6 F Confidence Interval for the Ratio of Two Variances

A research team took a random sample of 5 observations from a normally distributed random variable Y and observed that $\bar{y}_5 = 32.8$ and $s_Y^2 = 54.1$, where $\bar{y}_5$ was the average of the five observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$. A second research team took a random sample of 4 observations from a normally distributed random variable X and observed that $\bar{x}_4 = 57.6$ and $s_X^2 = 649.2$, where $\bar{x}_4$ was the average of the four observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$. Find the 95% confidence interval for $\frac{var(X)}{var(Y)}$.

USE $F_{4,3} = \dfrac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} = \dfrac{\sigma_X^2/\sigma_Y^2}{S_X^2/S_Y^2}$

FROM F TABLE

$$Pr\{F_{4,3} \le 15.1\} = 0.975$$

$$Pr\{F_{3,4} \le 9.98\} = 0.975$$

$$Pr\left\{\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \le 9.98\right\} = 0.975$$

INVERT EACH SIDE OF INEQUALITY

$$Pr\left\{\frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \ge \frac{1}{9.98}\right\} = 0.975$$

$$Pr\left\{\frac{\sigma_X^2/\sigma_Y^2}{S_X^2/S_Y^2} \ge \frac{1}{9.98}\right\} = 0.975$$

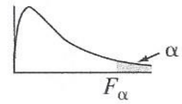

TABLE 8

Percentage points of the F distribution (df_2 between 1 and 6)

		df_1									
df_2	α	1	2	3	4	5	6	7	8	9	10
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32
	.10	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19
	.05	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
	.025	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6
	.01	4052.2	4999.5	5403.3	5624.6	5763.7	5859.0	5928.4	5981.0	6022.5	6055.8
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39
	.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
	.01	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
	.005	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4	199.4
	.001	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23
	.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
	.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
	.005	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69
	.001	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	129.2
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
	.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
	.005	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
	.01	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
	.005	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24	26.92
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
	.01	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
	.005	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41

Source: Computed by M. Longnecker using the R function qf($1 - \alpha, df_1, df_2$).
Additional values can be obtained using the same R function.

HENCE

$$Pr\left\{ \frac{1}{9.98} \le \frac{\sigma_x^2/\sigma_y^2}{S_x^2/S_y^2} \le 15.1 \right\} = 0.95$$

MULTIPLY EACH PART BY S_x^2/S_y^2:

$$Pr\left\{ \frac{1}{9.98} \frac{S_x^2}{S_y^2} \le \frac{\sigma_x^2}{\sigma_y^2} \le 15.1 \frac{S_x^2}{S_y^2} \right\} = 0.95$$

DATA FROM PROBLEM $\frac{S_x^2}{S_y^2} = \frac{649.2}{54.1} = 12.0$

HENCE LEFT ENDPOINT IS $\frac{1}{9.98}(12.0)$

RIGHT ENDPOINT IS $15.1\ (12.0)$

THE 95% CI FOR σ_x^2/σ_y^2 IS

$\boxed{1.20 \text{ TO } 181.2}$

TEST $H_0: (\sigma_x^2/\sigma_y^2) = 1$, VS $H_1: \frac{\sigma_x^2}{\sigma_y^2} \neq 1$.

AT $\alpha = .05$.

SINCE 1 IS NOT IN 95% CI (JUST BARELY),

$\boxed{\text{REJECT } H_0 \text{ AT } \alpha = .05.}$