

AA

1. An individual has one of four genotypes called A , B , C , and D , respectively, for a gene associated with disease X . The probability that an individual has genotype A is 0.4; the probability that an individual has genotype B is 0.3; the probability that an individual has genotype C is 0.2; and the probability that an individual has genotype D is 0.1. The probability that an individual with the A genotype is affected with disease X is 0.05. The probability that an individual with the B genotype is affected with disease X is 0.25. The probability that an individual with the C genotype is affected with disease X is 0.85. The probability that an individual with the D genotype is affected with disease X is 0.99.
- What is the probability that an individual is affected with disease X ? This part of the problem is worth 10 points.
 - Given that an individual has disease X , what is the probability that the individual is genotype C ? This part is worth 40 points.

$$A. P(X) = P(X|A)P(A) + P(X|B)P(B) + P(X|C)P(C) + P(X|D)P(D)$$

$$= (0.05)(0.4) + (0.25)(0.3) + (0.85)(0.2) + (0.99)(0.1)$$

$$P(X) = 0.364.$$

+10 CORRECT OR NOT
MINUS PTS FOR COPYING
OR COMPUTATION ERROR.

$$B. P(C|X) = \frac{P(C \cap X)}{P(X)} = \frac{P(X|C)P(C)}{P(X)}$$

$$= \frac{(0.85)(0.2)}{0.364} = 0.4670$$

$$P(C|X) = 0.4670.$$

IF $0 \leq P(C|X) \leq 1$,
+40 IF ANSWER CONSISTENT
WITH A.

BB

1. An individual has one of four genotypes called A , B , C , and D , respectively, for a gene associated with disease X . The probability that an individual has genotype A is 0.5; the probability that an individual has genotype B is 0.4; the probability that an individual has genotype C is 0.08; and the probability that an individual has genotype D is 0.02. The probability that an individual with the A genotype is affected with disease X is 0.1. The probability that an individual with the B genotype is affected with disease X is 0.2. The probability that an individual with the C genotype is affected with disease X is 0.90. The probability that an individual with the D genotype is affected with disease X is 0.99.
- What is the probability that an individual is affected with disease X ? This part of the problem is worth 10 points.
 - Given that an individual has disease X , what is the probability that the individual is genotype D ? This part is worth 40 points.

$$\begin{aligned} A. \quad P(X) &= P(X|A)P(A) + P(X|B)P(B) + P(X|C)P(C) \\ &\quad + P(X|D)P(D) \\ &= (0.1)(0.5) + (0.2)(0.4) + (0.90)(0.08) + (0.99)(0.02) \\ \boxed{P(X) = 0.2218} &\quad +10 \text{ CORRECT OR NOT} \\ &\quad \text{MINUS PTS FOR COPYING OR} \\ &\quad \text{COMPUTATION ERROR.} \end{aligned}$$

$$\begin{aligned} B. \quad P(D|X) &= \frac{P(D \cap X)}{P(X)} = \frac{P(X|D)P(D)}{P(X)} \\ &= \frac{(0.99)(0.02)}{(0.2218)} = 0.08927. \end{aligned}$$

$$\boxed{P(D|X) = 0.08927.} \quad +40$$

IF $0 \leq P(D|X) \leq 1$

GIVE CREDIT IF STUDENT ANSWER USES
STUDENT'S $P(X)$.

[illegible]

2. A research team took a sample of 7 observations from the random variable Y , which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_7 = 48.9$, where \bar{y}_7 is the average of the seven sampled observations and $s^2 = 83.9$ is the observed value of the unbiased estimate of σ^2 , based on the sample values (i.e., the divisor in the variance was $n - 1$). Test the null hypothesis $H_0: E(Y) = 60$ against $H_1: E(Y) \neq 60$. Use levels of significance 0.10, 0.05, and 0.01. This problem is worth 50 points.

$$DF = 7 - 1 = 6.$$

$$t_6 = \frac{48.9 - 60}{\sqrt{83.9/7}} = \frac{-11.1}{\sqrt{11.986}} = \frac{-11.1}{3.462}$$

$$= -3.206$$

α	Z_α	t_α	
0.10	1.645	1.943	REJECT
0.05	1.960	2.447	REJECT
0.01	2.326	3.707	ACCEPT.

REJECT $H_0: E(Y) = 60$ VS $H_1: E(Y) \neq 60$ AT $\alpha = .10$ AND $\alpha = .05$. ACCEPT AT $\alpha = .01$.

$\alpha = .10$ AND $\alpha = .05$. THERE
-40 NO DECISION OR INCONSISTENT DECISION
-100 INSTEAD OF t_6 .

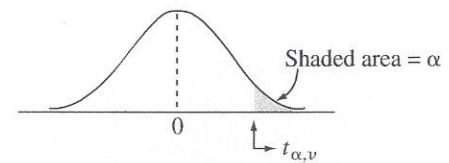
- 40 NO DECISION OR INCONSISTENT
- 33 USED NORMAL VALUES INSTEAD OF t_6
- 33 USED NORMAL VALUES INSTEAD OF t_6

-35 USED NORM. TEST.
-15 USED ONE SIDED TEST.

- 35 USED ONE SIDED TEST.
- 15 SUBSTANTIVE ERROR; E.G. $\sqrt{83.916}$

-15 SUBSTANTIVE
RATHER THAN $\sqrt{83917}$

-15 RATHER THAN
INCORRECT OR NO DEF.

**TABLE 2**Percentage points of Student's t distribution

df	Right-Tail Probability (α)								
	.40	.25	.10	.05	.025	.01	.005	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	.255	.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	.255	.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	.254	.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
inf.	.253	.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Source: Computed by M. Longnecker using the R function $qt(1 - \alpha, df)$.For level α two-tailed tests and $100(1 - \alpha)\%$ C.I.s use value in column headed by the number obtained by computing $\alpha/2$.

2. A research team took a sample of 3 observations from the random variable Y , which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_3 = 145.2$, where \bar{y}_3 is the average of the three sampled observations and $s^2 = 253.6$ is the observed value of the unbiased estimate of σ^2 , based on the sample values (i.e., the divisor in the variance was $n - 1$). Find the 99% confidence interval for $E(Y)$. This problem is worth 50 points.

99% CI FOR E(Y):

$$145.2 \pm 2.576, 2 \sqrt{\frac{253.6}{3}}$$

$$= 145.2 \pm 9.925 \sqrt{84.53}$$

$$= 145.2 \pm 9.925 (9.194)$$

~~$$= 145.2 \pm 91.25$$~~

~~99% CI FOR E(Y)~~

= 53.95 TO 236.45

-35 USED 1.960 FOR 9.925.

-35 USED 1.960 FOR
-15 USED 6.965 FOR 9.925
ERROR.

-15 SUBSTANTIVE ERROR.

SUBSTANTIVE ERROR.
E.G. USED $\sqrt{253.6}$ RATHER THAN.

$$\sqrt{\frac{2536}{3}}$$

-15 INCORRECT DF.

3. A research team took a random sample of 4 observations from a normally distributed random variable Y and observed that $\bar{y}_4 = 85.6$ and $s_Y^2 = 112.4$, where \bar{y}_4 was the average of the four observations sampled from Y and s_Y^2 was the unbiased estimate of $\text{var}(Y)$ (i.e., the divisor in the variance was $n - 1$). A second research team took a random sample of 2 observations from a normally distributed random variable X and observed that $\bar{x}_2 = 129.1$ and $s_X^2 = 142.8$, where \bar{x}_2 was the average of the two observations sampled from X and s_X^2 was the unbiased estimate of $\text{var}(X)$ (i.e., the divisor in the variance was $n - 1$). Find the 95% confidence interval for $E(X) - E(Y)$. This problem is worth 50 points.

$$S_p^2 = \frac{3(112) + 1(142.8)}{4} = \frac{478.8}{4} = 119.7$$

95% CI FOR $E(X) - E(Y)$ IS.

$$43.5 \pm 2.776 \sqrt{89.775}$$

$$43.5 \pm 26.30$$

THE 95% CI FOR $E(X) - E(Y)$ IS
 43.5 ± 26.30 ; IE. FROM 17.2 TO 69.8.

-35 USED 1.960 FOR 2.776.

-35 USED 1.100
-15 USED 2.132 FOR 2.776.

-15 SUBSTANTIVE ERROR.

-15 SUBSTANTIVE ERROR
NO PENALTY FOR UNEQUAL VARIANCE
PROCEDURE IF CORRECT.

3. A research team took a random sample of 5 observations from a normally distributed random variable Y and observed that $\bar{y}_5 = 35.6$ and $s_Y^2 = 71.4$, where \bar{y}_5 was the average of the five observations sampled from Y and s_Y^2 was the unbiased estimate of $\text{var}(Y)$ (i.e., the divisor in the variance was $n - 1$). A second research team took a random sample of 6 observations from a normally distributed random variable X and observed that $\bar{x}_6 = 24.2$ and $s_X^2 = 92.8$, where \bar{x}_6 was the average of the six observations sampled from X and s_X^2 was the unbiased estimate of $\text{var}(X)$ (i.e., the divisor in the variance was $n - 1$). Test the null hypothesis $H_0: E(X) = E(Y)$ against the alternative $H_1: E(X) \neq E(Y)$ at the 0.10, 0.05, and 0.01 levels of significance using the pooled variance t-test. This problem is worth 50 points.

$$S_p^2 = \frac{4(71.4) + 5(92.8)}{9} = \frac{749.6}{9} = 83.29$$

$$t_9 = \frac{35.6 - 24.2 - 0}{\sqrt{83.29 \left(\frac{1}{5} + \frac{1}{6} \right)}} = \frac{11.4}{\sqrt{30.54}}$$

$$= \frac{11.4}{5.526} = 2.063$$

α	Z_{α}	t_{α}	
.10	1.645	1.833	REJECT
.05	1.960	2.262	ACCEPT
.01	2.576	3.250	ACCEPT

REJECT $H_0: E(X) = E(Y)$ VS $H_1: E(X) \neq E(Y)$ AT $\alpha = .10$; ACCEPT THIS H_0 AT $\alpha = .05$ AND $\alpha = .01$.

- 40 NO DECISION OR INCONSISTENT DECISION.
- 35 USED NORMAL VALUES.
- 15 USED ONE SIDED TEST.
- 15 SUBSTANTIVE ERROR.
- 15 INCORRECT DF.

4. In a clinical trial, $2J$ patients suffering from an illness will be randomly assigned to one of two groups so that J will receive an experimental treatment and J will receive the best available treatment. The random variable X is the response of a patient to the experimental medicine, and the random variable B is the response of a patient to the best currently available treatment. Under the null hypothesis, both X and B are normally distributed with $\sigma_X = \sigma_B = 600$. The null hypothesis to be tested is that $H_0: E(X) - E(B) = 0$ against the alternative hypothesis $H_1: E(X) - E(B) > 0$ at the 0.025 level of significance.

- $$A. \sqrt{J} \geq \frac{\sqrt{2}(600)(1.960) + \sqrt{2}(600)(1.645)}{(200-0)}$$

$$\sqrt{5} \geq (\sqrt{2})(3)(1.960 + 1.645)$$

$$\sqrt{5} \approx (1.414)^3 (3.605)$$

$$\sqrt{5} \geq 15.29$$

$J \geq 233.9$.

$J \geq 233.9$.

J, THE NUMBER IN EACH GROUP, SHOULD BE 234 OR MORE

TOTAL NUMBER IN STUDY SHOULD BE 468 OR MORE

B.

-40 LOSE $\sqrt{2}$ IN $\sqrt{2}$ 600.

-20 NO 1.900

-20 NO 1.645.

-45 REPORT \sqrt{J} FOR J.

FF

4. In a clinical trial, $2J$ patients suffering from an illness will be randomly assigned to one of two groups so that J will receive an experimental treatment and J will receive the best available treatment. The random variable X is the response of a patient to the experimental medicine, and the random variable B is the response of a patient to the best currently available treatment. Under the null hypothesis, both X and B are normally distributed with $\sigma_X = \sigma_B = 800$. The null hypothesis to be tested is that $H_0: E(X) - E(B) = 0$ against the alternative hypothesis $H_1: E(X) - E(B) > 0$ at the 0.025 level of significance.

- What is the number J in each group that would have to be taken so that the probability of a Type II error for the test of the null hypothesis specified in the common section is 0.05 when $E(X) - E(B) = 200$ and $\sigma_X = 800$ and $\sigma_B = 800$. This part is worth 45 points.
- What is the total number of subjects for this clinical trial? This part is worth 5 points.

$$A. \quad \sqrt{J} \geq \frac{1.96 \sqrt{\sigma_0^2 + \sigma_0^2} + 1.645 \sqrt{\sigma_0^2 + \sigma_0^2}}{\Delta}$$

$$\sqrt{J} \geq \frac{1.960(\sqrt{2})(800) + 1.645 \sqrt{2}(800)}{200}$$

$$\sqrt{J} \geq \frac{\sqrt{2}(1.960 + 1.645)800}{200} = \sqrt{2}(3.605)4$$

$$\sqrt{J} \geq 20.39$$

$$J \geq 415.87$$

J , THE NUMBER IN EACH GROUP, SHOULD BE 416 OR MORE.

13. TOTAL NUMBER IN STUDY SHOULD BE 832 OR MORE.

-45 REPORT \sqrt{J} FOR J .

-40 LOSE $\sqrt{2}$ IN $\sqrt{2} 800$

-20 NO 1.960 IN FORMULA

-20 NO 1.645 IN FORMULA.

[illegible]

5. The random variable W is the winnings in one play of a game of chance. It is normally distributed with expected value \$10 and standard deviation \$100. Let the random variable S_n be $S_n = \sum_{i=1}^n W_i$.
 - a. What is the probability of winning money in one play of this game of chance? That is, what is $\Pr\{W > 0\}$? This part is worth 10 points.
 - b. Find the number of trials n such that $\Pr\{S_n \leq 0\} \leq .01$. This part is worth 40 points.

A. $W \sim N(10, 100^2)$

$$P_n\{W > 0\} = P_n\left\{\frac{W - EW}{\sigma_W} > \frac{0 - 10}{100}\right\}$$

$$= P_n\{Z > -0.1\} = 1 - \Phi(-0.1) = 1 - 0.4602$$

$$P_{\lambda}\{w > 0\} = 0.5398$$

B $P_n\{W > 0\} = 0.5510$
 $S_n = \sum W_i, \quad E(S_n) = nE(W) = 10n$
 $\sigma^2(S_n) = (100)^2 n$

$$S_n = \sum w_i, \quad \text{VAR}(S_n) = n \text{VAR}(w) = (100)^2 n.$$

$$\text{VAR}(S_n) = n \text{VAR}(W) = (100n)$$

$$P_n\{S_n \leq 0\} = P_n\left\{\frac{S_n - E(S_n)}{\sigma(S_n)} \leq \frac{0 - 10n}{100\sqrt{n}}\right\}$$

$$P_n\{S_n \leq 0\} = P_n\{Z \leq -0.10\sqrt{n}\}$$

$$P_L \{Z \leq -2.326\} = 0.01$$

CHOOSE n SO THAT

CHOOSE n SO THAT

$-0.10\sqrt{n} = -2.326, \sqrt{n} = 23.26, n = 541.078$

CHOOSE n TO BE 542 OR MORE.

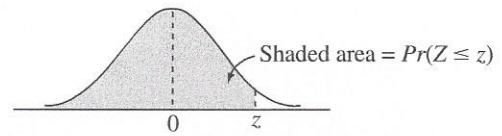
A RIGHT OR WRONG.

A RIGHT OR WRONG.
B. -35 FOR USING $\sqrt{n} = 23.26$ AS n .

- 20 NO 2.326.

TABLE 1

Standard normal curve areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

z	Area
-3.50	.00023263
-4.00	.00003167
-4.50	.00000340
-5.00	.00000029
$-\infty$.00000000

Source: Computed by M. Longnecker using the R function pnorm(z).

[illegible]

5. The random variable W is the winnings in one play of a game of chance. It is normally distributed with expected value \$25 and standard deviation \$200. Let the random variable S_n be $S_n = \sum_{i=1}^n W_i$.

- What is the probability of winning money in one play of this game of chance? That is, what is $\Pr\{W > 0\}$? This part is worth 10 points.
- Find the number of trials n such that $\Pr\{S_n \leq 0\} \leq .01$. This part is worth 40 points.

$$\begin{aligned} \text{A. } P_n\{W > 0\} &= P_n\left\{\frac{W - EW}{\sigma_W} > \frac{0 - 25}{200}\right\} \\ &= P_n\{Z > -0.125\} = 1 - 0.4503 = \boxed{0.5497} \end{aligned}$$

B. $S_m = \sum w_i$, $E(S_m) = mE(w) = 25m$

$$\text{VAR } S_n = n \text{VAR}(w) = (200)^2 n$$

$$\text{VAR } S_n = n \text{VAR}(W) = (200) \\ P_n \{ S_n \leq 0 \} = P_n \left\{ \frac{S_n - E(S_n)}{\sigma(S_n)} \leq \frac{0 - 25n}{200\sqrt{n}} \right\}$$

$$= P\{Z \leq -0.125\sqrt{n}\}.$$

SINCE $P\{Z \leq -2.320\} = 0.01$
 $0.125\sqrt{n} =$

SINCE $P_n \{ Z \leq -2.326 \}$
CHOOSE n SO THAT $-0.125\sqrt{n} = -2.326$

$$\sqrt{n} = 18.608$$

$$n = \underline{346.26}$$

$n = 346.26$
CHOOSE n TO BE 347 OR MORE!

A RIGHT OR WRONG.

A. RIGHT OR WRONG?

B. -35 FOR USING $\sqrt{n} = 18.608$ AS ANSWER

-20 NO: 52

2.32c:

6. The random variable Y has expected value $E(Y) = \mu$ and $\text{var}(Y) = \sigma^2 < \infty$. Let Y_1, Y_2, \dots, Y_n be a random sample of size n from Y . Let $T_n = \sum_{i=1}^n Y_i^2$. Find $E(T_n)$. This problem is worth 50 points.

End of the Examination

$$E(Y_i^2) = \text{VAR}(Y_i) + [E(Y_i)]^2 = \sigma^2 + \mu^2$$

$$E(T_n) = E\left(\sum_{i=1}^n Y_i^2\right) = \sum_{i=1}^n E(Y_i^2)$$

$$= \sum_{i=1}^n (\mu^2 + \sigma^2) = \boxed{n\mu^2 + n\sigma^2}$$