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AMS 315, Examination 1
March 8, 2018

Name: SOLUTION **ID:**

Directions: Write your name in the space provided. Work each problem in the space underneath the problem and on the back side of the page. This examination is worth 250 points. There are 6 problems, and the value of each problem is 40 points except for problem 3, which is worth 50 points. You may use only the paper in this form. If you un-staple your examination, please put your name on each sheet of the examination. You may use a calculator but not a computer or cell-phone. You may also use a ***single sheet of notes in your handwriting*** that is the size of the paper in this examination. You are on your honor not to use any other assistance during this examination. Do not make marks on the tables given to you to work this examination. Turn in both your paper, your notes, and your tables at the end of the examination. There will be no partial credit given for a problem unless you show your work. In the event of a fire alarm, please take your papers, exit the room, find a private place to work, and turn in your examination to me in my office (Math Tower 1-113) by 9:00 pm today. In this event, you are still on your honor not to give or receive assistance.

Since the course satisfies requirements for actuarial credentials, academic integrity standards will be enforced strictly.

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1. A research team took a random sample of 3 observations from a normally distributed random variable Y and observed that $\bar{y}_3 = 89.7$ and $s_Y^2 = 142.1$, where $\bar{y}_3$ was the average of the three observations sampled from Y and s_Y^2 was the unbiased estimate of $\text{var}(Y)$ (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 4 observations from a normally distributed random variable X and observed that $\bar{x}_4 = 28.2$ and $s_X^2 = 135.8$, where $\bar{x}_4$ was the average of the four observations sampled from X and s_X^2 was the unbiased estimate of $\text{var}(X)$ (i.e., the divisor in the variance was $n-1$). Calculate the 99% confidence interval for $E(X) - E(Y)$ using the pooled variance estimator. This problem is worth 40 points.

$$s_p^2 = \frac{2(142.1) + 3(135.8)}{5} = \frac{691.6}{5} = 138.32. \text{ ON 5 DF.}$$

99% CI FOR $E(X) - E(Y)$:

$$28.2 - 89.7 \pm 4.032 \sqrt{138.32} \sqrt{\tfrac{1}{3} + \tfrac{1}{4}}.$$

$$-61.5 \pm (4.032) \quad (11.761) \quad \sqrt{.5833}$$

$$-61.5 \pm 36.2$$

$$-97.7 \text{ TO. } -25.3$$

GRADING

-30 2.576 FOR 4.032.

-10 INCORRECT T VALUE FOR 4.032.

-5 +61.5 FOR -61.5.

-5 ARITHMETIC ERROR AFTER CORRECT FORMULA

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1. A research team took a random sample of 2 observations from a normally distributed random variable Y and observed that $\bar{y}_2 = 761.8$ and $s_Y^2 = 25.7$, where $\bar{y}_2$ was the average of the two observations sampled from Y and s_Y^2 was the unbiased estimate of $\text{var}(Y)$ (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 3 observations from a normally distributed random variable X and observed that $\bar{x}_3 = 751.8$ and $s_X^2 = 28.1$, where $\bar{x}_3$ was the average of the three observations sampled from X and s_X^2 was the unbiased estimate of $\text{var}(X)$ (i.e., the divisor in the variance was $n-1$). Test the null hypothesis $H_0 : E(X) = E(Y)$ against the alternative $H_1 : E(X) \neq E(Y)$ at the 0.10, 0.05, and 0.01 levels of significance using the pooled variance t-test. This problem is worth 40 points.

$$s_p^2 = \frac{1(25.7) + 2(28.1)}{3} = \frac{81.9}{3} = 27.3. \text{ ON 3 DF.}$$

$$t = \frac{761.8 - 751.8}{\sqrt{27.3\left(\frac{1}{2} + \frac{1}{3}\right)}} = \frac{10}{4.770} = 2.10.$$

$$27.3\left(\frac{1}{2} + \frac{1}{3}\right) = 22.75 = (4.770)^2$$

$|t| = 2.10 < 2.353$
ACCEPT AT
$\alpha = .10$

α	T_3	
.10	2.353	ACCEPT
.05	3.182.	ACCEPT
.01	5.841	ACCEPT

GRADING

-30 1.645, 1.960, 2.576 FOR 2.353, 3.182, 5.841.

-10 T VALUES OTHER THAN 2.353,...

-37 INCONSISTENT OR NO DECISION.

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2. A research team took a sample of 4 observations from the random variable Y, which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_4 = 72.1$, where $\bar{y}_4$ was the average of the four sampled observations and $s^2 = 68.2$ was the observed value of the unbiased estimate of σ^2 based on the sample values (i.e., the divisor in the variance was $n-1$). Find the 99% confidence interval for σ^2. This problem is worth 40 points. DF = 3.

$$\Pr\left\{ .07172 < \frac{(n-1)\, S_Y^2}{\sigma_Y^2} < 12.84 \right\} = 0.99$$

$$\Pr\left\{ \frac{(n-1)\, S_Y^2}{12.84} < \sigma_Y^2 < \frac{(n-1)\, S_Y^2}{.07172} \right\} = 0.99.$$

99% CI FOR σ^2: 15.93 TO 2852.8.

$(n-1)\, s_Y^2 = 3(68.2) = 204.6.$

GRADING

−10 FOR 2(68.2) RATHER THAN 3(68.2)

−10 NO .07172

−10 NO 12.84.

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2. A research team took a random sample of 5 observations from a normally distributed random variable Y and observed that $\bar{y}_5 = 31.4$ and $s_Y^2 = 738.7$, where $\bar{y}_5$ was the average of the five observations sampled from Y and s_Y^2 was the unbiased estimate of var(Y) (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 4 observations from a normally distributed random variable X and observed that $\bar{x}_4 = 12.5$ and $s_X^2 = 125.2$, where $\bar{x}_4$ was the average of the four observations sampled from X and s_X^2 was the unbiased estimate of var(X) (i.e., the divisor in the variance was $n-1$). Test the null hypothesis $H_0 : \text{var}(Y) = \text{var}(X)$ against the alternative $H_1 : \text{var}(Y) > \text{var}(X)$ at the 0.10, 0.05, and 0.01 levels of significance. This problem is worth 40 points.

$TS = \dfrac{S_Y^2}{S_X^2}$. UNDER H_0, $TS \sim F(4,3)$.

$ts = 5.90$.

α	$F(4,3)$.	
.10	5.34.	REJECT
.05	9.12	ACCEPT
.01	28.71	ACCEPT.

REJECT H_0 VAR(Y) = VAR(X) AT $\alpha = .10$,

ACCEPT AT .05 AND .01.

−20 FOR (3, 4) DF.

−20 FOR WRONG SIDE; I.E. $\dfrac{S_X^2}{S_Y^2}$.

−10 WRONG CRITICAL VALUES.

−37 NO OR INCONSISTENT DECISION!

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3. A research team collected data on $n = 378$ participants in a longitudinal study. Each participant was given a randomly selected dose of medicine. The average dose was 23.8 units, with an observed standard deviation of 5.5 units (the divisor in the underlying variance calculation was $n-1$). The cholesterol reduction of each participant to the medicine was determined by a blood test. The average reduction was 325.4 units with an observed standard deviation of 63.9 units (the divisor in the underlying variance calculation was $n-1$). The Pearson product moment correlation coefficient between the two variables was 0.48. The research team seeks to estimate the regression of the cholesterol reduction on the dosage of medicine.

   a. Complete the analysis of variance table for this regression and test the null hypothesis that the slope is zero at levels of significance 0.10, 0.05, and 0.01. (15 points)
   b. Find the estimated regression equation of the cholesterol reduction on the dosage of medication. Find the 99% confidence interval for the slope in this equation. (15 points). 2,589
   c. Use the least-squares prediction equation to estimate the reduction in cholesterol for participants whose dosage of medication was 35.0 units. Give the 99% confidence interval for the expected reduction in cholesterol for participants whose dosage was 35.0 units. (20 points)

IV = DOSE   DV = REDUCTION.

A. $TSS = 377(63.9)^2 = 1,539,370.17$ $\sum(x_i - \bar{x}_n)^2 = 377(5.5)^2 = 11404.25.$

$REGSS = r^2\,TSS = (.48)^2\,TSS = 354670.8872.$

$ERRSS = (1-r^2)\,TSS = (.7696)\,TSS = 1,184,699.283$

SOURCE	DF	SS	MS	F
REG	1	354,670.8872	354,670.8872	112.57.
ERROR	376	1,184,699.283	3,150.7960	
TOTAL	377	1,539,370.17		

α	$F(1,376)$.	
.10	2.719.	REJECT
.05	3.866	REJECT
.01	6.703	REJECT

B. $\hat{\beta}_0 = 325.4 - 5.58(23.8) = 192.596.$ $\hat{y}(x) = 192.596 + 5.58x$

$\hat{\beta}_1 = r\,\frac{s_{DV}}{s_{IV}} = 0.48\,\frac{63.9}{5.5} = 5.58$

99% CI FOR $\hat{\beta}_1$: $5.58 \pm 2.589\sqrt{\frac{3150.7960}{11404.25}}$

(4.22 TO 6.94) $5.58 \pm 2.589\sqrt{0.2763} = 5.58 \mp 1.36$

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C) 99% CI FOR $\beta_0 + 35\beta_1$

$\hat{Y}(35) = 192.596 + 1\ 195.3$

$= 387.90.$

$\hat{VAR}(\hat{Y}(x)) = MSE\left(\frac{1}{m} + \frac{(35-23.81)^2}{11404.25}\right)$

$= 3150.7960\left(\frac{1}{378} + \frac{(11.2)^2}{11404.25}\right)$

$= 3150.7960(.00265 + .0110).$

$= 3150.7960(.0136)$

$= 42.99 = (6.56)^2.$

99% CI FOR $\beta_0 + 35\beta_1$

$387.90 \pm 2.589(6.56) = 387.90 \pm 17.0$

370.9 TO 404.9

A. -10 EACH INCORRECT ANOVA TABLE.

-12 NO OR INCONSISTENT DECISION'

B. +5 CORRECT (OR CONSISTENT) $\hat{\beta}_1$.

-10 INCORRECT CI.

C. -20 USE PI FORMULA.

-5 COMP ERROR FOR $\hat{Y}(x)$.

-10 FORMULA ERRORS.

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3. A research team collected data on $n = 617$ participants in a longitudinal study. Each participant was given a randomly selected dose of medicine. The average dose was 138.8 units, with an observed standard deviation of 10.2 units (the divisor in the underlying variance calculation was $n-1$). The cholesterol reduction of each participant to the medicine was determined by a blood test. The average reduction was 75.4 units with an observed standard deviation of 18.9 units (the divisor in the underlying variance calculation was $n-1$). The Pearson product moment correlation coefficient between the two variables was 0.37. The research team seeks to estimate the regression of the cholesterol reduction on the dosage of medicine.

   a. Complete the analysis of variance table for this regression and test the null hypothesis that the slope is zero at levels of significance 0.10, 0.05, and 0.01. (15 points)
   b. Find the estimated regression equation of the cholesterol reduction on the dosage of medication. Find the 99% confidence interval for the slope in this equation. (15 points). 2.584.
   c. Use the least-squares prediction equation to estimate the reduction in cholesterol for a participant whose dosage of medication will be 160.0 units. Give the 99% prediction interval for the expected reduction in cholesterol for this participant whose dosage will be 160.0 units. (20 points)

DV = REDUCTION IV = DOSE.

A. TOT SS = $616(18.9)^2$ = 220041.36.

$\sum(x_i - \bar{x}_m)^2 = 616(10.2)^2$
$= 64088.64$

REGSS = r^2 TOTSS = $(.37)^2$ TSS = 30,123.66 ON 1 DF.

SSE = $(1-r^2)$ TOTSS = $(.8631)$ TSS. = 189,917.70.

SOURCE	DF	SS	MS	F
REG	1	30,123.66	30,123.66	97.55.
ERROR	615	189,917.70	308.81.	
TOTAL	616	220,041.36		

α	F(1, 615)	
.10	2.714	R
.05	3.857	R
.01	6.676	R.

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B) $\hat{Y}(x) = -19.82 + 0.686\,x.$

$$\hat{\beta}_1 = r\frac{S_{DV}}{S_{IV}} = 0.37\ \frac{18.9}{10.2} = 0.686.$$

$$\hat{\beta}_0 = 75.4 - 0.686\,(138.8) = -19.82.$$

$$99\%\ \text{CI FOR } \beta_1:\ 0.686 \pm 2.584\sqrt{\frac{308.81}{64088.64}}$$

$$0.686 \pm 2.584\,(.00482)^{1/2}$$

$$0.686 \pm 0.179.$$

99% CI FOR β_1: 0.507 TO 0.865.

C $\hat{Y}(160) = 89.94.$

$$\text{PRED MARGIN OF ERROR} = 2.584\sqrt{308.81}\sqrt{1+\frac{1}{n}+\frac{(x-\bar{x}_n)^2}{\sum(x_i-\bar{x}_n)^2}}$$

$$= 2.584\,(17.57)\sqrt{1+0.00162+\frac{(21.2)^2}{64088.64}}$$

$$= 2.584\,(17.57)\sqrt{1.00863}$$

$$= 45.60.$$

99% PI FOR $Y_F(160)$ IS 89.94 ± 45.60

SAME GRADING AS 3A.

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4. In a clinical trial, $2J$ patients suffering from an illness will be randomly assigned to one of two groups so that J will receive an experimental treatment and J will receive the best available treatment. The random variable X is the response of a patient to the experimental medicine, and the random variable B is the response of a patient to the best currently available treatment. Both X and B are normally distributed with $\sigma_X = \sigma_B = 150$. The null hypothesis to be tested is that $E(X) - E(B) = 0$ against the alternative that $E(X) - E(B) > 0$ at the 0.005 level of significance. What is the number J in each group that would have to be taken so that the probability of a Type II error for the test of the null hypothesis specified in the common section is 0.01 when $E(X) - E(B) = 100$ and $\sigma_X = \sigma_B = 150$? What is the total number of subjects for this clinical trial? This problem is worth 40 points.

$$\sqrt{J} \geq \frac{2.576\sqrt{2}(150) + 2.326\sqrt{2}(150)}{100}$$

$$= \frac{(4.902)\sqrt{2}\,150}{100} = 10.40.$$

$J \geq 108.1$ $\qquad J \geq 109$ PER GROUP.

$2J = 218$ IN STUDY.

−10 NO $2J$.

−10 NO 2.576

−10 NO 2.326.

−20 NO $\sqrt{2}$.

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4. A research team wishes to test the null hypothesis $H_0 : \rho = 0$ at $\alpha = 0.005$ against the alternative $H_1 : \rho > 0$ using Fisher's transformation of the Pearson product moment correlation coefficient as the test statistic. They have asked their consulting statistician for a sample size n such that $\beta = 0.01$ when $\rho = 0.3$ (that is, $\rho^2 = 0.09$). What is this value? (This question is worth 40 points).

$F(0.3) = \frac{1}{2} \ln\left(\frac{1.3}{0.7}\right) = \frac{1}{2} \ln(1.857) = 0.310$

$\sqrt{n-3} \geq \frac{2.576 + 2.326}{.310} = \frac{4.902}{.310} = 15.81.$

$n-3 \geq (15.81)^2 = 250.1$

$n \geq 254.$

+10 F(0.3)

−10 NO 2.576

−10 NO 2.326.

−20 MAJOR COMP ERROR

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5. The correlation matrix of the random variables Y_1, Y_2, Y_3 is
$\begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}$, $0 \le \rho < 1$, and each random variable has variance σ^2. Let $W_1 = Y_1 + Y_2$, and let $W_2 = Y_2 + Y_3$. Find the variance covariance matrix of (W_1, W_2). This problem is worth 40 points.

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\text{vcv}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \sigma^2 \begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 1+\rho & 1+\rho & 2\rho \\ 2\rho & 1+\rho & 1+\rho \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \sigma^2 \begin{bmatrix} 2(1+\rho) & 1+3\rho \\ 1+3\rho & 2(1+\rho) \end{bmatrix}$$

+10 FOR M.

−10 NO σ^2.

−20 NO $2(1+\rho)\sigma^2$ FOR VAR.

−20 NO $(1+3\rho)\sigma^2$ FOR COV.