Hermite Polynomials, and why I love them

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Wednesday, December 15th, 2021

The Motivation

Fast and Scalable Turbulent Flow Simulation with Two-Way Coupling
My project from PDE's 2020

The Math

$$H_{2n+1}(x) = \sum_{j=0}^{n} f(x_j) \cdot H_{n,j}(x) + \sum_{j=0}^{n} f'(x_j) \cdot \hat{H}_{n,j}(x)$$

- $H_{n,j} = [1 - 2(x - x_j)L'_{n,j}(x_j)]L^2_{n,j}(x)$
- $\hat{H}_{n,j} = (x - xj)]L^2_{n,j}(x)$
- $L_{n,j} = \prod_{i=0 \neq j}^{n} \frac{(x - x_i)}{(x_j - x_i)}$

So, R or Mathematica?

The Math

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I ended up choosing Mathematica.

How I did it, and what went wrong

The Steps

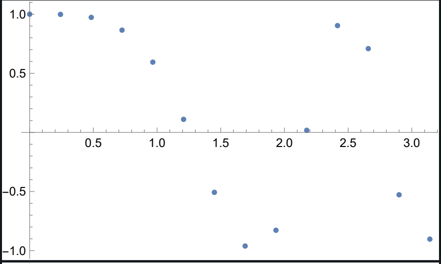
1. Define a function and take its derivative
2. Evaluate the function at some points
3. Define the $L, H, and \hat{H}$ functions
4. Plug them all in and you have you approximation

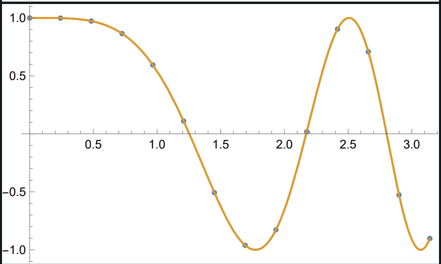
The Missteps

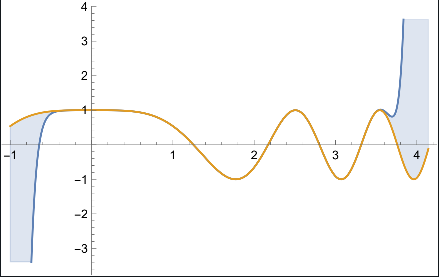
1. Assuming Mathematica can parse book notation
2. Confusing variables with tables i.e. $x \neq x_j$
3. Failing to trust that whacky looking output will be correct when evaluated

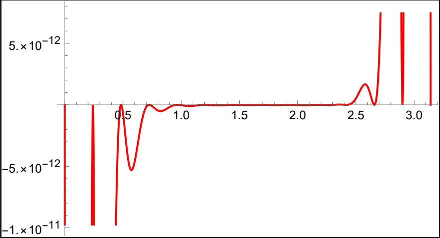
Results

Now I have code that can do this.

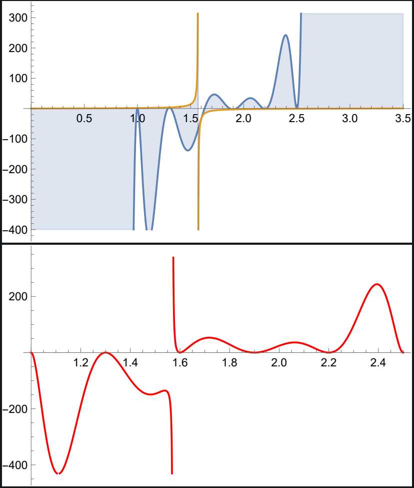






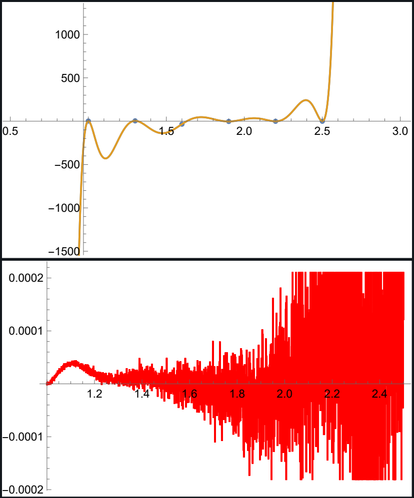


Lessons Learned (Picture time)



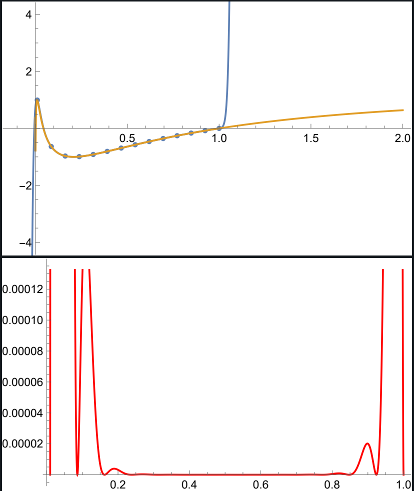
- 5th order approximation of $\tan(x)$
- Tip, continuously differentiable functions work better

Lessons Learned (Picture time)



- 5th order approximation of the previous approximation
- I'd love to make this more robust so it could deal with piecewise functions

Thanks!



- 13th order approximation of $\sin(\ln(x))$
- Thanks for listening

Citations

Burden, R.L. and Faires, J.D. *Numerical Analysis* Ninth Edition, 2010 Cengage Learning ISBN 9780538733519

Wei Li, Yixin Chen, Mathieu Desbrun, Changxi Zheng, and Xiaopei Liu. 2020. *Fast and Scalable Turbulent Flow Simulation with Two-Way Coupling.* ACM Trans. Graph. 39, 4, Article 47 (July 2020). https://doi.org/10.1145/3386569.3392400