Applications
Projects
Software
Vehicle
Example
References

Interactive vehicle simulation

Models, numerics and software for heavy machinery

Martin Servin

UMIT Research Lab

December 19, 2013

Interactive vehicle simulation

- Applications
- Project examples
- Common software and methods
- Elements of a vehicle model
- A wheel loader model

Applications of vehicle simulation

- Operator training

  Wheel loader

- Conceptual design and marketing

  Mining modularity concept

- Research and development - design, control & HMI

  Mars rover simulation

  VTI simulator

Project examples

- Simovate - Algoryx, Atlas Copco, Volvo CE

  Wheel loader

- Long Tracked Bogie (LTB) - Vimek

  LTB

- Bogie and dynamic terrain - Komatsu Forest & Olofsfors

  Elastic terrain

  Elasto-plastic terrain

Common software tools & methods

- Block-diagram modeling (casual)

  MATLAB & SIMULINK,...

- Equation based modeling (acasual), e.g., Modelica

  AMEsim, Dymola, MapleSim, OpenModelica,...

- Multibody dynamics software

  MSC Adams,...

- Physics engine

  Chrono Engine,...

Elements of vehicles

- Mechanical components
- Joints
- Drivetrain
- Suspension
- Steering
- Control system
- Operator interface

Mechanics

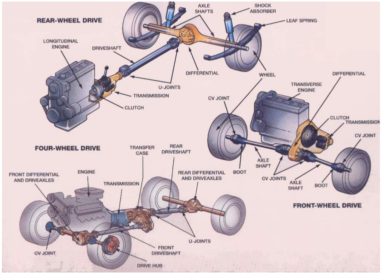
Structure and kinematics

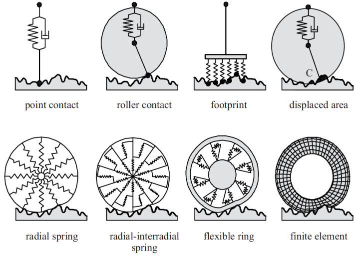
- Chassis - front and rear
- Articulation
- Axis - front and rear
- Bogie, wheel, tracks
- Cabin
- Crane/arm and tool

Drivetrain

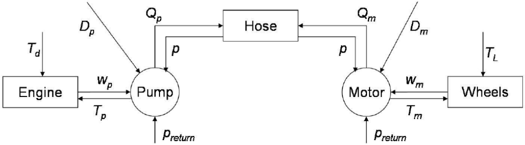
Delivers power from engine to ground

- Engine
- Clutch or torque converter
- Gearbox (transmission)
- U-joints, CV-joints
- Drive shafts
- Differential
- Wheel (Pacejka magic formula, Wong)





Hydraulic drivetrain



Hydrostatic transmission

- Engine: $\tau_e(t), \omega_e(t)$
- Pump: ω_p
- Hose: p
- Motor: ω_m
- Wheel: $\omega_w, \tau_w(t)$

In reality - valves, multiple motors and cylinders.

$$(J_p + J_d)\dot{w}_p = T_d - T_{fp} - T_p,$$
$$T_{fp} = \mu_p \omega_p + K_p,$$
$$T_p = \frac{D_p \Delta p}{\eta_{tp}}.$$

$$(J_m + J_\omega)\,\dot{w}_m = T_m - T_{fm} - T_L,$$
$$T_m = D_m \Delta p \eta_{tm},$$
$$T_{fm} = \mu_m \omega_m + K_m sign(\omega_m).$$

$$p = \frac{\beta}{V}\int_0^t (Q_p - Q_m)\, d\tau,$$
$$\Delta p = p - p_{return},$$
$$Q_p = D_p \omega_p \eta_{vp},$$
$$Q_m = \frac{\omega_p D_m}{\eta_{vm}}.$$

Nonsmooth multibody dynamics

Multibody system equations of motion (x, v), $v = dx/dt$

$$M\frac{dv}{dt} = -\nabla U(x) + F(x, v, t) + G^T\lambda \quad (1)$$

Constraints

$$g(x) = 0 \quad (2)$$
$$g(v) = 0 \quad (3)$$
$$g(x, v, t) = 0 \quad (4)$$
$$g(x) \geqslant 0 \quad (5)$$

Complementarity conditions: $0 \leqslant Gv \perp \lambda \geqslant 0$, $\lambda_t \leqslant \mu|\lambda_n|$

Discrete events: $Gv_{-} = -eGv_{-}$

Applications
Projects
Software
Vehicle
Example
References

Time-discrete multibody dynamics

Position $\boldsymbol{q}$, velocity $\dot{\boldsymbol{q}}$, mass $\boldsymbol{M}$, position constraints $\boldsymbol{g}(\boldsymbol{q}) = 0$, Jacobian $\boldsymbol{G} = \partial \boldsymbol{g}/\partial \boldsymbol{q}$, velocity constraints $\bar{\boldsymbol{G}}\dot{\boldsymbol{q}} - \boldsymbol{w} = 0$.

The SPOOK stepper reads

$$\boldsymbol{q}_{n+1} = \boldsymbol{q}_n + h\dot{\boldsymbol{q}}_{n+1} \tag{6}$$

$$\begin{bmatrix} \boldsymbol{M} & -\boldsymbol{G}^T & -\bar{\boldsymbol{G}}^T \\ \boldsymbol{G} & \boldsymbol{\Sigma} & 0 \\ \bar{\boldsymbol{G}} & 0 & \bar{\boldsymbol{\Sigma}} \end{bmatrix} \begin{pmatrix} \dot{\boldsymbol{q}}_{n+1} \\ \boldsymbol{\lambda} \\ \bar{\boldsymbol{\lambda}} \end{pmatrix} = \begin{pmatrix} M\dot{\boldsymbol{q}}_n + h\boldsymbol{f}_n \\ -\frac{4}{h}\Upsilon g + \Upsilon G\dot{\boldsymbol{q}}_n \\ \boldsymbol{\omega}_n \end{pmatrix} \tag{7}$$

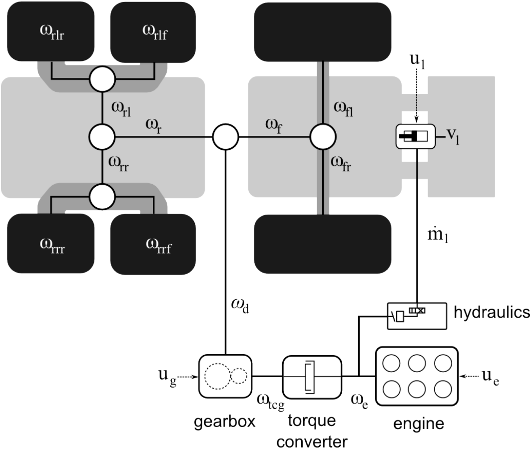
$$\boldsymbol{\Sigma} = \frac{4}{h^2}\,\mathrm{diag}\left(\frac{\varepsilon_1}{1+4\frac{\tau_1}{h}}, \frac{\varepsilon_2}{1+4\frac{\tau_2}{h}}, \dots\right) \tag{8}$$

$$\bar{\boldsymbol{\Sigma}} = \frac{1}{h}\,\mathrm{diag}\left(\gamma_1, \gamma_2, \dots\right) \tag{9}$$

$$\Upsilon = \mathrm{diag}\left(\frac{1}{1+4\frac{\tau_1}{h}}, \frac{1}{1+4\frac{\tau_2}{h}}, \dots\right) \tag{10}$$

- Applications
- Projects
- Software
- Vehicle
- Example
- References

Example: wheel loader model



Example: wheel loader model

ω_e	engine speed
ω_{tcg}	torque converter to gearbox
ω_d	main drive shaft
ω_f	front drive shaft
ω_{fl}	front left axle and wheel
ω_{fr}	front right axle and wheel
ω_r	rear drive shaft
ω_{rl}	rear left axle
ω_{rlf}	rear left bogie front wheel
ω_{rlr}	rear left bogie rear wheel
ω_{rr}	rear right axle
ω_{rrf}	rear right bogie front wheel
ω_{rrr}	rear right bogie rear wheel

Engine dynamics model

$$I_e \dot{\omega}_e = T_e(u_e, \omega_e) + G_{tc}^T \lambda_{tc} + G_h^T \lambda_h$$

where engine torque and friction are

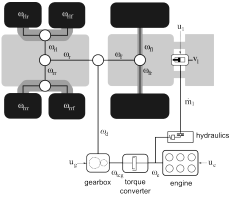
$$T_e(u_e, \omega_e) = T_{ig} - T_{efr}$$

$$T_{ig} = c_{ig} u_e$$

$$T_{efr} = c_{efr0} + c_{efr1}\omega_e + c_{efr2}\omega_e^2$$

with engine signal $u_e \in [0, 1]$ and constants c_x

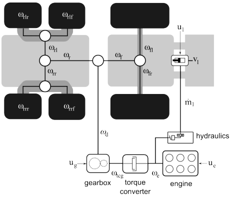
Torque converter



$$
\begin{aligned}
0 &= g_{\mathrm{tc}} \equiv \alpha(r_\omega)\omega_{\mathrm{tcg}} - \omega_{\mathrm{e}} + \gamma^{-1}(r_\omega)\lambda_{\mathrm{tc}} \\
&= \underbrace{\begin{bmatrix} -1 & \alpha(r_\omega) \end{bmatrix}}_{\boldsymbol{G}_{\mathrm{tc}}} \begin{pmatrix} \omega_{\mathrm{e}} \\ \omega_{\mathrm{tcg}} \end{pmatrix} + \gamma(r_\omega)\lambda_{\mathrm{tc}}
\end{aligned}
$$

torque multiplication $\alpha(r_\omega)$, speed ratio $r_\omega = \frac{\omega_{\mathrm{tcg}}}{\omega_{\mathrm{e}}}$

Gearbox



$$0 = g_{gb} \equiv \omega_d - \eta_{gb}\omega_{tcg} = \underbrace{\begin{bmatrix} -\eta_{gb} & 1 \end{bmatrix}}_{\boldsymbol{G}_{gb}} \begin{pmatrix} \omega_{tcg} \\ \omega_d \end{pmatrix}$$

gear ratio coefficient $\eta_{gb}(u_g)$

Differentials

Main, rear and front differential

$$0 = g_\mathrm{d} \equiv \eta_\mathrm{d}(\omega_\mathrm{f} + \omega_\mathrm{r}) - \omega_\mathrm{d} = \underbrace{\begin{bmatrix} -1 & \eta_\mathrm{d} & \eta_\mathrm{d} \end{bmatrix}}_{\boldsymbol{G}_\mathrm{d}} \begin{pmatrix} \omega_\mathrm{d} \\ \omega_\mathrm{f} \\ \omega_\mathrm{r} \end{pmatrix}$$

$$0 = g_\mathrm{f} \equiv \eta_\mathrm{f}(\boldsymbol{n}_\mathrm{fl}^T \boldsymbol{\omega}_\mathrm{fl} + \boldsymbol{n}_\mathrm{fr}^T \boldsymbol{\omega}_\mathrm{fr}) - \omega_\mathrm{f} = \begin{bmatrix} -1 & \eta_\mathrm{f}\boldsymbol{n}_\mathrm{fl}^T & \eta_\mathrm{f}\boldsymbol{n}_\mathrm{f3}^T \end{bmatrix} \begin{pmatrix} \omega_\mathrm{f} \\ \boldsymbol{\omega}_\mathrm{fl} \\ \boldsymbol{\omega}_\mathrm{fr} \end{pmatrix}$$

$$0 = g_\mathrm{r} \equiv \eta_\mathrm{r}(\omega_\mathrm{rl} + \omega_\mathrm{rr}) - \omega_\mathrm{r} = \begin{bmatrix} -1 & \eta_\mathrm{r} & \eta_\mathrm{r} \end{bmatrix} \begin{pmatrix} \omega_\mathrm{r} \\ \omega_\mathrm{rl} \\ \omega_\mathrm{rr} \end{pmatrix}$$

with gear coefficient η

Applications
Projects
Software
Vehicle
Example
References

Differential locking

$$0 = g_{\text{d-d}} \equiv \omega_{\text{f}} - \omega_{\text{r}} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{pmatrix} \omega_{\text{f}} \\ \omega_{\text{r}} \end{pmatrix}$$

$$0 = g_{\text{f-d}} \equiv \omega_{\text{fl}} - \omega_{\text{fr}} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{pmatrix} \omega_{\text{fl}} \\ \omega_{\text{fr}} \end{pmatrix}$$

$$0 = g_{\text{r-d}} \equiv \omega_{\text{rl}} - \omega_{\text{rr}} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{pmatrix} \omega_{\text{rl}} \\ \omega_{\text{rr}} \end{pmatrix}$$

Full drivetrain constraint

$$
\begin{bmatrix}
-1 & \alpha(r_\omega) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\eta_{\mathrm{gb}} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & \eta_{\mathrm{d}} & 0 & 0 & \eta_{\mathrm{d}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & \eta_{\mathrm{f}}\boldsymbol{n}_{\mathrm{fl}}^T & \eta_{\mathrm{f}}\boldsymbol{n}_{\mathrm{fr}}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & \eta_{\mathrm{r}} & 0 & 0 & 0 & \eta_{\mathrm{r}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \eta_{\mathrm{bl}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \boldsymbol{n}_{\mathrm{rlf}}^T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \boldsymbol{n}_{\mathrm{rlr}}^T & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \eta_{\mathrm{br}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \boldsymbol{n}_{\mathrm{rrf}}^T \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \boldsymbol{n}_{\mathrm{rrr}}^T & 0
\end{bmatrix}
\begin{pmatrix}
\omega_{\mathrm{e}} \\ \omega_{\mathrm{tcg}} \\ \omega_{\mathrm{d}} \\ \omega_{\mathrm{f}} \\ \boldsymbol{\omega}_{\mathrm{fl}} \\ \boldsymbol{\omega}_{\mathrm{fr}} \\ \omega_{\mathrm{r}} \\ \omega_{\mathrm{rl}} \\ \omega_{\mathrm{bl}} \\ \boldsymbol{\omega}_{\mathrm{rlf}} \\ \boldsymbol{\omega}_{\mathrm{rlr}} \\ \omega_{\mathrm{rr}} \\ \omega_{\mathrm{br}} \\ \boldsymbol{\omega}_{\mathrm{rrf}} \\ \boldsymbol{\omega}_{\mathrm{rrr}}
\end{pmatrix}
$$

Simplified drivetrain constraint

Eliminate axle variables and assuming no bogies

$$
\left[\begin{array}{c|ccc|cccc}
-1 & \alpha(r_\omega) & 0 & 0 & & & & \\
\hline
 & -\eta_{\mathrm{gb}} & \eta_{\mathrm{d}} & \eta_{\mathrm{d}} & & & & \\
\hline
 & 0 & -1 & 0 & \eta_{\mathrm{f}}\boldsymbol{n}_{\mathrm{f}}^T & \eta_{\mathrm{f}}\boldsymbol{n}_{\mathrm{f}}^T & 0 & 0 \\
 & 0 & 0 & -1 & 0 & 0 & \eta_{\mathrm{r}}\boldsymbol{n}_{\mathrm{rl}}^T & \eta_{\mathrm{r}}\boldsymbol{n}_{\mathrm{r}}^T
\end{array}\right]
\left(\begin{array}{c}
\omega_{\mathrm{e}} \\
\hline
\omega_{\mathrm{tcg}} \\
\omega_{\mathrm{f}} \\
\omega_{\mathrm{r}} \\
\hline
\boldsymbol{\omega}_{\mathrm{fl}} \\
\boldsymbol{\omega}_{\mathrm{fr}} \\
\boldsymbol{\omega}_{\mathrm{rl}} \\
\boldsymbol{\omega}_{\mathrm{rr}}
\end{array}\right)
$$

Collect in blocks

$$
\begin{bmatrix}
\boldsymbol{G}_{\mathrm{e}} & \boldsymbol{G}_{\mathrm{ed}} & 0 & 0 \\
0 & \boldsymbol{G}_{\mathrm{d}} & 0 & 0 \\
0 & \boldsymbol{G}_{\mathrm{dw}} & \boldsymbol{G}_{\mathrm{w}} & 0
\end{bmatrix}
\begin{pmatrix}
\boldsymbol{\omega}_{\mathrm{e}} \\
\boldsymbol{\omega}_{\mathrm{d}} \\
\boldsymbol{\omega}_{\mathrm{w}}
\end{pmatrix}
$$

Full equations of motions

$$
\left[\begin{array}{cccc|cccc}
\boldsymbol{I}_{\mathrm{e}} & 0 & 0 & 0 & \boldsymbol{G}_{\mathrm{e}}^{T} & 0 & 0 & 0 \\
0 & \boldsymbol{I}_{\mathrm{d}} & 0 & 0 & \boldsymbol{G}_{\mathrm{ed}}^{T} & \boldsymbol{G}_{\mathrm{d}}^{T} & \boldsymbol{G}_{\mathrm{dw}}^{T} & 0 \\
0 & 0 & \boldsymbol{I}_{\mathrm{w}} & 0 & 0 & 0 & \boldsymbol{G}_{\mathrm{w}}^{T} & \boldsymbol{G}_{\mathrm{wc}}^{T} \\
0 & 0 & 0 & \boldsymbol{M}_{\mathrm{c}} & 0 & 0 & 0 & \boldsymbol{G}_{\mathrm{c}}^{T} \\
\hline
\boldsymbol{G}_{\mathrm{e}} & \boldsymbol{G}_{\mathrm{ed}} & 0 & 0 & \boldsymbol{\Sigma}_{\mathrm{e}} & 0 & 0 & 0 \\
0 & \boldsymbol{G}_{\mathrm{d}} & 0 & 0 & 0 & \boldsymbol{\Sigma}_{\mathrm{d}} & 0 & 0 \\
0 & \boldsymbol{G}_{\mathrm{dw}} & \boldsymbol{G}_{\mathrm{w}} & 0 & 0 & 0 & \boldsymbol{\Sigma}_{\mathrm{w}} & 0 \\
0 & 0 & \boldsymbol{G}_{\mathrm{wc}} & \boldsymbol{G}_{\mathrm{c}} & 0 & 0 & 0 & \boldsymbol{\Sigma}_{\mathrm{c}}
\end{array}\right]
\begin{pmatrix}
\boldsymbol{\omega}_{\mathrm{e}} \\ \boldsymbol{\omega}_{\mathrm{d}} \\ \boldsymbol{\omega}_{\mathrm{w}} \\ \boldsymbol{v}_{\mathrm{c}} \\ \boldsymbol{\lambda}_{\mathrm{ed}} \\ \boldsymbol{\lambda}_{\mathrm{d}} \\ \boldsymbol{\lambda}_{\mathrm{dw}} \\ \boldsymbol{\lambda}_{\mathrm{c}}
\end{pmatrix}
=
\begin{pmatrix}
\boldsymbol{I}_{\mathrm{e}}\boldsymbol{\omega}_{\mathrm{e},n} + h\boldsymbol{T}_{\mathrm{e}}(u_{\mathrm{e}}, \boldsymbol{\omega}_{\mathrm{e},n}) \\
\boldsymbol{I}_{\mathrm{d}}\boldsymbol{\omega}_{\mathrm{d},n} \\
\boldsymbol{I}_{\mathrm{w}}\boldsymbol{\omega}_{\mathrm{w},n} + h\boldsymbol{T}_{\mathrm{w}} \\
\boldsymbol{M}_{\mathrm{c}}\boldsymbol{v}_{\mathrm{c},n} + h\boldsymbol{f}_{\mathrm{c}} \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0} \\
-\frac{4}{h}\Upsilon \boldsymbol{g}_{\mathrm{c}} + \Upsilon \boldsymbol{G}_{\mathrm{c}}\boldsymbol{v}_{textc,n}
\end{pmatrix}
$$

with $\boldsymbol{I}_{\mathrm{d}} = \mathrm{diag}(I_{\mathrm{d}}, I_{\mathrm{r}}, I_{\mathrm{f}})$

Applications
Projects
Software
Vehicle
Example
References

The full time stepper

The drivetrain is part of the full system

$$\begin{bmatrix} \boldsymbol{M} & -\boldsymbol{G}^T & -\bar{\boldsymbol{G}}^T \\ \boldsymbol{G} & \boldsymbol{\Sigma} & 0 \\ \bar{\boldsymbol{G}} & 0 & \bar{\boldsymbol{\Sigma}} \end{bmatrix} \begin{pmatrix} \dot{\boldsymbol{q}}_{n+1} \\ \boldsymbol{\lambda} \\ \bar{\boldsymbol{\lambda}} \end{pmatrix} = \begin{pmatrix} M\dot{\boldsymbol{q}}_n + h\boldsymbol{f}_n \\ -\frac{4}{h}\Upsilon g + \Upsilon G\dot{\boldsymbol{q}}_n \\ \boldsymbol{\omega}_n \end{pmatrix} \quad (11)$$

$$\boldsymbol{\Sigma} = \frac{4}{h^2}\,\mathrm{diag}\left(\frac{\varepsilon_1}{1+4\frac{\tau_1}{h}}, \frac{\varepsilon_2}{1+4\frac{\tau_2}{h}}, \dots\right) \quad (12)$$

$$\bar{\boldsymbol{\Sigma}} = \frac{1}{h}\,\mathrm{diag}\left(\gamma_1, \gamma_2, \dots\right) \quad (13)$$

$$\Upsilon = \mathrm{diag}\left(\frac{1}{1+4\frac{\tau_1}{h}}, \frac{1}{1+4\frac{\tau_2}{h}}, \dots\right) \quad (14)$$

References

- C. Lacoursiére. Regularized, stabilized, variational methods for multibodies. In The 48th Scandinavian Conference on Simulation and Modeling (SIMS 2007).
- V. Nezhadali and L. Eriksson. Modeling and optimal control of a wheel loader in the lift-transport section of the short loading cycle. In 7th IFAC Symposium on Advances in Automotive Control, Tokyo, Japan, 2013.
- J. Wong. Terramechanics and off-road vehicle engineering: terrain behaviour, off-road vehicle performance and design. Amsterdam, Butterworth-Heinemann, 2010.