4 Two sample n

In a clinical trial, $2J$ patients suffering from an illness will be randomly assigned to one of two groups so that J will receive an experimental treatment and J will receive the best available treatment. The random variable X is the response of a patient to the experimental medicine, and the random variable B is the response of a patient to the best currently available treatment. Both X and B are normally distributed with $\sigma_X = \sigma_B = 500$. The null hypothesis to be tested is that $H_0: E(X) - E(B) = 0$ against the alternative hypothesis $H_1: E(X) - E(B) > 0$ at the 0.005 level of significance.

a. What is the number J in each group that would have to be taken so that the probability of a Type II error for the test of the null hypothesis specified in the common section is 0.01 when $E(X) - E(B) = 200$ and $\sigma_X = 600$ and $\sigma_B = 500$. This part is worth 45 points.
b. What is the total number of subjects for this clinical trial? This part is worth 5 points.

A. PARAMETERS OF DISTRIBUTIONS.

TREATMENT	NULL E(·)	NULL VAR(·)	ALTERNATIVE E(·)	ALTERNATIVE VAR(·)
X	μ_0	$(500)^2$	$\mu_0 + 200$	$(600)^2$
B	μ_0	$(500)^2$	μ_0	$(500)^2$

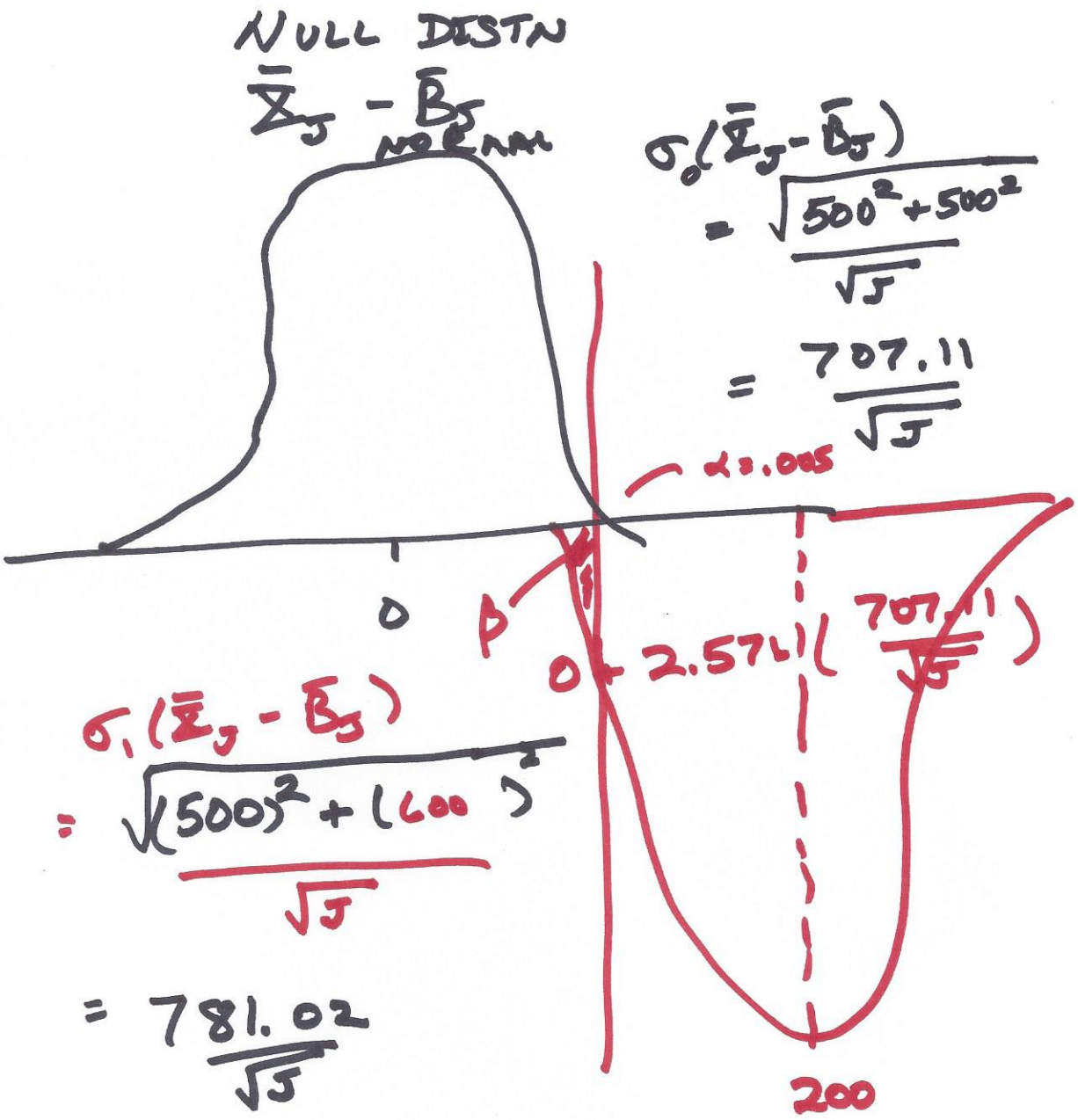
$\bar{X}_J - \bar{B}_J$ DISTRIBUTION.

NULL

$N(0, \frac{1}{J}(500^2 + 500^2))$.

ALTERNATIVE

$N(200, \frac{1}{J}((600)^2 + (500)^2))$.


NULL DISTN
$\bar{X}_J - \bar{B}_J$
NORMAL
$\sigma_0(\bar{X}_J - \bar{B}_J)$
$= \frac{\sqrt{500^2+500^2}}{\sqrt{J}}$
$= \frac{707.11}{\sqrt{J}}$
$\alpha = .005$
0
β
$0 + 2.576 \left(\frac{707.11}{\sqrt{J}}\right)$
$\sigma_1(\bar{X}_J - \bar{B}_J)$
$= \frac{\sqrt{(500)^2+(600)^2}}{\sqrt{J}}$
$= \frac{781.02}{\sqrt{J}}$
200
ALT DISTN
OF $\bar{X}_J - \bar{B}_J$

GENERAL RESULT

$$\beta = P_{\mu_1}\{\text{ACCEPT } H_0\}.$$

$$= P_{\mu_1}\left\{\bar{X}_J - \bar{B}_J < 0 + 2.57\left(\frac{707.11}{\sqrt{J}}\right)\right\}$$

$$= P_{\mu}\left\{\frac{\bar{X}_J - \bar{B}_J - E_1(\bar{X}_J - B_J)}{\sigma_1(\bar{X}_J - B_J)} < \frac{0 + 2.57\left(\frac{707.11}{\sqrt{J}}\right) - 200}{781.02/\sqrt{J}}\right\}$$

$$= P_{\mu}\left\{Z < \frac{0 + 2.57\left(\frac{707.11}{\sqrt{J}}\right) - 200}{781.02/\sqrt{J}}\right\}$$

CHOOSE J SO THAT

$$\beta = P_{\mu}\left\{Z < \frac{2.57\left(\frac{707.11}{\sqrt{J}}\right) - 200}{781.02/\sqrt{J}}\right\}$$

$$= P_{\mu}\{Z < -2.32\}$$

$$= .01.$$

CHOOSE J SO THAT

$$\frac{2.576\left(\frac{707.11}{\sqrt{J}}\right) - 200}{781.02/\sqrt{J}} = -2.326.$$

NOW SOLVE. OR USE FORMULA

$$\sqrt{J} \geq \frac{|\beta_1|\sqrt{\sigma_{x0}^2 + \sigma_{\delta 0}^2} + |\beta_0|\sqrt{\sigma_{x1}^2 + \sigma_{\delta 1}^2}}{|(E_1(\Sigma) - E_1(\Delta)) - (E_0(\Sigma) - E_0(\Delta))|}$$

$$\sqrt{J} \geq \frac{2.576(707.11) + 2.326(781.02)}{|200 - 0|}$$

$$\sqrt{J} \geq \frac{1821.51 + 1816.65}{200} = \frac{3633.16}{200}$$

$$\sqrt{J} \geq 18.19; \quad J \geq 330.9; \quad \boxed{J \geq 331.}$$

B 331 IN X AND 331 IN B IS

$$\boxed{\text{TOTAL} = 662}$$