AMS 361: Applied Calculus IV

Homework 3

Assignment Date: When available in Brightspace
Due Date: See Brightspace
Submission to: Brightspace (1 PDF)
Grades: See individual problems

Problem 3.1 (10 points): Find the GS of
$$(x - ye^y)y' = 1$$

Problem 3.2 (10 points): Find the GS of
$$\left(\frac{x}{y} + \ln x\right)y' + \left(\frac{y}{x} + \ln y\right) = 0$$

Problem 3.3 (10 points): Find the GS of
$$y' = 2\frac{xy + y^2}{x^2 + 1}$$

Ch2: Math Models

Problem 3.4 (20 points): The inside of a container is an ellipsoid (as shown)
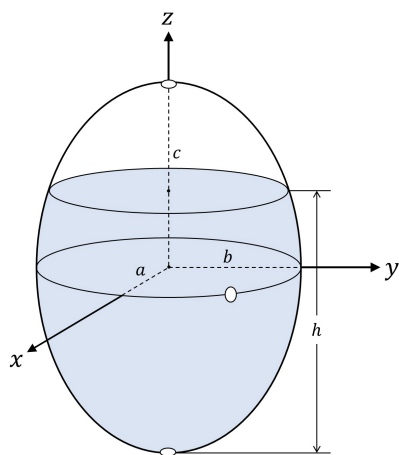$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$
Three holes are made (one at the south pole, and one at the equator, for leaking, one

at the north pole for air intake) to enable the same draining constant k for the leaking holes. The draining process follows the IVP
$$\begin{cases} \dfrac{A(h)dh}{dt} = -k\sqrt{h} \\ h(t = 0) = h_0 \end{cases}$$
introduced by Torricelli in 1643. In the DE, $A(h)$ is the cross-section area at the height (from the leaking point) h.

Derive the formula for the time needed to empty a tank of initial liquid height $h_0 = \frac{3c}{2}$. Again, the height is measured from the bottom hole.

Note: *Both leaking holes are at work while draining the upper half tank and, naturally, only one hole leaks while draining the lower half.*