EE309 TERM PROJECT

STABILITY PREDICTON -1

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Locating the Source of Oscillation Using Two-Tier Dynamic Mode Decomposition Integrating Early-Stage Energy (Stability Prediction)

Objective:

The objective of this project is to locate the source of oscillations in a power system. We have used an improved Dynamic Mode Decomposition (DMD) technique that integrates early-stage energy information. We aim to replicate and validate the results presented in the IEEE paper "*Locating the Source of Oscillation with Two-Tier Dynamic Mode Decomposition Integrating Early-Stage Energy.”*

Methodology:

Dynamic Mode Decomposition (DMD)

- Dynamic Mode Decomposition (DMD) is a powerful data-driven technique used for analysing complex dynamical systems.
- It helps extract spatio-temporal coherent structures (modes) from high-dimensional time-series data, making it a popular tool in fluid dynamics, control systems, power systems, and many other engineering applications.
- Given sequential snapshots of data collected over time, DMD decomposes this dataset into a set of modes, each associated with a specific growth/decay rate and oscillation frequency.
- This allows us to predict future system behavior, identify dominant patterns, and locate the source of instabilities or oscillations.

Mathematically, DMD seeks to identify a linear operator A that maps the system from one time step to the next:

$$\mathbf{X' = AX}$$

Where:

- X is the present measurement matrix.
- X’ is the future measurement matrix.

DMD through Singular Value Decomposition (SVD)

Dynamic Mode Decomposition can be derived using Singular Value Decomposition (SVD), which serves as a data dimensionality reduction method. SVD decomposes the measurement matrix X as:

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$$\boxed{\begin{aligned} & \mathbf{X} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^*, \\ \text{where } & \mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \cdots\ \mathbf{u}_n], \mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2\ \cdots\ \mathbf{v}_m], \\ & \boldsymbol{\Sigma} = [\hat{\boldsymbol{\Sigma}}\ 0]^T, \hat{\boldsymbol{\Sigma}} = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_m) \end{aligned}}$$

- U and V are unitary matrices containing the left and right singular vectors
- Σ is a diagonal matrix with singular values,* denotes the conjugate transpose.

Truncated SVD: Further reduces dimensionality by keeping only the most significant r modes.

$$\text{Truncated SVD :}\quad \mathbf{X} = \tilde{\mathbf{U}}\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{V}}^*$$

According to Eckart-Young's theorem, the truncated SVD provides the best rank-r approximation of X.

$$\text{argmin}_{\tilde{\mathbf{X}}} \|\mathbf{X} - \tilde{\mathbf{X}}\| \approx \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^*$$

Computing the DMD Operator

To compute the DMD operator without explicitly calculating A, we project A onto the reduced space defined by Ũ.

$$\tilde{\mathbf{A}} = \tilde{\mathbf{U}}^*\mathbf{A}\tilde{\mathbf{U}} = \tilde{\mathbf{U}}^*\mathbf{X}'\tilde{\mathbf{V}}\tilde{\mathbf{\Sigma}}^{-1}$$

If we let the eigenvalues and eigenvectors of Ã as Λ and W, i.e., ÃW = WΛ.

$$\mathbf{X}' = \tilde{\mathbf{U}}\mathbf{W}\mathbf{\Lambda}\mathbf{W}^{-1}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^* = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Gamma}$$

Here,
Λ contains the DMD eigenvalues (growth/decay rates and oscillation frequencies) and p = ŨW denotes spatio-temporal coherency and $\Gamma = \mathbf{W}^{-1}\tilde{\Sigma}\tilde{\mathbf{V}}^*$ denotes the temporal distribution of the mode amplitudes.

Improved Dynamic Mode Decomposition (DMD)

Inspired by DMD with control (DMDc), which separates external control impacts from inherent system dynamics by incorporating control input Y.

The improved DMD introduces the initial measurement state Xini as an input. This enhancement allows the model to focus solely on the oscillatory dynamics while decoupling the stationary base state.

X’ = AX + BY

$$\mathbf{X}' = \mathbf{G}\boldsymbol{\Omega}$$

$$\text{where} \quad \mathbf{G} = [\mathbf{A}\ \mathbf{B}],\ \boldsymbol{\Omega} = \begin{bmatrix} \mathbf{X} \\ \mathbf{X}_{ini} \end{bmatrix}$$

For any discrete time k, the system evolves as:

$$\mathbf{X}_{k+1} = \mathbf{A}\mathbf{X}_k + \mathbf{B}\mathbf{X}_{ini} = \mathbf{X}_{ini} + \mathbf{f}(k)$$

Here, f(k) represents the oscillation dynamics evolving over time. At k=1, the relationship becomes:

$$\mathbf{X}(2) = \mathbf{A}\mathbf{X}_{ini} + \mathbf{B}\mathbf{X}_{ini} = \mathbf{X}_{ini} + \mathbf{f}(1)$$

which derives $BX_{ini} = (I-A)X_{ini} + f(1)$

Substituting back, we get the modified state relationship:

$$\mathbf{A} \cdot (\mathbf{X}_k - \mathbf{X}_{ini}) = \mathbf{f}(k) - \mathbf{f}(1)$$

SVD-Based Decomposition:

Apply Singular Value Decomposition (SVD) to the input matrix: $\boldsymbol{\Omega} = \tilde{\mathbf{U}}\tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{V}}^*$

The least-squares solution for G is computed as: $\mathbf{G} \approx \mathbf{X}'\tilde{\mathbf{V}}\tilde{\boldsymbol{\Sigma}}^{-1}\tilde{\mathbf{U}}^* = \mathbf{X}'\tilde{\mathbf{V}}\tilde{\boldsymbol{\Sigma}}^{-1}\left[\tilde{\mathbf{U}}_1^*\ \tilde{\mathbf{U}}_2^*\right] = \left[\bar{\mathbf{A}}\ \bar{\mathbf{B}}\right]$

The final approximations of the system dynamics matrix are: $\tilde{\mathbf{A}} = \mathbf{U}^*\bar{\mathbf{A}}\mathbf{U} = \mathbf{U}^*\mathbf{X}'\tilde{\mathbf{V}}\tilde{\boldsymbol{\Sigma}}^{-1}\tilde{\mathbf{U}}_1^*\mathbf{U}$

$$\tilde{\mathbf{B}} = \mathbf{U}^*\bar{\mathbf{B}} = \mathbf{U}^*\mathbf{X}'\tilde{\mathbf{V}}\tilde{\boldsymbol{\Sigma}}^{-1}\tilde{\mathbf{U}}_2^*$$

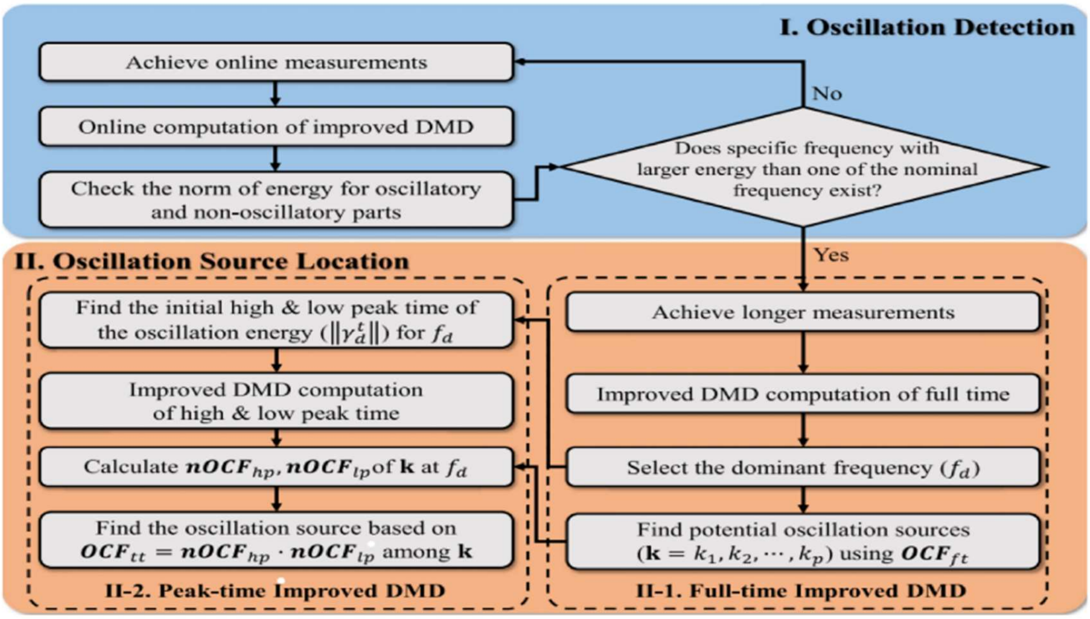
According to the eigen-decomposition of Ã, the DMD mode and temporal distribution can be derived as :

$$\mathbf{\Phi} = \mathbf{X}'\tilde{\mathbf{V}}\tilde{\mathbf{\Sigma}}^{-1}\tilde{\mathbf{U}}_1^*\mathbf{U}\mathbf{W} = [\phi_1 \ \phi_2 \ \cdots \ \phi_{2l}]$$

$$\mathbf{\Gamma} = \mathbf{W}^{-1}\mathbf{U}^*\tilde{\mathbf{U}}_1\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^* = [\gamma_1^T \ \gamma_2^T \ \cdots \ \gamma_{2l}^T]^T$$

where $\tilde{\mathbf{A}} = \mathbf{W}\mathbf{\Lambda}\mathbf{W}^{-1}$.

Proposed oscillation source location framework



Computation of full time Oscillation Contribution factor(OCF)

$$\boxed{\begin{aligned}
\mathbf{OCF}_{ft} &= |\mathbf{MM}_v^{ft} \cdot \mathbf{MM}_\theta^{ft}| \\
\mathbf{MM}_v^{ft} &= \frac{\mathfrak{Re}\{\phi_{d,v}\} \cdot \mathfrak{Im}\{\phi_{d,v}\}}{\sqrt{\mathfrak{Re}\{\phi_{d,v}\}^2 + \mathfrak{Im}\{\phi_{d,v}\}^2}}, \\
\mathbf{MM}_\theta^{ft} &= \frac{\mathfrak{Re}\{\phi_{d,\theta}\} \cdot \mathfrak{Im}\{\phi_{d,\theta}\}}{\sqrt{\mathfrak{Re}\{\phi_{d,\theta}\}^2 + \mathfrak{Im}\{\phi_{d,\theta}\}^2}}
\end{aligned}}$$

Where the subscript ft means the full-time, and the mode magnitudes are separated into $\mathbf{MM}_v^{ft}$ and $\mathbf{MM}_\theta^{ft}$

Normalisation

$$nOCF_k^{hp,lp} = \frac{OCF_k^{hp,lp}}{max(OCF_{k\in\mathbf{k}}^{hp,lp})}$$

where $\mathbf{OCF}_{hp,lp} =$

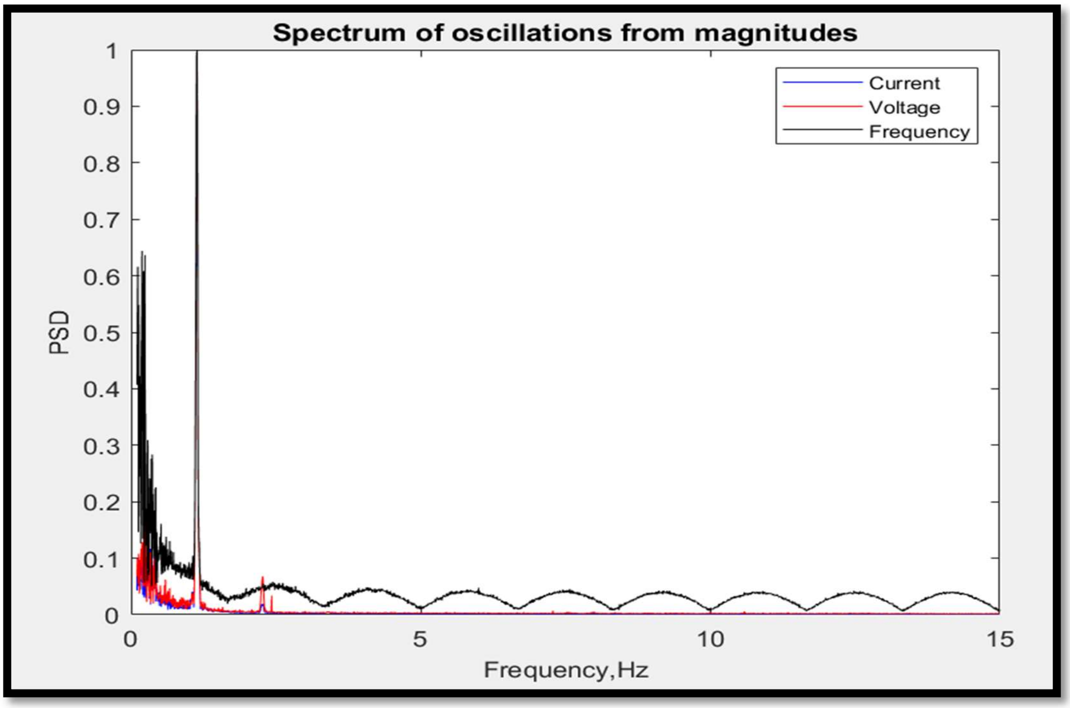
$$|(\mathbf{MM}_v^{ft} - \mathbf{MM}_v^{hp,lp}) \cdot (\mathbf{MM}_\theta^{ft} - \mathbf{MM}_\theta^{hp,lp}|.$$

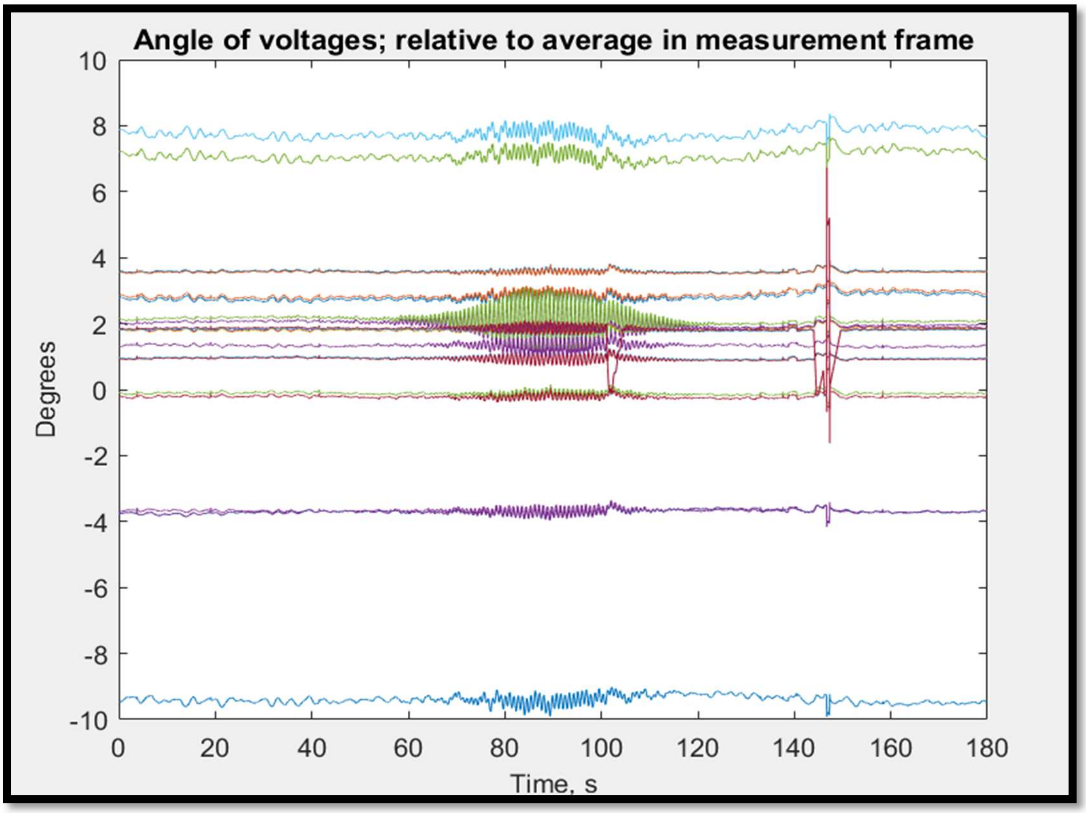
By aggregating the high and low peak-time factors, the pro- posed final oscillation contribution factor can be given by

$$\mathrm{OCF}_{tt} = (\mathrm{nOCF}_{hp}) \cdot (\mathrm{nOCF}_{lp})$$

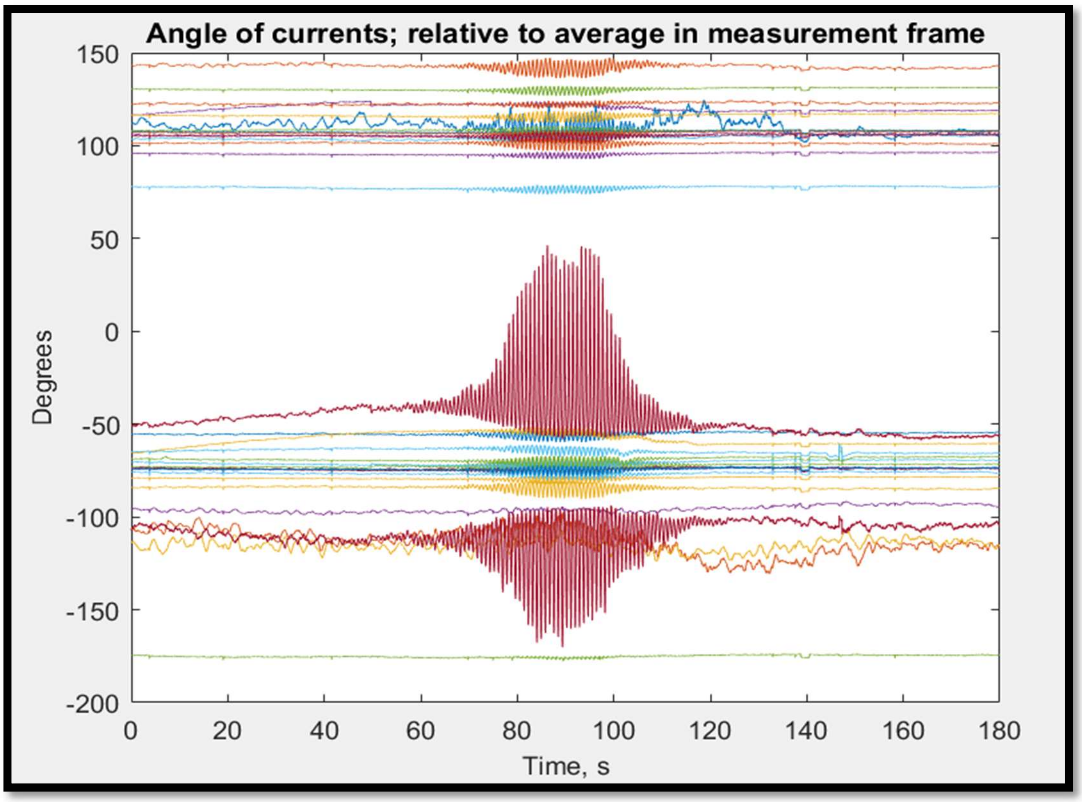
Simulation Results:

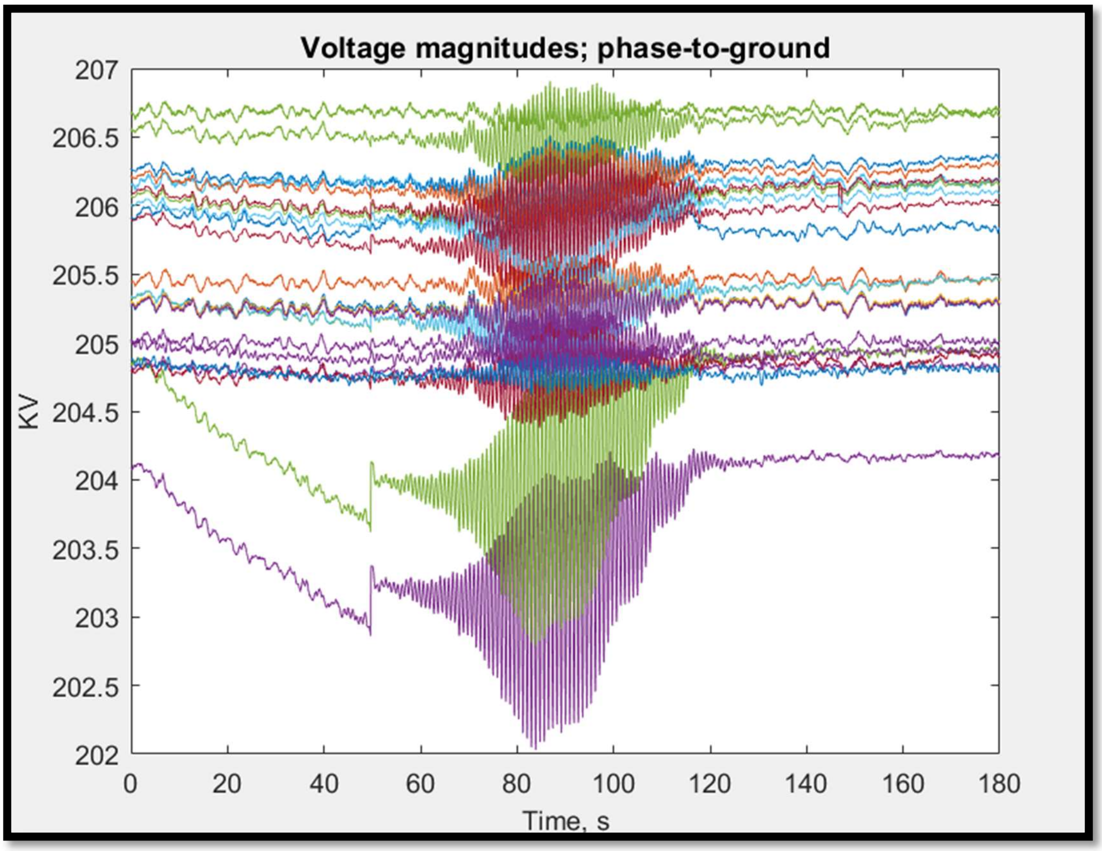
Input PMU parameter plots for ISO-NE-cases:



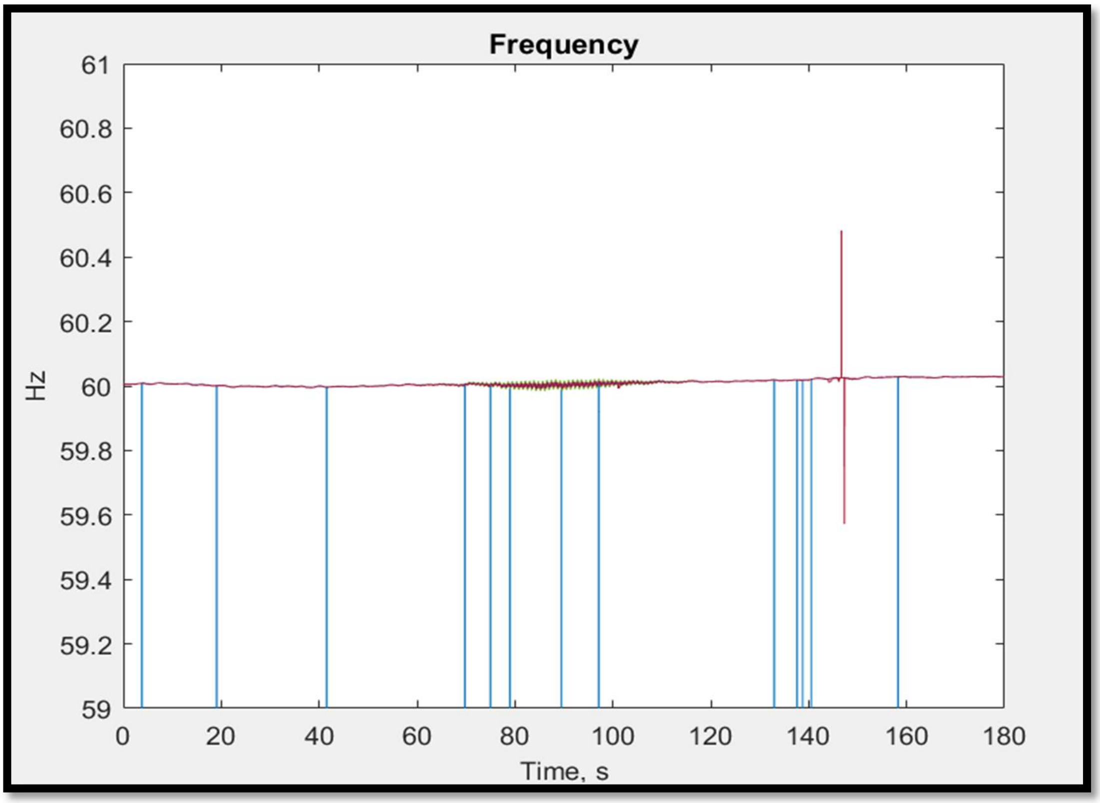

Angle of voltages; relative to average in measurement frame
Degrees
Time, s
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
20
40
60
80
100
120
140
160
180

V

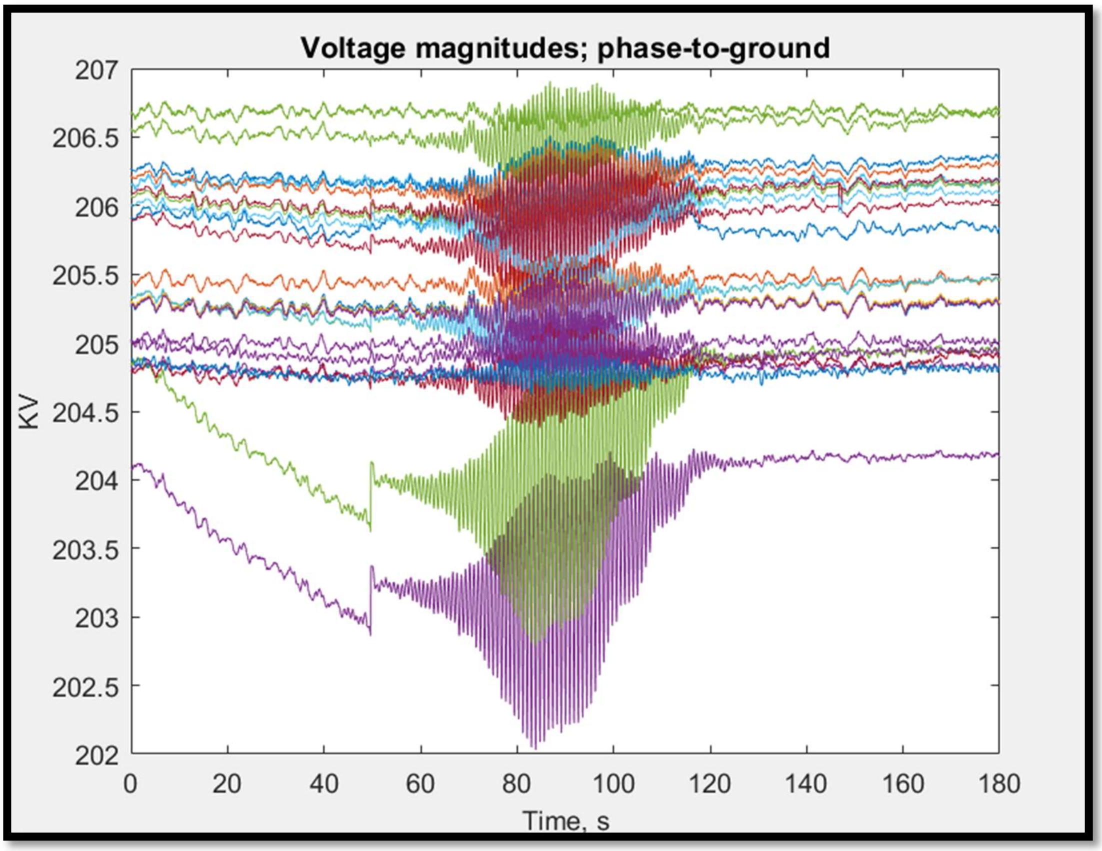

Angle of currents; relative to average in measurement frame
Degrees
Time, s
150
100
50
0
-50
-100
-150
-200
0
20
40
60
80
100
120
140
160
180

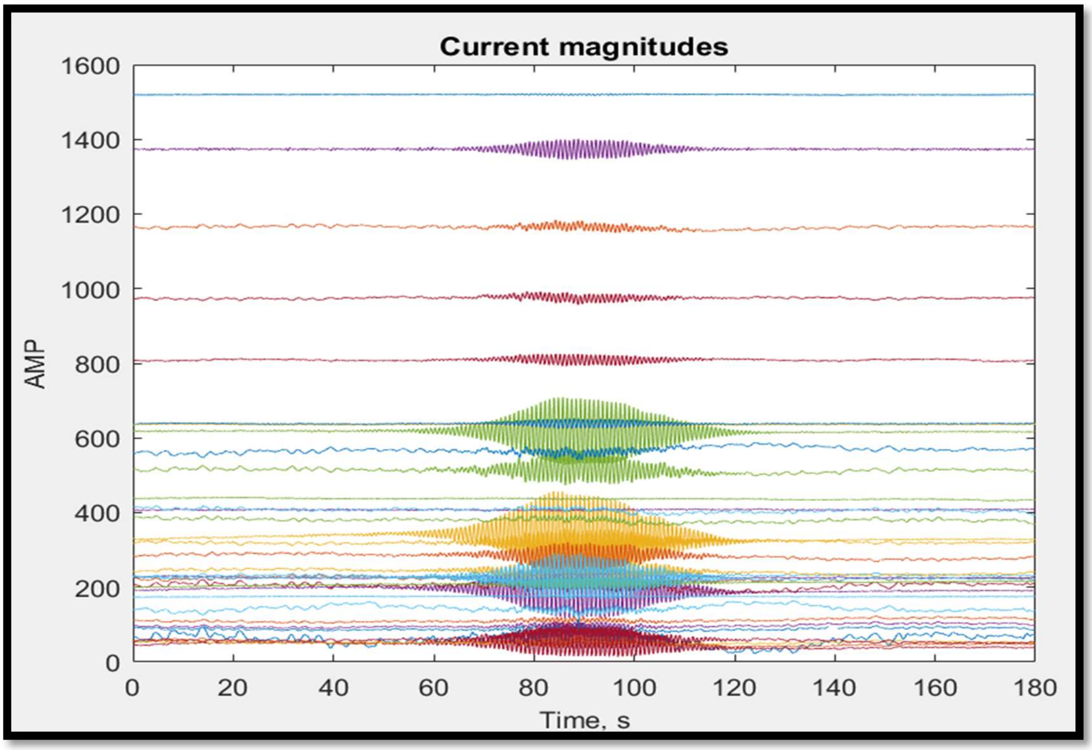


Voltage magnitudes; phase-to-ground
KV
Time, s
207
206.5
206
205.5
205
204.5
204
203.5
203
202.5
202
0
20
40
60
80
100
120
140
160
180

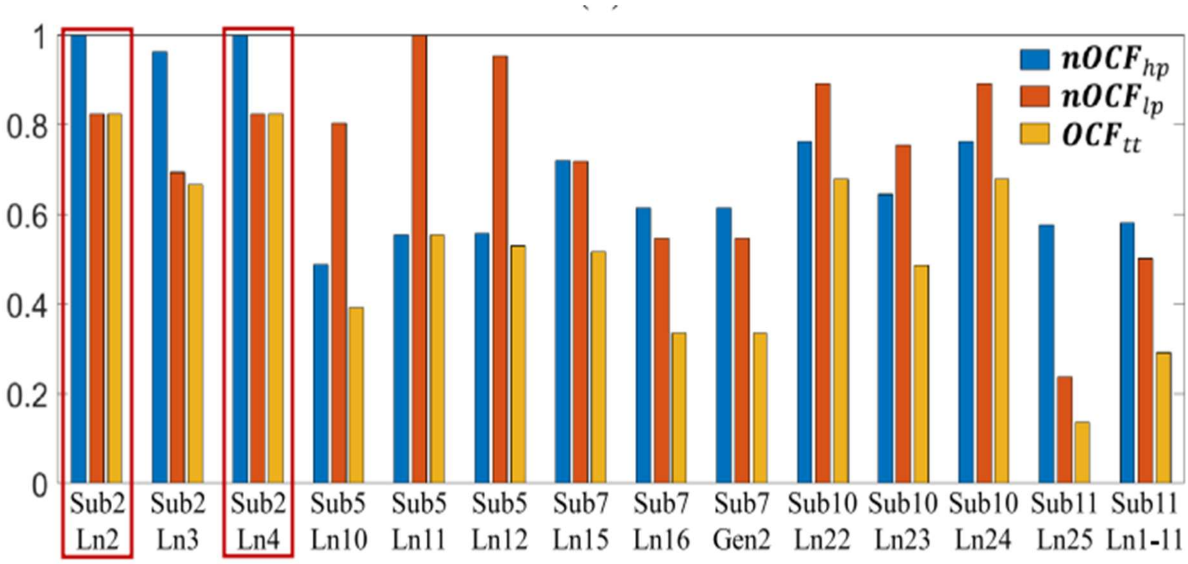


Frequency
Hz
Time, s
61
60.8
60.6
60.4
60.2
60
59.8
59.6
59.4
59.2
59
0
20
40
60
80
100
120
140
160
180

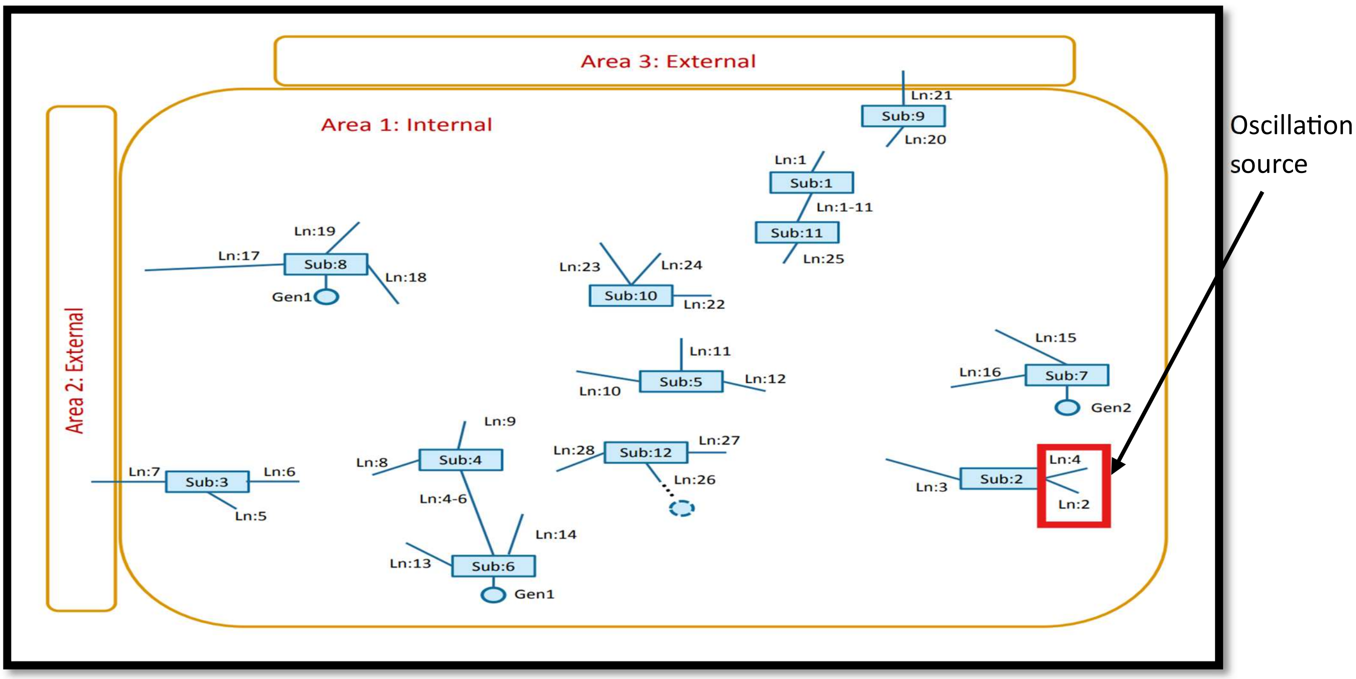

Voltage magnitudes; phase-to-ground
KV
207
206.5
206
205.5
205
204.5
204
203.5
203
202.5
202
0
20
40
60
80
100
120
140
160
180
Time, s


Current magnitudes
AMP
1600
1400
1200
1000
800
600
400
200
0
0
20
40
60
80
100
120
140
160
180
Time, s

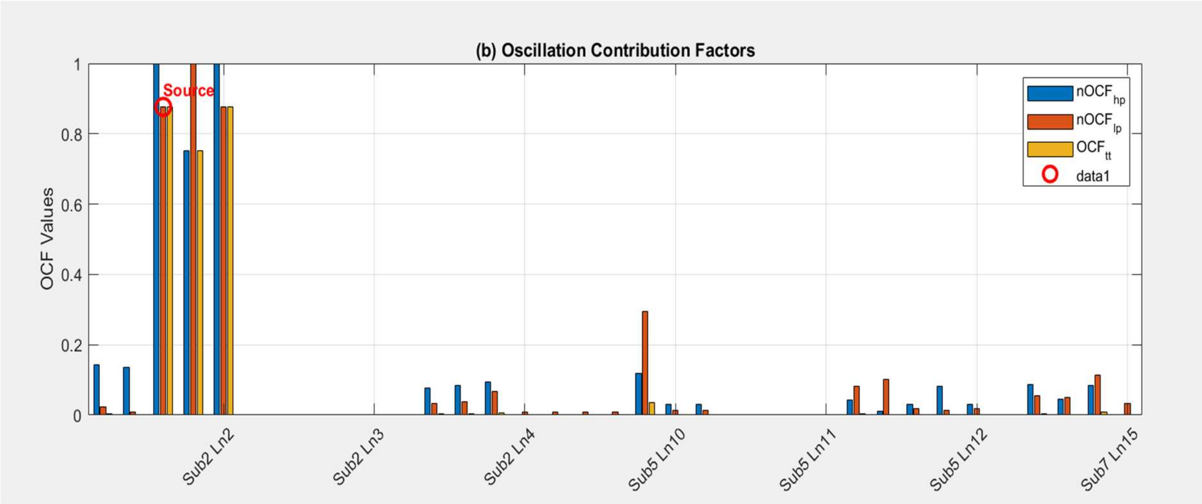
Calculated OCF plots:



Obtained Source of Oscillation MAP:



OCF of all the substations and lines:



Discussion:

We have applied the Two-Tier DMD analysis in the real case scenario of ISO-NE case3 PMU measurement. After performing the simulation on MATLAB 2024 we obtained the OCF (tt), OCF (hp), OCF (lp). And One substation with highest value of OCF becomes the oscillation source. We also analysed the initial PMU data to check the oscillations. Contribution factors on the dominant frequency can be found in Fig. 12(b), where the largest factor is observed at both line 2 and line 4 of substation 2 with the same magnitude. Hence, we can infer that the source would be located on the bus connected at both line 2 and line 4, i.e., substation 2 or another unknown bus. However, notice that the OCF (tt) value of line 3, connected to the west of substation 2, is less than those of lines 2 and 4, connected to the east. Therefore, the source may be located east of substation 2, and the oscillation has been propagated from east to west.

Conclusion:

This article proposes an improved DMD-based approach to locate the oscillation source beyond the conventional usage of DMD to identify the oscillation modes. Motivated by the structural limitation of the original DMD in locating the forced oscillations with unusual patterns, we have reformulated the DMD with control (DMDc) by replacing the control input with the initial state of the measurements and successfully demonstrated the performance. Furthermore, the two-tier structure and corresponding normalized contribution factor help to enhance the accuracy and robustness by analyzing the spatial distribution of the energy in the initial stage of the oscillation. Case studies on various oscillation data represent the promising results of the proposed method, even with partial observability. However, the proposed method has been implemented with the assumption that all data are synchronized and treated in a centralized manner. The distributed implementation of the

algorithm and the loss of data synchronization need to be considered in the future. Furthermore, significant efforts need to be exerted to investigate the sub-synchronous oscillations, especially with the fast deployment of inverter-based resources.

References:

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[7] J. L. Proctor, S. L. Brunton, and J. N. Kutz, “Dynamic mode decomposition with control,” SIAM J. Appl. Dynamical Syst., vol. 15, no. 1, pp. 142–161, 2016.

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