DATE / / SUBJECT :

سوال ۱:

1) $x[-n] = \left(\frac{1}{5}\right)^n u[n-1] \Rightarrow X(e^{-j\omega}) = e^{-j\omega} \times \frac{1}{1-\frac{1}{5}e^{-j\omega}} \Rightarrow$

$$\Rightarrow X(e^{j\omega}) = e^{j\omega} \times \frac{1}{1-\frac{1}{5}e^{j\omega}}$$

$\rightarrow$) $u[n+2] - u[n-3] = x[n] \Rightarrow \cancel{X(e^{j\omega})} =$

$$\Rightarrow X(e^{j\omega}) = e^{2j\omega}\left\{\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{k=+\infty} \pi\delta(\omega-2\pi k)\right\}$$

$$- e^{-3j\omega}\left\{\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega-2\pi k)\right\} =$$

$$= \left(e^{2j\omega} - e^{-3j\omega}\right)\left\{\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega-2\pi k)\right\}$$

2) $x[n] = 2\delta(-2n+4) \rightsquigarrow x[-n] = 2\delta(2n+4) \rightsquigarrow$

$\rightsquigarrow x[-\frac{n}{2}] = 2\delta(n+4) \rightsquigarrow \cancel{X(e^{-jk\omega})} =$

$\hookrightarrow X(e^{-2j\omega}) = 2e^{4j\omega} \times 1 \Rightarrow X(e^{j\omega}) = 2e^{-2j\omega}$

DATE / / SUBJECT :

$$x[n] = \sin\left(\frac{5\pi n}{3}\right) + \cos\left(\frac{7\pi n}{3}\right) \rightsquigarrow$$

$$\rightsquigarrow X(e^{j\omega}) = \frac{\pi}{j}\left\{\sum_{k=-\infty}^{+\infty}\left[\delta\left(\omega - \frac{5\pi}{3} - 2\pi k\right) - \delta\left(\omega + \frac{5\pi}{3} - 2\pi k\right)\right]\right\}$$

$$+ \pi\left\{\sum_{k=-\infty}^{+\infty}\left[\delta\left(\omega - \frac{7\pi}{3} - 2\pi k\right) + \delta\left(\omega + \frac{5\pi}{3} - 2\pi k\right)\right]\right\}$$

$$x[n] = \begin{cases} 0{,}5 + 0{,}5\cos\left(\frac{\pi}{N} n\right) & |n| \leqslant N \\ 0 & \text{otherwise} \end{cases}$$

$$\cancel{x[n] = \{0{,}5\,(u[n\ \ x[n] = 0{,}5\left(1 + \frac{1}{2}\left\{e^{j\frac{\pi}{N}n} + e^{-j\frac{\pi}{N}n}\right\}\right)(u[\pi n}$$

$$x[n] = 0{,}5\left(1 + \frac{1}{2}\left\{e^{j\frac{\pi}{N}n} + e^{-j\frac{\pi}{N}n}\right\}\right)\left(u[-n] - u[n]\right)$$

DATE / / SUBJECT :

9) $x[n] = \dfrac{\sin(\frac{\pi}{4}n)}{\pi n} * \dfrac{\sin(\frac{\pi}{4}(n-8))}{\pi(n-8)} \Rightarrow$

$$\Rightarrow X(e^{j\omega}) = F\left\{\frac{\sin(\frac{\pi}{4}n)}{\pi n}\right\} \times F\left\{\frac{\sin(\frac{\pi}{4}(n-8))}{\pi(n-8)}\right\} =$$

$$\cancel{= \tfrac{1}{4} \times 1 \times 1 \times e^{-8j\omega}}$$

$$\Rightarrow X(e^{j\omega}) = \begin{cases} e^{-8j\omega} & 0 \leq |\omega| \leq \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \leq |\omega| \leq \pi \end{cases}$$

periodic with period (2π)

~~عکس تبدیل فوریه سیگنال های زیر را حساب کنید~~

T) $X(e^{j\omega}) = \cos^2(\omega) + \sin^2(3\omega) =$

$$= \frac{1}{2}\{1 + \cos(2\omega) + 1 - \cos(6\omega)\} = \cancel{1 + \tfrac{1}{4}\{e^{2j}}$$

$$= 1 + \frac{1}{4}\{e^{2j\omega} + e^{-2j\omega} - e^{6j\omega} - e^{-6j\omega}\} \Rightarrow$$

$$\Rightarrow x[n] = \delta[n] + \frac{1}{4}\{\delta[n+2] + \delta[n-2] - \delta[n+6] - \delta[n-6]\}$$

DATE / / SUBJECT :

ب)
$$X(e^{j\omega}) = \begin{cases} e^{j\omega/2} & \text{for } \pi \geq \omega \geq -\pi \\ 0 & \text{else} \end{cases}$$

DATE / / SUBJECT :

1) $X(e^{j\omega}) = \cos(2\omega) + j(\sin 2\omega) = e^{j2\omega} \Rightarrow$

$\Rightarrow x[n] = \delta[n+2]$

2) $X(e^{j\omega}) = \cos(\omega) + j\sin(\omega/2) =$

$= \frac{1}{2}\left(e^{\omega j} + e^{-\omega j}\right) + \frac{1}{2}\left(e^{\omega j/2} - e^{-\omega j/2}\right) \Rightarrow$

$\Rightarrow x[n] = \frac{1}{2}\left\{\delta[n+1] + \delta[n-1] + \delta[n+\frac{1}{2}] - \delta[n-\frac{1}{2}]\right\}$

9) $e^{j\omega_0 n} x[n] \longrightarrow X\left(e^{(j(\omega-\omega_0))}\right)$

$X(e^{j\omega}) = \begin{cases} 1 & \pi/4 < |\omega| < 3\pi/4 \\ 0 & \text{otherwise} \end{cases}$

$X\left(e^{j(\omega-\omega_0)}\right) = \begin{cases} 1 & 0 < |\omega| < \pi/2 \\ 0 & \text{otherwise} \end{cases} \rightsquigarrow$

$\rightsquigarrow e^{j\pi/4 n} x[n] = \frac{\sin(\pi/2\, n)}{\pi n} \Rightarrow$

$\Rightarrow x[n] = e^{-j\pi/4 n} \frac{\sin(\pi/2\, n)}{\pi n}$

DATE / / SUBJECT :

سؤال ② :

آ)

$$X(e^{j\omega}) = 2e^{-j\omega} \times \frac{1}{1-0{,}25e^{-2j\omega}}$$

$$\cancel{F^{-1}\left\{\frac{1}{1-\frac{1}{4}e^{-2j\omega}}\right\} = \left(\frac{1}{4}\right)^n u[n]}$$

$$F^{-1}\left\{\frac{1}{1-\frac{1}{4}e^{-j\omega}}\right\} = \left(\frac{1}{4}\right)^n u[n] \Rightarrow$$

$$\Rightarrow F^{-1}\left\{\frac{1}{1-\frac{1}{4}e^{-2j\omega}}\right\} = \begin{cases}\left(\frac{1}{4}\right)^{n/2} u\left[\frac{n}{2}\right] & \text{if } \frac{n}{2}\in\mathbb{Z}\\ 0 & \text{otherwise}\end{cases}$$

$$\cancel{\Rightarrow x[n] = \begin{cases}\left(\frac{1}{4}\right)^{n/2} u\left[\frac{n}{2}\right] & \text{if } n \text{ is multiple of } 2\\ 0 & \text{otherwise}\end{cases}}$$

$$\Rightarrow x[n] = \begin{cases}2\left(\frac{1}{4}\right)^{\frac{n-1}{2}} u\left[\frac{n-1}{2}\right] & \text{if } n-1 \text{ is multiple of } 2\\ 0 & \text{otherwise}\end{cases}$$

DATE / / SUBJECT :

$$X(e^{jw}) = \frac{6 - 2e^{-jw} + 0{,}5e^{-2jw}}{(1-0{,}25e^{-2jw})(1-0{,}25e^{-jw})} =$$

$$= \frac{6}{(1-0{,}25e^{-2jw})(1-0{,}25e^{-jw})} - \frac{2e^{-jw}}{1-0{,}25e^{-2jw}}$$

$$= \frac{A + Be^{-jw}}{1-0{,}25e^{-2jw}} + \frac{C}{1-0{,}25e^{-jw}} - \frac{2e^{-jw}}{1-0{,}25e^{-2jw}} \Rightarrow$$

$$\Rightarrow A + C = 6 \ , \ -0{,}25A + B = 0$$

$$, \ -0{,}25B - 0{,}25C = 0 \Rightarrow B + C = 0$$

$$\Rightarrow \left.\begin{array}{l} A + C = 6 \\ -0{,}25A - C = 0 \end{array}\right\} \Rightarrow 0{,}75A = 6 \Rightarrow A = 8 \Rightarrow C = -2$$

$$\Rightarrow B = 2$$

$$\Rightarrow X(e^{jw}) = \frac{8}{1-0{,}25e^{-2jw}} - \frac{2}{1-0{,}25e^{-jw}} \Rightarrow$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

$$x_1[n] \begin{cases} 8 \times (1/4)^{n/2} u[n/2] & \text{if } n/2 \text{ is multiple of } 2 \\ 0 & \text{otherwise} \end{cases}$$

$$x_2[n] = \left\{ -2 \times (1/4)^{n} u[n] \right.$$

DATE / / SUBJECT :

سوال ۱۶ (3) :

$x[n] \rightarrow X(e^{j\omega})$

$y[n] = \cancel{x} \; x\left[\frac{n}{2}\right] = x_{(2)}[n] + x_{(2)}[n-1]$

$$x_{(2)}[n] \begin{cases} x\left[\frac{n}{2}\right] & \text{if } n = \text{multiple of } k \\ 0 & \text{otherwise} \end{cases}$$

$\cancel{x_{(2)}[n-1]} \quad F(x_{(2)}[n]) = X_1(e^{j\omega}) = X(e^{2j\omega})$

$F(x_{(2)}[n-1]) = e^{-j\omega} X_1(e^{j\omega})$

$F(y[n]) = (1+e^{-j\omega}) X(e^{2j\omega})$

DATE / / SUBJECT :

سؤال 5 : الف)

$y[n] + \frac{1}{2} y[n-1] = \cos[n]$

$$H(e^{jw}) = 1 + \frac{e^{-jw}}{(2-0{,}5e^{-jw})(1+0{,}25e^{-jw})} =$$

$$= 1 + \frac{e^{-jw}}{2\left(1-\frac{1}{16}e^{-2jw}\right)} =$$

$$= 1 + \frac{e^{-jw}}{2-\frac{1}{8}e^{-2jw}} \Rightarrow$$

$$\Rightarrow H(e^{jw}) = \frac{2+e^{-jw}-\frac{1}{8}e^{-2jw}}{2-\frac{1}{8}e^{-2jw}} = \frac{X(e^{jw})}{Y(e^{jw})} \Rightarrow$$

$$\Rightarrow 2y[n] - \frac{1}{8}y[n-2] = 2x[n] + x[n-1] - \frac{1}{8}x[n-2]$$

DATE / / SUBJECT :

(2

$$h[n] = \delta[n] + 2 \times (0,5)^n u[n] + (-0,5)^n u[n] \Rightarrow$$

$$\Rightarrow H(e^{j\omega}) = 1 + \frac{2}{1-0,5e^{-j\omega}} + \frac{1}{1+0,5e^{-j\omega}} =$$

$$= 1 + \frac{3+0,5e^{-j\omega}}{1-0,25e^{-2j\omega}} =$$

$$= \frac{5+0,5e^{-j\omega}-0,25e^{-2j\omega}}{1-0,25e^{-2j\omega}} = \frac{X(e^{j\omega})}{Y(e^{j\omega})} \Rightarrow$$

$$\Rightarrow y[n] - \frac{1}{4} y[n-2] = 5x[n] + \frac{1}{2} x[n-1] - \frac{1}{4} x[n-2]$$

DATE / / SUBJECT :

سوال 6)

آ)

Time Shifting:

$$x[n] \xleftrightarrow{F} X(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$

$$F(x[n-n_0]) = \sum_{n=-\infty}^{+\infty} x[n-n_0] e^{-j\omega n} \qquad u := n-n_0 \Rightarrow$$

$$\Rightarrow F(x[n-n_0]) = \sum_{u=-\infty}^{+\infty} x[u] e^{-j\omega(u+n_0)} =$$

$$= e^{-j\omega n_0} \sum_{u=-\infty}^{+\infty} x[u] e^{-j\omega u} = e^{-j\omega n_0} X(e^{j\omega}) \Rightarrow$$

$$\Rightarrow x[n-n_0] \xleftrightarrow{F} e^{-j\omega n_0} X(e^{j\omega})$$

Convolution Property:

ب)

$$x[n] \xleftrightarrow{F} X(e^{j\omega}) \qquad y[n] \xleftrightarrow{F} Y(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n} \qquad Y(e^{j\omega}) = \sum_{n=\infty}^{+\infty} y[n] e^{-j\omega n}$$

$$F(x[n] * y[n]) = \sum_{n=-\infty}^{+\infty} x[n] * y[n] \; e^{-j\omega n}$$

DATE / / SUBJECT :

$$= \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} x[m]\, y[n-m]\, e^{-jwn} =$$

$$= \sum_{m=-\infty}^{+\infty} x[m] \sum_{n=-\infty}^{+\infty} y[n-m]\, e^{-jwn} = \cancel{\sum_{m=-\infty}^{+\infty} e}$$

$$= \sum_{m=-\infty}^{+\infty} x[m] \times e^{-jwm}\, Y(e^{jw}) = X(e^{jw})\, Y(e^{jw})$$

$$x[n] * y[n] \xleftrightarrow{F} X(e^{jw})\, Y(e^{jw})$$

The Multiplication Property

C2

$$\cancel{Ye^{j}} \quad y[n] = x_1[n]\, x_2[n]$$

$$Y(e^{jw}) = \sum_{n=-\infty}^{+\infty} x_1[n]\, x_2[n]\, e^{-jwn} \rightsquigarrow$$

$$x_1[n] = \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\theta})\, e^{j\theta n}\, d\theta \Rightarrow$$

$$\Rightarrow Y(e^{jw}) = \sum_{n=-\infty}^{+\infty} x_2[n] \times \left\{ \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\theta})\, e^{j\theta n}\, d\theta \right\} e^{-jwn}$$

$$= \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\theta}) \times \left\{ \sum_{n=-\infty}^{+\infty} x_2[n]\, e^{-j(w-\theta)n} \right\} d\theta$$

$$\Rightarrow Y(e^{jw}) = \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\theta})\, X_2(e^{j(w-\theta)})\, d\theta$$

DATE / / SUBJECT :

Time Expansion Property:

$$x_{(k)}[n] = \begin{cases} x[n/k] & \text{if } n \text{ is multiple of } k \\ 0 & \text{otherwise} \end{cases}$$

$$X_{(k)}(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x_{(k)}[n] e^{-j\omega n} = \sum_{r=-\infty}^{+\infty} x_{(k)}[rk] e^{-j\omega rk}$$

$$= \sum_{r=-\infty}^{+\infty} x[n] e^{-j\omega rk} = X(e^{jk\omega})$$

سؤال 7 :

T)

$y[n] - ay[n-1] + by[n-2] = \delta[n]$

$n=0 \leadsto y[0] = 1$

$n=1 \leadsto y[1] = ay[0] = 3/4 \Rightarrow a = 3/4$

$n=2 \leadsto y[2] = ay[1] - by[0] = \cancel{a \times 3}\ 3/4 \times 3/4 - b = 7/16$

$\Rightarrow b = 1/8$

$n=3 \leadsto y[3] = ay[2] - by[1] = 3/4 \times 7/16 - 1/8 \times 3/4 =$

$\Rightarrow y[3] = \frac{15}{64}$ ✓

ج

$n=4 \leadsto y[4] = ay[3] - by[2] = 3/4 \times 15/64 - 1/8 \times 7/16 =$

$= \frac{45}{256} - \frac{7}{128} = \frac{31}{256}$

$\Rightarrow y[n] - 3/4\, y[n-1] + 1/8\, y[n-2] = x[n]$

$\Rightarrow H(e^{j\omega}) =$

$$H(e^{j\omega}) = \frac{1}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-2j\omega}} = \frac{8}{(2-e^{-j\omega})(4-e^{-j\omega})} =$$

$$= 4 \times \left(\frac{1}{2-e^{-j\omega}} - \frac{1}{4-e^{-j\omega}} \right) =$$

$$= \frac{2}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{1-\frac{1}{4}e^{-j\omega}} \Rightarrow$$

$$\Rightarrow h[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$$