19 Contrasts, Sum of Squares of a Contrast, Scheffe Confidence Interval

Common Information for Questions 1, 2, and 3

A research team studied *Y*, the protein production of a laboratory animal, and how *Y* was affected by the dose of medicine. The research team sought to set the dose of the medicine so that $E(Y)$ is minimized. They used four doses of the medicine: 0, 1, 2, and 3 units respectively. They randomly assigned 35 animals to dosage 0, 35 to dosage 1, 35 to dosage 2, and 35 to dosage 3. They observed that the average values of *Y* at each dosage were $y_{0\bullet} = 217, y_{1\bullet} = 181,\ y_{2\bullet} = 118,\ and\ y_{3\bullet} = 68$, where $y_{i.}$ was the average of the observations taken with dosage $i = 0,1,2,3$, respectively. They also observed that $s_0^2 = 26{,}425, s_1^2 = 19{,}564,\ s_2^2 = 31{,}950$, and $s_3^2 = 22{,}917$, where s_i^2 was the unbiased estimate of the variance for the observations taken with dosage $i = 0,1,2,3$, respectively.

1. Complete the analysis of variance table for these results; that is, be sure to specify the degrees of freedom, sums of squares, mean squares, F-test, and your conclusion. Test the null hypothesis that all treatment means are equal using significance levels 0.10, 0.05, and 0.01.
2. Find the estimated linear contrast, the sum of squares due to the linear contrast and the 99% Scheffe confidence interval for the linear contrast. The coefficients of the linear contrast are $-3, -1, 1, 3$.
3. What is the optimal setting of dosage, and how do you document it?

End of Application of Common Information

1. FROM VIDEO 17,

ANOVA TABLE

SOURCE	DF	SS	MS	F
DOSE	3	459,600	153,230	6.077
(PURE) ERROR	136	3,429,104	25,214	
TOTAL	139	3,888,794		

$F_{.99,3,136} = 3.929$

SINCE $F_{OVERALL} = 6.077 > 3.929$

REJECT $H_0: E(Y_{1j}) = E(Y_{2j}) = E(Y_{3j}) = E(Y_{4j})$

VS $H_1: E(Y_{ij}) \neq E(Y_{i'j'})$ FOR $i \neq i'$. AT $\alpha = .01$.

2. FIND $\hat{\lambda}_{LINEAR}$ (CHAPTER 9)

$$\hat{\lambda}_{LINEAR} = -3(\bar{Y}_{1\cdot}) - 1(\bar{Y}_{2\cdot}) + 1(\bar{Y}_{3\cdot}) + 3(\bar{Y}_{4\cdot})$$

$$= -3(217) - (181) + (118) + 3(68) = \boxed{-510}$$

FIND $SS_{LINEAR} = \dfrac{(\hat{\lambda}_{LINEAR})^2}{[(-3)^2 + (-1)^2 + (1)^2 + 3^2]/35}$

$$= \frac{(-510)^2}{20/35} = \boxed{455,175.0 \text{ ON } 1 \text{ DF.}}$$

FIND 99% SCHEFFE CI FOR λ_{LINEAR}:

$$\hat{\lambda}_{LINEAR} \mp \sqrt{3F_{.99,3,136}}\sqrt{MSE\left(\frac{\sum a_i^2}{J}\right)}$$

$$= -510 \pm \sqrt{3(3.929)}\sqrt{25,214\left(\frac{20}{35}\right)}$$

$$= -510 \pm 3.433(120.03)$$

$$= \boxed{-510 \pm 412.1 = -922.1 \text{ TO } -97.9}$$

THIS EXCLUDES 0. AND DOCUMENTS A SIGNIFICANT NEGATIVE LINEAR RELATION

3. **DOSE 3 OR HIGHER MINIMIZES $E(Y_{ij})$** AS SHOWN BY SIGNIFICANT NEGATIVE $\hat{\lambda}_{LINEAR}$ FROM SCHEFFE CI.

RECALL THE 99% LSD IS $t_{2.576,136}\sqrt{MSE\left(\frac{1}{J}+\frac{1}{J}\right)}$

$$= 2.612\sqrt{25,214\left(\frac{2}{35}\right)} = 2.612(37.96) = 99.1.$$

$E(Y_{4j}) - E(Y_{3j})$ IS NOT SIGNIFICANTLY DIFFERENT FROM ZERO (DIFFERENCE IS -50); $E(Y_{4j}) - E(Y_{2j})$ IS.