Bayes' Theorem Problem

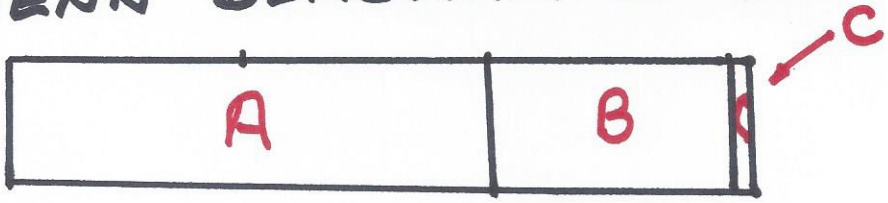
This is problem 1 from the Chapter 4 study guide.

1. An individual has one of three genotypes called A, B, and C, respectively, for a gene associated with disease X. The probability that an individual has genotype A is 0.64; the probability that an individual has genotype B is 0.32; and the probability that an individual has genotype C is 0.04. The probability that an individual with the A genotype is affected with disease X is 0.05. The probability that an individual with the B genotype is affected with disease X is 0.80. The probability that an individual with the C genotype is affected with disease X is 0.99.
    a. What is the probability that an individual is affected with disease X?
    b. Given that an individual has disease X, what is the probability that the individual is genotype B?

A. USE LAW OF TOTAL PROBABILITY.

AN INDIVIDUAL MAY HAVE GENOTYPE A, B, OR C.

VENN DIAGRAM OF GENOTYPES.



$P(\text{AFFECTED} \cap A) = P(\text{AFFECTED} | A)\, P(A)$

$= (0.05)(0.64) = 0.032.$

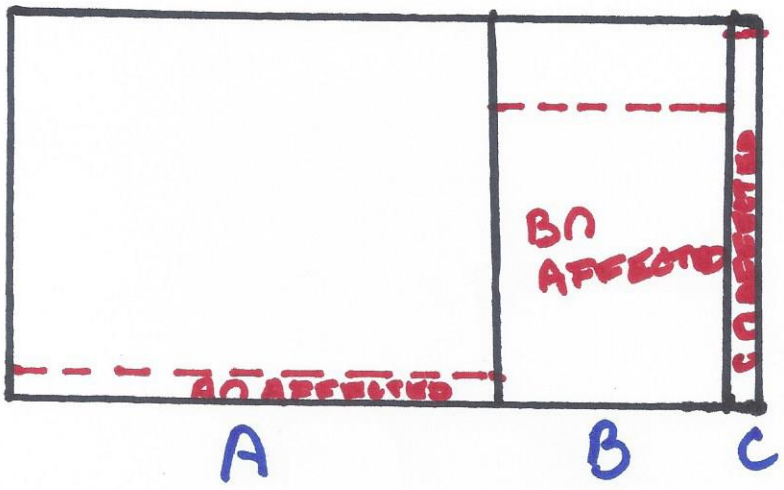
$P(\text{AFFECTED} \cap B) = P(\text{AFFECTED} | B)\, P(B)$

$= (0.80)(0.32) = 0.256.$

$P(\text{AFFECTED} \cap C) = P(\text{AFFECTED} | C)\, P(C)$

$= (0.99)(0.04) = 0.0396.$

VENN DIAGRAM GENOTYPES AND CONDITIONAL PROBABILITIES



$$P(\text{AFFECTED}) = (0.05)(0.64) + (0.80)(0.32) + (0.99)(0.04)$$

$$= 0.032 + 0.256 + 0.0396$$

$$= \boxed{0.3276.}$$

PREVALENCE OF DISEASE IS 32.76%.

B. BAYES' THEOREM TO FIND $P(B \mid \text{AFFECTED})$.

DEFN: $P(B \mid \text{AFFECTED}) = \dfrac{P(B \cap \text{AFFECTED})}{P(\text{AFFECTED})}$

BAYES' THEOREM

$$P(B \mid \text{AFFECTED}) = \frac{P(\text{AFFECTED} \mid B)\, P(B)}{P(\text{AFFECTED})}.$$

$$= \frac{(0.80)(0.32)}{0.3276}$$ ← ONE OF TERMS IN PART A.

$$= \frac{0.256}{0.3276} = \boxed{0.7814.}$$

78% OF AFFECTEDS HAVE GENOTYPE B.

EXTRA

COULD HAVE ASKED FOR

$$P(A \mid \text{AFFECTED}) = \frac{(0.05)(0.64)}{0.3276}$$

$$= 0.0977$$

$$P(C \mid \text{AFFECTED}) = \frac{(0.99)(0.04)}{0.3276}$$

$$= \frac{0.0396}{0.3276} = 0.1209.$$

NOTE

$$P(A \mid \text{AFFECTED}) + P(B \mid \text{AFFECTED}) + P(C \mid \text{AFFECTED})$$

$$= 0.0977 + 0.7814 + 0.1209$$

$$= 1.00.$$