S2022X1A1AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA

An individual in a population can have only one of three genotypes: *A*, *B*, or *C*. The probability that an individual has genotype *A* is 0.64; the probability that an individual has genotype *B* is 0.32; and the probability that an individual has genotype *C* is 0.04. The probability that an individual has condition *X* differs by genotype. The probability that an individual with genotype *A* has condition *X* is 0.01. The probability that an individual with genotype *B* has condition *X* is 0.90. The probability that an individual with genotype *C* has condition *X* is 0.90.

a. What is the probability that an individual in the population has condition *X*? (10 points)
b. What is the probability that an individual with conditon *X* has genotype *A*? (40 points)

A1A $P(X) = P(X|A)P(A) + P(X|B)P(B) + P(X|C)P(C)$

$= (0.01)(.64) + (0.90)(0.32) + (0.90)(0.04)$

$= 0.0064 + 0.288 + 0.036 = 0.3304.$

$P(X) = 0.3304$

A1B. $P(A|X) = \dfrac{P(A \cap X)}{P(X)} = \dfrac{P(X|A)P(A)}{P(X)} = \dfrac{(0.01)(.64)}{0.3304}$

$= \dfrac{.0064}{0.3304} = 0.01937.$

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A research team wished to estimate the amount of weight reduction that followed from a diet regimen followed by a participant for 6 weeks. They ran a pilot study with 6 participants labelled *A*, *B*, *C*, *D*, *E*, and *F*.

a. Find the 95% confidence interval for the expected weight reduction using the data in the table below. (40 points)

b. Should the research team accept or reject the null hypothesis that the expected weight reduction is zero against the alternative that the expected weight reduction is not zero at the 0.05 level of significance? (10 points)

Participant	Starting Weight (pounds)	Ending Weight (pounds)	D_i
A	221	215	6
B	243	247	-4
C	231	225	6
D	208	204	4
E	237	223	14
F	197	195	2

$\bar{d}_6 = \frac{6-4+6+4+14+2}{6} = \frac{28}{6} = 4.667.$

$d_i - \bar{d}_6$: 1.333, -8.667, 1.333, -0.667, 9.333, -2.667

$\Sigma(d_i - \bar{d}_6) = -0.002,$

$\Sigma(d_i - \bar{d}_6)^2 = 173.333$

$s_D^2 = \frac{\Sigma(d_i-\bar{d}_6)^2}{5} = \frac{173.333}{5} = 34.67.$

95% CI FOR $E(D) = 4.667 \pm 2.571\sqrt{\frac{34.67}{6}}$

$= 4.667 \pm 6.180 = -1.51$ TO 10.85

A) 95% CI FOR $E(D)$ IS -1.51 LB TO 10.85 LB.

B.) SINCE 0 IS IN 95% CI ACCEPT H_0: $E(D) = 0$ VS

H_1 $E(D) \neq 0$ AT $\alpha = .05$.

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2. A research team took a random sample of 4 observations from a normally distributed random variable Y and observed that $\bar{y}_4 = 201.3$ and $s_Y^2 = 164.8$, where $\bar{y}_4$ was the average of the four observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$ (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 6 observations from a normally distributed random variable X and observed that $\bar{x}_6 = 183.7$ and $s_X^2 = 198.2$, where $\bar{x}_6$ was the average of the six observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$ (i.e., the divisor in the variance was $n-1$).
   a. Find the 99% confidence interval for $E(X) - E(Y)$. This part is worth 40 points.
   b. What is the correct decision for the test $H_0: E(X) - E(Y) = 0$ against the alternative $H_1: E(X) - E(Y) \neq 0$ at the 0.01 level of significance. This part is worth 10 points.

A. $S_p^2 = \dfrac{3(164.8) + 5(198.2)}{8} = \dfrac{1485.4}{8} = 185.675$ ON 8DF

$SE(\bar{X}_6 - \bar{Y}_4) = \sqrt{S_p^2\left(\frac{1}{4}+\frac{1}{6}\right)} = \sqrt{184.675\left(\frac{1}{4}+\frac{1}{6}\right)} = \sqrt{77.36}$

$= 8.796.$

99% CI FOR $E(X) - E(Y)$ IS

$\bar{x}_6 - \bar{y}_4 \pm 3.355(8.796) = -17.6 \pm 29.5$

$= -47.1$ TO $+11.9$.

B. SINCE 0 IS IN 99% CI FOR $E(X) - E(Y)$,

ACCEPT $H_0: E(X) - E(Y) = 0$ VS $H_1: E(X) - E(Y) \neq 0$

AT $\alpha = .01$.

C

3. A research team took a sample of 7 observations from the random variable Y, which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_7 = 143.9$, where $\bar{y}_7$ was the average of the seven sampled observations and $s^2 = 166.7$ was the observed value of the unbiased estimate of σ^2 based on the sample values. Test the null hypothesis $H_0: \sigma^2 = 100$ against the alternative hypothesis $H_1: \sigma^2 \neq 100$ at the 0.10, 0.05, and 0.01 levels of significance. This problem is worth 50 points.

$$TS = \frac{(n-1) S_y^2}{\sigma_0^2} = \frac{6(166.7)}{100} = 10.00 \text{ ON } 6 \text{ DF.}$$

FOR $\alpha = .10, .05, .01,$

$$P_r\{\lambda < \chi_6^2 < \rho\} = 1-\alpha:$$

α	λ	ρ	
.10	1.635	12.59	ACCEPT
.05	1.237	14.45	ACCEPT
.01	0.6757	18.55	ACCEPT

SINCE $TS = 10.0$ ON 6 DF AND $P\{\chi_6^2 > 12.59\} = .05$

ACCEPT $H_0: \sigma^2 = 100$ VS $H_1: \sigma^2 \neq 100$ AT $\alpha = .10$

(AND $\alpha = .05$ AND $\alpha = .01$)

2. A research team took a random sample of 5 observations from a normally distributed random variable Y and observed that $\bar{y}_5 = 182.2$ and $s_Y^2 = 218.3$, where $\bar{y}_5$ was the average of the five observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$ (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 7 observations from a normally distributed random variable X and observed that $\bar{x}_7 = 159.7$ and $s_X^2 = 245.2$, where $\bar{x}_7$ was the average of the seven observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$ (i.e., the divisor in the variance was $n-1$). Test the null hypothesis $H_0: E(X) = E(Y)$ against the alternative $H_1: E(X) \neq E(Y)$ at the 0.10, 0.05, and 0.01 levels of significance using the pooled variance t-test. This problem is worth 50 points.

$$S_p^2 = \frac{4(218.3) + 6(245.2)}{10} = \frac{2344.4}{10} = 234.44 \text{ ON 10 DF}$$

$$SE(\bar{X}_7 - \bar{Y}_5) = \sqrt{S_p^2\left(\frac{1}{5} + \frac{1}{7}\right)} = \sqrt{234.44(0.3429)}$$

$$= \sqrt{80.379} = 8.965$$

$$t_{10} = \frac{159.7 - 182.2}{8.965} = \frac{-22.5}{8.965} = -2.510$$

α	Z_α	t_{10}	
.10	1.645	1.832	R
.05	1.960	2.228	R
.01	2.576	3.169	A

REJECT $H_0: E(X) = E(Y)$ VS $H_1: E(X) \neq E(Y)$ AT $\alpha = .10$ AND $\alpha = .05$; ACCEPT AT $\alpha = .01$

3D.

3. A research team took a sample of 5 observations from the random variable Y, which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_5 = 143.9$, where $\bar{y}_5$ was the average of the five sampled observations and $s^2 = 586.2$ was the observed value of the unbiased estimate of σ^2 based on the sample values.

a. Find the 95% confidence interval for σ^2. This part is worth 40 points.

b. What is the correct decision for the test $H_0: \sigma^2 = 100$ against the alternative $H_1: \sigma^2 \neq 100$ at the 0.05 level of significance. This part is worth 10 points

$$TS = \frac{(n-1)S^2}{\sigma^2} = \frac{4S^2}{\sigma^2} \quad \text{ON} \quad 4 \text{ DF}$$

$$P\left\{0.4844 < \frac{4S^2}{\sigma^2} < 11.14\right\} = 0.95$$

$$P\left\{\frac{4S^2}{11.14} < \sigma^2 < \frac{4S^2}{0.4844}\right\} = 0.95$$

THEN LEFT END POINT $= \frac{4(586.2)}{11.14} = 210.5$

AND RIGHT END POINT $= \frac{4(586.2)}{0.4844} = 4840.6$

SINCE $\sigma^2 = 100$ IS NOT IN 95% CI

REJECT H_0 $\sigma^2 = 100$ VS H_1 $\sigma^2 \neq 100$ AT $\alpha = .05$.

THE 95% CI FOR σ^2 IS 210.5 TO 4,840.6.

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A research team took a random sample of 5 observations from a normally distributed random variable Y and observed that $\bar{y}_5 = 31.2$ and $s_Y^2 = 25.2$, where $\bar{y}_5$ was the average of the five observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$. A second research team took a random sample of 6 observations from a normally distributed random variable X and observed that $\bar{x}_6 = 148.9$ and $s_X^2 = 433.4$, where $\bar{x}_6$ was the average of the six observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$.

a. Find the 99% confidence interval for $\frac{var(X)}{var(Y)}$. This part is worth 40 points.

b. What is the correct decision for the test $H_0: \frac{var(X)}{var(Y)} = 1$ against the alternative $H_1: \frac{var(X)}{var(Y)} \neq 1$ at the 0.01 level of significance. This part is worth 10 points

$$TS = \frac{S_Y^2/\sigma_Y^2}{S_X/\sigma_X^2} \sim F(4,5)$$

$$F_{.005,4,5} = 15.56$$

$$F_{.005,5,4} = 22.46$$

$$P\left\{ \frac{1}{22.46} < \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} < 15.56 \right\} = 0.99$$

$$P\left\{ \frac{1}{22.46}\frac{S_X^2}{S_Y^2} < \frac{\sigma_X^2}{\sigma_Y^2} < 15.56\frac{S_X^2}{S_Y^2} \right\} = 0.99$$

THEN LEFT END POINT OF 99% CI FOR $\frac{\sigma_X^2}{\sigma_Y^2}$ IS $\frac{1}{22.46}\frac{433.4}{25.2} = 0.766$

THE RIGHT END POINT IS $15.56\frac{433.4}{25.2} = 267.6$

A THE 99% CI FOR $\frac{\sigma_X^2}{\sigma_Y^2}$ IS 0.766 TO 267.6.

B. SINCE 1 IS IN THE 99% CI FOR $\frac{\sigma_X^2}{\sigma_Y^2}$, ACCEPT H_0

$\frac{\sigma_X^2}{\sigma_Y^2} = 1$ VS H_1 $\frac{\sigma_X^2}{\sigma_Y^2} \neq 1$ AT $\alpha = .01$

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A research team took a random sample of 6 observations from a normally distributed random variable Y and observed that $\bar{y}_6 = 241.2$ and $s_Y^2 = 238.2$, where $\bar{y}_6$ was the average of the six observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$. A second research team took a random sample of 7 observations from a normally distributed random variable X and observed that $\bar{x}_7 = 948.9$ and $s_X^2 = 1{,}191.4$, where $\bar{x}_7$ was the average of the seven observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$.

a. Find the 95% confidence interval for $\frac{var(X)}{var(Y)}$. This part is worth 40 points.

b. What is the correct decision for the test $H_0: \frac{var(X)}{var(Y)} = 1$ against the alternative $H_1: \frac{var(X)}{var(Y)} \neq 1$ at the 0.05 level of significance. This part is worth 10 points

$$TS = \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \sim F(5,6)$$

$$F_{.025,5,6} = 5.99$$

$$F_{.025,6,5} = 6.98$$

$$P\left\{\frac{1}{6.98} < \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} < 5.99\right\} = 0.95$$

$$P\left\{\frac{1}{6.98}\frac{S_X^2}{S_Y^2} < \frac{\sigma_X^2}{\sigma_Y^2} < 5.99\frac{S_X^2}{S_Y^2}\right\} = 0.95$$

THE LEFT END POINT OF 95% CI FOR $\frac{\sigma_X^2}{\sigma_Y^2}$ IS $\frac{1}{6.98} \cdot \frac{1191.4}{238.2} = 0.717$

THE RIGHT END POINT IS $5.99\left(\frac{1191.4}{238.2}\right) = 29.96.$

A THE 95% CI FOR $\frac{\sigma_X^2}{\sigma_Y^2}$ IS 0.717 to 29.96.

B. THE 95% CI INCLUDES 1. ACCEPT H_0! $\frac{\sigma_X^2}{\sigma_Y^2} = 1$

VS $H_1: \frac{\sigma_X^2}{\sigma_Y^2} \neq 1$ AT $\alpha = .05$

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5 In a clinical trial, *2J* patients suffering from an illness will be randomly assigned to one of two groups so that *J* will receive an experimental treatment and *J* will receive the best available treatment. The random variable *X* is the response of a patient to the experimental medicine, and the random variable *B* is the response of a patient to the best currently available treatment. The null hypothesis to be tested is that $H_0: E(X) - E(B) = 0$ against the alternative hypothesis $H_1: E(X) - E(B) > 0$ at the 0.025 level of significance. Under H_0, both *X* and *B* are normally distributed with $\sigma_X = \sigma_B = 500$.

a. What is the number *J* in each group that would have to be taken so that the probability of a Type II error for the test of the null hypothesis specified in the common section is 0.05 when $E(X) - E(B) = 400$, $\sigma_X = 700$, and $\sigma_B = 500$. This part is worth 45 points.

b. What is the total number of subjects for this clinical trial? This part is worth 5 points.

A. $$\sqrt{J} \geq \frac{|z_\alpha| \sqrt{\sigma_{X0}^2 + \sigma_{B0}^2} + |z_\beta| \sqrt{\sigma_{X1}^2 + \sigma_{B1}^2}}{\Delta}$$

$$\sqrt{J} \geq \frac{1.960 \sqrt{500^2 + 500^2} + 1.645 \sqrt{(700)^2 + (500)^2}}{400 - 0}$$

$$\sqrt{J} \geq \frac{1385.93 + 1415.08}{400} \doteq 7.003$$

$J \geq 49.03.$

USE 50 IN EACH GROUP

B. TOTAL NUMBER OF RESPONDENTS IS

50 + 50 = 100.

S2022X1H5HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH

5 In a clinical trial, *2J* patients suffering from an illness will be randomly assigned to one of two groups so that *J* will receive an experimental treatment and *J* will receive the best available treatment. The random variable *X* is the response of a patient to the experimental medicine, and the random variable *B* is the response of a patient to the best currently available treatment. The null hypothesis to be tested is that $H_0\colon E(X) - E(B) = 0$ against the alternative hypothesis $H_1\colon E(X) - E(B) > 0$ at the 0.005 level of significance. Under H_0, both *X* and *B* are normally distributed with $\sigma_X = \sigma_B = 600$.

a. What is the number *J* in each group that would have to be taken so that the probability of a Type II error for the test of the null hypothesis specified in the common section is 0.01 when $E(X) - E(B) = 800,\ \sigma_X = 900$, and $\sigma_B = 600$. This part is worth 45 points.

b. What is the total number of subjects for this clinical trial? This part is worth 5 points.

A. $$\sqrt{J} \geq \frac{|z_\alpha|\sqrt{\sigma_{X0}^2 + \sigma_{B0}^2} + |z_\beta|\sqrt{\sigma_{X1}^2 + \sigma_{B1}^2}}{\Delta}$$

$$\sqrt{J} \geq \frac{2.576\sqrt{600^2 + 600^2} + 2.326\sqrt{(900)^2 + 600^2}}{800 - 0}$$

$$\sqrt{J} \geq \frac{2185.81 + 2515.95}{800} = 5.877$$

$J \geq 34.54$

USE $J \geq 35$ PARTICIPANTS PER GROUP.

B. TOTAL NUMBER OF PARTICIPANTS IS 70 OR MORE.

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6. The random variable W is the winnings in one play of a game of chance. It is normally distributed with expected value \$5 and standard deviation \$200. Let the random variable S_n be $S_n = \sum_{i=1}^{n} W_i$.

a. What is the probability of winning money in one play of this game of chance? That is, what is $\Pr\{W > 0\}$? This part is worth 10 points.

b. When $n = 100$, what is $\Pr\{S_{100} \le 0\}$? This part is worth 40 points.

End of the Examination

A. $\Pr\{W > 0\} = \Pr\left\{\frac{W - EW}{\sigma_W} > \frac{0-5}{200}\right\}$

$= \Pr\left\{Z > -\frac{5}{200}\right\} = 1 - \Phi(-0.025) = \cancel{1}\, 0.4900$

$= 0.51$

B. $E(S_{100}) = 100\, E(W) = 500$

$VAR(S_{100}) = 100\,(200)^2 = 4{,}000{,}000 = (2{,}000)^2.$

$\Pr\{S_{100} \le 0\} = \Pr\left\{\frac{S_{100} - E(S_{100})}{\sigma(S_{100})} \le \frac{0-500}{2000}\right\}.$

$= \Pr\{Z \le -0.25\} = \Phi(-0.25) = 0.4013$

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6. The random variable W is the winnings in one play of a game of chance. It is normally distributed with expected value \$10 and standard deviation \$50. Let the random variable S_n be $S_n = \sum_{i=1}^{n} W_i$.

a. What is the probability of winning money in one play of this game of chance? That is, what is $\Pr\{W > 0\}$? This part is worth 10 points.

b. When $n = 200$, what is $\Pr\{S_{200} \leq 0\}$? This part is worth 40 points.

End of the Examination

A. $Pr\{W > 0\} = Pr\left\{\frac{W - E(W)}{\sigma_W} > \frac{0 - 10}{50}\right\}$

$= 1 - Pr\{W \leq -0.20\} = 1 - \Phi(-0.20)$

$= 1 - 0.4207 = 0.5793$

B. $E(S_{200}) = 200\ E(W) = 200(10) = 2000$

$VAR(S_{200}) = 200\ VAR(W) = 200(50)^2 = 500{,}000$

$= 707.107.$

$Pr\{S_{200} \leq 0\} = Pr\left\{\frac{S_{200} - E(S_{200})}{\sigma(S_{200})} < \frac{0 - 2000}{707.107}\right\}$

$= Pr\{Z < -2.828\} = \Phi(-2.828) = 0.00234.$