16 Sequential Regression Test Problem

Common Information for Questions 1, 2 and 3

A research team sought to estimate the model $E(Y) = \beta_0 + \beta_1 x + \beta_2 w$. The variable Y is a measure of educational achievement of a participant observed at age 25 (where a larger number indicates more achievement); the variable x is the measure of the of the participant's education achievement at age 15 (where a larger number indicates greater achievement); and the variable w is a measure of the participant's educational achievement at age 20 (where a larger number indicates greater achievement). They observed values of y, x, and w on 672 participants. They found that the variance of Y was 230.2; the variance of x was 184.2; and the variance of w was 208.2. The correlation between Y and w was 0.55; the correlation between Y and x was 0.37; and the correlation between x and w was 0.65.

1. Compute the partial correlation coefficients $r_{Yw \cdot x}$ and $r_{Yx \cdot w}$.

2. Is a mediation model or an explanation model a better explanation of the observed results?

3. Compute the analysis of variance table for the multiple regression analysis of Y. Include the sum of squares due to the regression on x and the sum of squares due to the regression on w after including x. Test the null hypothesis that $\beta_2 = 0$ at the 0.10, 0.05, and 0.01 levels of significance.

End of application of common information

SOLUTION.

1. $r_{Yw \cdot x} = \dfrac{r_{Yw} - r_{Yx} r_{wx}}{\sqrt{(1 - r_{Yx}^2)(1 - r_{wx}^2)}} = \dfrac{0.55 - (0.37)(0.65)}{\sqrt{(1 - .37^2)(1 - .65^2)}}$

$= \dfrac{0.3095}{\sqrt{(0.8631)(0.5775)}} = \dfrac{0.3095}{0.70600} = \boxed{0.43838.}$
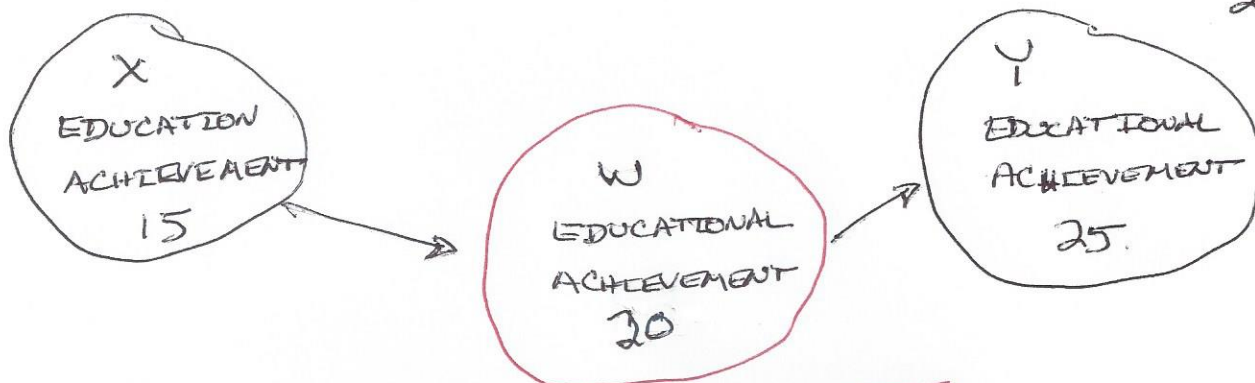
$r_{Yx \cdot w} = \dfrac{0.37 - (0.55)(0.65)}{\sqrt{(1 - .55^2)(1 - (0.65)^2)}} = \dfrac{0.0125}{0.63467}$

$= \boxed{0.01970}$

$\text{se}(r_{Yx \cdot w}) \cong \sqrt{\dfrac{1}{n-3}} = \sqrt{\dfrac{1}{669}} = \boxed{0.0387.}$

THE KEY VARIABLE IS w, NOT x.

2. TYPE OF CAUSAL MODEL.

2.

$$\boxed{\text{MEDIATION MODEL}}$$

3. SEQUENTIAL TEST

$$\text{TOTAL SS} = (n-1)\left(\hat{SD}^2_{DV}\right) = (n-1)(\hat{VAR}\,DV)$$

$$= 671\,(230.2) = 154,464.2 \quad \text{ON} \quad 671\,DF$$

NOTE $\hat{VAR}\,x = 184.2$ AND $\hat{VAR}\,w = 208.2$

$$\text{SS REG}(x) = \left(r_{yx}\right)^2 \text{TOT SS}$$

$$= (0.37)^2 \text{TOTSS}$$

$$= 21,146.15 \quad \text{ON} \quad 1\,DF$$

SS NOT EXPLAINED BY x IS.

$$\text{TOT SS} - \text{SS REG}(x) = 154,4642 - 21,146.15$$

$$= 133,318.05$$

THIS IS SSE FROM CH 11 ANALYSIS.

$$\text{SS REG}(w|x) = \left(r_{yw\cdot x}\right)^2 (\text{TOT SS} - \text{SS REG}(x))$$

$$= (0.43838)^2 (133,318.05)$$

$$= 25,620.67 \quad \text{ON} \quad 1\,DF.$$

SS ERROR = TOTAL SS − SS REG(x) − SS REG(w|x)   3.

$$= 154{,}464.2 − 21{,}146.15 − 25{,}620.67$$

$$= 107{,}697.38 \quad \text{ON } 669 \text{ DF.}$$

ANOVA TABLE

SOURCE	DF	SS	MS
REG(x)	1	21,146.15	21,146.15
REG(w|x)	1	25,620.67	25,620.67
ERROR	669	107,697.38	160.98
TOTAL	671	154,464.2	

$$F_{w|x} = \frac{\text{SS REG}(w|x)/1}{\text{MSE}} = \frac{25{,}620.67}{160.98} = 159.15$$

α	$F(1,669)$	$F(1,\infty)$	DECISION
.10	2.713	2.71	REJECT
.05	3.855	3.84	REJECT
.01	6.673	6.63	REJECT

REJECT $H_{0(w|x)}$ $\beta_2 = 0$ VS $H_{1(w|x)}$ $\beta_2 \neq 0$

AT $\alpha = 0.01$ (AND 0.05 AND 0.10).