SPURIOUS CORRELATION: A CAUSAL INTERPRETATION*

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To test whether a correlation between two variables is genuine or spurious, additional variables and equations must be introduced, and sufficient assumptions must be made to identify the parameters of this wider system. If the two original variables are causally related in the wider system, the correlation is "genuine."

EVEN in the first course in statistics, the slogan "Correlation is no proof of causation!" is imprinted firmly in the mind of the aspiring statistician or social scientist. It is possible that he leaves the course (and many subsequent courses) with no very clear ideas as to what *is* proved by correlation, but he never ceases to be on guard against "spurious" correlation, that master of imposture who is always representing himself as "true" correlation.

The very distinction between "true" and "spurious" correlation appears to imply that while correlation in general may be no proof of causation, "true" correlation does constitute such proof. If this is what is intended by the adjective "true," are there any operational means for distinguishing between true correlations, which do imply causation, and spurious correlations, which do not?

A generation or more ago, the concept of spurious correlation was examined by a number of statisticians, and in particular by G. U. Yule [8]. More recently important contributions to our understanding of the phenomenon have been made by Hans Zeisel [9] and by Patricia L. Kendall and Paul F. Lazarsfeld [1]. Essentially, all these treatments deal with the three variable case—the clarification of the relation between two variables by the introduction of a third. Generalizations to n variables are indicated but not examined in detail.

Meanwhile, the main stream of statistical research has been diverted into somewhat different (but closely related) directions by Frisch's work on confluence analysis and the subsequent exploration of the "identification problem" and of "structural relations" at the hands of Haavelmo, Hurwicz, Koopmans, Marschak, and many others.[1] This work has been carried on at a level of great generality. It has now reached a point where it can be used to illuminate the concept of

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[1] See Koopmans (2) for a survey and references to the literature.

spurious correlation in the three-variable case. The bridge from the identification problem to the problem of spurious correlation is built by constructing a precise and operationally meaningful definition of causality—or, more specifically, of causal ordering among variables in a model.[2]

1. STATEMENT OF THE PROBLEM

How do we ordinarily make causal inferences from data on correlations? We begin with a set of observations of a pair of variables, x and y. We compute the coefficient of correlation, r_{xy}, between the variables and whenever this coefficient is significantly different from zero we wish to know what we can conclude as to the causal relation between the two variables. If we are suspicious that the observed correlation may derive from "spurious" causes, we introduce a third variable, z, that, we conjecture, may account for this observed correlation. We next compute the partial correlation, $r_{xy \cdot z}$, between x and y with z "held constant," and compare this with the zero order correlation, r_{xy}. If $r_{xy \cdot z}$ is close to zero, while r_{xy} is not, we conclude that either: (a) z is an intervening variable—the causal effect of x on y (or vice versa) operates through z; or (b) the correlation between x and y results from the joint causal effect of z on both those variables, and hence this correlation is spurious. It will be noted that in case (a) we do not know whether the causal arrow should run from x to y or from y to x (via z in both cases); and in any event, the correlations do not tell us whether we have case (a) or case (b).

The problem may be clarified by a pair of specific examples adapted from Zeisel.[3]

I. The data consist of measurements of three variables in a number of groups of people: x is the percentage of members of the group that is married, y is the average number of pounds of candy consumed per month per member, z is the average age of members of the group. A high (negative) correlation, r_{xy}, was observed between marital status

[2] Simon (6) and (7). See also Orcutt (4) and (5). I should like, without elaborating it here, to insert the *caveat* that the concept of causal ordering employed in this paper does not in any way solve the "problem of Hume" nor contradict his assertion that all we can ever observe are covariations. If we employ an ontological definition of cause—one based on the notion of the "necessary" connection of events—then correlation cannot, of course, prove causation. But neither can anything else prove causation, and hence we can have no basis for distinguishing "true" from "spurious" correlation. If we wish to retain the latter distinction (and working scientists have not shown that they are able to get along without it), and if at the same time we wish to remain empiricists, then the term "cause" must be defined in a way that does not entail objectionable ontological consequences. That is the course we shall pursue here.

[3] Zeisel [9], pp. 192–95. Reference to the original source will show that in this and the following example we have changed the variables from attributes to continuous variables for purposes of exposition.

and amount of candy consumed. But there was also a high (negative) correlation, r_{yz}, between candy consumption and age; and a high (positive) correlation, r_{xz}, between marital status and age. However, when age was held constant, the correlation $r_{xy \cdot z}$, between marital status and candy consumption was nearly zero. By our previous analysis, either age is an intervening variable between marital status and candy consumption; or the correlation between marital status and candy consumption is spurious, being a joint effect caused by the variation in age. "Common sense"—the nature of which we will want to examine below in detail—tells us that the latter explanation is the correct one.

II. The data consist again of measurements of three variables in a number of groups of people: x is the percentage of female employees who are married, y is the average number of absences per week per employee, z is the average number of hours of housework performed per week per employee.[4] A high (positive) correlation, r_{xy}, was observed between marriage and absenteeism. However, when the amount of housework, z was held constant, the correlation $r_{xy \cdot z}$ was virtually zero. In this case, by applying again some common sense notions about the direction of causation, we reach the conclusion that z is an intervening variable between x and y: that is, that marriage results in a higher average amount of housework performed, and this, in turn, in more absenteeism.

Now what is bothersome about these two examples is that the same statistical evidence, so far as the coefficients of correlation are concerned, has been used to reach entirely different conclusions in the two cases. In the first case we concluded that the correlation between x and y was spurious; in the second case that there was a true causal relationship, mediated by the intervening variable z. Clearly, it was not the statistical evidence, but the "common sense" assumptions added afterwards, that permitted us to draw these distinct conclusions.

2. CAUSAL RELATIONS

In investigating spurious correlation we are interested in learning whether the relation between two variables persists or disappears when we introduce a third variable. Throughout this paper (as in all ordinary correlation analyses) we will assume that the relations in question are linear, and without loss of generality, that the variables are measured from their respective means.

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[4] Zeisel [9], pp. 191–92.

Now suppose we have a system of three variables whose behavior is determined by some set of linear mechanisms. In general we will need three mechanisms, each represented by an equation—three equations to determine the three variables. One such set of mechanisms would be that in which each of the variables *directly influenced* the other two. That is, in one equation x would appear as the dependent variable, y and z as independent variables; in the second equation y would appear as the dependent variable, x and z as the independent variables; in the third equation, z as dependent variable, x and y as independent variables.[5]

The equations would look like this:

$$\text{(I)}\quad \begin{array}{ll} (2.1) & x + a_{12}y + a_{13}z = u_1, \\ (2.2) & a_{21}x + y + a_{23}z = u_2, \\ (2.3) & a_{31}x + a_{32}y + z = u_3, \end{array}$$

where the u's are "error" terms that measure the net effects of all other variables (those not introduced explicitly) upon the system. We refer to $A = \|a_{ij}\|$ as the *coefficient matrix* of the system.

Next, let us suppose that not all the variables directly influence all the others—that some independent variables are absent from some of the equations. This is equivalent to saying that some of the elements of the coefficient matrix are zero. By way of specific example, let us assume that $a_{31} = a_{32} = a_{21} = 0$. Then the equation system (I) reduces to:

$$\text{(II)}\quad \begin{array}{ll} (2.4) & x + a_{12}y + a_{13}z = u_1, \\ (2.5) & y + a_{23}z = u_2, \\ (2.6) & z = u_3. \end{array}$$

By examining the equations (II), we see that a change in u_3 will change the value of z directly, and the values of x and y indirectly; a change in u_2 will change y directly and x indirectly, but will leave z unchanged; a change in u_1 will change only x. Then we may say that y *is causally dependent on* z in (II), and that x is causally dependent on y and z.

If x and y were correlated, we would say that the correlation was genuine in the case of the system (II), for $a_{12} \neq 0$. Suppose, instead, that the system were (III):

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[5] The question of how we distinguish between "dependent" and "independent" variables is discussed in Simon (7), and will receive further attention in this paper.

$$\text{(III)} \quad \begin{aligned} x + a_{13}z &= u_1, && (2.7) \\ y + a_{23}z &= u_2, && (2.8) \\ z &= u_3. && (2.9) \end{aligned}$$

In this case we would regard the correlation between x and y as spurious, because it is due solely to the influence of z on the variables x and y. Systems (II) and (III) are, of course, not the only possible cases, and we shall need to consider others later.

3. THE *a priori* ASSUMPTIONS

We shall show that the decision that a partial correlation is or is not spurious (does not or does indicate a causal ordering) can in general only be reached if *a priori* assumptions are made that certain *other* causal relations do *not* hold among the variables. This is the meaning of the "common sense" assumptions mentioned earlier. Let us make this more precise.

Apart from any statistical evidence, we are prepared to assert in the first example of Section 1 that the age of a person does *not* depend upon either his candy consumption or his marital status. Hence z cannot be causally dependent upon either x or y. This is a genuine empirical assumption, since the variable "chronological age" really stands, in these equations, as a surrogate for physiological and sociological age. Nevertheless, it is an assumption that we are quite prepared to make on evidence apart from the statistics presented. Similarly, in the second example of Section 1, we are prepared to assert (on grounds of other empirical knowledge) that marital status is not causally dependent upon either amount of housework or absenteeism.[6]

The need for such *a priori* assumption follows from considerations of elementary algebra. We have seen that whether a correlation is genuine or spurious depends on which of the coefficients, a_{ij}, of A are zero, and which are non-zero. But these coefficients are not observable nor are the "error" terms, u_1, u_2 and u_3. What we observe is a sample of values of x, y, and z.

Hence, from the standpoint of the problem of statistical estimation, we must regard the $3n$ sample values of x, y, and z as numbers given by observation, and the $3n$ error terms, u_i, together with the six coefficients, a_{ij}, as variables to be estimated. But then we have $(3n+6)$

[6] Since these are empirical assumptions it is conceivable that they are wrong, and indeed, we can imagine mechanisms that would reverse the causal ordering in the second example. What is argued here is that these assumptions, right or wrong, are implicit in the determination of whether the correlation is true or spurious.

variables ($3n$ u's and six a's) and only $3n$ equations (three for each sample point). Speaking roughly in "equation-counting" terms, we need six more equations, and we depend on the *a priori* assumptions to provide these additional relations.

The *a priori* assumptions we commonly employ are of two kinds:

(1) *A priori* assumptions that certain variables are not directly dependent on certain others. Sometimes such assumptions come from knowledge of the time sequence of events. That is, we make the general assumption about the world that if y precedes x in time, then $a_{21}=0$—x does not directly influence y.

(2) *A priori* assumptions that the errors are uncorrelated—i.e., that "all other" variables influencing x are uncorrelated with "all other" variables influencing y, and so on. Writing $E\ (u_iu_j)$ for the expected value of u_iu_j, this gives us the three additional equations:

$$E(u_1u_2) = 0; \qquad E(u_1u_3) = 0; \qquad E(u_2u_3) = 0.$$

Again it must be emphasized that these assumptions are "a priori" only in the sense that they are not derived from the statistical data from which the correlations among x, y, and z are computed. The assumptions are clearly empirical.

As a matter of fact, it is precisely because we are unwilling to make the analogous empirical assumptions in the two-variable case (the correlation between x and y alone) that the problem of spurious correlation arises at all. For consider the two-variable system:

$$\text{(IV)}\quad \begin{array}{ll} (3.1) & x + b_{12}y = v_1 \\ (3.2) & y = v_2 \end{array}$$

We suppose that y precedes x in time, so that we are willing to set $b_{21}=0$ by an assumption of type (1). Then, if we make the type (2) assumption that $E(v_1v_2)=0$, we can immediately obtain a unique estimate of b_{12}. For multiplying the two equations, and taking expected values, we get:

$$E(xy) + b_{12}E(y^2) = E(v_1v_2) = 0. \tag{3.3}$$

Whence

$$b_{12} = -\frac{E(xy)}{E(y^2)} = -\frac{\sigma_y}{\sigma_x}r_{xy}. \tag{3.4}$$

It follows immediately that (sampling questions aside) b_{12} will be zero or non-zero as r_{12} is zero or non-zero. *Hence correlation is proof of causa-*

tion in the two-variable case if we are willing to make the assumptions of time precedence and non-correlation of the error terms.

If we suspect the correlation to be spurious, we look for a common component, z, of v_1 and v_2 which might account for their correlation:

$$v_1 \equiv u_1 - a_{13}z, \tag{3.5a}$$

$$v_2 \equiv u_2 - a_{23}z. \tag{3.5b}$$

Substitution of these relations in (IV) brings us back immediately to systems like (II). This substitution replaces the unobservable v's by unobservable u's. Hence, we are not relieved of the necessity of postulating independence of the errors. We are more willing to make these assumptions in the three-variable case because we have explicitly removed from the error term the component z which we suspect is the source, if any, of the correlation of the v's.

Stated otherwise, introduction of the third variable, z, to test the genuineness or spuriousness of the correlation between x and y, is a method for determining whether in fact the v's of the original two variable system were uncorrelated. But the test can be carried out only on the assumption that the unobservable error terms of the three variable system are uncorrelated. If we suspect this to be false, we must further enlarge the system by introduction of a fourth variable, and so on, until we obtain a system we are willing to regard as "complete" in this sense.

Summarizing our analysis we conclude that:

(1) Our task is to determine which of the six off-diagonal matrix coefficients in a system like (I) are zero.

(2) But we are confronted with a system containing a total of nine variables (six coefficients and three unobservable errors), and only three equations.

(3) Hence we must obtain six more relations by making certain *a priori* assumptions.

(a) Three of these relations may be obtained, from considerations of time precedence of variables or analogous evidence, in the form of direct assumptions that three of the a_{ij} are zero.

(b) Three more relations may be obtained by assuming the errors to be uncorrelated.

4. SPURIOUS CORRELATION

Before proceeding with the algebra, it may be helpful to look a little more closely at the matrix of coefficients in systems like (I), (II), and

(III), disregarding the numerical values of the coefficients, but considering only whether they are non-vanishing (X), or vanishing (0). An example of such a matrix would be

$$\begin{Vmatrix} X & 0 & 0 \\ X & X & X \\ 0 & 0 & X \end{Vmatrix}.$$

In this case x and z both influence y, but not each other, and y influences neither x nor z. Moreover, a change in u_2—u_1 and u_3 being constant—will change y, but not x or z; a change in u_1 will change x and y, but not z; a change in u_3 will change z and y, but not x. Hence the causal ordering may be depicted thus:

$$\begin{tikzcd} x \arrow[dr] & & z \arrow[dl] \\ & y & \end{tikzcd}$$

In this case the correlation between x and y is "true," and not spurious.

Since there are six off-diagonal elements in the matrix, there are $2^6=64$ possible configurations of X's and 0's. The *a priori* assumptions (1), however, require 0's in three specified cells, and hence for each such set of assumptions there are only $2^3=8$ possible distinct configurations. If (to make a definite assumption) x does not depend on y, then there are three possible orderings of the variables (z, x, y; x, z, y; x, y, z), and consequently $3\cdot 8=24$ possible configurations, but these 24 configurations are not all distinct. For example, the one depicted above is consistent with either the ordering (z, x, y) or the ordering (x, z, y).

Still assuming that x does not depend on y, we will be interested, in particular, in the following configurations:

$$\underset{(\alpha)}{\begin{Vmatrix} X & 0 & 0 \\ X & X & X \\ 0 & 0 & X \end{Vmatrix}} \qquad \underset{(\beta)}{\begin{Vmatrix} X & 0 & X \\ X & X & 0 \\ 0 & 0 & X \end{Vmatrix}} \qquad \underset{(\gamma)}{\begin{Vmatrix} X & 0 & 0 \\ X & X & 0 \\ X & 0 & X \end{Vmatrix}}$$

$$\underset{(\delta)}{\begin{Vmatrix} X & 0 & X \\ 0 & X & X \\ 0 & 0 & X \end{Vmatrix}} \qquad \underset{(\epsilon)}{\begin{Vmatrix} X & 0 & 0 \\ 0 & X & X \\ X & 0 & X \end{Vmatrix}}$$

In Case α, either x may precede z, or z, x. In Cases β and δ, z precedes x; in Cases γ and ϵ, x precedes z. The causal orderings that may be inferred are:

$$\begin{tikzcd} x \arrow[dr] & & z \arrow[dl] \\ & y & \end{tikzcd} \qquad \begin{tikzcd} z \arrow[d] \\ x \arrow[d] \\ y \end{tikzcd} \qquad \begin{tikzcd} & x \arrow[dl] \arrow[dr] & \\ y & & z \end{tikzcd} \qquad \begin{tikzcd} & z \arrow[dl] \arrow[dr] & \\ x & & y \end{tikzcd} \qquad \begin{tikzcd} x \arrow[d] \\ z \arrow[d] \\ y \end{tikzcd}$$

(α) (β) (γ) (δ) (ϵ)

The two cases we were confronted with in our earlier examples of Section 1 were δ and ϵ, respectively. Hence, δ is the case of spurious correlation due to z; ϵ the case of true correlation with z as an intervening variable.

We come now to the question of which of the matrices that are consistent with the assumed time precedence is the correct one. Suppose, for definiteness, that z precedes x, and x precedes y. Then $a_{12} = a_{31} = a_{32} = 0$; and the system (I) reduces to:

$$\begin{aligned} &(4.1) & x \qquad + a_{13}z &= u_1, \\ &(4.2) & a_{21}\,x + y + a_{23}z &= u_2, \\ &(4.3) & z &= u_3. \end{aligned}$$

Next, we assume the errors to be uncorrelated:

$$(4.4) \qquad E(u_1u_2) = E(u_1u_3) = E(u_2u_3) = 0.$$

Multiplying equations (4.1)–(4.3) by pairs, and taking expected values we get:

$$(4.5) \quad a_{21}E(x^2) + E(xy) + a_{23}E(xz) + a_{13}\left[a_{21}E(xz) + E(yz) + a_{23}E(z^2)\right] = E(u_1u_2) = 0,$$

$$(4.6) \qquad E(xz) + a_{13}E(z^2) = E(u_1u_3) = 0,$$

$$(4.7) \qquad a_{21}E(xz) + E(yz) + a_{23}E(z^2) = E(u_2u_3) = 0.$$

Because of (4.7), the terms in the bracket of (4.5) vanish, giving:

$$(4.8) \qquad a_{21}E(x^2) + E(xy) + a_{23}E(xz) = 0.$$

Solving for $E\,(xz)$, $E\,(yz)$ and $E\,(xy)$ we find:

$$\begin{aligned} &(4.9) & E(xz) &= -\,a_{13}E(z^2), \\ &(4.10) & E(yz) &= (a_{13}a_{21} - a_{23})E(z^2), \\ &(4.11) & E(xy) &= a_{13}a_{23}E(z^2) - a_{21}E(x^2). \end{aligned}$$

Case α: Now in the matrix of case α, above, we have $a_{13}=0$. Hence:

(4.12a) $E(xz) = 0$; (4.12b) $E(yz) = -a_{23}E(z^2)$,

(4.12c) $E(xy) = -a_{21}E(x^2)$.

Case β: In this case, $a_{23}=0$, hence,

(4.13a) $E(xz) = -a_{13}E(z^2)$; (4.13b) $E(yz) = a_{13}a_{21}E(z^2)$;

(4.13c) $E(xy) = -a_{21}E(x^2)$;

from which it also follows that:

$$E(xy) = E(x^2)\frac{E(yz)}{E(xz)}. \tag{4.14}$$

Case δ: In this case, $a_{21}=0$. Hence,

(4.15a) $E(xz) = -a_{13}E(z^2)$; (4.15b) $E(yz) = -a_{23}E(z^2)$;

(4.15c) $E(xy) = a_{13}a_{23}E(z^2)$;

and we deduce also that:

$$E(xy) = \frac{E(xz)E(yz)}{E(z^2)}. \tag{4.16}$$

We have now proved that $a_{13}=0$ implies (4.12a); that $a_{23}=0$ implies (4.14); and that $a_{21}=0$ implies (4.16). We shall show that the converse also holds.

To prove that (4.12a) implies $a_{13}=0$ we need only set the left-hand side of (4.9) equal to zero.

To prove that (4.14) implies that $a_{23}=0$ we substitute in (4.14) the values of the cross-products from (4.9)–(4.11). After some simplification, we obtain:

$$a_{23}[E(x^2) - a_{13}^2E(z^2)] = 0. \tag{4.17}$$

Now since, from (4.1)

$$E(x^2) - E(u_1^2) + 2a_{13}E(zu_1) = a_{13}^2E(z^2), \tag{4.18}$$

and since, by multiplying (4.3) by u_1, we can show that $E(zu_1)=0$, the second factor of (4.17) can vanish only in case $E(u_1^2)=0$. Excluding this degenerate case, we conclude that $a_{23}=0$.

To prove that (4.16) implies that $a_{21}=0$, we proceed in a similar manner, obtaining:

$$a_{21}[E(x^2) - a_{13}^2E(z^2)] = 0, \tag{4.19}$$

from which we can conclude that $a_{21}=0$.

We can summarize the results as follows:

1) If $E(xz)=0$, $E(yz)\neq 0$, $E(xy)\neq 0$, we have Case α

2) If none of the cross-products is zero, and

$$E(xy) = E(x^2)\frac{E(yz)}{E(xz)},$$

we have Case β.

3) If none of the cross-products is zero, and

$$E(xy) = \frac{E(xz)E(yz)}{E(z^2)},$$

we have Case δ.

We can combine these conditions to find the conditions that two or more of the coefficients a_{13}, a_{23}, a_{21} vanish:

4) If $a_{13}=a_{23}=0$, we find that:
$E(xz)=0$, $E(yz)=0$. Call this Case $(\alpha\beta)$.

5) If $a_{13}=a_{21}=0$, we find that:
$E(xz)=0$, $E(xy)=0$. Call this Case $(\alpha\delta)$.

6) If $a_{23}=a_{21}=0$, we find that:
$E(yz)=0$, $E(xy)=0$. Call this Case $(\beta\delta)$.

7) If $a_{13}=a_{23}=a_{21}=0$, then
$E(xz)=E(yz)=E(xy)=0$. Call this Case $(\alpha\beta\delta)$.

8) If none of the conditions (1)–(7) are satisfied, then all three coefficients a_{13}, a_{23}, a_{21} are non-zero. Thus, by observing which of the conditions (1) through (8) are satisfied by the expected values of the cross products, we can determine what the causal ordering is of the variables.[7]

We can see also, from this analysis, why the vanishing of the partial correlation of x and y is evidence for the spuriousness of the zero-order correlation between x and y. For the numerator of the partial correlation coefficient $r_{xy\cdot z}$, we have:

$$N(r_{xy.z}) = \frac{E(xy)}{\sqrt{E(x^2)E(y^2)}} - \frac{E(xz)E(yz)}{E(z^2)\sqrt{E(x^2)E(y^2)}}. \tag{4.20}$$

We see that the condition for Case δ is precisely that $r_{xy\cdot z}$ vanish while none of the coefficients, r_{xy}, r_{xz}, r_{yz} vanish. From this we conclude

[7] Of course, the expected values are not, strictly speaking, observables except in a probability sense. However, we do not wish to go into sampling questions here, and simply assume that we have good estimates of the expected values.

that the first illustrative example of Section 1 falls in Case δ, as previously asserted. A similar analysis shows that the second illustrative example of Section 1 falls in Case ϵ.

In summary, our procedure for interpreting, by introduction of an additional variable z, the correlation between x and y consists in making the six *a priori* assumptions described earlier; estimating the expected values, $E(xy)$, $E(xz)$, and $E(yz)$; and determining from their values which of the eight enumerated cases holds. Each case corresponds to a specified arrangement of zero and non-zero elements in the coefficient matrix and hence to a definite causal ordering of the variables.

5. THE CASE OF EXPERIMENTATION

In sections (3)–(4) we have treated u_1, u_2 and u_3 as random variables. The causal ordering among x, y, and z can also be determined without *a priori* assumptions in the case where u_1, u_2, and u_3 are controlled by an experimenter. For simplicity of illustration we assume there is time precedence among the variables. Then the matrix is triangular, so that $a_{ij} \neq 0$ implies $a_{ji} = 0$; and $a_{ij} \neq 0$, $a_{jk} \neq 0$ implies $a_{ki} = 0$.

Under the given assumptions at least three of the off-diagonal a's in (I) must vanish, and the equations and variables can be reordered so that all the non-vanishing coefficients lie on or below the diagonal. If (with this ordering) u_2 or u_3 are varied, at least the variable determined by the first equation will remain constant (since it depends only on u_1). Similarly, if u_3 is varied, the variables determined by the first and second equations will remain constant.

In this way we discover which variables are determined by which equations. Further, if varying u_i causes a particular variable other than the ith to change in value, this variable must be causally dependent on the ith.

Suppose, for example, that variation in u_1 brings about a change in x and y, variation in u_2 a change in y, and variation in u_3 a change in x, y, and z. Then we know that y is causally dependent upon x and z, and x upon z. But this is precisely the Case β treated previously under the assumption that the u's were stochastic variables.

6. CONCLUSION

In this paper I have tried to clarify the logical processes and assumptions that are involved in the usual procedures for testing whether a correlation between two variables is true or spurious. These procedures begin by imbedding the relation between the two variables in a larger three-variable system that is assumed to be self-contained, except for stochastic disturbances or parameters controlled by an experimenter.

Since the coefficients in the three-variable system will not in general be identifiable, and since the determination of the causal ordering implies identifiability, the test for spuriousness of the correlation requires additional assumptions to be made. These assumptions are usually of two kinds. The first, ordinarily made explicit, are assumptions that certain variables do *not* have a causal influence on certain others. These assumptions reduce the number of degrees of freedom of the system of coefficients by implying that three specified coefficients are zero.

The second type of assumption, more often implicit than explicit, is that the random disturbances associated with the three-variable system are uncorrelated. This assumption gives us a sufficient number of additional restrictions to secure the identifiability of the remaining coefficients, and hence to determine the causal ordering of the variables.

REFERENCES

[1] Kendall, Patricia L., and Lazarsfeld, Paul F., "Problems of Survey Analysis," in Merton and Lazarsfeld (eds.), *Continuities in Social Research*, The Free Press, 1950, 133–96.

[2] Koopmans, Tjalling C., "Identification Problems in Economic Model Construction," *Econometrica* 17: 125–44 (April 1949), reprinted as Chapter II in *Studies in Econometric Methods*, Cowles Commission Monograph 14.

[3] ———, "When Is an Equation System Complete for Statistical Purposes?" Chapter 17 in *Statistical Inference in Dynamic Economic Models*, Cowles Commission Monograph 10.

[4] Orcutt, Guy H., "Toward Partial Redirection of Econometrics," *The Review of Economics and Statistics*, 34 (1952), 195–213.

[5] ———, "Actions, Consequences, and Causal Relations," *The Review of Economics and Statistics*, 34 (1952), 305–14.

[6] Simon, Herbert A., "On the Definition of the Causal Relation," *The Journal of Philosophy*, 49 (1952), 517–28.

[7] ———, "Causal Ordering and Identifiability." Chapter III in *Studies in Econometric Methods*, Cowles Commission Monograph 14.

[8] Yule, G. Udny, *An Introduction to the Theory of Statistics*, Charles Griffin and Co., 10th ed., 1932, Chapters 4, 12. (Equivalent chapters will be found in all subsequent editions of Yule and Yule and Kendall, through the 14th.)

[9] Zeisel, Hans, *Say It With Figures*, New York, Harper and Brothers, 1947.