

Chapter 5 One sample t test and confidence interval

A research team took a sample of 5 observations from the random variable Y , which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_5 = 56.2$, where \bar{y}_5 is the average of the five sampled observations and $s^2 = 53.6$ is the observed value of the unbiased estimate of σ^2 , based on the sample values.

1. Test the null hypothesis $H_0: E(Y) = 45$ against $H_1: E(Y) \neq 45$. Use levels of significance 0.10, 0.05, and 0.01.
2. Find the 99% confidence interval for $E(Y)$.

$$\bar{Y}_5 \sim N(\mu, \frac{\sigma^2}{5}); \quad H_0: E(Y) = \mu_0.$$

$$H_1: E(Y) \neq \mu_0.$$

$$Z = \frac{\bar{Y}_5 - \mu_0}{\sigma / \sqrt{5}}; \quad \text{IF } \sigma \text{ KNOWN}$$

REJECT H_0 AT $\alpha = 0.10$ IF

$$|Z| > 1.645$$

AT $\alpha = 0.05$

$$\text{IF } |Z| > 1.960$$

AT $\alpha = 0.01$

$$\text{IF } |Z| > 2.576.$$

σ NOT KNOWN: USE S.

2.

$$T_4 = \frac{\bar{Y}_5 - \mu_0}{S/\sqrt{5}} \text{ IS } T_4 \text{ UNDER } H_0.$$

REJECT H_0 IF

$$|T_4| > t_{1.645, 4} \quad \text{FOR } \alpha = 0.10$$

$$|T_4| > t_{1.960, 4} \quad \text{FOR } \alpha = 0.05$$

$$|T_4| > t_{2.576, 4} \quad \text{FOR } \alpha = 0.01.$$

T STRETCH
TWO SIDED TEST.

α	Z_α	$t_{\alpha, 4}$
0.10	1.645	2.132
0.05	1.960	2.776
0.01	2.576	4.604.

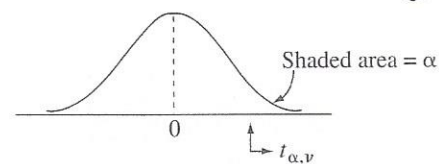


TABLE 2

Percentage points of Student's t distribution

df	Right-Tail Probability (α)								
	.40	.25	.10	.05	.025	.01	.005	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	.289	.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	.277	.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	.255	.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	.255	.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	.255	.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	.254	.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
inf.	.253	.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Source: Computed by M. Longnecker using the R function $qt(1 - \alpha, df)$.For 2-tailed tests and C.I.s use value in column headed by $\alpha/2$.

4.

CALCULATIONS

$$t_4 = \frac{56.2 - 45}{\sqrt{53.6} / \sqrt{5}} = \frac{11.2}{7.321 / 2.236}$$

$$= \frac{11.2}{3.274} = 3.421.$$

↑
STANDARD ERROR
OF \bar{y}_5

DECISIONS. $H_0: E(Y) = 45$ $H_1: E(Y) \neq 45$

$3.421 > 2.132 \Rightarrow$ REJECT H_0
AT $\alpha = .10$

$3.421 > 2.776 \Rightarrow$ REJECT H_0
AT $\alpha = .05$

$3.421 < 4.604 \Rightarrow$ ACCEPT H_0
AT $\alpha = .01$

99% CONFIDENCE INTERVAL ⁵
FOR $E(Y)$

STANDARD ERROR
OF \bar{y} 5.

$$56.2 \pm 4.604 (3.274)$$

↑
55

↑
4 DF STRETCH
OF 2.576

$$56.2 \pm 15.07$$

THE 99% CI FOR $E(Y)$ IS
41.13 TO 71.27