18 Delta Method

The random variable $Y, Y > 0$, has $E(Y) = \theta$ and $var(Y) = \theta^4, \theta > 0$. Find the approximate mean and variance of $W = Y^p, p \neq 0$. For what value of p is the approximate variance of W constant?

$E(W) \cong (EY)^p = \theta^p, \quad p \neq 0.$

NOTE $w = y^p \quad f'(w) = p(y^{p-1})$.

$VAR(w) = VAR(y^p)$

$VAR(y^p) \cong (f'(EY))^2 VAR(Y) = (p\theta^{p-1})^2 \theta^4$

$VAR(w) = VAR(y^p) \cong p^2 \theta^{2p-2} \theta^4 = p^2 \theta^{2p+2}$

NOTE $\theta^{2p+2} = 2$ WHEN $2p+2 = 0$.

THAT IS, WHEN $p = -1$.

y^{-1} HAS APPROXIMATE VARIANCE EQUAL TO A CONSTANT:

$VAR(y^{-1}) \cong [(-1)\theta^{-2}]^2 \theta^4 = 1$

NOTE $w = y^{-1}$, $f'(w) = (-1)y^{-2}$

$f'(\theta) = -1(\theta^{-2})$

ALTERNATIVE.

$VAR(Y) = \theta^4 = (\sigma_Y)^2; \quad \sigma_Y = \theta^2$.

$\ln \sigma_Y = 2 \ln \theta = 2 \ln(EY)$.

THE VARIANCE STABILIZING TRANSFORMATION IS y^{1-m}, WHERE $m = 2$.

THAT IS, USE y^{-1}