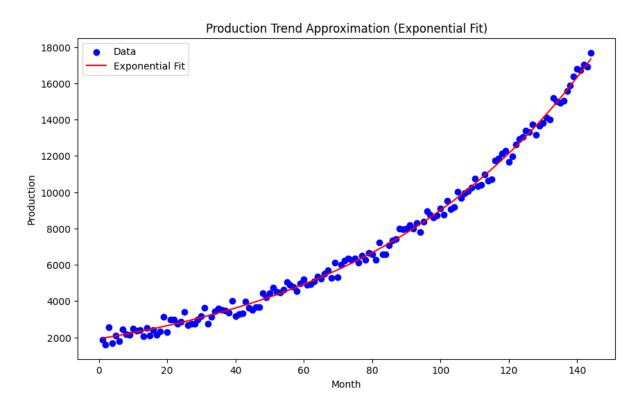
To convert the mathematical model from problem number 1 to numerical form (approximation), we must first know the estimated parameters (a, b, and c) of the mathematical model. I used the curve fit function from scipy.optimize to estimate the values of a, b, and c.



- From the exponential graph of this data trend, we know that the mathematical model is $a.e^{bx} + c$
- After that, we need to estimate the parameters a, b, and c using the curve_fit function from scipy.optimize
- After the parameters a, b, and c are estimated using the curve_fit function from scipy.optimize, we get the values a = 2139.452, b = 0.015, c = -220.005
- After we know the parameters a, b, and c. Then the numerical form (approximation) from the mathematical model $a \cdot e^{bx} + c$ is **2139.452** . $e^{0.015x} 220.005$

As shown in the picture below

```
Exponential function parameters: a= 2139.452, b= 0.015, c=-220.005

Exponential approximation = 2139.452 * e^(0.015 * x) + -220.005

Mean Absolute Error (MAE): 244.001

Root Mean Squared Error (RMSE): 285.571

Exponential Fit MAPE: 4.987%
```

Accuracy About the Conversion

The accuracy of the conversion results from the exponential regression mathematical model can be seen from the results of the MAE, RMSE, and MAPE metrics and can be compared with the results of polynomials of degree 2 and degree 3, here are the results:

```
Polynomial degree 2 parameters: a = 0.706, b = -1.680, c = 2349.630

Polynomial degree 2 approximation = 0.706 * x^2 + -1.680 * x + 2349.630

Mean Absolute Error (MAE): 292.626

Root Mean Squared Error (RMSE): 361.509

Mean Absolute Percentage Error (MAPE): 5.964%

Polynomial degree 3 parameters: a = 0.004, b = -0.134, c = 47.224, d = 1748.507

Polynomial degree 3 approximation = 0.004 * x^3 + -0.134 * x^2 + 47.224 * x + 1748.507

Mean Absolute Error (MAE): 245.304

Root Mean Squared Error (RMSE): 288.436

Mean Absolute Percentage Error (MAPE): 4.988%
```

1. Mean Absolute Error (MAE)

The MAE of polynomial degree 2 is **292.626** while MAE polynomial degree 3 is **245.304**, and MAE exponential is **244.001**, which is the lowest among the other 2 methods. From the data we know that:

- The MAE for the polynomial degree 2 is 292.626, which means that the average prediction error of the conversion is about 292.626 units.
- The MAE for the polynomial degree 3 is 245.304, which means that the average prediction error of the conversion is about 245.304 units.
- The MAE for the exponential is 244.001, which means that the average prediction error of the conversion is about 244.001 units.

From the MAE results above, it can be concluded that the conversion of the mathematical model from the exponential regression method to its numerical form is more accurate than the other 2 methods. Which on average, the conversion results deviate from the actual production value by about 244 units.

2. Root Mean Squared Error (RMSE)

The RMSE of polynomial degree 2 is **361.509** while RMSE polynomial degree 3 is **288.436**, and RMSE exponential is **285.571**, which is the lowest among the other 2 methods. From the data we know that:

- The RMSE for the polynomial degree 2 is 361.509, which means that the average prediction error of the conversion is about 361.509 units.
- The RMSE for the polynomial degree 3 is 288.436, which means that the average prediction error of the conversion is about 288.436 units.
- The RMSE for the exponential is 285.571, which means that the average prediction error of the conversion is about 285.571 units.

From the RMSE results above, it can be concluded that the conversion of the mathematical model from the exponential regression method to its numerical form is more accurate than the other 2 methods. Which on average, the conversion results deviate from the actual production value by about 286 units.

3. Mean Absolute Percentage Error (MAPE)

The MAPE of polynomial degree 2 is **5.964%** while MAPE polynomial degree 3 is **4.988%**, and MAPE exponential is **4.987%**, which is the lowest among the other 2 methods. From the data we know that:

- The MAPE for polynomial degree 2 is 5.964%, this means that the average relative error of the model's prediction to the true value is about 5.964%.
- The MAPE for polynomial degree 3 is 4.988%, this means that the average relative error of the model's prediction to the true value is about 4.988%.
- The MAPE for exponential regression is 4.987%, this means that the average relative error of the model's prediction to the true value is about 4.987%.

From the MAPE results above, it can be concluded that the conversion of the mathematical model from the exponential regression method to its numerical form is more accurate than the other 2 methods. Which on average, the conversion results deviate from the actual production value by about 4.987%.