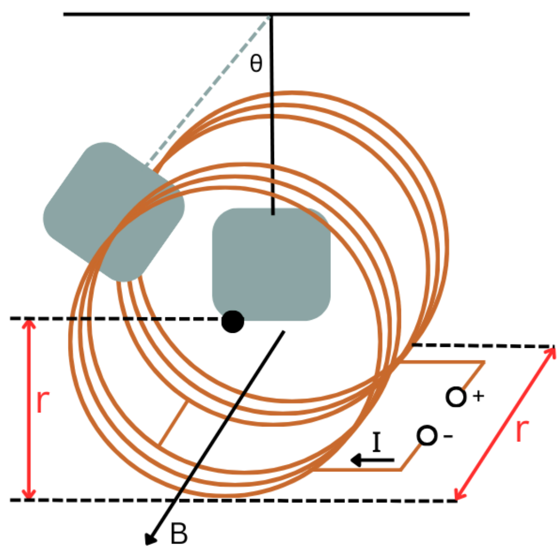
How does the magnetic field of a Helmholtz coil affect the period of a pendulum with a conductive bob?

Background research:

A swinging pendulum is an example of simple harmonic motion. Like all objects in this motion, pendulums have periods of oscillation in which they complete one cycle. Period of an object moving in simple harmonic motion can be increased or decreased by various factors.
In this experiment, a pendulum with a conductive metal bob (aluminum) was placed within a Helmholtz coil to see how the period changes as the magnetic field strength of the coils is varied.

Figure 1 (on the right) shows a pendulum moving freely within a Helmholtz coil.



Helmholtz coils are two identical circular coils, arranged symmetrically along a common axis. These coils are separated by a distance equal to the radius of the coil circle, a unique arrangement that produces a nearly uniform magnetic field at the midpoint between them. The magnetic field generated by each coil adds constructively in the middle, creating a stronger field. Away from the center, the magnetic field from each coil begins to vary slightly. However, the distance being equal to the radius of the coils causes the magnetic fields from both coils to overlap in a way that balances out any uneven parts of the field.

The magnetic field at the center of the Helmholtz coil can be calculated by:

$$B = \mu 0 \frac{8\, I\, n}{\sqrt{125}\, r}$$

Where μ0 is the permeability of free space, n is the number of turns in each coil, I is the current in the wire, and r is the radius of each coil. From this equation, it can be seen that the magnetic field strength of the coils is linearly proportional to the current passing through the wire. Using Ohm's law, the current in the wire can be calculated by:

$$V = IR$$

Where V is the applied voltage and R is the resistance of the wire. This shows that current is also proportional to the applied voltage. It can be found from the two above equations that the magnetic field produced in the coils is proportional to the current, which in turn depends on the voltage applied across the coils. As a result, the magnetic field can be varied by connecting the wire of the coils to a power supply and changing the voltage applied to the wire.

When placed within a magnetic field, a conductive material produces its own magnetic field. The magnetized bob will then interact with the external magnetic field, which can influence the pendulum's motion.

An eddy current is a loop of electric current induced within conductors by a changing magnetic field in the conductor by the relative motion of the conductor in the magnetic field. Eddy currents can produce significant drag, called magnetic damping, on the motion of the pendulum. As the metal plate enters and leaves the magnetic field, it experiences a force opposing its motion.
According to Lenz's law, the orientation of the induced current in the sheet will produce a magnetic field that counteracts the change in magnetic flux.

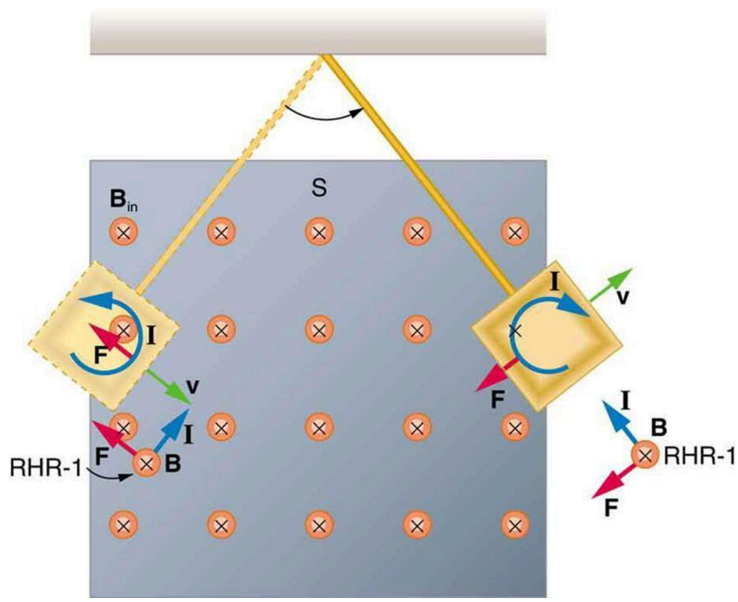


Figure 2: As the sheet enters the magnetic field from the left, the magnetic flux within the sheet will increase which causes an eddy current to set up. Lenz's law tells us that the current must flow in a direction such that the magnetic field produced within the sheet opposes the direction of the original magnetic field. Using the right-hand rule, it can be found that a counterclockwise current will produce a magnetic field that points out of the page.

Based on these findings, I hypothesize that as the magnetic field strength is increased, the period of oscillation will increase, meaning the pendulum will oscillate at a lower frequency due to the opposing force caused by the magnetic field.

Variables:

The **independent variable** is the magnetic field strength of the coils, measured in tesla (T). It will be varied by varying the voltage applied by the power supply. The voltage will be varied by increments of 2 volts, with 8 values from 6 to 20 volts which are the limits of the power supply.

The **dependent variable** is the period (T) of oscillation, a single period, measured in seconds (s). Since the bob of the pendulum is conductive, it can interact with the magnetic field produced by the coils, which causes the motion of the pendulum and hence its period to change

Controlled variables are listed below:

- **Pendulum length_** The length of the pendulum affects the period of oscillation. As you increase the string length, the period will affectionately increase. To keep the period only dependent on the magnetic field strength, the same length for the pendulum was used.

- **Coil geometry_** The coil geometry has a direct impact on the magnetic field it can produce. Slight changes in the structure of the coils such as their distance of separation or the plane at which each circle coil lies can alter the magnetic field strength. Keeping this variable constant was achieved by taping the two coil structures to a base, which ensures that the coils are kept the same distance from each other and are parallel to each other.

- **Wire temperature_** Wire resistivity will increase with heating during prolonged use. This would decrease the current flowing through the wire due to its inverse proportionality. The reduced current would decrease the magnetic field strength as they are directly proportional. To ensure a constant temperature, the trials were separated by time intervals of 5 minutes. It is assumed that 5 minutes is enough time for the wire to reach thermal equilibrium with the room temperature, thus obtaining its original temperature.

- **Air resistance_** Air resistance can significantly affect the pendulum's period, as it depends on the shape, surface area, and velocity of the object. To ensure consistent air resistance across trials, the same aluminum sheet was used to maintain a constant shape and surface area. The pendulum was also released from the same angle in each trial to keep its velocity consistent. The initial release angle was controlled by elevating the rod until it made contact with a horizontal pole placed on top of the coil structure (Figure 6).

Methodology:

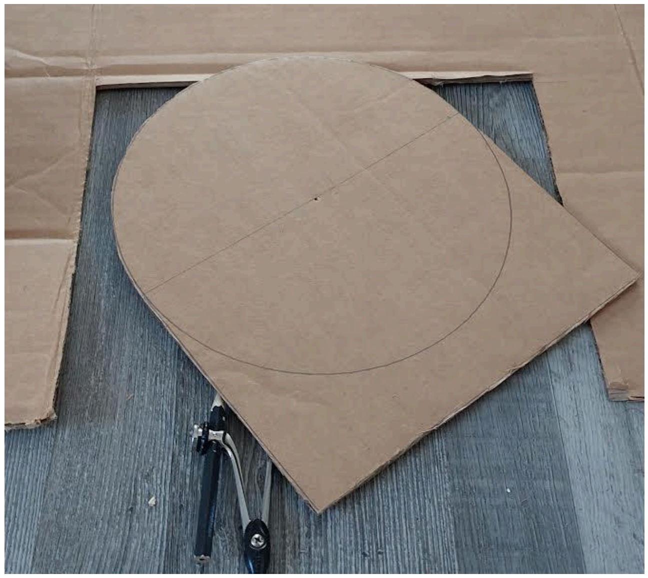


Figure 3: Early stage of building the coils. The figure shows how the structure was stabilized by keeping the base non-curved.

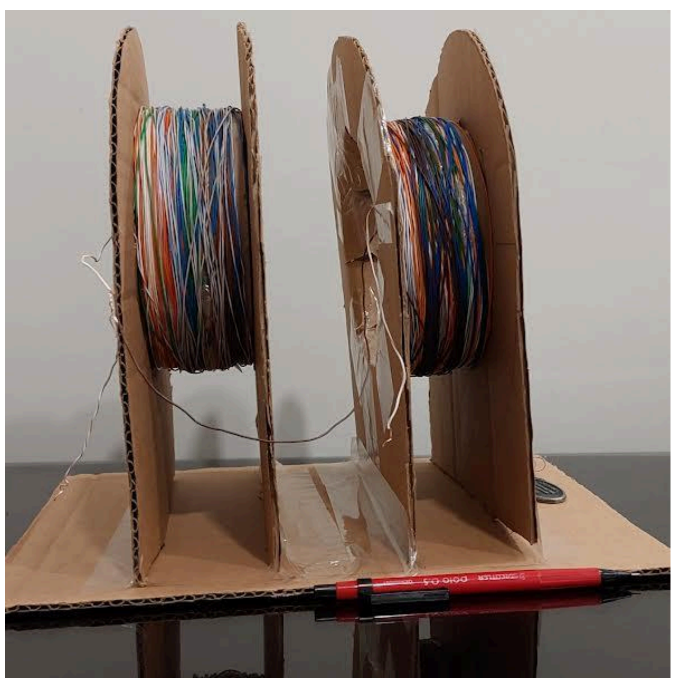


Figure 4: The Helmholtz coil without a pendulum. Later, two cardboard sheets will be added around the structure on which the pendulum will be set up.

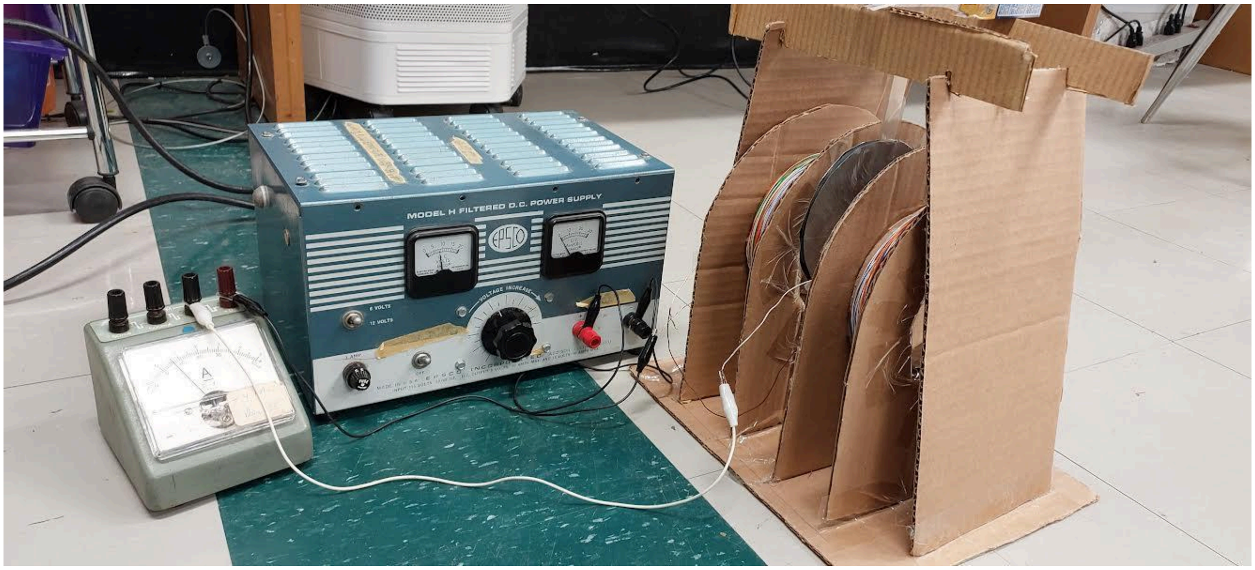


Figure 5: The experiment setup. The Helmholtz coil is connected to the power supply using alligator clips. An ammeter is connected in series to ensure constant current is flowing through the wire.

<u>Apparatus:</u>

1. DC Power supply (Adjustable)
2. Ammeter
3. Aluminum sheet
4. Wooden rod (vibrates back and forth)
5. PVC-insulated copper wire
6. alligator clips
7. Cardboard cylinder (To coil the wire around)
8. Cardboard sheet (Base and walls of the coils)
9. Ruler
10. Scissors and tape
11. Cellphone (Timer)

Figure 6: Aluminum sheet being released. The initial angle of release is kept constant by releasing the pendulum from the

horizontal rod on top of the structure in each trial.

Procedure:

1. First, a cardboard cylinder with a radius of approximately 7 cm was cut to have a height of 5 cm. The same process was repeated to produce another identical cylinder.

2. A circle with an 11 cm radius was drawn on strong cardboard. At first, I planned to cut the entire circle but changed my mind for stability. Instead, I cut half of the circle, leaving the rest uncut, creating a flat piece with a curved top and rectangular base to prevent wobbling (see Figure 3). This process was repeated to make three identical shapes.

3. Each cylinder was taped to two pieces of cardboard surrounding it from the sides. This produced two structures for coils.

4. used two insulated wires, each 4.5 meters long, with 8 strands inside. I separated the strands, resulting in 16 individual 4.5-meter strands. Then, I connected the ends of the strands to form a single long wire. This step, though seemingly unnecessary, was intended to increase the wire length, strengthening the magnetic field produced by the coils. The final wire length was about 72 meters (16 strands × 4.5 meters = 72 meters).

5. After the wire was ready, it was coiled around the two coils. For the Helmholtz coil to produce a nearly uniform magnetic field, the number of turns must be equal on each coil. The radius of each coil is 0.065m, which makes the circumference of the circle equal to:

   $$P = 2\pi r = 2\pi\,(0.065\text{m}) \approx 0.44\text{m}.$$

   To find the total number of turns, one can divide the total wire length by the circumference which gives:

   $$n = \frac{72m}{0.44m} \approx 163 \text{ turns}$$

   As a result, since there were two coils, I did 80 turns in each coil with some left over to cover the distance between the two coils and to connect the alligator clips to the coils.

6. After the coils were ready, I taped them to a piece of cardboard to maintain a constant distance between them during each trial. The separation between the coils was 6.5 cm, equal to their radius.

7. The final step was adding the pendulum. I attached two flat cardboard walls to the structure, placed a third piece on top, and hung the pendulum from it. The bob consists of

two circular aluminum sheets around a rod. Flat sheets were chosen over a spherical bob to maximize eddy currents by increasing the surface area exposed to the magnetic field.

8. The two end wires of the coil were connected to the two ends of an adjustable DC power supply. I released the pendulum with the same voltage supply three times and varied the voltage 8 times.

Safety considerations:

1. Avoid touching the coils during the trials since the wire heats up as the current passes.
2. Keeping the voltage within the safe limit.

Qualitative observations:

After the structure was set up, I turned on the power supply. Then, the ticks on the ammeter changed, showing that there was a current in the wire. No visible changes occurred to the motion of the pendulum; it kept oscillating the same way with each increase of voltage. After each trial, I turned on the power supply for approximately 5 minutes to avoid the wire heating up. Still, even after that amount of time, the wire was rather warm when I grabbed it. Furthermore, as the pendulum was oscillating, it caused the whole structure to vibrate. Making it unstable. To avoid the unwanted wobble, I placed heavy books on its base to minimize the vibration of the structure.

Analysis:

Sample calculation of the magnetic field strength in trial 2:

$$B = \mu 0 \frac{8 I n}{\sqrt{125}\, r}$$

$$B = (4\pi \times 10^{-7}) \frac{(8)\ (1.2 \pm 0.5\ A)(160)}{(\sqrt{125})(0.065 \pm 0.005 m)}$$

$$B = 2.7mT + \%\Delta Current + \%\Delta distance$$

$$B = 2.7\ mT + 41.7\ \% + 7.7\%$$

$$B = 2.7\, mT \pm 49.4\%$$

$$B = 2.7 \pm 1.33\, mT$$

Data collection:

Data can further be analyzed by graphing it. I graph the magnetic field strength against the period of the pendulum and I find a seemingly linear relationship between the two. Here is the data used for the graph:

Trial	Voltage (V) ± 0.5 V	Current (A) ±0.5 A	Magnetic Field Strength(mT)	T_1 ±0.033 s	T_2 ±0.033 s	T_3 ±0.033 s	T_{avg} ±0.033 s
1	6.0	0.8	1.8 ± 1.2	0.786	0.785	0.783	0.785
2	8.0	1.2	2.7 ± 1.3	0.787	0.789	0.789	0.787
3	10.0	1.4	3.0 ± 1.3	0.789	0.791	0.789	0.790
4	12.0	1.6	3.6 ± 1.4	0.790	0.793	0.792	0.792
5	14.0	1.9	4.3 ± 1.5	0.793	0.795	0.795	0.794
6	16.0	2.2	4.9 ± 1.5	0.797	0.796	0.795	0.796
7	18.0	2.5	5.5 ± 1.5	0.798	0.797	0.798	0.798
8	20.0	2.7	6.1 ± 1.6	0.800	0.800	0.799	0.800

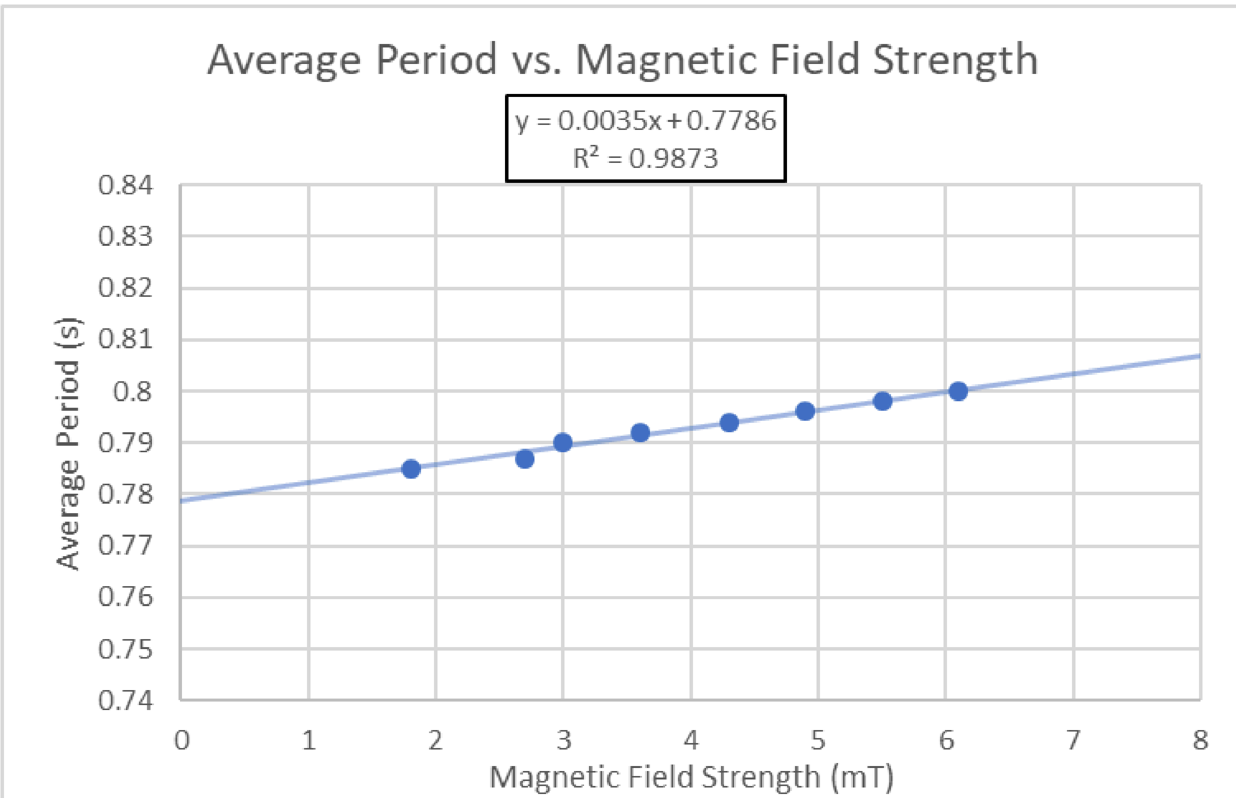
Table 1: Results of the period of oscillation with each magnetic field strength

The uncertainty in voltage and current is half of the smallest measurement interval, which is half a volt for voltage and half an amp for current. The average human reaction time is approximately ± 0.25 seconds[1]. Since I used the stopwatch to time both the release and completion of the pendulum's swing, the total error would be ± 0.5 seconds. Given that the period of one oscillation was just over half a second, this error would result in a large relative error. To minimize this, I measured the time for 15 cycles, which allowed me to calculate the uncertainty in time measurement as

[1] Carson, Oliver. “How Fast Is Realtime? Human Perception and Technology.” *PubNub*, 8 Mar. 2024, www.pubnub.com/blog/how-fast-is-realtime-human-perception-and-technology/. Accessed 19 Dec. 2024.

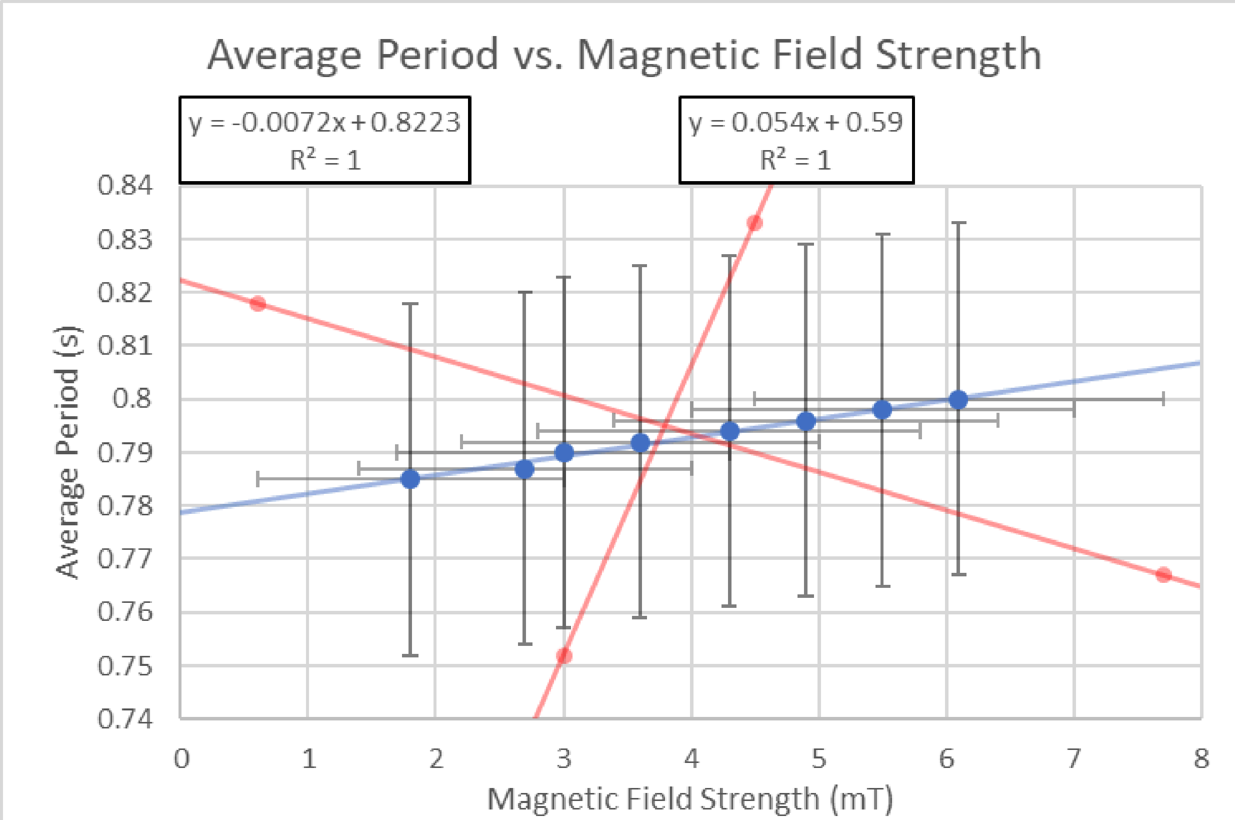
$$\Delta Period = \frac{0.500s}{15\ cycles} \approx 0.033s$$

Thus, the uncertainty in the period of one oscillation is approximately ± 0.033 seconds. Below is the graph of the data from Table 1.



Graph 1: Magnetic field strength against the period of oscillation

As seen in Graph 1, the magnetic field strength and the average period have a seemingly linear relationship. In the following graph, the vertical error bars show the uncertainty of the period of one oscillation, and the horizontal error bars show the uncertainty of the magnetic field strength.



Graph 2: Maximum and Minimum gradients for Graph 1

Using the error bars of the leftmost and rightmost data points, the maximum and minimum error lines were graphed and their respective equations were obtained. Having the equation of the maximum and minimum slope, one can calculate the error in slope using the average of the two slopes:

Slope of the trend line = $\frac{0.800 - 0.785}{6.1 - 1.8} = 0.0035 \ \frac{s}{mT}$

<u>Error in slope:</u>

Maximum slope = $\frac{(0.800+0.033)-(0.785-0.033)}{(6.1-1.6)-(1.8+1.2)} = 0.05400\frac{s}{mT}$

Minimum slope = $\frac{(0.800-0.033)-(0.785+0.033)}{(6.1+1.6)-(1.8-1.2)} = -0.00718\frac{s}{mT}$

Error in slope = $\frac{Slope\ max - Slope\ min}{2} = \frac{0.054-(-0.00718)}{2} = 0.03059\frac{s}{mT}$

Slope with error = $0.0035 \pm 0.0306 \frac{s}{mT}$ (874.3% error)

Conclusion:

The purpose of this experiment was to determine how the magnetic field strength of a Helmholtz coil would affect the period of a conductive pendulum's oscillation. After having collected and processed the data, from both mathematical and graphical evidence, it can be concluded that:

$$\mathbf{B} \propto \mathbf{T}_{\mathbf{pendulum}}$$

A linear relation was found between the magnetic field strength of the coils and the period of the pendulum. As the magnetic field strength increases, the pendulum oscillates at a slower rate, finishing one cycle at a longer time, thus increasing its period. However, it must be noted that the change in the period of the pendulum was small, a small range of period was obtained, which is due to the fact that the magnetic field produced by the coils was on an order of negative thousand, making it so weak to apply a large enough opposing force to significantly increase the period of oscillation. This limited range of obtained data restricts the conclusion made only true for the obtained range. Furthermore, Some relationships may only become apparent over a broader range. This limitation could make such trends invisible, leading to the wrong conclusion that their relationship is simply linear.

Assumptions made during this experiment:

The main reason that I decided to use a Helmholtz coil for this experiment was to be able to generate a uniform magnetic field at the midpoint between the coils. The assumption that the produced magnetic field was uniform is quite valid since precise measurement tools such as rulers were used in the process of building the coils. However, since no magnetic field sensor was used to ensure this, there are uncertainties regarding this which will be discussed later.

Another assumption made was that 5 minutes is enough time for the wire to cool down to its original temperature. The assumption that the wire in each trial requires the same amount of time to obtain its original temperature was not valid. It is because at higher applied voltages, the wire undergoes a higher change in temperature, so it requires a longer amount of time to reach thermal equilibrium with the room temperature.

Accuracy:
There is no accepted value for this experiment. However, the accuracy of this experiment can be considered high because the experimental results align well with expected trends, as indicated by the linear relationship between the magnetic field strength and the period of oscillation. Even though no theoretical or true values are available for direct comparison, the consistency of the observed trend with physical principles and expectations suggests that the experimental design and data collection methods were able to capture the key factors influencing the system.

Precision:
In contrast to its accuracy, the precision of the experiment is notably low, as evidenced by the large percent error (874.3%) and variability in the measurements. This suggests that random errors played a dominant role in the experiment. The lack of precision in the results indicates that the data points were scattered and lacked consistency across trials.

Evaluation:

Like all experiments, there have been inaccuracies and errors throughout this experiment that influenced the data obtained.

Systematic errors:

Magnetic field uniformity:
While a Helmholtz coil is designed to produce a nearly uniform magnetic field at the center of the coils, small imperfections in the coil geometry and distance of separation of the coils might have caused non-uniform fields, systematically affecting the pendulum's motion. If the pendulum passes through weak regions of the magnetic field, the damping force would be smaller, causing the pendulum to oscillate at a higher frequency, making the period smaller. Strong regions have the opposite effect on the period. However, since each trial is done with the same coils, the regions of strong and weak magnetic fields are consistent, the non-uniformity of the magnetic field should not have a significant effect on the period. Still, even though this does not affect the period, we would be able to correctly find out at what magnetic field strength a specific period occurs since the formula for the magnetic field strength of a Helmholtz coil assumes a uniform magnetic field. This problem could have been improved by using a magnetic field sensor to ensure the magnetic field in the coils is consistent. Furthermore, I realized that since I was doing the experiment in a room with computers and electronic devices, they could have distorted the magnetic field due to their ferromagnetic materials. Even my cellphone which I used to measure the period of oscillation could have interacted with the magnetic field produced by the coils,

pulling it towards itself. Hence, the experiment could have been done in a room without electronic devices to ensure minimal magnetic field interaction with external materials. I also could have kept my phone at a distance from the coils to ensure minimal interaction between them.

Wire temperature:

In this experiment, the trials were done quite successively, with about 5 minutes of interval between each. The assumption that the wire reached thermal equilibrium with the room temperature in such a small amount of time was another systematic error since the wire was still warm when I began a new trial. Especially at higher applied voltages, the wires heated up significantly. As the wire heats up, its resistance increases due to the temperature dependence of resistivity ($\rho = \rho_0 (1+\alpha(\Delta T)$, where α is the temperature coefficient of resistance.). According to Ohm's law, increased resistance reduces the current flowing through the coil assuming constant voltage, which lowers the magnetic field strength. The decrease in the magnetic field strength causes the damping force to be smaller, not influencing the pendulum's motion as it would if it had a lower temperature, so the pendulum can oscillate at a higher frequency, having a lower period. As a result, the periods found experimentally would be smaller than their theoretical values. To address this error, trials could have been conducted with long enough time intervals in between to ensure the wire cools down to its original temperature. Furthermore as addressed previously, even if 5 minutes cools down the wire with lower voltages, it will definitely take longer for the wire to cool down if a higher voltage is applied.

Random errors:

Timing measurements:

Timing the period of oscillation manually was the major error in this experiment. In the case of measuring the period of oscillation of the pendulum, even small inaccuracies in timing can lead to noticeable discrepancies, especially considering the fact that the period of a single oscillation was so small. Hitting the timer too fast would make the values found for the period to be too small. Hitting it too fast has the opposite effect. To improve the accuracy of the timing measurements, several solutions could be implemented. The most effective improvement would be to use an automated timing system, such as light gates or sonic motion detectors, which can detect when the pendulum crosses a specific point and automatically start or stop the times. This would completely eliminate the issue of human reaction time. Alternatively, video recording the pendulum's motion and analyzing it frame-by-frame using video analysis software would allow for more precise timing measurements.

Lateral movement of the pendulum:

The pendulum rod was tightly fixed at its pivot point, causing it to move left and right along its axis. This lateral movement introduces additional motion that affects the pendulum's oscillations.f the pendulum moves laterally, its motion deviates from the ideal 2D plane. This increases the overall path length of its swing and hence the period of oscillation that was experimentally found. To fix this problem, rubber bands can be placed around the pivot point to prevent the rod from shifting left and right. The rubber bands would act as a stabilizing force, ensuring that the pendulum rod remains in place during oscillation and that the motion stays purely rotational. However, it is crucial not to place the rubber bands too close to the pivot point. If they are positioned too near to the pivot, they could introduce unwanted friction as the pendulum swings.

Extension:

This investigation provided insights into the interactions between magnetic fields and oscillatory motion, showing that a Helmholtz coil's magnetic field can indeed influence the period of a pendulum. This investigation particularly focused on an aluminum sheet being the bob of the pendulum. Future investigations could explore the effect of different pendulum materials or configurations to further analyze this phenomenon and its potential applications in magnetic damping systems or oscillatory devices in magnetic environments.

Reference

Magnetic field of two Helmholtz coils. (n.d.). https://virtuelle-experimente.de/en/b-feld/b-feld/helmholtzspulenpaar.php#:~:text=To%20setup%20a%20Helmholtz%20coil,a%20nearly%20uniform%20magnetic%20field.

Wikipedia contributors. (2024, September 3). *Helmholtz coil*. Wikipedia. https://en.wikipedia.org/wiki/Helmholtz_coil

GradePod. (n.d.). *100 IB Physics IA Ideas | GradePod [New in 2024]*. https://www.gradepod.com/blog/100-ib-physics-ia-ideas

Luke Donforth. (2016, January 14). *Helmholtz coil investigation of the Earth's magnetic field* [Video]. YouTube. https://www.youtube.com/watch?v=yBgPsnB0rUI

Cross reference AWG to MM2. (n.d.). Multi/Cable Corporation. https://www.multicable.com/resources/reference-data/cross-reference-awg-to-mm2/

OpenStax. (n.d.). *8.4 Eddy currents and magnetic damping*. Pressbooks. https://pressbooks.bccampus.ca/introductorygeneralphysics2phys1207/chapter/23-4-eddy-currents-and-magnetic-damping/

Libretexts. (2023, December 28). *6.7: Helmholtz Coils*. Physics LibreTexts. https://phys.libretexts.org/Bookshelves/Electricity_and_Magnetism/Electricity_and_Magnetism_(Tatum)/06%3A_The_Magnetic_Effect_of_an_Electric_Current/6.07%3A_Helmholtz_Coils

NickyDee. (2020, June 6). *Adding more lines to an Excel line graph* [Video]. YouTube. https://www.youtube.com/watch?v=4mcZtuIeUvE

TeachExcel. (2020, March 12). *Change the vertical y-axis start or end point in Excel - Excel Quickie 37* [Video]. YouTube. https://www.youtube.com/watch?v=r5RwETjFZNI