12 Sample Size in a Correlation Study

A research team wishes to test the null hypothesis H_0: $\rho = 0$ at $\alpha = 0.005$ against the alternative H_1: $\rho > 0$ using Fisher's transformation of the Pearson product moment correlation coefficient as the test statistic. They have asked their consulting statistician for a sample size n such that $\beta = 0.01$ when $\rho = 0.15$. What is this value?

FIRST FIND $F(\rho_1)$:

$$F(0.15) = \frac{1}{2} \ln\left(\frac{1+0.15}{1-0.15}\right)$$

$$= \frac{1}{2} \ln(1.3529) = \frac{1}{2}(.30228)$$

$$= 0.1511.$$ PARTIAL CREDIT.

NOTE THAT $F(0.15)$ IS SLIGHTLY BIGGER THAN 0.15.

SECOND USE FORMULA: CHOOSE n

$$\sqrt{n-3} \geq \frac{|z_\alpha| + |z_\beta|}{F(\rho_1) - F(\rho_0)}.$$

SINCE $\rho_0 = 0$, $F(\rho_0) = 0$.

$\alpha = .005 \Rightarrow |z_\alpha| = 2.576$

$\beta = .01$, $|z_\beta| = 2.326$

$$\sqrt{n-3} \geq \frac{2.576(1) + 2.326}{0.1511} = 32.43.$$

$$n - 3 \geq (32.43)^2 = 1051.9.$$

$$n \geq 1054.9.$$

$$\boxed{n \geq 1055.}$$

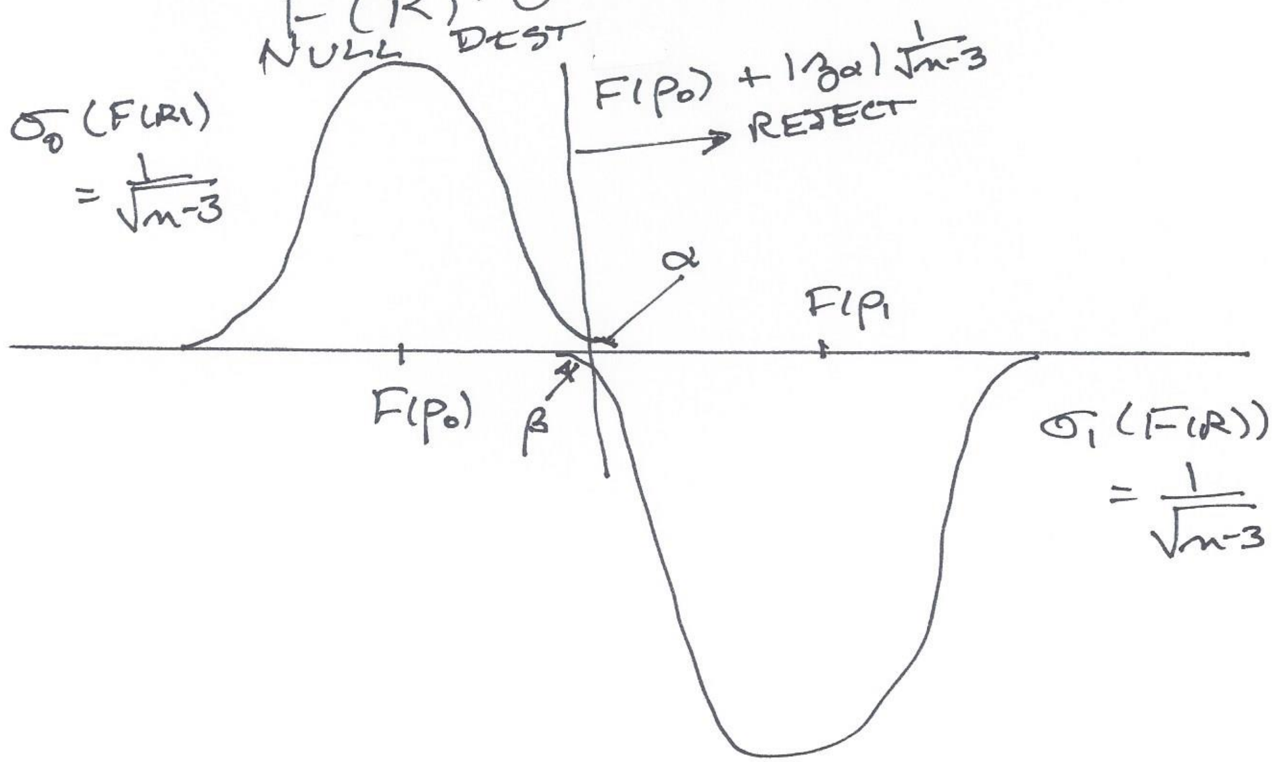
DERIVATION
OF FORMULA.

NULL DISTRIBUTION OF
F(R) WHEN $\rho = \rho_0$

$$F(R) \overset{APPX}{\sim} N\left(\frac{1}{2} \ln\left(\frac{1+\rho_0}{1-\rho_0}\right), \frac{1}{n-3}\right).$$

ALTERNATIVE DISTRIBUTION OF
F(R) WHEN $\rho = \rho_1$.

$$F(R) \overset{APPX}{\sim} N\left(\frac{1}{2} \ln\left(\frac{1+\rho_1}{1-\rho_1}\right), \frac{1}{n-3}\right)$$



ALT DIST OF F(R).

$\beta = Pr_1\{\text{ACCEPT } H_0\}.$

$$\beta = Pr_1 \{ F(R) < F(\rho_0) + |z_\alpha| \sqrt{\tfrac{1}{n-3}} \}$$

$$= Pr \{ F(R) - F(\rho_1) < F(\rho_0) + |z_\alpha| \sqrt{\tfrac{1}{n-3}} - F(\rho_1) \}$$

$$= Pr \{ \frac{F(R) - F(\rho_1)}{\sigma_1(F(R))} < \frac{F(\rho_0) - F(\rho_1) + |z_\alpha \sqrt{\tfrac{1}{n-3}}}{\sqrt{\tfrac{1}{n-3}}} \}.$$

BUT $\beta = Pr \{ Z < -|z_\beta| \}$

$$\beta = Pr \{ Z < \frac{F(\rho_0) - F(\rho_1) + |z_\alpha| \sqrt{\tfrac{1}{n-3}}}{\sqrt{\tfrac{1}{n-3}}} \}.$$

HENCE CHOOSE n SO THAT

$$-|z_\beta| = \frac{F(\rho_0) - F(\rho_1) + |z_\alpha| \sqrt{\tfrac{1}{n-3}}}{\sqrt{\tfrac{1}{n-3}}}$$

$$-|z_\beta| \sqrt{\tfrac{1}{n-3}} = F(\rho_0) - F(\rho_1) + |z_\alpha| \sqrt{\tfrac{1}{n-3}}$$

$$F(\rho_1) - F(\rho_0) = |z_\alpha| \sqrt{\tfrac{1}{n-3}} + |z_\beta| \sqrt{\tfrac{1}{n-3}}$$

$$\sqrt{n-3} = \frac{|z_\alpha| \cdot \sqrt{1} + |z_\beta| \sqrt{1}}{F(\rho_1) - F(\rho_0)}.$$