3 Two sample t-test and confidence interval

A research team took a random sample of 3 observations from a normally distributed random variable Y and observed that $\bar{y}_3 = 25.6$ and $s_Y^2 = 51.4$, where $\bar{y}_3$ was the average of the three observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$ (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 5 observations from a normally distributed random variable X and observed that $\bar{x}_5 = 39.1$ and $s_X^2 = 62.8$, where $\bar{x}_5$ was the average of the five observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$ (i.e., the divisor in the variance was $n-1$). Test the null hypothesis $H_0: E(X) = E(Y)$ against the alternative $H_1: E(X) \neq E(Y)$ at the 0.10, 0.05, and 0.01 levels of significance using the pooled variance t-test.

$$\bar{X}_5 - \bar{Y}_3 \sim N\left(E(X) - E(Y), \frac{\sigma_X^2}{5} + \frac{\sigma_Y^2}{3}\right).$$

Alternative problem:

Using the data from the problem above, find the 99% confidence interval for $E(X) - E(Y)$.

ASSUMING $\sigma_X^2 = \sigma_Y^2 = \sigma^2$ KNOWN.

$$Z = \frac{\bar{X}_5 - \bar{Y}_3 - 0}{\sigma\sqrt{\frac{1}{5} + \frac{1}{3}}} \sim N(0,1).$$

σ^2 NOT KNOWN: USE S_p^2 TO ESTIMATE σ^2.

$$S_p^2 = \frac{4(62.8) + 2(51.4)}{6} = \frac{354}{6}$$

$$= 59.0 \text{ ON } 6 \text{ DF.}$$

$$t_6 = \frac{(39.1 - 25.6) - 0}{\sqrt{59.0}\sqrt{0.5333}} = \frac{13.5}{5.610} = 2.406$$

T-STRETCH

α	z_α	$t_{\alpha,6}$	
.10	1.645	1.943	REJECT
.05	1.960	2.447	ACCEPT
.01	2.576	3.707	ACCEPT.

99% CONFIDENCE INTERVAL
FOR E(X) - E(Y).

$$39.1 - 25.6 \pm 3.707\sqrt{59.0}\sqrt{0.5333}$$

$$\bar{x}_5 - \bar{y}_3 \pm t_{2.576,6}\sqrt{S_p^2}\sqrt{\frac{1}{n_x}+\frac{1}{n_y}}$$

ORDER MATTERS

T STRETCH

$$13.5 \pm 3.707\,\boxed{(5.610)}$$

STANDARD ERROR OF $\bar{x}_5 - \bar{y}_3$.

$$\boxed{\begin{array}{c} 13.5 \pm 20.8 \\ -7.3 \text{ TO } 34.3 \end{array}}$$

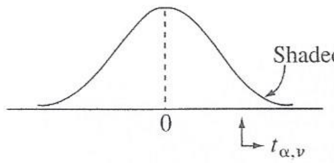

TABLE 2

Percentage points of Student's t distribution

	Right-Tail Probability (α)								
df	**.40**	**.25**	**.10**	**.05**	**.025**	**.01**	**.005**	**.001**	**.0005**
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	.255	.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	.255	.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	.254	.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
inf.	.253	.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Source: Computed by M. Longnecker using the R function $qt\,(1-\alpha,\text{df})$.

For 2-tailed tests and C.I.s use value in column headed by $\alpha/2$.