5 Chi-square Confidence Interval for Variance

A research team took a sample of 7 observations from the random variable Y, which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_7 = 93.4$, where $\bar{y}_7$ was the average of the 7 sampled observations, and $s^2 = 47.5$ was the observed value of the unbiased estimate of σ^2, based on the sample values. Find the 99% confidence interval for σ^2.

Alternate problem

A research team took a sample of 7 observations from the random variable Y, which had a normal distribution $N(\mu, \sigma^2)$. They observed $\bar{y}_7 = 93.4$, where $\bar{y}_7$ was the average of the 7 sampled observations, and $s^2 = 47.5$ was the observed value of the unbiased estimate of σ^2, based on the sample values. Test the null hypothesis that $\sigma^2 = 200$ against the alternative that $\sigma^2 > 200$. Use levels of significance 0.10, 0.05, and 0.01.

99% CI FOR σ^2

S^2 HAS $n-1=6$ DF.

THAT IS, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_6$

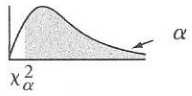
FROM TABLE 7

$Pr\{\chi^2_6 < 0.6757\} = .005$

$Pr\{\chi^2_6 > 18.55\} = .005.$

$Pr\{0.6757 < \chi^2_6 < 18.55\} = 0.99.$

TABLE 7
Percentage points of the chi-square distribution



	Right-Tail Probability (α)					
df	**.999**	**.995**	**.99**	**.975**	**.95**	**.90**
1	.000002	.000039	.000157	.000982	.003932	.01579
2	.002001	.01003	.02010	.05064	.1026	.2107
3	.02430	.07172	.1148	.2158	.3518	.5844
4	.09080	.2070	.2971	.4844	.7107	1.064
5	.2102	.4117	.5543	.8312	1.145	1.610
6	.3811	.6757	.8721	1.237	1.635	2.204
7	.5985	.9893	1.239	1.690	2.167	2.833
8	.8571	1.344	1.646	2.180	2.733	3.490
9	1.152	1.735	2.088	2.700	3.325	4.168
10	1.479	2.156	2.558	3.247	3.940	4.865
11	1.834	2.603	3.053	3.816	4.575	5.578
12	2.214	3.074	3.571	4.404	5.226	6.304
13	2.617	3.565	4.107	5.009	5.892	7.042
14	3.041	4.075	4.660	5.629	6.571	7.790
15	3.483	4.601	5.229	6.262	7.261	8.547
16	3.942	5.142	5.812	6.908	7.962	9.312
17	4.416	5.697	6.408	7.564	8.672	10.09
18	4.905	6.265	7.015	8.231	9.390	10.86
19	5.407	6.844	7.633	8.907	10.12	11.65
20	5.921	7.434	8.260	9.591	10.85	12.44
21	6.447	8.034	8.897	10.28	11.59	13.24
22	6.983	8.643	9.542	10.98	12.34	14.04
23	7.529	9.260	10.20	11.69	13.09	14.85
24	8.085	9.886	10.86	12.40	13.85	15.66
25	8.649	10.52	11.52	13.12	14.61	16.47
26	9.222	11.16	12.20	13.84	15.38	17.29
27	9.803	11.81	12.88	14.57	16.15	18.11
28	10.39	12.46	13.56	15.31	16.93	18.94
29	10.99	13.12	14.26	16.05	17.71	19.77
30	11.59	13.79	14.95	16.79	18.49	20.60
40	17.92	20.71	22.16	24.43	26.51	29.05
50	24.67	27.99	29.71	32.36	34.76	37.69
60	31.74	35.53	37.48	40.48	43.19	46.46
70	39.04	43.28	45.44	48.76	51.74	55.33
80	46.52	51.17	53.54	57.15	60.39	64.28
90	54.16	59.20	61.75	65.65	69.13	73.29
100	61.92	67.33	70.06	74.22	77.93	82.36
120	77.76	83.85	86.92	91.57	95.70	100.62
240	177.95	187.32	191.99	198.98	205.14	212.39

Source: Computed by M. Longnecker using the R function qchisq($1 - \alpha$, df).

For level α two-tailed tests and $100(1 - \alpha)\%$ C.I.s use value in columns headed by the numbers obtained by computing $1 - \frac{\alpha}{2}$ and $\frac{\alpha}{2}$.

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TABLE 7
(continued)

Right-Tail Probability (α)						
.10	.05	.025	.01	.005	.001	df
2.706	3.841	5.024	6.635	7.879	10.83	1
4.605	5.991	7.378	9.210	10.60	13.82	2
6.251	7.815	9.348	11.34	12.84	16.27	3
7.779	9.488	11.14	13.28	14.86	18.47	4
9.236	11.07	12.83	15.09	16.75	20.52	5
10.64	12.59	14.45	16.81	18.55	22.46	6
12.02	14.07	16.01	18.48	20.28	24.32	7
13.36	15.51	17.53	20.09	21.95	26.12	8
14.68	16.92	19.02	21.67	23.59	27.88	9
15.99	18.31	20.48	23.21	25.19	29.59	10
17.28	19.68	21.92	24.72	26.76	31.26	11
18.55	21.03	23.34	26.22	28.30	32.91	12
19.81	22.36	24.74	27.69	29.82	34.53	13
21.06	23.68	26.12	29.14	31.32	36.12	14
22.31	25.00	27.49	30.58	32.80	37.70	15
23.54	26.30	28.85	32.00	34.27	39.25	16
24.77	27.59	30.19	33.41	35.72	40.79	17
25.99	28.87	31.53	34.81	37.16	42.31	18
27.20	30.14	32.85	36.19	38.58	43.82	19
28.41	31.41	34.17	37.57	40.00	45.31	20
29.62	32.67	35.48	38.93	41.40	46.80	21
30.81	33.92	36.78	40.29	42.80	48.27	22
32.01	35.17	38.08	41.64	44.18	49.73	23
33.20	36.42	39.36	42.98	45.56	51.18	24
34.38	37.65	40.65	44.31	46.93	52.62	25
35.56	38.89	41.92	45.64	48.29	54.05	26
36.74	40.11	43.19	46.96	49.64	55.48	27
37.92	41.34	44.46	48.28	50.99	56.89	28
39.09	42.56	45.72	49.59	52.34	58.30	29
40.26	43.77	46.98	50.89	53.67	59.70	30
51.81	55.76	59.34	63.69	66.77	73.40	40
63.17	67.50	71.42	76.15	79.49	86.66	50
74.40	79.08	83.30	88.38	91.95	99.61	60
85.53	90.53	95.02	100.43	104.21	112.32	70
96.58	101.88	106.63	112.33	116.32	124.84	80
107.57	113.15	118.14	124.12	128.30	137.21	90
118.50	124.34	129.56	135.81	140.17	149.45	100
140.23	146.57	152.21	158.95	163.65	173.62	120
268.47	277.14	284.80	293.89	300.18	313.44	240

$$Pr\left\{0.6757 < \frac{(n-1)S^2}{\sigma^2} < 18.55\right\} = 0.99.$$

INVERT INEQUALITY:

$$Pr\left\{\frac{1}{18.55} < \frac{\sigma^2}{(n-1)S^2} < \frac{1}{0.6757}\right\} = 0.99.$$

$$Pr\left\{\frac{(n-1)S^2}{18.55} < \sigma^2 < \frac{(n-1)S^2}{0.6757}\right\} = 0.99.$$

LEFT END POINT OF CI FOR σ^2 IS.

$$\frac{(n-1)S^2}{18.55} = \frac{6(47.5)}{18.55} = 15.36$$

RIGHT END POINT IS

$$\frac{(n-1)S^2}{0.6757} = \frac{6(47.5)}{0.6757} = 421.78$$

THE 99% CI FOR σ^2 IS 15.36 TO 421.78.

ALTERNATIVE PROBLEM

$H_0: \sigma^2 = 200$ VS $H_1: \sigma^2 > 200$

REMEMBER $\frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2_{n-1}$

IF H_0 TRUE ($\sigma^2 = \sigma_0^2$),

REJECT H_0 IF $\frac{(n-1)S^2}{\sigma_0^2}$ LARGE.

HERE, $\frac{(n-1)S^2}{\sigma_0^2} = \frac{6(47.5)}{200} = 1.425$

$P_{H_0}\{\chi^2_6 > 10.64\} = 0.10$ ACCEPT H_0 AT $\alpha = .10$

$P_{H_0}\{\chi^2_6 > 12.59\} = 0.05$ ACCEPT H_0 AT $\alpha = .05$

$P_{H_0}\{\chi^2_6 > 16.81\} = 0.01$ ACCEPT H_0 AT $\alpha = .01$.

ACCEPT $H_0: \sigma^2 = 200$ VS $H_1: \sigma^2 > 200$ AT $\alpha = .10$ (AND .05 AND .01).