

# 14 Explanation Model

A research team sought to estimate the model  $E(Y) = \beta_0 + \beta_1x + \beta_2w$ . The variable  $Y$  was the measure of the task proficiency of an employee observed 6 years after training; the variable  $w$  was the employee's education level observed 2 years after training; and the variable  $x$  was the measure of the task proficiency of an employee observed immediately after training. They observed values of  $y$ ,  $x$ ,  $w$  on  $n = 534$  employees. They found that the standard deviation of  $Y$ , where the variance estimator used division by  $n - 1$ , was 54.9; the standard deviation of  $x$  was 45.2; and the standard deviation of  $w$  was 53.6. The correlation between  $Y$  and  $w$  was 0.41; the correlation between  $Y$  and  $x$  was 0.81; and the correlation between  $x$  and  $w$  was 0.51. Total 110 points

1. Compute the partial correlation coefficient  $r_{Yx.w}$ . (15 points).
2. Compute the partial correlation coefficient  $r_{Yw.x}$ . (15 points).
3. Which of the following is the best causal model for this data? (20 points)
  - a. Explanation model.
  - b. Mediation model.
  - c. Neither a nor b.

$$\begin{aligned}
 1. \quad r_{Yx.w} &= \frac{r_{Yx} - r_{Yw} r_{xw}}{\sqrt{(1 - r_{Yw}^2)(1 - r_{xw}^2)}} \\
 &= \frac{0.81 - (0.41)(0.51)}{\sqrt{(1 - (0.41)^2)(1 - (0.51)^2)}} \\
 &= \frac{0.81 - 0.2091}{\sqrt{(1 - 0.1681)(1 - 0.2601)}} \\
 &= \frac{0.6009}{\sqrt{(0.8319)(0.7399)}} = \frac{0.6009}{\sqrt{0.61552}} \\
 &= \frac{0.6009}{0.78455} = \boxed{0.76591}
 \end{aligned}$$

2.  $r_{Y_{W \cdot X}} = \frac{r_{Y_W} - r_{Y_X} r_{W_X}}{\sqrt{(1 - r_{Y_X}^2)(1 - r_{W_X}^2)}}$  2.

$$= \frac{0.41 - (0.81)(0.51)}{\sqrt{(1 - (0.81)^2)(1 - (0.5)^2)}}$$

$$= \frac{0.41 - 0.4131}{\sqrt{(1 - 0.6561)(1 - 0.2601)}}$$

$$= \frac{-0.0031}{\sqrt{(0.3439)(0.7399)}} = \frac{-0.0031}{\sqrt{0.25445}}$$

$$= \frac{-0.0031}{0.50443} = \boxed{-0.00615}$$

3. RECALL STANDARD ERROR OF  $r_{Y_{W \cdot X}}$

$$\approx \frac{1}{\sqrt{n-3}} = \frac{1}{\sqrt{534-3}} = \frac{1}{\sqrt{531}} = 0.043$$

SINCE  $r_{Y_{W \cdot X}}$  IS CLOSE TO 0

X IS THE KEY VARIABLE.

NOW LOOK AT TIME ORDER.  
OF OPERATION OF VARIABLES

3.

X  
PROFICIENCY  
AT  
TRAINING

Y  
PROFICIENCY  
6 YEARS  
AFTER  
TRAINING

W  
ED LEVEL  
2 YEARS

AFTER  
TRAINING

IDENTIFY KEY VARIABLE

SINCE KEY VARIABLE OPERATES FIRST,  
THIS IS AN EXPLANATION MODEL.

THE VARIABLE W (ED LEVEL 2 YEARS  
AFTER TRAINING) DOES NOT HAVE ANY  
ASSOCIATION WITH PROFICIENCY 6 YEARS  
AFTER TRAINING WHEN PROFICIENCY  
AFTER TRAINING IS CONTROLLED.