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Please affirm that you comply with the following statement:

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the appropriate office and that I will be subjected to the maximum possible penalty permitted under University guidelines.

Sign and date that you comply with this statement before the first question that you answer:

1. A firm has three production lines called *A*, *B*, and *C*, respectively. Production line *A* produces 0.5 of the firm's total output. The fraction of line *A* products that are defective is 0.005. Production line *B* produces 0.3 of the firm's total output. The fraction of line *B* products that are defective is 0.01. Production line *C* produces 0.2 of the firm's total output. The fraction of line *C* products that are defective is 0.06.
   a. What is the probability that a randomly selected product is defective? This part of the problem is worth 10 points.
   b. Given that a product is defective, what is the probability that it was produced by line *A*? This part is worth 40 points.

A. $P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)$

$= (.005)(0.5) + (0.01)(0.3) + (0.06)(0.2)$

$= .0025 + 0.003 + 0.012 = \boxed{0.0175}$

B. $P(A|D) = \dfrac{P(A \cap D)}{P(D)} = \dfrac{P(D|A)P(A)}{P(D)}$

$= \dfrac{(0.005)(.5)}{0.0175} = \boxed{0.143.}$

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Sign and date that you comply with this statement before the first question that you answer:

1. A firm has three production lines called *A*, *B*, and *C*, respectively. Production line *A* produces 0.6 of the firm's total output. The fraction of line *A* products that are defective is 0.004. Production line *B* produces 0.3 of the firm's total output. The fraction of line *B* products that are defective is 0.002. Production line *C* produces 0.1 of the firm's total output. The fraction of line *C* products that are defective is 0.001.
   a. What is the probability that a randomly selected product is defective? This part of the problem is worth 10 points.
   b. Given that a product is defective, what is the probability that it was produced by line *A*? This part is worth 40 points.

A. $P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)$

$= (0.004)(0.6) + (0.002)(0.3) + (0.001)(0.1)$

$= 0.0024 + 0.0006 + 0.0001 = \boxed{0.0031}$

B. $P(A|D) = \dfrac{P(A \cap D)}{P(D)} = \dfrac{P(D|A)P(A)}{P(D)}$

$= \dfrac{0.0024}{0.0031} = \boxed{0.774}$

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2. A research team ran a pilot study of a mathematics curriculum. They gave 5 consenting participants (called *A*, *B*, *C*, *D*, and *E*) a test before the participants completed the curriculum (called the pre-test) and then gave the same participants a post-test after completion of the curriculum with the results in the table below. Test the null hypothesis that the expected improvement (post-test)-(pre-test) is equal to zero against the alternative that the expected improvement is not equal to zero. Use levels of significance 0.10, 0.05, and 0.01. This problem is worth 50 points.

Participant	Pre-test Score	Post-test Score
A	43	51
B	38	42
C	29	25
D	52	66
E	46	50

3. A research team took a random sample of 2 observations from a normally distributed random variable Y and observed that $\bar{y}_2 = 48.6$ and $s_Y^2 = 37.4$, where $\bar{y}_2$ was the average of the 2 observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$ (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 4 observations from a normally distributed random variable X and observed that $\bar{x}_4 = 89.1$ and $s_X^2 = 42.8$, where $\bar{x}_4$ was the average of the 4 observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$ (i.e., the divisor in the variance was $n-1$). Find the 99% confidence interval for $E(X) - E(Y)$. This problem is worth 50 points.

2.

PARTICIPANT	POST - PRE	POST - PRE - $\bar{d}_5$	$(\text{POST}-\text{PRE}-\bar{d}_5)^2$
A	51 - 43 = 8	2.8	7.84
B	42 - 38 = 4	-1.2	1.44
C	25 - 29 = -4	-9.2	84.64
D	66 - 52 = 14	8.8	77.44
E	50 - 46 = 4	-1.2	1.44
Σ	26	0	172.80

$\bar{d}_5 = \frac{26}{5} = 5.2 \qquad s_D^2 = \frac{\Sigma(d_i - \bar{d}_5)^2}{4} = \frac{172.80}{4} = 43.2 = (6.573)^2$

$t_4 = \frac{\bar{d}_5 - 0}{s_D/\sqrt{5}} = \frac{5.2 - 0}{6.573 / 2.236} = 1.769$

α	$t_{\alpha/14}$	
.10	2.132	ACCEPT
.05	2.776	ACCEPT
.01	4.604	ACCEPT

ACCEPT $H_0: E(D) = 0$ VS $H_1: E(D) \neq 0$ AT $\alpha = .10$ (AND $\alpha = .05$ AND $\alpha = .01$). THERE WAS NO SIGNIFICANT IMPROVEMENT ($\alpha = .10$) IN POST-TEST SCORE COMPARED TO PRE-TEST SCORE.

$$3C \quad S_p^2 = \frac{(2-1)\,37.4 + (4-1)\,42.8}{2+4-2} = \frac{165.8}{4} = 41.45 = (6.438)^2$$

$S_p = 6.438$ WITH 4 DF.

99% CI FOR $E(X) - E(Y)$ IS

$$\bar{x}_4 - \bar{y}_2 \pm t_{.005,4}\, S_p \sqrt{\tfrac{1}{2} + \tfrac{1}{4}}$$

$$89.1 - 48.6 \pm (4.604)(6.438)(0.8660)$$

$$40.5 \pm 25.67 = \boxed{14.8 \text{ TO } 66.2}$$

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2. A research team ran a pilot study of a mathematics curriculum. They gave 6 consenting participants (called *A*, *B*, *C*, *D*, *E*, and *F*) a test before the participants completed the curriculum (called the pre-test) and then gave the same participants a post-test after completion of the curriculum with the results in the table below. Find the 99% confidence interval for the expected improvement (post-test)-(pre-test). This problem is worth 50 points.

Participant	Pre-test Score	Post-test Score
A	43	51
B	38	42
C	29	25
D	52	66
E	46	44
F	48	62

3. A research team took a random sample of 7 observations from a normally distributed random variable *Y* and observed that $\bar{y}_7 = 48.6$ and $s_Y^2 = 27.4$, where $\bar{y}_7$ was the average of the 7 observations sampled from *Y* and s_Y^2 was the unbiased estimate of $var(Y)$ (i.e., the divisor in the variance was $n-1$). A second research team took a random sample of 5 observations from a normally distributed random variable *X* and observed that $\bar{x}_5 = 28.0$ and $s_X^2 = 32.7$, where $\bar{x}_5$ was the average of 5 observations sampled from *X* and s_X^2 was the unbiased estimate of $var(X)$ (i.e., the divisor in the variance was $n-1$). Test the null hypothesis $H_0: E(X) = E(Y)$ against the alternative $H_1: E(X) \neq E(Y)$ at the 0.10, 0.05, and 0.01 levels of significance using the pooled variance t-test. This problem is worth 50 points.

2D

PARTICIPANT	POST-PRE	POST-PRE-$\bar{d}_6$	$(\text{POST-PRE}-\bar{d}_6)^2$
A	51-43=8	2.333	5.443
B	42 -38=4	-1.667	2.779
C	25 - 29=-4	-9.667	93.451
D	66 - 52=14	8.333	69.439
E	44-46=-2	-7.667	58.783
F	62-48=14	8.333	69.439
	34	-0.002	299.333

$\bar{d}_6 = \frac{34}{6} = 5.667$ $\quad s_D^2 = \frac{\Sigma(d_x - \bar{d}_6)^2}{5} = \frac{299.333}{5} = 59.867$

$= (7.737)^2$

$t_5 = \frac{\bar{d}_6 - 0}{s_D/\sqrt{6}} = \frac{5.667-0}{(7.737)/2.449} = 1.794$

99% CI FOR E(POST) - E(PRE) IS

$5.667 \pm 4.032(7.737)\sqrt{\frac{1}{6}} = 5.667 \pm (4.032)(7.737)(0.4082)$

$= 5.667 \pm 12.735 = \boxed{-7.1 \text{ TO } 18.4}$

3D $S_p^2 = \frac{(7-1)(27.4) + (5-1)(32.7)}{5+7-2} = \frac{295.2}{10}$

$= 29.52 = (5.433)^2$

$t_{10} = \frac{\bar{x}_5 - \bar{y}_7 - 0}{S_p \sqrt{\frac{1}{5} + \frac{1}{7}}} = \frac{28.0 - 48.6 - 0}{5.433 \sqrt{\frac{1}{5} + \frac{1}{7}}}$

$= \frac{-20.6}{5.433(.5855)} = \frac{-20.6}{3.181} = -6.475.$

α	Z	t_{10}
.10	1.645	1.812
.05	1.960	2.228
.01	2.576	3.169.

REJECT $H_0: E(\bar{X}-\bar{Y}) = 0$ VS
$H_1: E(\bar{X}-\bar{Y}) \neq 0$ AT $\alpha = .01$
(AND $\alpha = .05$ AND $\alpha = .10$)

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4. In a clinical trial, 200 patients suffering from an illness will be randomly assigned to one of two groups so that 100 receive an experimental treatment and 100 receive the best available treatment. The random variable X is the response of a patient to the experimental medicine, and the random variable B is the response of a patient to the best currently available treatment. Under the null hypothesis, the random variables X and B are normally distributed with $\sigma_X = \sigma_B = 250$. Under the alternative hypothesis, the random variables X and B are normally distributed with $\sigma_X = 300$ and $\sigma_B = 250$ The null hypothesis to be tested is that $E(X) - E(B) = 0$ against the alternative that $E(X) - E(B) > 0$ at the 0.005 level of significance. What is the probability of a Type II error for the test of the null hypothesis when $E(X) - E(B) = 100$? This problem is worth 50 points.

NULL DIST OF $\overline{X}_{100} - \overline{B}_{100}$ IS $N\left(0, \frac{(250)^2}{100} + \frac{(250)^2}{100}\right)$

$= N(0, 1250 = (35.36)^2)$

ALT DIST OF $\overline{X}_{100} - \overline{B}_{100}$ IS $N\left(100, \frac{(300)^2}{100} + \frac{(250)^2}{100}\right)$

$= N(100, 1525 = (39.05)^2)$.

REJECT H_0 WHEN $\overline{X}_{100} - \overline{B}_{100} > 0 + 2.576(35.36) = 91.09$.

$\beta = Pr_1\{\text{ACCEPT } H_0\} = Pr_1\{\overline{X}_{100} - \overline{B}_{100} < 91.09\}$

$= Pr\left\{\frac{\overline{X}_{100} - \overline{B}_{100} - E_1(\overline{X}_{100} - \overline{B}_{100})}{\sigma_1(\overline{X}_{100} - \overline{B}_{100})} < \frac{91.09 - 100}{39.05} = \frac{-8.91}{39.05} = -0.23\right\}$

$= \Phi(-0.23) = 0.4090$

$\boxed{\beta = 0.4090}$

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4. In a clinical trial, 500 patients suffering from an illness will be randomly assigned to one of two groups so that 250 receive an experimental treatment and 250 receive the best available treatment. The random variable X is the response of a patient to the experimental medicine, and the random variable B is the response of a patient to the best currently available treatment. Under the null hypothesis, the random variables X and B are normally distributed with $\sigma_X = \sigma_B = 600$. Under the alternative hypothesis, the random variables X and B are normally distributed with $\sigma_X = 800$ and $\sigma_B = 600$ The null hypothesis to be tested is that $E(X) - E(B) = 0$ against the alternative that $E(X) - E(B) > 0$ at the 0.025 level of significance. What is the probability of a Type II error for the test of the null hypothesis when $E(X) - E(B) = 150$? This problem is worth 50 points.

THE NULL DISTRIBUTION OF $\bar{X}_{250} - \bar{B}_{250}$

IS $N\left(0, \frac{(600)^2}{250} + \frac{(600)^2}{250}\right) = N(0, 2880 = (53.67)^2)$.

THE ALTERNATIVE DISTRIBUTION OF

$\bar{X}_{250} - \bar{B}_{250}$ IS $N\left(150, \frac{(800)^2}{250} + \frac{(600)^2}{250}\right)$

$= N(150, 4000 = (63.25)^2)$.

REJECT H_0 WHEN $\bar{X}_{250} - \bar{B}_{250} > 0 + 1.960(53.67)$.

REJECT H_0 WHEN $\bar{X}_{250} - \bar{B}_{250} > 105.19$.

$\beta = \Pr_1\{\text{ACCEPT } H_0\} = \Pr_1\{\bar{X}_{250} - \bar{B}_{250} < 105.19\}$

$= \Pr\left\{\frac{\bar{X}_{250} - \bar{B}_{250} - E_1(\bar{X}_{250} - \bar{B}_{250})}{\sigma_1(\bar{X}_{250} - \bar{B}_{250})} < \frac{105.19 - 150}{63.25}\right\}$

$= \Pr\left\{Z < \frac{-44.81}{63.25} = -0.71\right\} = \Phi(-0.71)$

$= 0.2389$.

$\boxed{\beta = 0.2389}$

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5. In a clinical trial, $2J$ patients suffering from an illness will be randomly assigned to one of two groups so that J will receive an experimental treatment and J will receive the best available treatment. The random variable X is the response of a patient to the experimental medicine, and the random variable B is the response of a patient to the best currently available treatment. Under the null hypothesis, both X and B are normally distributed with $\sigma_X = \sigma_B = 1{,}000$. The null hypothesis to be tested is that $H_0: E(X) - E(B) = 0$ against the alternative hypothesis $H_1: E(X) - E(B) > 0$ at the 0.025 level of significance.
    a. What is the number J in each group that would have to be taken so that the probability of a Type II error for the test of the null hypothesis specified in the common section is 0.01 when $E(X) - E(B) = 200$ and $\sigma_X = 1{,}200$ and $\sigma_B = 1{,}000$. This part is worth 45 points.
    b. What is the total number of subjects for this clinical trial? This part is worth 5 points.

a) $\sqrt{J} \geq \dfrac{|z_\alpha|\sqrt{\sigma_{0X}^2 + \sigma_{0B}^2} + |z_\beta|\sqrt{\sigma_{1X}^2 + \sigma_{1B}^2}}{|E_1 - E_0|}$

$\sqrt{J} \geq \dfrac{1.960\sqrt{2(1000)^2} + 2.326\sqrt{(1200)^2 + (1000)^2}}{|200 - 0|}$

$\sqrt{J} \geq \dfrac{2771.86 + 3633.33}{200} = \dfrac{6405.19}{200} = 32.026$

$J \geq (32.026)^2$; $\boxed{J > 1026.}$

b) NEED 2(1026) PARTICIPANTS = $\boxed{2052}$ TOTAL PARTICIPANTS!

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5 In a clinical trial, *2J* patients suffering from an illness will be randomly assigned to one of two groups so that *J* will receive an experimental treatment and *J* will receive the best available treatment. The random variable *X* is the response of a patient to the experimental medicine, and the random variable *B* is the response of a patient to the best currently available treatment. Under the null hypothesis, both *X* and *B* are normally distributed with $\sigma_X = \sigma_B = 200$. The null hypothesis to be tested is that $H_0: E(X) - E(B) = 0$ against the alternative hypothesis $H_1: E(X) - E(B) > 0$ at the 0.005 level of significance.

a. What is the number *J* in each group that would have to be taken so that the probability of a Type II error for the test of the null hypothesis specified in the common section is 0.01 when $E(X) - E(B) = 300$ and $\sigma_X = 300$ and $\sigma_B = 200$. This part is worth 45 points.
b. What is the total number of subjects for this clinical trial? This part is worth 5 points.

a) $$\sqrt{J} \geq \frac{|z_\alpha| \sqrt{\sigma_{0X}^2 + \sigma_{0B}^2} + |z_\beta| \sqrt{\sigma_{1X}^2 + \sigma_{1B}^2}}{|E_1 - E_0|}$$

$$\sqrt{J} \geq \frac{2.576 \sqrt{(200)^2 + (200)^2} + (2.326) \sqrt{(300)^2 + (200)^2}}{300}$$

$$\sqrt{J} \geq \frac{(2.576)(282.84) + 2.326(360.56)}{300}$$

$$\sqrt{J} \geq \frac{728.60 + 838.65}{300} = \frac{1567.25}{300} = 5.224$$

$$J \geq (5.224)^2 = 27.29.$$

USE $\boxed{J = 28}$ OR MORE PARTICIPANTS

b) USE $2 \times 28 = \boxed{56}$ TOTAL PARTICIPANTS

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6. A research team collects a random sample of n observations: $y_1, y_2, \cdots, y_n$. The sample average is $\bar{y}_n = \frac{\sum_{i=1}^{n} y_i}{n}$. Prove or disprove: $\sum_{i=1}^{n}(y_i - \bar{y}_n)^2 = \sum_{i=1}^{n} y_i^2 - n\bar{y}_n^2$. This problem is worth 60 points.

End of the Examination

THE RELATION IS TRUE. NOTE $\sum_{i=1}^{n} y_i = (n\bar{y}_n)$.

$$\sum_{i=1}^{n} (y_i - \bar{y}_n)^2 = \sum_{i=1}^{n} (y_i^2 - 2\bar{y}_n y_i + \bar{y}_n^2)$$

$$= \sum_{i=1}^{n} y_i^2 - \sum_{i=1}^{n} 2\bar{y}_n y_i + \sum_{i=1}^{n} (\bar{y}_n)^2$$

$$= \sum_{i=1}^{n} y_i^2 - 2\bar{y}_n \sum_{i=1}^{n} y_i + n(\bar{y}_n)^2$$

$$= \sum y_i^2 - 2\bar{y}_n (n\bar{y}_n) + n(\bar{y}_n)^2$$

$$= \sum y_i^2 - 2n(\bar{y}_n)^2 + n(\bar{y}_n)^2$$

$$= \sum y_i^2 - n(\bar{y}_n)^2$$