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5. The correlation matrix of the random variables Y_1, Y_2, Y_3 is

$$\begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}, \ 0 \le \rho < 1, \text{ and each random variable has variance } \sigma^2. \text{ Let}$$

$W_1 = Y_1 - Y_2$, and let $W_2 = Y_2 - Y_3$. Find the variance covariance matrix of (W_1, W_2). This problem is worth 40 points.

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix} \sigma^2 = \sigma^2 \begin{bmatrix} 1-\rho & \rho-1 & 0 \\ 0 & 1-\rho & \rho-1 \end{bmatrix}$$

$$vcv(W) = \sigma^2 \begin{bmatrix} 2(1-\rho) & \rho-1 \\ \rho-1 & 2(1-\rho). \end{bmatrix}$$

SAME GRADING AS 5B

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6. The random variable Y has expected value $E(Y) = \mu_Y$ and $\text{var}(Y) = \sigma_Y^2 < \infty$, and the constant θ is a real number $-\infty < \theta < \infty$. Prove or disprove that $E[(Y-\theta)^2] = \sigma_Y^2 + (\theta - \mu_Y)^2$. This problem is worth 40 points.

End of Examination

THE RESULT IS TRUE

SOLUTION 1:

RECALL $E(W^2) = VAR(W) + [E(W)]^2$. THEN

$$E[(Y-\theta)^2] = VAR(Y-\theta) + [E(Y-\theta)]^2$$

$$= VAR(Y) + [\mu_Y - \theta]^2.$$

SOLUTION 2:

$$E[(Y-\theta)^2] = E[(Y-\mu_Y+\mu_Y-\theta)^2]$$

$$= E[(Y-\mu_Y)^2 + 2(Y-\mu_Y)(\mu_Y-\theta) + (\mu_Y-\theta)^2]$$

$$= E[(Y-\mu_Y)^2] + 2(\mu_Y-\theta)[E(Y-\mu_Y)] + (\mu_Y-\theta)^2$$

$$= VAR(Y) + 2(\mu_Y-\theta)\,0 + (\mu_Y-\theta)^2$$

$$= VAR(Y) + (\theta-\mu_Y)^2$$

+10 CORRECT FIRST EXPANSION