SOLUTION

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Common Information for Questions 1, 2, and 3

A research team sought to estimate the model $E(Y) = \beta_0 + \beta_1 x + \beta_2 w$. The variable Y is the participant's BMI (body mass index, a measure of obesity with higher value indicating greater obesity) observed at age 35; the variable x is a measure of the education expertise achieved by the participant at age 25; and the variable w is the participant's reported income (in thousands of dollars) at age 30. They observed values of y, x, and w on 873 participants. They found that the sample standard deviation of Y was 6.37. The correlation between Y and w was ***-0.43***; the correlation between Y and x was ***-0.59***; and the correlation between x and w was 0.71.

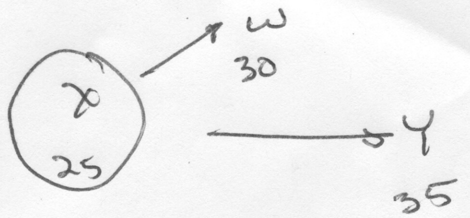
1. Compute the partial correlation coefficients $r_{Yw \bullet x}$ and $r_{Yx \bullet w}$ (20 points)

$$r_{Yw.x} = \frac{-0.43-(-0.59)(0.71)}{\sqrt{(1-(-0.59)^2)(1-.71^2)}} = \frac{-0.011}{\sqrt{(.6519)(.4959)}}$$

$$= \frac{-0.0111}{\sqrt{.32328}} = \frac{-.0111}{.56857} = -.01952$$

$$r_{Yxw} = \frac{-0.59-(-0.43)(0.71)}{\sqrt{(1-(.43)^2)(1-.71^2)}} = \frac{-.2847}{\sqrt{(.8151)(.4959)}}$$

$$= \frac{-.2847}{\sqrt{.40421}} = \frac{-.2847}{.63577} = -.4478$$



EXPLANATION

SOLUTION

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1. Compute the partial correlation coefficients $r_{Yw\bullet x}$ and $r_{Yx\bullet w}$ (20 points)

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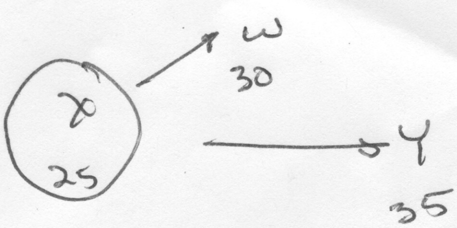

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2. Compute the analysis of variance table for the multiple regression analysis of Y. Include the sum of squares due to the regression on w and the sum of squares due to the regression on x after including w. Test the null hypothesis that $\beta_1 = 0$, (β_1 is the coefficient of x) at the 0.10, 0.05, and 0.01 levels of significance. (40 points)

$$TSS = 872(6.37)^2 = 35383.0568$$

$$SSREG(w) = (-0.43)^2 TSS = 6542.3272$$

$$TSS - SSREG(w) = 28840.7296$$

$$SS(x|w) = (r_{y(x \cdot w)})^2 (TSS - SSREG(w))$$

$$= (-.4478)^2 (28840.7296)$$

$$= 5783.2827$$

	DF	SS	MS
REG(w)	1	6542.3272	6542.3272
REG(x|w)	1	5783.2827	5783.2827
ERROR	870	23057.4469	26.5028
TOTAL	872	35383.0568	

$$F_{x|w} = \frac{5783.2827}{26.5028} = 218.21$$

α	F(1,870)	
.01	6.664	REJECT
.05	3.852	REJECT
.10	2.711	REJECT

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3. Is a mediation model or an explanation model a better explanation of the observed results? You must support your choice with results from your analyses to receive credit for this question. (20 points).

End of application of common information

EXPLANATION.

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4. A research team studied Y, the protein production of a laboratory animal, and how Y was affected by the dose of medicine. They used four doses of medicine: 0, 1, 2, and 3 units respectively. They randomly assigned 9 animals to each dosage and observed that the average values of Y at each dosage were $y_{0\bullet} = 192.6, y_{1\bullet} = 239.4, y_{2\bullet} = 278.7$, and $y_{3\bullet} = 333.3$ where $y_{i.}$ was the average of the observations taken with dosage $i = 0,\ldots,3$, respectively. They also observed that $s_0^2 = 2250.9, s_1^2 = 1878.3, s_2^2 = 2030.4$, and $s_3^2 = 2164.5$ where s_i^2 was the unbiased estimate of the variance for the observations taken with dosage $i = 0,\ldots,3$, respectively. The research team seeks to maximize $E(Y)$.

a. Complete the balanced one way analysis of variance table for these results; that is, be sure to specify the degrees of freedom, sum of squares, mean square, F-test, and your conclusion. Use the 0.10, 0.05, and 0.01 levels of significance. (30 points)

b. Find the estimate of the linear contrast and its sum of squares. The coefficients of the linear contrast are $-3,-1,1,3$. (10 points)

c. Which setting of dosage is optimal? (10 points) The three parts of this problem are worth 50 points.

0	192.6	2250.9	9	−68.4
1	239.4	1878.3	9	−21.6
2	278.7	2030.4	9	17.7
3	333.3	2164.5	9	72.3
Σ	1044.	8324.1	36	0
AVE	261	2081.025 ON 32DF		

MSE = 2081.025

SSE = 66592.8 ON 32DF.

$SSTR = 9[(-68.4)^2 + (-21.6)^2 + (17.7)^2 + (72.3)^2]$

$= 9(10685.7) = 96171.3$

SOURCE	DF	SS	MS	F	α	F(3,32)
DOSE	3	96171.3	32057.1	15.40	.10	2.263 R
ERROR	32	66592.8	2081.025		.05	2.901 R
TOTAL	35	162764.1			.01	4.459 R

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b) $\hat{\lambda}_{LIN} = -3(192.6) - 239.4 + 278.7 + 3(333.3)$

$= 461.4.$

$SS_{LIN} = \frac{(\hat{\lambda}_{LIN})^2}{2019} = 95\,800.482$

c) HIGHEST DOSE OPT.

USE HIGHEST POSSIBLE DOSE BECAUSE

$E(Y_{ij})$ IS INCREASING IN i

PROTECTED T CI FOR $\alpha_3 - \alpha_2$ IS

$333.3 - 278.7 \pm 2.738 \sqrt{2081.025\left(\frac{1}{9} + \frac{1}{9}\right)}$

$54.6 \pm 2.738(21.50) = 54.6 \pm 58.88.$

ALMOST EXCLUDES 0.

SOLUTION

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Common Information for Questions 5 and 6

A research team studied Y, the protein production of a laboratory animal, and how Y was affected by the dose of medicine. The research team sought to maximize $E(Y)$. They used four doses of medicine: 0, 1, 2, and 3 units respectively. They randomly assigned 12 animals to each dosage and observed that the average values of Y at each dosage were $y_{0\bullet} = 10.5, y_{1\bullet} = 88.6, y_{2\bullet} = 91.2,$ and $y_{3\bullet} = 9.7$ where $y_{i.}$ was the average of the observations taken with dosage $i = 0,\ldots,3$, respectively. They also observed that $s_0^2 = 250.1, s_1^2 = 208.7, s_2^2 = 225.6,$ and $s_3^2 = 240.5$ where s_i^2 was the unbiased estimate of the variance for the observations taken with dosage $i = 0,\ldots,3$, respectively. They correctly calculated that the total sum of squares was 86634.78 and that the mean squared error was $MSE = 231.225$. They also correctly calculated that the values of the linear, quadratic and cubic contrasts were $\hat{\lambda}_{Lin} = 0.2$, $\hat{\lambda}_{Quad} = -159.6$, and $\hat{\lambda}_{Cubic} = -8.6$.

5. Report the analysis of variance table for the linear regression of Y on dosage. Use the sum of squares due to the linear contrast as the sum of squares for the regression of Y on dosage. Test the null hypothesis that there is no linear association at the 0.10, 0.05, and 0.01 levels of significance. (40 points)

$$SS_{LIN} = \frac{(\hat{\lambda}_{LIN})^2}{20/12} = .024.$$

SOURCE	DF	SS	MS	F
LINEAR	1	0.024	0.024	.0000129
ERROR	46	86634.756	1883.36	
TOTAL	47	86634.78		

ACCEPT Ho NO LINEAR ASSOCIATION AT $\alpha = .10$

(α .05, .01).

α	$F(1, 46)$	
.10	2.818	A
.05	4.052	A
.01	7.220	A

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6. Test the null hypothesis that the linear model is adequate at the 0.10, 0.05, and 0.01 levels of significance. Report the analysis of variance table including the sum of squares due to lack of fit. Test whether there is evidence that the linear model has lack of fit at the 0.10, 0.05, and 0.01 levels. (40 points)

End of Common Information

SOURCE	DF	SS	MS
LINEAR	1	0.024	
LACK OF FIT	2	76460.852	38230.428
PURE ERROR	44	10173.9	231.225
TOTAL	47	86634.78	

$$F_{LOF} = \frac{38230.428}{231.225} = 165.339$$

REJECT H_0 LINEAR MODEL ADEQUATE AT $\alpha = .01$

(α .05 α .10).

$$SS_Q = \frac{(159.6)^2}{4/12} = 76416.48$$

$$SS_C = \frac{(8.6)^2}{20/12} = 44.376$$

F (2, 44).

α	
.10	2.427
.05	3.209
.01	5.123

SOLUTION

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7. The correlation matrix of the random variables Y_1, Y_2, Y_3, Y_4 is

$$\begin{pmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{pmatrix}, \quad 0 < \rho < 1, \text{ and each random variable has variance } \sigma^2.$$

Let $W_1 = Y_1 + Y_2 + Y_3 + Y_4$, and let $W_2 = -Y_1 - Y_2 + Y_3 + Y_4$. Find the variance covariance matrix of (W_1, W_2). This question is worth 40 points.

End of Examination

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$M\Sigma = \sigma^2 \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{bmatrix} = \sigma^2 \begin{bmatrix} 1+3\rho & 1+3\rho & 1+3\rho & 1+3\rho \\ -1+\rho & -1+\rho & 1-\rho & 1-\rho \end{bmatrix}$$

$$M\Sigma M^T = \sigma^2 \begin{bmatrix} 1+3\rho & 1+3\rho & 1+3\rho & 1+3\rho \\ -1+\rho & -1+\rho & 1-\rho & 1-\rho \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 4+12\rho & 0 \\ 0 & 4-4\rho \end{bmatrix}$$