AMS 315, Fall Semester 2021

Second examination grading criteria

General grading of examination 2

1. Take off 2 points for minor computational errors and 5 points for more serious errors. An answer that has a substantive error in a calculation should have a 20-point deduction. For example, an incorrect number of degrees of freedom.
2. A decision to accept or reject a null hypothesis that is inconsistent with the calculations of the problem should have a 35-point deduction. The objective of the course is to train each student to make consistent decisions. Computations that are incorrect should be penalized as discussed in point 2.

Grading of specific problems

1. Confidence interval for σ^2: -20 for degree of freedom error; -10 left endpoint inconsistent with reported degrees of freedom; -10 right endpoint inconsistent with reported degrees of freedom; -15 square variance in computation of endpoints of CI; -50 one-sample confidence interval for $E(Y)$.
2. C: Confidence interval for σ_X^2/σ_Y^2 : -20 for each degree of freedom error; -10 left endpoint inconsistent with reported degrees of freedom; -10 right endpoint inconsistent with reported degrees of freedom; -10 correct confidence interval for σ_Y^2/σ_X^2; -50 wrong procedure.
D: Confidence interval for ρ: +15 for correct F(0.48). An additional +10 for correct 99% CI for $F(\rho)$. -25 for incorrect transformations of the endpoints. -50 for a wrong procedure.
3. Regression Problem: A: -15 for incorrect dependent variable—only deduct once for this error; that is no further deductions for consistent work; -15 each incorrect degree of freedom; -10 for each incorrect sum of squares—do not deduct for consistent errors; -20 inconsistent decision about hypothesis; B: -10 incorrect slope; -15 incorrect standard error of slope; -10 wrong $|z_\alpha|$; C: -20 for confidence interval reported in prediction interval problem, and vice versa.
4. Sample size for a correlation study problem: +15 for correct $F(\rho_1)$; -20 incorrect $|z_\alpha|$; -20 incorrect $|z_\beta|$; -30 forget to square answer; -5 forget to add 3.
5. $vcv(MY)$: Give +15 points for correct M; -10 omitting σ^2; -20 for each incorrect variance; -10 for each incorrect covariance (the two covariances should be equal).
6. Finding an OLS estimate. Point values are given for each point. Deduct full points for creative algebra.

AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA

You are on your honor not to use any other assistance during this examination.
Please affirm in the space before your answer to question 1 that you have not sought assistance from other students or other live sources and that you have not given assistance to other students, followed by your dated signature.

1. A research team took a sample of 8 observations from the random variable Y, which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_8 = 98.7$, where $\bar{y}_8$ was the average of the 8 sampled observations, and $s^2 = 253.1$ was the observed value of the unbiased estimate of σ^2, based on the sample values. Find the 99% confidence interval for σ^2. This problem is worth 50 points.

$DF = n - 1 = 7.$

$$Pr\{0.9893 \le \chi^2_7 \le 20.28\} = 0.99$$

$$Pr\left\{0.9893 \le \frac{\sum (Y_i - \bar{Y}_8)^2}{\sigma^2} \le 20.28\right\} = 0.99$$

$$Pr\left\{\frac{0.9893}{7S^2} \le \frac{1}{\sigma^2} \le \frac{20.28}{7S^2}\right\} = 0.99$$

$$Pr\left\{\frac{7S^2}{20.28} \le \sigma^2 \le \frac{7S^2}{0.9893}\right\} = 0.99.$$

HENCE 99% CI FOR σ^2 IS $\frac{7(253.1)}{20.28} = 87.36$

TO $\frac{7S^2}{0.9893} = 1790.86.$

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

You are on your honor not to use any other assistance during this examination.
Please affirm in the space before your answer to question 1 that you have not sought assistance from other students or other live sources and that you have not given assistance to other students, followed by your dated signature.

1. A research team took a sample of 12 observations from the random variable Y, which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_{12} = 158.9$, where $\bar{y}_{12}$ was the average of the 12 sampled observations, and $s^2 = 875.6$ was the observed value of the unbiased estimate of σ^2, based on the sample values. Find the 95% confidence interval for σ^2. This problem is worth 50 points.

DF = 12 − 1 = 11.

$$Pr\{3.816 \le \chi^2_{11} \le 21.92\} = 0.95.$$

$$Pr\left\{3.816 \le \frac{\sum(Y_i - \bar{Y}_{12})^2}{\sigma^2} \le 21.92\right\} = 0.95.$$

$$Pr\left\{\frac{3.816}{11S^2} \le \frac{1}{\sigma^2} \le \frac{21.92}{11S^2}\right\} = 0.95$$

$$Pr\left\{\frac{11S^2}{21.92} \le \sigma^2 \le \frac{11S^2}{3.816}\right\} = 0.95.$$

HENCE 95% CI FOR σ^2 IS $\frac{11(875.6)}{21.92} = 439.4$

TO $\frac{11(875.6)}{3.816} = 2524.0$

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

2. A research team took a random sample of 4 observations from a normally distributed random variable Y and observed that $\bar{y}_4 = 231.2$ and $s_Y^2 = 1{,}138.7$, where $\bar{y}_4$ was the average of the four observations sampled from Y and s_Y^2 was the unbiased estimate of $var(Y)$. A second research team took a random sample of 5 observations from a normally distributed random variable X and observed that $\bar{x}_5 = 491.8$ and $s_X^2 = 2{,}891.3$, where $\bar{x}_5$ was the average of the five observations sampled from X and s_X^2 was the unbiased estimate of $var(X)$. Find the 99% confidence interval for $\frac{var(X)}{var(Y)}$. This problem is worth 50 points.

$$TS = \frac{S_Y^2}{S_X^2} \sim F(3,4)$$

$$Pr\left\{ \frac{1}{46.19} \le \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \le 24.26 \right\} = 0.99$$

$$Pr\left\{ \frac{1}{46.19} \le \left(\frac{S_Y^2}{S_X^2}\right)\left(\frac{\sigma_X^2}{\sigma_Y^2}\right) \le 24.26 \right\} = 0.99.$$

$$Pr\left\{ \frac{1}{46.19}\left(\frac{S_X^2}{S_Y^2}\right) \le \frac{\sigma_X^2}{\sigma_Y^2} \le 24.26\left(\frac{S_X^2}{S_Y^2}\right) \right\} = 0.99$$

HENCE THE 99% CI FOR $\frac{\sigma_X^2}{\sigma_Y^2}$ IS

$$\frac{1}{46.19}\left(\frac{2891.3}{1138.7}\right) = 0.055 \text{ TO } 24.26\left(\frac{S_X^2}{S_Y^2}\right) = 61.60.$$

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

2. A research team took a random sample of 348 observations from a bivariate normally distributed random variable (Y, X), with population correlation coefficient ρ. They observed $\bar{y}_{348} = 281.1$, with an observed standard deviation of 21.7 (the divisor in the underlying variance calculation was $n-1$). They observed $\bar{x}_{348} = 78.2$, with an observed standard deviation of 41.5 (the divisor in the underlying variance calculation was also $n-1$). The Pearson product moment correlation coefficient between the two variables was 0.48. Find the 99% confidence interval for ρ. This problem is worth 50 points.

$$F(0.48) = \frac{1}{2}\ln\left(\frac{1+0.48}{1-0.48}\right) = \frac{1}{2}\ln\left(\frac{1.48}{0.52}\right)$$

$$= \frac{1}{2}\ln(2.8462) = \frac{1.0460}{2} = 0.5230$$

99% CI FOR $F(\rho)$ IS $0.5230 \pm 2.576\sqrt{\frac{1}{348-3}}$

$$= 0.5230 \pm 2.576(0.0538) = 0.5230 \pm 0.1387$$

$= 0.3843$ TO 0.6617

LEFT ENDPOINT: $\text{EXP}(2(.3843)) = \text{EXP}(0.7686) = 2.1567.$

$$r_L = \frac{2.1567 - 1}{2.1567 + 1} = \frac{1.1567}{3.1567} = 0.366.$$

RIGHT ENDPOINT: $\text{EXP}(2(0.6617)) = \text{EXP}(1.3294) = 3.7788$

$$r_R = \frac{3.7788 - 1}{3.7788 + 1} = \frac{2.7788}{4.7788} = 0.581.$$

THE 99% CI FOR ρ IS (0.366 TO 0.581).

EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE

3. A research team studied the response of a participant to a dosage of medication. Dosages were randomly assigned to participants. The research team then measured each participant's response for $n = 658$ participants. The average response was 548.3, with an observed standard deviation of 128.4 (the divisor in the underlying variance calculation was $n-1$). The average dosage was 74.1, with an observed standard deviation of 29.5 (the divisor in the underlying variance calculation was also $n-1$). The correlation coefficient between the two variables was 0.76. The team sought to estimate the regression of participant response on the dosage of medication.
   a. Complete the analysis of variance table for the regression of participant response on the dosage of medicine given the participant. Test the null hypothesis that the slope of this regression is zero at levels of significance 0.10, 0.05, and 0.01. This part is worth 30 points.
   b. Find the estimated regression equation of participant response on dosage. Find the 95% confidence interval for the slope in this equation. 20 points.
   c. Use the least-squares equation to estimate the response for participants whose assigned dosage was 125.0 What is the 95% confidence interval for the expected value of this response? This part is worth 20 points.

A. $TSS = (n-1)\, SD_{DV}^2 = (657)(128.4)^2 = 10{,}831{,}669.92.$

$\Sigma(x_i - \bar{x}_n)^2 = (n-1)\, SD_{IV}^2 = (657)(29.5)^2 = 571{,}754.25.$

$REG\ SS = r^2\, TSS = (0.76)^2\, TSS = 6{,}256{,}372.546$ ON 1 DF.

$SSE = (1-r^2)\,TSS = (1-0.76^2)\,TSS = 0.4224\,TSS$

$= 4{,}575{,}297.374. \quad MSE = \frac{SSE}{n-2} = \frac{SSE}{656} = 6974.54.$

$F = \frac{MSREG}{MSERR} = \frac{6{,}256{,}372.546/1}{6974.54} = 897.03.$

α	$F(1,656)$	$F(1,\infty)$
.10	2.713 R	2.71
.05	3.856 R	3.84.
.01	6.674. R	6.64

ANOVA TABLE

SOURCE	DF	SS	MS	F.
REGRESSION	1	6,256,372.55	6,256,372.55	897.03
ERROR	656	4,575,297.37	6,974.54	
TOTAL	657	10,831,669.92		

REJECT $H_0: \beta_1 = 0$ VS $H_1\ \beta_1 \neq 0$ AT $\alpha = .01$ (AND .05, AND .10).

E3B

$$\hat{\beta}_1 = r \frac{SD_{DV}}{SD_{IV}} = 0.76 \frac{128.4}{29.5} = 3.308$$

$$\hat{y}(x) = \bar{y}_n + \hat{\beta}_1(x - \bar{x}_n) = 548.3 + 3.308(x - 74.1)$$

$$= [548.3 - 3.308(74.1)] + 3.308x$$

$$= 303.2 + 3.308x$$

95% CI FOR β_1 IS $3.308 \pm 1.964 \sqrt{\frac{MSE}{\Sigma(x_i - \bar{x}_n)^2}}$

$$= 3.308 \pm 1.964 \sqrt{\frac{6974.54}{571{,}754.25}} = 3.308 \pm 1.964\,(0.110)$$

$$= 3.308 \pm 0.216$$

$$= 3.09 \text{ TO } 3.52.$$

E3C

$$\hat{y}(125) = 548.3 + 3.308(125 - 74.1)$$

$$= 548.3 + 3.308(50.9) = 716.7$$

$$SE(\hat{y}(125)) = \sqrt{MSE\left(\frac{1}{658} + \frac{(50.9)^2}{571{,}754.25}\right)}$$

$$= \sqrt{(6{,}974.54)(0.00152 + 0.00453)}$$

$$= \sqrt{42.20} = 6.496.$$

95% CI FOR $\beta_0 + 125\beta_1$ IS

$$716.7 \pm 1.964(6.496) = 716.7 \pm 12.8$$

$$= 703.9 \text{ TO } 729.5.$$

FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF

3. A research team studied the response of a participant to a dosage of medication. Dosages were randomly assigned to participants. The research team then measured each participant's response for $n = 415$ participants. The average response was 464.5, with an observed standard deviation of 147.4 (the divisor in the underlying variance calculation was $n-1$). The average dosage was 213.6, with an observed standard deviation of 25.7 (the divisor in the underlying variance calculation was also $n-1$). The correlation coefficient between the two variables was 0.45. The team sought to estimate the regression of participant response on the dosage of medication.

a. Complete the analysis of variance table for the regression of participant response on the dosage of medicine given the participant. Test the null hypothesis that the slope of this regression is zero at levels of significance 0.10, 0.05, and 0.01. This part is worth 30 points.

b. Find the estimated regression equation of participant response on dosage. Find the 99% confidence interval for the slope in this equation. 20 points.

c. Use the least-squares equation to estimate the response for a participant whose dosage was 275.0 What is the 99% prediction interval for this participant's response? This part is worth 20 points.

A. $TSS = (n-1)\, SD_{DV}^2 = 414\,(147.4)^2 = 8{,}994{,}878.64.$

$\Sigma (x_i - \bar{x}_n)^2 = (n-1)\, SD_{IV}^2 = 414\,(25.7)^2 = 273{,}442.86$

$REG\,SS = r^2\, TSS = (0.45)^2\, TSS = 1{,}821{,}462.93$

$SSE = (1-r^2)\,TSS = (0.7975)\, TSS = 7{,}173{,}415.72$

$MSE = \dfrac{SSE}{n-2} = \dfrac{SSE}{413} = 17{,}369.05.$

$F = \dfrac{MSREG}{MSE} = \dfrac{1{,}821{,}462.93/1}{17{,}369.05} = 104.87.$

α	$F(1,413)$		$F(1,\infty)$.
.10	2.718	R	2.71
.05	3.864	R	3.84
.01	6.697	R	6.64

ANOVA TABLE

SOURCE	DF	SS	MS	F
REGRESSION	1	1,821,462.93	1,821,462.93	104.87
ERROR	413	7,173,415.72	17,369.05	
TOTAL	414	8,994,878.64		

REJECT $H_0: \beta_1 = 0$ VS $H_1: \beta_1 \neq 0$ AT $\alpha = .01$ (AND .05 AND .10).

F3B. $\hat{\beta}_1 = r\frac{SD_{DV}}{SD_{IV}} = 0.45\frac{147.4}{25.7} = 2.58.$

$\hat{Y}(x) = \bar{y}_m + \hat{\beta}_1(x - \bar{x}_m) = 464.5 + 2.58(x - 213.6)$

$= [465 - 2.58(213.6)] + 2.58x$

$= (-86.6) + 2.58x$

99% CI FOR β_1: $\hat{\beta}_1 \pm 2.588\sqrt{\frac{MSE}{\Sigma(x_i - \bar{x}_m)^2}}$

$= 2.58 \pm 2.588\sqrt{\frac{17,369.05}{273,442.86}}$

$= 2.58 \pm 2.588(0.252) = 2.58 \pm 0.65$

$= 1.93$ TO 3.23.

F3C: $\hat{Y}(275) = 464.5 + 2.58(275 - 213.6)$

$= 464.5 + 2.58(61.4) = 464.5 + 158.4 = 622.9.$

$PSE = \sqrt{MSE\left(1 + \frac{1}{m} + \frac{(x - \bar{x}_m)^2}{\Sigma(x_i - \bar{x}_m)^2}\right)}$

$= \sqrt{17,369.05\left(1 + 0.00241 + \frac{(61.4)^2}{273,442.86}\right)}$

$= \sqrt{17,369.05(1 + 0.00241 + 0.01379)}$

$= \sqrt{17,369.05(1.0162)} = 132.9.$

99% PI FOR $Y_F(275)$ IS. $622.9 \mp 2.588(132.9)$

$= 622.9 \pm 343.8 = 279.1$ TO 966.7.

GGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGGG

4. A research team wishes to test the null hypothesis $H_0\colon\ \rho = 0$ at $\alpha = 0.025$ against the alternative $H_1\colon\ \rho > 0$ using Fisher's transformation of the Pearson product moment correlation coefficient as the test statistic. They have asked their consulting statistician for a sample size n such that $\beta = 0.05$ when $\rho = 0.30$. What is this value? This problem is worth 50 points.

$$F(0.30) = \frac{1}{2}\ln\left(\frac{1+.3}{1-.3}\right) = \frac{1}{2}\ln\left(\frac{1.3}{0.7}\right) = \frac{1}{2}(0.6190) = 0.3095.$$

$$\sqrt{n-3} \geq \frac{1.960(1) + 1.645(1)}{|.3095 - 0|} = \frac{3.605}{0.3095} = 11.65$$

$$n - 3 \geq (11.65)^2 = 135.7$$

$$n \geq 139.$$

HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH

4. A research team wishes to test the null hypothesis H_0: $\rho = 0$ at $\alpha = 0.005$ against the alternative H_1: $\rho > 0$ using Fisher's transformation of the Pearson product moment correlation coefficient as the test statistic. They have asked their consulting statistician for a sample size n such that $\beta = 0.01$ when $\rho = 0.15$. What is this value? This problem is worth 50 points.

$$F(0.15) = \frac{1}{2} \ln\left(\frac{1.15}{0.85}\right) = \frac{0.30228}{2} = 0.151.$$

$$\sqrt{n-3} \geq \frac{2.576(1) + 2.326(1)}{0.151 - 0} = \frac{4.902}{0.151} = 32.46$$

$$n - 3 \geq 1053.9$$

$$\boxed{n \geq 1057}$$

JJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJJ

5. The correlation matrix of the random variables $(Y_1, Y_2, Y_3, Y_4)^T$ is $\begin{pmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & \tau & 1 \end{pmatrix}$, $0 < \rho, \tau < 1$, and each random variable has variance σ^2. Let $W_1 = -Y_1 - Y_2 + Y_3 + Y_4$, and let $W_2 = Y_1 + 2Y_2 + 2Y_3 + Y_4$. Find the variance covariance matrix of $(W_1, W_2)^T$. This problem is worth 50 points.

$$M = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

$$\text{vcv}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix} \sigma^2 \begin{pmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & \tau & 1 \end{pmatrix} M^T$$

$$= \sigma^2 \begin{bmatrix} -1-\rho & -1-\rho & 1+\tau & 1+\tau \\ 1+2\rho & 2+\rho & 2+\tau & 1+2\tau \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -1 & 2 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 4+2\rho+2\tau & 0+3\tau-3\rho \\ 0+3\tau-3\rho & 10+4\rho+4\tau \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 4+2\rho+2\tau & 3\tau-3\rho \\ 3\tau-3\rho & 10+4\tau+4\rho \end{bmatrix}$$

KKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKKK

5. The correlation matrix of the random variables $(Y_1, Y_2, Y_3, Y_4)^T$ is $\begin{pmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & \tau & 1 \end{pmatrix}$, $0 < \rho, \tau < 1$, and each random variable has variance σ^2. Let $W_1 = Y_1 + Y_2 + Y_3 + Y_4$, and let $W_2 = 4Y_1 + 3Y_2 + 2Y_3 + Y_4$. Find the variance covariance matrix of $(W_1, W_2)^T$. This problem is worth 50 points.

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$$\mathrm{vcv}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 1 \end{bmatrix} \sigma^2 \begin{bmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & T & 1 \end{bmatrix} M^T$$

$$= \sigma^2 \begin{bmatrix} 1+\rho & 1+\rho & 1+T & 1+T \\ 4+3\rho & 3+4\rho & 2+T & 1+2T \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 4 + 2\rho + 2T & 10 + 7\rho + 3T \\ 10 + 7\rho + 3T & 30 + 24\rho + 4T \end{bmatrix}$$

LLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL

6. A research team collected data $(y_i, x_i), i = 1, \ldots, n$, for n participants. Here y_i was a sample of size 1 from Y_i, which was normally distributed with $E(Y_i) = \beta x_i$ and variance σ^2. The n Y observations are independent. They seek to fit the model $E(Y_i) = \beta x_i$, using Ordinary Least Squares by minimizing the function
$$SS(b) = \sum_{i=1}^{n} w_i (y_i - b x_i)^2.$$
   a. Find $\frac{\partial}{\partial b}[SS(b)]$. This part is worth 10 points.
   b. Specify the normal equation whose solution is $\hat{\beta}$. This part is worth 10 points.
   c. What is the ordinary least squares estimate $\hat{\beta}$? This part is worth 10 points.
   d. Find $E(\hat{\beta})$ and $var(\hat{\beta})$. This part is worth 20 points.

End of Examination

A. $\frac{\partial}{\partial b}[SS(b)] = \sum w_i \frac{\partial}{\partial b}[(y_i - b x_i)^2]$

$= \sum -2 w_i x_i (y_i - b x_i).$

B. $\sum w_i x_i (y_i - \hat{\beta} x_i) = 0$

C. $\sum w_i x_i y_i = (\sum w_i x_i^2)\hat{\beta}$

$\hat{\beta} = \frac{\sum w_i x_i y_i}{\sum w_i x_i^2}.$

D. $E(\hat{\beta}) = \frac{\sum w_i x_i E(Y_i)}{\sum w_i x_i^2} = \frac{\sum w_i x_i \beta x_i}{\sum w_i x_i^2} = \beta.$

$VAR(\hat{\beta}) = VAR\left(\sum \left(\frac{w_i x_i}{\sum w_i x_i^2}\right) Y_i\right)$

$= \sum \frac{(w_i x_i)^2}{(\sum w_i x_i^2)^2} \cdot \sigma^2 = \frac{\sum (w_i^2 x_i^2)}{(\sum w_i x_i^2)^2} \cdot \sigma^2.$