X3S2022P1AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA

A research team wished to choose the dosage of medicine so that *Y*, the response to the medicine was as large as possible. They studied four doses: 1units, 2 units, 3 units, and 4 units. They ran a balanced one-way layout with 15 observations with dosage 1, 15 with dosage 2, 15 with dosage 3, and 15 with dosage 4, They observed that $y_{1\bullet} = 19.8,\ y_{2\bullet} = 12.6,\ y_{3\bullet} = 43.2,$ and $y_{4\bullet} = 24.4,$ where $y_{i\bullet}$ was the average of the observations taken with dosage $i =$ 1,2,3,4 respectively. They also observed that $s_1^2 = 1{,}648, s_2^2 = 812,\ s_3^2 = 2{,}082,$ and $s_4^2 = 1{,}062,$ where s_i^2 was the unbiased estimate of the variance for the observations taken with dosage $i =$ 1,2,3,4 respectively.

a. Complete the analysis of variance table for these results; that is, be sure to specify the degrees of freedom, sum of squares, mean square, and F-test. Use significance levels set to 0.10, 0.05, and 0.01. (40 points)
b. Find the quadratic contrast, the sum of squares due to the quadratic contrast, and the 99% Scheffe confidence interval for the quadratic contrast. The coefficients of the quadratic contrast are $1, -1. -1, 1.$ (30 points)
c. What is the 99% Tukey W confidence interval for $E(Y_{3j} - Y_{2j})$? What is the optimal setting of the concentration level? What statistical support is there for your answer? (20 points)

A. $y_{\bullet\bullet} = \dfrac{19.8 + 12.6 + 43.2 + 24.4}{4} = 25.0$

$y_{i\bullet} - y_{\bullet\bullet} = (19.8 - 25) = -5.2, -12.4, 18.2, -0.6$

SS TREAT $= J \sum (y_{i\bullet} - y_{\bullet\bullet})^2 = 15(27.04 + 153.76 + 331.24 + 0.36)$

$= 15(512.4) = 7686.0$ ON 3 DF

MS TREAT $=$ SS TREAT$/(I-1) = 7686.0/3 = 2562.$

MS ERR $= (1648 + 812 + 2082 + 1062)/4 = 5604/4 = 1401.0$ ON 56 DF

SS ERR $= 56(\text{MSE}) = 78{,}456.$

ANOVA TABLE

SOURCE	DF	SS	MS	F
TREATMENT (DOSE)	3	7,686.0	2,562.0	1.83.
(PURE) ERROR	56	78,456.0	1,401.0	
TOTAL	59	86,142.0		

α	F(3, 56)		F(3,60)
.10	2.184	ACCEPT	2.18
.05	2.769	ACCEPT	2.76
.01	4.152	ACCEPT	4.13

ACCEPT H_0: ALL TREATMENT MEANS EQUAL VS $H_1: \mu_i \neq \mu_{i'}$ $i \neq i'$ AT $\alpha = .10$ (AND $\alpha = .05$ AND $\alpha = .01$)

$\eta^2 = \dfrac{\text{SS TREAT}}{\text{SS TOTAL}} = \dfrac{7{,}686}{86{,}142} = 0.089.$

B. $\hat{\lambda}_Q = 1(19.8) - 1(12.6) - 1(43.2) + 24.4 = -11.6$

$SS_Q = \dfrac{(\hat{\lambda}_Q)^2}{4/15} = 504.6$

99% SCHEFFE FOR λ_Q:

$-11.6 \pm \sqrt{3(4.152)}\sqrt{1401\left(\frac{4}{15}\right)}$

$-11.6 \pm 3.529\ (19.33) = -11.6 \pm 68.2$

$= -79.8$ TO 56.6.

C. TUKEY 99% CI FOR $E(Y_{3j} - Y_{2j})$

$= (43.2 - 12.6) \pm 4.61\sqrt{\dfrac{MSE}{15}}$

$= 30.6 \pm 4.61\sqrt{\dfrac{1401}{15}} = 30.6 \pm 4.61\ (9.66).$

$= 30.6 \pm 44.6 = -14.0$ TO 75.2.

NO OPTIMAL SETTING BECAUSE H_0:

ALL TREATMENT MEANS EQUAL WAS ACCEPTED.

X3S2022P1*BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB*

A research team wished to choose the dosage of medicine so that Y, the response to the medicine was as small as possible. They studied four doses: 1 unit, 2 unit, 3 units, and 4 units. They ran a balanced one-way layout with 8 observations with dosage 1, 8 with dosage 2, 8 with dosage 3, and 8 with dosage 4, They observed that $y_{1\bullet} = 49.8$, $y_{2\bullet} = 42.6$, $y_{3\bullet} = 28.2$, and $y_{4\bullet} = 18.6$, where $y_{i\bullet}$ was the average of the observations taken with dosage $i = 1,2,3,4$ respectively. They also observed that $s_1^2 = 304, s_2^2 = 212$, $s_3^2 = 284$, and $s_4^2 = 252$, where s_i^2 was the unbiased estimate of the variance for the observations taken with dosage $i = 1,2,3,4$ respectively.

a. Complete the analysis of variance table for these results; that is, be sure to specify the degrees of freedom, sum of squares, mean square, and F-test. Use significance levels set to 0.10, 0.05, and 0.01. (40 points)
b. Find the quadratic contrast, the sum of squares due to the quadratic contrast, and the 99% Scheffe confidence interval for the quadratic contrast. The coefficients of the quadratic contrast are $1, -1. -1, 1$. (30 points)
c. What is the 99% Tukey W confidence interval for $E(Y_{1j} - Y_{4j})$? What is the optimal setting of the concentration level? What statistical support is there for your answer? (20 points)

A. $y_{\bullet\bullet} = \dfrac{49.8 + 42.6 + 28.2 + 18.6}{4} = \dfrac{139.2}{4} = 34.8.$

$y_{i\bullet} - y_{\bullet\bullet} = (49.8 - 34.8 = 15.0), 7.8, -6.6, -16.2$

$\Sigma(y_{i\bullet} - y_{\bullet\bullet})^2 = 225.00 + 60.84 + 43.56 + 262.44) = 591.84.$

$SS_{TREAT} = 8(591.84) = 4734.72$ ON 3 DF

$MS_{TREAT} = 4734.72/3 = 1578.24.$

$MS\ ERR = \dfrac{304 + 212 + 284 + 252}{4.} = 263$ ON 28 DF.

$SS\ ERR = 28(263) = 7364.$

ANOVA TABLE

SOURCE	DF	SS	MS	F
TREATMENT (DOSE)	3	4734.72	1578.24	6.00
(PURE) ERROR	28	7364.00	263.0	
TOTAL	31	12,098.72		

$\eta^2 = \dfrac{SS\ TREAT}{SS\ TOTAL} = 0.39$

REJECT H_0: ALL MEANS EQUAL VS. H_1: $\mu_i \neq \mu_{i'}$, $i \neq i'$ AT $\alpha = .01$ (AND $\alpha = .05$ AND $\alpha = .10$).

α	$F(3,28)$	
.10	2.291	REJECT
.05	2.947	REJECT
.01	4.568	REJECT

X3 2022P1B CONTINUED

B. $\hat{\lambda}_Q = 1(49.8) - 1(42.6) - 1(28.2) + 1(18.6) = -2.4.$

$SS_Q = \dfrac{(\hat{\lambda}_Q)^2}{4/8} = 11.52.$

99% SCHEFFE CI FOR $\mu_1 - \mu_2 - \mu_3 + \mu_4$ IS.

$-2.4 \pm \sqrt{3(4.568)}\,\sqrt{MSE\left(\frac{4}{8}\right)}$

$= -2.4 \pm (3.702)(11.47) = -2.4 \pm 42.5$

$= -44.85$ TO $40.1.$

C. 99% TUKEY CI FOR $E(Y_{1j} - Y_{4j})$ IS.

$(49.8 - 18.6) \pm 4.830\sqrt{\dfrac{263.0}{8}} = 31.2 \pm (4.83)(5.73)$

$= 31.2 \pm 27.7 = 3.5$ TO $58.9.$

TABLE VALUE FOR (4, 30) IS 4.80.

OPTIMAL SETTING IS 4 UNITS OR MORE

THERE IS A STRONG NEGATIVE LINEAR

ASSOCIATION: $\hat{\lambda}_L = -3(49.8) - 1(42.6) + 1(28.2) + 3(18.6).$

$= -108;\ SS_L = 4665.6$ WITH

$SS_{TREAT} = 4734.72.$

X3S2022P234CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C. A research team studied how Y, the protein production of a laboratory animal, could be minimized by choice of dosage of a medicine. They used four doses: 1, 2, 3, and 4 units. They randomly assigned 7 animals to 1 unit of dosage, 7 to 2 units, 7 to 3 units, and 7 to 4 units. The average values of Y at each dosage were $y_{1\bullet} = 794$, $y_{2\bullet} = 682$, $y_{3\bullet} = 614$, and $y_{4\bullet} = 582$, where $y_{i.}$ was the average of the observations taken with dosage $i = 1,2,3,4$. The within dosage variances were $s_1^2 = 8{,}804$, $s_2^2 = 7{,}048$, $s_3^2 = 10{,}466$, and $s_4^2 = 6{,}124$. They found that $y_{\bullet\bullet} = 668$ and that the average s_i^2 was 8,110.5. The total sum of squares was 379,340. The coefficients of the linear contrast were $-3,-1,1,3$;, and $\hat{\lambda}_{Lin} = -704.0$ The coefficients of the quadratic contrast were $1,-1,-1,1$; and $\hat{\lambda}_{Quad} = 80.0$. The coefficients of the cubic contrast were $-1,3,-3,1$; and $\hat{\lambda}_{Cubic} = -8.0$.

2. What are the values of the sum of squares due to the linear contrast, the sum of squares dues to the quadratic contrast, and the sum of squares due to the cubic contrast? What is the sum of squares for treatments? (40 points)
3. Find the analysis of variance table for the linear regression of Y on dosage, using the sum of squares due to the linear contrast as the sum of squares for the regression of Y on dosage. Test the null hypothesis that there is no linear association at the 0.10, 0.05, and 0.01 levels of significance. (40 points)
4. Test the null hypothesis that the linear model is adequate at the 0.10, 0.05, and 0.01 levels of significance. Report the analysis of variance table including the sum of squares due to lack of fit. What dosage appears to be optimal? (50 points).

End of application of common information

2. $\hat{\lambda}_L = -704$. $SS_L = \dfrac{(\hat{\lambda}_L)^2}{20/7} = 173{,}465.6$ ON 1 DF

$\hat{\lambda}_Q = 80$ $SS_Q = \dfrac{(\hat{\lambda}_Q)^2}{4/7} = 11{,}200$ ON 1 DF.

$\hat{\lambda}_C = -8$ $SS_C = \dfrac{(\hat{\lambda}_C)^2}{20/7} = 22.4$

$SS_{TREAT} = SS_{TOTAL} - SS_{ERROR} = 379{,}340 - 24(MSE)$

$= 379{,}340 - 194{,}652 = 184{,}688$

$SS_{TREAT} = SS_L + SS_Q + SS_C = 173{,}465.6 + 11{,}200 + 22.4$

$= 184{,}688.$

3. ANOVA TABLE

SOURCE	DF	SS	MS	F
LINEAR REGRESSION	1	173,465.6	173,465.6	21.9
ERROR	26	205,874.4	7,918.2	
TOTAL	27	379,340		

α	$F(1,26)$	
.10	2.909	REJECT
.05	4.225	REJECT
.01	7.721	REJECT

REJECT H_0: NO LINEAR ASSOCIATION VS H_1 LINEAR ASSOCIATION AT $\alpha = .01$ (AND $\alpha = .05$ AND $\alpha = .10$)

X3S 2022 P4.

4.

ANOVA TABLE

SOURCE	DF	SS	MS
LINEAR	1	173,465.6	
LACK OF FIT	2	11,222.4	5,611.2
(PURE) ERROR	24	194,652	8,110.5
TOTAL	27	379,340.0	

$SS_{LOF} = SS_q + SS_c = 11,200 + 22.4 = 11,222.4$ ON 2 DF

$F_{LOF} = \frac{5,611.2}{8,110.5} = 0.69$

α	$F(2,24)$	
.10	2.538	ACCEPT
.05	3.403	ACCEPT
.01	5.614	ACCEPT

ACCEPT H_0 LINEAR MODEL ADEQUATE VS H_1 LINEAR MODEL NOT ADEQUATE AT $\alpha = .10$ (AND $\alpha = .05$ AND $\alpha = .01$).

OPTIMAL DOSAGE IS 4 UNITS OR HIGHER, IF POSSIBLE. THE LINEAR TREND IS SIGNIFICANTLY NEGATIVE

X3S2022P234DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

D. A research team studied how *Y*, the protein production of a laboratory animal, could be maximized by choice of dosage of a medicine. They used four doses: 1, 2, 3, and 4 units. They randomly assigned 18 animals to 1 unit of dosage, 18 to 2 units, 18 to 3 units, and 18 to 4 units. The average values of *Y* at each dosage were $y_{1\bullet} = 546$, $y_{2\bullet} = 708$, $y_{3\bullet} = 582$, and $y_{4\bullet} = 104$, where $y_{i.}$ was the average of the observations taken with dosage $i =$ 1,2,3,4. The within dosage variances were $s_1^2 = 310{,}026, s_2^2 = 146{,}208,\ s_3^2 = 124{,}722,$ and $s_4^2 = 172{,}864$. They found that $y_{\bullet\bullet} = 485$ and that the average s_i^2 was 188,455. The total sum of squares was 16,559,300. The coefficients of the linear contrast were $-3, -1, 1, 3$;, and $\hat{\lambda}_{Lin} = -1452.0$ The coefficients of the quadratic contrast were $1, -1, -1, 1$; and $\hat{\lambda}_{Quad} = -640.0$. The coefficients of the cubic contrast were $-1, 3, -3, 1$; and $\hat{\lambda}_{Cubic} = -64.0$.

2. What are the values of the sum of squares due to the linear contrast, the sum of squares dues to the quadratic contrast, and the sum of squares due to the cubic contrast? What is the sum of squares for treatments? (40 points)
3. Find the analysis of variance table for the linear regression of *Y* on dosage, using the sum of squares due to the linear contrast as the sum of squares for the regression of *Y* on dosage. Test the null hypothesis that there is no linear association at the 0.10, 0.05, and 0.01 levels of significance. (40 points)
4. Test the null hypothesis that the linear model is adequate at the 0.10, 0.05, and 0.01 levels of significance. Report the analysis of variance table including the sum of squares due to lack of fit. What dosage appears to be optimal? (50 points).

End of application of common information

2. $\hat{\lambda}_L = (-1452.0)$, $SS_L = (\hat{\lambda}_L)^2/[20/18] = 1{,}897{,}473.6$

$\hat{\lambda}_Q = (-640.0)$, $SS_Q = (\hat{\lambda}_Q)^2/[4/18] = 1{,}843{,}200.0$

$\hat{\lambda}_c = (-64)$, $SS_c = (\hat{\lambda}_c)^2/[20/18] = 3{,}686.4.$

$SS_{TREAT} = SS_{TOTAL} - SS_{PE} = 16{,}559{,}300 - 68(188{,}455)$

$= 3{,}744{,}360$

$SS_{TREAT} = SS_L + SS_Q + SS_c = 1{,}897{,}473.6 + 1{,}843{,}200.0 + 3{,}686.4$

$= 3{,}744{,}360$

3.

LINEAR REGRESSION
ANOVA

SOURCE	DF	SS	MS
LIN REGRESSION	1	1,897,473.6	1,897,473.6
ERROR	70	14,661,826.4.	209,459.7
TOTAL	71	16,559,300	

$F_{LINEAR} = 1{,}897{,}473.6 / 209{,}459.7 = 9.06$

α	$F(1,70)$	
.10	2.779	REJECT
.05	3.978	REJECT
.01	7.011	REJECT

$F(1,60) = 7.08$
.01

$F_{.01}(1,90) = 6.93.$

REJECT H_0 NO LINEAR ASSOCIATION VS. H_1 LINEAR ASSOCIATION AT $\alpha = 0.01$ (AND $\alpha = .05$ AND $\alpha = .10$),

4. LACK OF FIT
ANOVA TABLE

SOURCE	DF	SS	MS
LINEAR REG	1	1,897,4736	
LACK OF LINEAR FIT	2	1,846,886.4	923,443.2
PURE ERROR	68	12,814,940	188,455.0
TOTAL	71	16,559,300	

$$F_{LOF} = \frac{923,443.2}{188,455.0} = 4.90 \text{ ON } (2,68) \text{ DF.}$$

$F_{.01}(2,60) = 4.98$
$F_{.01}(2,90) = 4.85$

α	$F(2,68)$	
.10	2.382	REJECT
.05	3.132	REJECT
.01	4.932	ACCEPT (BARELY)

REJECT H_0: LINEAR MODEL IS ADEQUATE VS H_1 LINEAR MODEL ADEQUATE AT $\alpha = .05$ (AND $\alpha = .10$)

ACCEPT H_0: LINEAR MODEL ADEQUATE AT $\alpha = .01$ (BARELY).

HIGHEST MEANDOSAGE IS 2 UNITS WITH OBSERVED MEAN = 708. THE 99% LSD IS 383.5 $\doteq t_{2.576,68}\sqrt{MSE\left(\frac{2}{18}\right)}$

ONLY DOSAGE 1 & 4, DOSAGE 2 AND 4, AND DOSAGE 3 AND 4 APPEAR DIFFERENT.

DOSAGE 4 IS THE WORST DOSE....

X3S2022P5*EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE*

5. The random variable $Y, Y > 0$, has $E(Y) = \theta$ and $var(Y) = \theta^{1.5}, \theta > 0$. Find the approximate mean and variance of $W = Y^p, p \neq 0$. For what value of p is the approximate variance of W constant? (50 points).

$f(y) = y^p$.

$f'(y) = p y^{p-1}$ $\quad f'(EY) = p\theta^{p-1}$; $E(W) \cong \theta^p$.

$VAR(W) \cong (f'(EY))^2 VAR(Y)$

$VAR(W) \cong p^2 \theta^{2p-2} \theta^{1.5}$

$VAR(W) \cong$ CONSTANT WHEN $2p-2+1.5=0$

$2p = 0.5$

$\boxed{p = 0.25.}$

LET $W = Y^{0.25}$; $E(W) = \theta^{0.25}$. $\quad f(y) = y^{0.25}$

$f'(y) = 0.25\, y^{-0.75}$.

$VAR(W) \cong (f'(\theta))^2\, VAR(Y) = (0.25)^2 (\theta^{-0.75})^2 \theta^{1.5}$

$VAR(W) \cong (0.25)^2$.

X3S2022P5FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF

The random variable $Y, Y > 0$, has $E(Y) = \theta$ and $var(Y) = \theta^{2.5}, \theta > 0$. Find the approximate mean and variance of $W = Y^p, p \neq 0$. For what value of p is the approximate variance of W constant? (50 points).

$f(y) = y^p.$

$f'(y) = py^{p-1}$; $f'(EY) = p\theta^{p-1}.$

$E(W) \cong \theta^p$

$VAR(W) \cong (f'(EY))^2 \, VAR\,Y$

$VAR(W) \cong p^2 \theta^{2p-2} \theta^{2.5}.$

VAR(W) IS CONSTANT APPROXIMATELY

WHEN $2p - 2 + 2.5 = 0$, $2p = -.5$

$\boxed{p = -0.25.}$

CONSIDER $W = Y^{-0.25}$; $f(y) = y^{-0.25}$, $f'(y) = -0.25y^{-1.25}$

$E(W) \cong \theta^{-0.25}$

$VAR(W) \cong (-0.25)^2 (\theta^{-1.25})^2 \theta^{2.5} = (0.25)^2.$

X3S2022P6*GGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGG*

6. The random variables Y_1, Y_2, Y_3 are independent and normally distributed but not identical. The distribution of Y_1 is $N(\mu+\alpha_1, \sigma^2)$; the distribution of Y_2 is $N(\mu+\alpha_2, \sigma^2)$; and the distribution of Y_3 is $N(\mu+\alpha_3, \sigma^2)$, with $\alpha_1+\alpha_2+\alpha_3 = 0$. Let $\bar{Y}_3 = \frac{Y_1+Y_2+Y_3}{3}$. Find $E(\sum_{i=1}^{3}(Y_i - \bar{Y}_3)^2$. This problem is worth 65 points.

End of the Examination

RECALL "COMPUTATIONAL" FORMULA

$$\sum_{i=1}^{n}(Y_i-\bar{Y}_n)^2 = \sum(Y_i^2) - n\bar{Y}_n^2.$$

THEN $\sum_{i=1}^{3}(Y_i-\bar{Y}_3)^2 = \sum_{i=1}^{3} Y_i^2 - 3\bar{Y}_3^2.$

$$E(Y_i^2) = VAR(Y_i) + (EY_i)^2 = \sigma^2 + (\mu+\alpha_i)^2$$

$$E\left[\sum_{i=1}^{3} Y_i^2\right] = \sum_{i=1}^{3}(\sigma^2 + (\mu+\alpha_i)^2)$$

$$= 3\sigma^2 + \sum(\mu^2 + 2\mu\alpha_i + \alpha_i^2)$$

$$= 3\sigma^2 + 3\mu^2 + 2\mu\sum\alpha_i + \sum\alpha_i^2$$

$$= 3\sigma^2 + 3\mu^2 + \sum\alpha_i^2.$$

$$E(\bar{Y}_3) = E\left(\frac{Y_1+Y_2+Y_3}{3}\right) = \frac{\mu+\alpha_1+\mu+\alpha_2+\mu+\alpha_3}{3}$$

$$= \frac{3\mu+\sum\alpha_i}{3} = \mu.$$

$$VAR(\bar{Y}_3) = \frac{\sigma^2}{3}.$$

$$-3E[(\bar{Y}_3)^2] = -3\left(\frac{\sigma^2}{3} + \mu^2\right)$$

$$E\sum_{i=1}^{3}(Y_i-\bar{Y}_n)^2 = 3\sigma^2 + 3\mu^2 + \sum\alpha_i^2 - \sigma^2 - 3\mu^2$$

$$= \boxed{2\sigma^2 + \sum_{i=1}^{3}\alpha_i^2.}$$