SOLUTION

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

Common Information for Questions 1, 2, and 3

A research team sought to estimate the model $E(Y) = \beta_0 + \beta_1 x + \beta_2 w$. The variable Y is the participant's BMI (body mass index, a measure of obesity with a higher value indicating greater obesity) observed at age 35; the variable x is the measure of the of the participant's years of education at age 25; and the variable w is the measure of the healthiness of the participant's life style at age 30 (where a larger number indicates healthier life style). They observed values of y, x, and w on 421 participants. They found that the variance of Y was 38.4. The correlation between Y and w was ***-0.61***; the correlation between Y and x was ***-0.31***; and the correlation between x and w was 0.48.

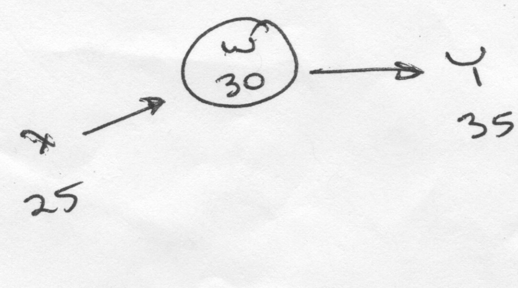
1. Compute the partial correlation coefficients $r_{Yw\bullet x}$ and $r_{Yx\bullet w}$ (20 points)

$$r_{Yw.x} = \frac{-0.61 - (\cancel{-0.38}\ -0.31)(0.48)}{\sqrt{(1-(-.31)^2)(1-.48^2)}} = \frac{-.4612}{\sqrt{(.9039)(.7696)}}$$

$$= \frac{-.4612}{\sqrt{.69564}} = \frac{-.4612}{.83405} = -0.55296$$

$$r_{Yx.w} = \frac{-.31 - (-0.61)(0.48)}{\sqrt{(1-(-.61)^2)(1-.48^2)}} = \frac{-.0172}{\sqrt{(.6279)(.7696)}}$$

$$= \frac{-.0172}{\sqrt{0.48323}} = \frac{-.0172}{.69515} = -.0247$$



MEDIATION

SOLUTION

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

2. Compute the analysis of variance table for the multiple regression analysis of Y. Include the sum of squares due to the regression on w and the sum of squares due to the regression on x after including w. Test the null hypothesis that $\beta_1 = 0$ (β_1 is the regression coefficient of x) at the 0.10, 0.05, and 0.01 levels of significance.

$TSS = 420(38.4) = 16128$ (NOTE 619315.2.

$SS\,REG(w) = (-.61)^2\,TSS = 6001.229.$

$TSS - SS\,REG(w) = 10126.771.$

$SS\,REG(x|w) = (-.0247)^2(10126.771) = 6.178$

SOURCE	DF	SS	MS
w	1	6001.229	6001.229
$x \mid w$	1	6.178	6.178
ERROR	418	10120.593	24.212
TOTAL	420	16128	

$$F_{x|w} = \frac{6.178}{24.212} = 0.255.$$

ACCEPT $H_0: \beta_1 = 0$ AT $\alpha = .10$

α	$F(1, 418)$.	
.10	2.718	A
.05	3.864	A
.01	6.696	A

SOLUTION

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

3. Is a mediation model or an explanation model a better explanation of the observed results? You must support your choice with results from your analyses to receive credit for this question. (20 points).

End of application of common information

MEDIATION

SOLUTION

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

4. A research team studied Y, a measure of the blood chemistry of a laboratory animal, and how Y was affected by the dose of medicine. The research team seeks to maximize $E(Y)$. They used four doses of medicine: 0, 1, 2, and 3 units respectively. They randomly assigned 9 animals to each dosage and observed that the average values of Y at each dosage were $y_{0\bullet} = 31.5, y_{1\bullet} = 265.8, y_{2\bullet} = 273.6$, and $y_{3\bullet} = 29.1$ where $y_{i.}$ was the average of the observations taken with dosage $i = 0,\ldots,3$, respectively. They also observed that $s_0^2 = 2501.4, s_1^2 = 2087.7, s_2^2 = 2256.1$, and $s_3^2 = 2405.8$ where s_i^2 was the unbiased estimate of the variance for the observations taken with dosage $i = 0,\ldots,3$, respectively.

a. Complete the balanced one way analysis of variance table for these results; that is, be sure to specify the degrees of freedom, sum of squares, mean square, F-test, and your conclusion. Use the 0.10, 0.05, and 0.01 levels of significance. (30 points)
b. Find the estimated quadratic contrast and its sum of squares. The coefficients of the linear contrast are $1,-1,-1,1$. (10 points)
c. Which setting of dosage is optimal? (10 points) The three parts of this problem are worth 50 points.

A)

i	$y_{i\cdot}$	$s_{i\cdot}^2$	$y_{i\cdot}-y_{\cdot\cdot}$	J
0	31.5	2501.4	−118.5	9
1	265.8	2087.7	115.8	9
2	273.6	2256.1	123.6	9
3	29.1	2405.8	−120.9	9
Σ	600	9251.	0.	
	150	2312.75 = MSE		

$SSTR = 9(-118.5^2 + 115.8^2 + 123.6^2 + (-120.9)^2)$

$= 9\ (57\ 345.66) = 516\ 110.94.$

α	$F(3,32)$	
.01	4.459	R
.05	2.901	R
.10	2.263	R

SOURCE	DF	SS	MS	F	REJECT AT
TREAT	3	516,110.94	172,036.98	74.386	$\alpha = .01\ (.05, .10)$.
ERROR	32	74,008.00	2312.75		
TOTAL	35	590118.99			

B) $\hat{\lambda}_Q = 31.5 - 265.8 - 273.6 + 29.1 = -478.8 \qquad SS_Q = \dfrac{(\hat{\lambda}_Q)^2}{4/9} = 515811.24$

C) DOSAGE BETWEEN 1 & 2.

SOLUTION

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

Common Information for Questions 5 and 6

A research team studied Y, the protein production of a laboratory animal, and how Y was affected by the dose of medicine. The research team sought to maximize $E(Y)$. They used four doses of medicine: 0, 1, 2, and 3 units respectively. They randomly assigned 15 animals to each dosage and observed that the average values of Y at each dosage were $y_{0\bullet} = 64.2, y_{1\bullet} = 79.8$, $y_{2\bullet} = 92.9$, and $y_{3\bullet} = 111.1$ where $y_{i.}$ was the average of the observations taken with dosage $i = 0,\ldots,3$, respectively. They also observed that $s_0^2 = 250.1, s_1^2 = 208.7, s_2^2 = 225.6$, and $s_3^2 = 240.5$ where s_i^2 was the unbiased estimate of the variance for the observations taken with dosage $i = 0,\ldots,3$, respectively. They correctly calculated that the total sum of squares was 30758.1 and that the mean squared error was $MSE = 231.225$. They also correctly calculated that the values of the linear, quadratic and cubic contrasts were $\hat{\lambda}_{Lin} = 153.8$, $\hat{\lambda}_{Quad} = 2.6$, and $\hat{\lambda}_{Cubic} = 7.6$.

5. Construct the analysis of variance table for the linear regression of Y on dosage. Use the sum of squares due to the linear contrast as the sum of squares for the regression of Y on dosage. Test the null hypothesis that there is no linear association at the 0.10, 0.05, and 0.01 levels of significance. (40 points)

SOURCE	DF	SS	MS	F
LINEAR	1	17740.83	17740.83	79.046.
ERROR	58	13017.27	224.436	
TOTAL	59	30758.1.		

$\hat{\lambda}_L = 153.8. \qquad SS_L = \dfrac{(\hat{\lambda}_L)^2}{20/15} = 17740.83$

α	$F(1,58)$.	
.10	2.794	REJECT
.05	4.007	REJECT
.01	7.093	REJECT.

SOLUTION

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

6. Test the null hypothesis that the linear model is adequate at the 0.10, 0.05, and 0.01 levels of significance. Report the analysis of variance table, including the sum of squares due to lack of fit. Test whether there is evidence that the linear model has lack of fit at the 0.10, 0.05, and 0.01 levels. (40 points)

End of Application of Common Information

SOURCE	DF	SS	MS
REG	1	17740.83	
LACK OF FIT	2	68.67	34.335
PURE ERROR	56	12948.6	231.225
TOTAL	59	30758.1	

$$F_{LOF} = \frac{34.335}{231.225} = 0.148.$$

α	F (2, 56).	
.10	2.400	ACCEPT
.05	3.162.	ACCEPT
.01	5.006	ACCEPT

SOLUTION

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

7. The correlation matrix of the random variables Y_1, Y_2, Y_3, Y_4 is $\begin{pmatrix} 1 & \gamma & \gamma & \gamma \\ \gamma & 1 & \gamma & \gamma \\ \gamma & \gamma & 1 & \gamma \\ \gamma & \gamma & \gamma & 1 \end{pmatrix}$, $0 < \gamma < 1$, and each random variable has variance σ^2. Let $W_1 = Y_1 + Y_2 + Y_3 + Y_4$, and let $W_2 = -Y_1 - Y_2 + Y_3 + Y_4$. Find the variance covariance matrix of (W_1, W_2). This question is worth 40 points.

End of Examination

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$\text{vcv}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \sigma^2 \begin{bmatrix} 4 + 12\gamma & 0 \\ 0 & 4 - 4\gamma \end{bmatrix}$$