UMEÅ UNIVERSITY
Department of Computing Science
Department of Physics

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Exam in Visual Interactive Simulation D (5DV058) (7.5 ECTS credits)

Location: Sal 2 Östra paviljongerna. Date: 2010-06-04 Time: 9:00-13:00. Bokn.nr. 111834.
Calculator and math formulas allowed. Max 48 credit points.

Always start a new assignment on a new sheet of paper. If solving a problem involving math and derivations, make sure you write down all important steps and not just the final result.
N.B. many of the subtasks are independent, so give them all a try even if you fail on the first.
Good luck! ☺

1. **Knowledge check (14p)**
   - **a.** What are holonomic and non-holonomic constraints, respectively? How do they affect the effective number of degrees of freedom in a rigid body system? (2p)
   - **b.** Briefly describe the principles of broad phase collision detection methods: sweep-and-prune, mapping to an explicit grid, spatially hashed grids, octree. (4p)
   - **c.** Why is it hard to accurately simulate a rigid body with a highly inhomogeneous inertia tensor, i.e. a very long rod? Explain how this problem often is approximated away, and how this approximation limits the realism of the simulation. (2p)
   - **d.** Why are quaternions better for representing rotating rigid bodies than e.g. Euler angles are? What is the point of regularly applying Gram-Schmidt orthonormalization to a quaternion in a simulation? How is it done? (2p)
   - **e.** In SPH, smoothed averages are computed as $< A_i >= \sum_j \frac{m_j}{\rho_j} A_j K(r_{ij})$. What is the smoothed average of the density, ρ_i, itself? What is the smoothed average of unity at particle i? Explain when and why the smoothed value of unity may differ from unity (that is, $< 1_i > \neq 1$) , and show how this deviation can be used to renormalize the density and other smoothed averages. (4p)

2. **One-dimensional contact problem (16p)**
   - **a.** Consider a particle on a plane. The plane is defined to be at the origin and we only consider motion perpendicular to the plane so we are dealing with a 1D problem. The particle has a mass m, a velocity v and its position coordinate q directly measures the penetration (negative q means there is a penetration). Gravity also acts in this direction. A constraint is applied to avoid penetration, and this is modelled with SPOOK,

$$(GM^{-1}G^T + \Sigma)\lambda = -a\, q - bGV - hGM^{-1}f$$

$$a = \frac{4}{h(1+4d)}$$

$$b = \frac{4d}{1+4d}$$

$$\epsilon = \frac{4}{h^2 k(1+4d)}$$

     Describe in words the meaning of: G, $\Sigma, h, k, d, \lambda,$ and each of the three terms on the right hand side of the equation. (4p)

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b. Reduce the SPOOK equation to the simplest possible form for our explicit case of a 1D particle on a plane, and also solve it so you get an expression for computing λ. (2p)

c. Once the value of λ is computed, we can use it to integrate the velocity of the particle using,

$$v(t+h) = v(t) + \frac{\lambda}{m} + \frac{hf}{m}$$

Now consider a particle that has come to rest. Derive the expression that gives the size of the penetration when the particles has come to rest just below the plane, and $v(t+h) = v(t) = 0$, that is: What is q for an equilibrated particle when the constraint force precisely counteracts gravity? (2p)

d. What is the size of the equilibrated penetration depth when $\epsilon = 0$? (1p)

e. Explain why a finite regularization is an important improvement to: the contact model of real materials: the solver; the collision detection in relation to the time integration. (3p)

f. Now consider the case of two (line shaped bodies) bodies of length 2R, with one resting on the plane, and the other resting on top of the first one. Write down these two equations, and solve for the two λ's. (4p)

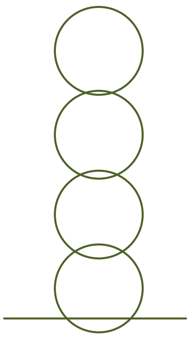
3. Penalty methods for rigid body contacts (9p)

Consider a vertical pile of N contacting rigid body spheres, of radius *R*, and mass *m*, resting on an inert plane. Each contact is modeled with a spring of stiffness k. Assume uniform gravitational acceleration of magnitude, *g*, pointing downward.

a. Compute the total compression of the pile as a function of N, for the configuration where the pile is at equilibrium. (4p)

Hint: reduce the problem to one dimension and consider the bodies as line shaped without rotations. Draw a picture displaying the system and all the relevant forces.

Gauss' formula: $\sum_{n=1}^{M} n = M(M+1)/2$, will come in handy.



b. What is the maximum penetration size? (2p)

c. Assume the pile contains N = 100 bodies with m = 0.1 kg and R=0.05 m. We want the maximum total compression of the pile to be max 3%. What does the value of the spring constant need to be? (2p)

d. If you're using Leapfrog for the integration, estimate the maximum size of the timestep that will make this simulation stable. (1p)

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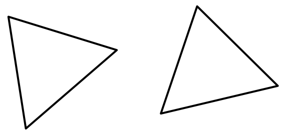
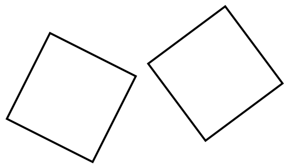
4. Intersection test/find (9p)

a. For a 2D case of two triangles, the separating axis theorem can be used to do an intersection test. How should the axes be chosen and how many tests are required at most? (3p)



b. Do the same type of test for the 2D case of two squares. How many tests are needed if this is done naively without using symmetries, and how many can this be reduced to by exploiting symmetries? (3p)



c. Explain the principles for computing intersection points, penetration depth and contact normals for these 2D boxes. What types of feature contacts can exist? (3p)