rect}\left(\frac{\omega}{6}\right) \Rightarrow$$

$$\Rightarrow X(j\omega) = \frac{1}{6} \operatorname{rect}\left(\frac{\omega-2}{6}\right)$$

i) $X(j\omega) = 3\delta(\omega-1) + 2j\delta(\omega-2) + 3\delta(\omega+1) - 2j\delta(\omega+2) \Rightarrow$

$$\Rightarrow x(t) = \frac{3}{2\pi}\left\{e^{jt} + e^{-jt}\right\} + \frac{2j}{2\pi}\left\{e^{2jt} - e^{-2jt}\right\} =$$

$$= \frac{3}{\pi}\operatorname{Cos}(t) - \frac{2}{\pi}\operatorname{Sin}(2t)$$

ii) $X(j\omega) = \dfrac{\operatorname{Sin}\left(\frac{\omega}{2}\right)}{j\omega+2} e^{-2j\omega} = \dfrac{\cancel{4}\,1}{2j(2+j\omega)} \dfrac{e^{-2j\omega}}{\cancel{2}} \left\{e^{\frac{\omega j}{2}} - e^{-\frac{j\omega}{2}}\right\}$

$$= \frac{1}{2j}\left\{\frac{e^{-\frac{3\omega j}{2}}}{2+j\omega} - \frac{e^{-\frac{5\omega j}{2}}}{2+j\omega}\right\}$$

$$e^{-2(t-t_0)} \cdot u(t-t_0) \xrightarrow{F.} \frac{e^{-j\omega t_0}}{2+j\omega} \Rightarrow$$

$$\Rightarrow x(t) = \frac{1}{2j}\left\{e^{-2(t-3/2)} u(t-3/2) - e^{-2(t-5/2)} u(t-5/2)\right\}$$

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iii) $X(j\omega) = \dfrac{9+3j\omega}{(2+j\omega)(4+j\omega)} = \dfrac{3}{2}\left\{\dfrac{6+2j\omega}{(2+j\omega)(4+j\omega)}\right\} =$

$= \dfrac{3}{2} \times \dfrac{1}{2+j\omega} + \dfrac{3}{2} \times \dfrac{1}{4+j\omega} \Rightarrow$

$\Rightarrow x(t) = \dfrac{3}{2}\left\{e^{-2t} + e^{-4t}\right\} u(t)$

سوال 2)

الف) $\hat{X}(j\omega) = \int_{-1}^{1} e^{-j\omega t} dt = \dfrac{2\sin(\omega)}{\omega} = \dfrac{2}{\pi} \operatorname{sinc}\left(\dfrac{\omega}{\pi}\right)$

ب) تناوب هر بازه برابر با 4 است در نتیجه داریم:

$x(t) = \sum_{k=-\infty}^{+\infty} \hat{x}(t-4k)$

ج) $x(t) = \sum_{k=-\infty}^{+\infty} \hat{x}(t-4k) \Rightarrow$

$\Rightarrow X(j\omega) = \sum_{k=-\infty}^{+\infty} \hat{X}(j\omega) e^{-4kj\omega} = \sum_{k=-\infty}^{+\infty} \dfrac{2\sin(\omega)}{\omega} \times e^{-4kj\omega}$

د) $H(j\omega) = F\left\{\dfrac{2\sin(\omega)}{\omega}\right\} = F\left\{\dfrac{2}{\pi} \operatorname{sinc}\left(\dfrac{\omega}{\pi}\right)\right\} =$

$= \dfrac{2}{\pi} \operatorname{rect}\left(\dfrac{\pi\omega}{2}\right)$

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ه)

$$Y(j\omega) = H(j\omega) \times X(j\omega) = 2\,rect\left(\frac{\omega\pi}{2}\right) \times \sum_{k=-\infty}^{+\infty} \frac{2\sin\omega}{\omega} e^{-4jk\omega}$$

سؤال سوم)

$$e^{-|t|}u(t) \xrightarrow{F} \frac{2}{1+\omega^2} \Rightarrow \frac{2}{1+t^2} \xrightarrow{F} 2\pi e^{-|\omega|} \Rightarrow$$

$$\Rightarrow \frac{1}{\pi} \times \frac{1}{1+t^2} \xrightarrow{F} e^{-|\omega|} \Rightarrow$$

$$x(t) = \frac{1}{\pi} \times \frac{1}{1+t^2} \Rightarrow X(j\omega) = e^{-|\omega|} \Rightarrow$$

$$\Rightarrow X(j\pi/4) = e^{-\pi/4} = 0{,}456$$

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سوال ۴ (4

$$x(t) = \begin{cases} 3 & -1 \leq t \leq 0 \\ 3-t & 0 \leq t \leq 2 \\ t-1 & 2 \leq t \leq 4 \\ 3 & 4 \leq t \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

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$$X(0) = \int_{-\infty}^{+\infty} x(t) e^{-0t} dt = \int_{-\infty}^{+\infty} x(t) dt = \cancel{15}\ 14$$

(ب

$$x_2(t) := x(t+2)$$

سیگنال $x_2(t)$ یک سیگنال حقیقی و زوج است در نتیجه

$$\angle X_2(j\omega) = 0, \pi$$

$$X(j\omega) = e^{-2j\omega} X_2(j\omega) \Rightarrow$$

$$\Rightarrow \angle X(j\omega) = -2\omega, \pi - 2\omega$$

(ج

$$X^2(j\omega) = X(j\omega) \times X(j\omega) \Rightarrow F^{-1}\{X^2(j\omega)\} = x(t) * x(t)$$

$$x_2(t) := x(t) * x(t) \Rightarrow$$

$$\Rightarrow \int_{-\infty}^{+\infty} X^2(j\omega) d\omega = 2\pi x_2(0) = 2\pi \times 2 \int_0^1 3(3-t) dt = 30\pi$$

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Using Parseval Theorem: $\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{+\infty} |x(t)|^2 dt =$ (د

$= \frac{212\pi}{3}$

$\int_{-\infty}^{+\infty} X(j\omega) e^{2j\omega} d\omega = 2\pi x(2) = 2\pi$ (ه

$X^2(j\omega) = |X^2(j\omega)|^2 \Rightarrow X^2(j\omega) = X^*(j\omega) X(j\omega) \Rightarrow$ (و

$\Rightarrow X^*(j\omega) = X(j\omega)$

X باید تماماً حقیقی باشد.

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سوال 5)

$x(t)$ یک سیگنال حقیقی است.

$$\cancel{X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt}$$

$$x_e(t) \xrightarrow{\mathcal{F}} Re\{X(j\omega)\}$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} Re\{X(j\omega)\} e^{j\omega t} d\omega = |t| e^{-|t|} = x_e(t)$$

با توجه به تعریف سیگنال زوج داریم:

$$\frac{x(t) + x(-t)}{2} = |t| e^{-|t|} \qquad x(-t) = 0 \Rightarrow$$

$$\Rightarrow x(t) = 2te^{-t}u(t)$$

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سؤال ٦)

الف)

$$\frac{dy(t)}{dt} + 20y(t) = \int_{-\infty}^{+\infty} x(\tau)\, z(t-\tau)\, d\tau - x(t) =$$

$$= x(t) * z(t) - x(t) \Rightarrow$$

$$\xrightarrow{F.} j\omega Y(j\omega) + 20\, Y(j\omega) = X(j\omega) Z(j\omega) - X(j\omega) \quad \text{I}$$

$$z(t) = e^{-t} u(t) + 2\delta(t) \xrightarrow{F.} Z(j\omega) = \frac{1}{1+j\omega} + 2$$

$$\text{I} \Rightarrow H(\omega j) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2+j\omega}{(1+j\omega)(20+j\omega)}$$

ب)

$$H(j\omega) = \frac{1}{19} \times \frac{1}{1+j\omega} + \frac{18}{19} \times \frac{1}{20+j\omega} \Rightarrow$$

$$\Rightarrow h(t) = \frac{1}{19} e^{-t} u(t) + \frac{18}{19} e^{-20t} u(t)$$

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سوال ۷)

به صورت کلی داریم که

$$X(j\omega) = |X(j\omega)| e^{j\angle X(j\omega)}$$

که در این سوال داریم : $|X(j\omega)| = \pi$ و $\angle X(j\omega) = -\omega$

حال سیگنال $z(t) = x(t+1)$ را تعریف می‌کنیم.

با توجه به اینکه تبدیل فوریه این سیگنال حقیقی و زوج است

$x^2(t+1)$ نیز حقیقی و ~~زوج خواهد~~ زوج خواهد بود.

$$x^2(t+1) \xrightarrow{F} Y(j\omega) e^{j\omega}$$

$$\angle \{Y(j\omega) e^{j\omega}\} = 0 , \pi \Rightarrow$$

$$\Rightarrow \angle Y(j\omega) = -\omega \text{ و } \pi - \omega$$