08 Confidence Interval for Univariate Linear Regression Slope

A research team conducted a longitudinal study of participants between 25 and 30 years of age. They measured each participant's level of education at age 25. They also measured each participant's earnings at age 30. The team collected data on $n = 314$ participants. The average level of education at age 25 was 15.3, with an observed standard deviation of 3.1 (the divisor in the underlying variance calculation was $n - 1$). The average earnings (in thousands of dollars) was 54.9, with an observed standard deviation of 14.9 (the divisor in the underlying variance calculation was $n-1$). The Pearson product moment correlation coefficient between the two variables was 0.76. The research team seeks to estimate the regression of participant earnings at age 30 on participant education at age 25.

b. Find the estimated regression of participant earnings at age 30 on participant education at age 25. Find the 95% confidence interval for the slope in this equation. This part is worth 25 points.

ANOVA TABLE FROM Q1

SOURCE	DF	SS	MS	F
REGRESSION	1	40,136.92	40,136.92	4126.63
ERROR	312	29,352.21	94.08	
TOTAL	313	69,489.13		

$$\sum (x_i - \bar{x}_m)^2 = 313(3.1)^2 = 3007.93$$

$$\hat{\beta}_1 = r \frac{SD_{DV}}{SD_{IV}} = 0.76\left(\frac{14.9}{3.1}\right) = 3.653$$

$$\hat{y}(x) = \bar{y}_m + \hat{\beta}_1(x - \bar{x}_m)$$

$$= 54.9 + 3.653(x - 15.31) \quad \text{OR}$$

$$= -0.991 + 3.653x$$

$$= (\bar{y}_m - \hat{\beta}_1 \bar{x}_m) + \hat{\beta}_1 x$$

RECALL $\text{VAR}(\hat{\beta}_1) = \sigma^2_{y|x} / \sum (x_i - \bar{x}_m)^2$

MODEL $Y_i = \beta_0 + \beta_1 x_i + \sigma_{Y|x} Z_i$

95% CI FOR β_1

$$\hat{\beta}_1 \pm t_{1.960,\, n-2} \sqrt{\frac{MSE}{\sum (x_i - \bar{x}_m)^2}}$$

$$= 3.653 \pm 1.968 \sqrt{\frac{94.08}{3007.93}}$$

$$= 3.653 \pm 1.968 \sqrt{0.03128}$$

$$= 3.653 \pm 1.968 (0.1769)$$

$$= 3.653 \pm 0.348$$

$$= 3.305 \text{ TO } 4.00.$$

I: $H_0: \beta_1 = 3.0, \quad \alpha = .05$

$H_1: \beta_1 \neq 3.0$

REJECT H_{0I}

II $H_0: \beta_1 = 3.5 \quad \alpha = .05$

$H_1: \beta_1 \neq 3.5$

ACCEPT H_{0II}

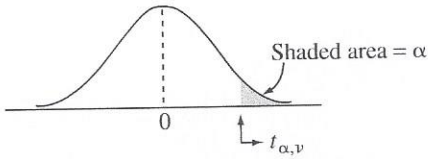

TABLE 2
Percentage points of Student's t distribution

	Right-Tail Probability (α)								
df	**.40**	**.25**	**.10**	**.05**	**.025**	**.01**	**.005**	**.001**	**.0005**
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	.255	.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	.255	.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	.254	.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
inf.	.253	.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Source: Computed by M. Longnecker using the R function $qt(1 - \alpha, \text{df})$.

For level α two-tailed tests and $100(1 - \alpha)\%$ C.I.s use value in column headed by the number obtained by computing $\alpha/2$.