1. **(a)** Find the distance between points $P = (-1, 1)$ and $Q = (11, -4)$.
(b) Find the midpoint of the line segment joining the points P and Q.

2. **(a)** Find the distance between points $P = (-3, 2)$ and $Q = (-1, 5)$.
(b) Find the midpoint of the line segment joining the points P and Q.

3. **(a)** Find the distance between points $P = (5, -3)$ and $Q = (-4, 7)$.
(b) Find the midpoint of the line segment joining the points P and Q.

4. **(a)** Find the distance between points $P = (8, 3)$ and $Q = (-8, 15)$.
(b) Find the midpoint of the line segment joining the points P and Q.

5. Let $P = (3, 5)$, $Q = (7, -1)$ and $\vec{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Compute the following quantities:

(a) $\text{Dist}(P, Q)$ **(b)** $\overrightarrow{PQ}$ **(c)** $P + \vec{v}$ **(d)** $Q - \vec{v}$ **(e)** $P - Q$

6. Let $P = (4, 7)$, $Q = (5, -2)$ and $\vec{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. Compute the following quantities:

(a) $\text{Dist}(P, Q)$ **(b)** $\overrightarrow{PQ}$ **(c)** $P + \vec{v}$ **(d)** $Q - \vec{v}$ **(e)** $P - Q$

7. Let $P = (-2, 3)$, $Q = (2, -4)$ and $\vec{v} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$. Compute the following quantities:

(a) $\text{Dist}(P, Q)$ **(b)** $\overrightarrow{PQ}$ **(c)** $P + \vec{v}$ **(d)** $Q - \vec{v}$ **(e)** $P - Q$

8. Let $P = (2, -3)$, $Q = (5, 1)$ and $\vec{v} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$. Compute the following quantities:

(a) $\text{Dist}(P, Q)$ **(b)** $\overrightarrow{PQ}$ **(c)** $P + \vec{v}$ **(d)** $Q - \vec{v}$ **(e)** $P - Q$

9. Let $P = (1, -5)$, $\vec{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$. Compute the following quantities:

(a) $P + \vec{v}$ **(b)** $P + \vec{w}$ **(c)** $\vec{v} + \vec{w}$ **(d)** $P + \vec{v} + \vec{w}$ **(e)** $\vec{w} - \vec{v}$ **(f)** $\text{proj}_{\vec{v}}(\vec{w})$

10. Let $P = (3, -4)$, $\vec{v} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$. Compute the following quantities:

(a) $P + \vec{v}$ **(b)** $P + \vec{w}$ **(c)** $\vec{v} + \vec{w}$ **(d)** $P + \vec{v} + \vec{w}$ **(e)** $\vec{w} - \vec{v}$ **(f)** $\text{proj}_{\vec{v}}(\vec{w})$

11. Let $P = (-2, 3)$, $\vec{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$. Compute the following quantities:

(a) $P+\vec{v}$ **(b)** $P+\vec{w}$ **(c)** $\vec{v}+\vec{w}$ **(d)** $P+\vec{v}+\vec{w}$ **(e)** $\vec{w}-\vec{v}$ **(f)** $\text{proj}_{\vec{v}}(\vec{w})$

12. Let $P = (2, -7)$, $\vec{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$. Compute the following quantities:

(a) $P+\vec{v}$ **(b)** $P+\vec{w}$ **(c)** $\vec{v}+\vec{w}$ **(d)** $P+\vec{v}+\vec{w}$ **(e)** $\vec{w}-\vec{v}$ **(f)** $\text{proj}_{\vec{v}}(\vec{w})$

13. Let $\vec{v} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$. Compute the following quantities:

(a) $\vec{v}+\vec{w}$ **(b)** $\vec{w}-\vec{v}$ **(c)** $3\vec{v}-2\vec{w}$ **(d)** $\frac{1}{2}(\vec{v}+\vec{w})$ **(e)** $\|\vec{w}\|$ **(f)** $\text{proj}_{\vec{w}}(\vec{v})$

14. Let $\vec{v} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$. Compute the following quantities:

(a) $\vec{v}+\vec{w}$ **(b)** $\vec{w}-\vec{v}$ **(c)** $3\vec{v}-2\vec{w}$ **(d)** $\frac{1}{2}(\vec{v}+\vec{w})$ **(e)** $\|\vec{w}\|$ **(f)** $\text{proj}_{\vec{w}}(\vec{v})$

15. Let $\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$. Compute the following quantities:

(a) $\vec{v}+\vec{w}$ **(b)** $\vec{w}-\vec{v}$ **(c)** $3\vec{v}-2\vec{w}$ **(d)** $\frac{1}{2}(\vec{v}+\vec{w})$ **(e)** $\|\vec{w}\|$ **(f)** $\text{proj}_{\vec{w}}(\vec{v})$

16. Let $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$. Compute the following quantities:

(a) $\vec{v}+\vec{w}$ **(b)** $\vec{w}-\vec{v}$ **(c)** $3\vec{v}-2\vec{w}$ **(d)** $\frac{1}{2}(\vec{v}+\vec{w})$ **(e)** $\|\vec{w}\|$ **(f)** $\text{proj}_{\vec{w}}(\vec{v})$

17. Let $\vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Find: **(a)** $2\vec{v}-3(\vec{u}+\vec{w})$ **(b)** $\|\vec{v}\|$

(c) a unit vector in the direction of $\vec{v}$. **(d)** a unit vector perpendicular to $\vec{v}$.

18. Let $\vec{u} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$. Find: **(a)** $2\vec{v}-3(\vec{u}+\vec{w})$ **(b)** $\|\vec{v}\|$

(c) a unit vector in the direction of $\vec{v}$. **(d)** a unit vector perpendicular to $\vec{v}$.

19. Let $\vec{u}=\begin{bmatrix}1\\-3\end{bmatrix}$, $\vec{v}=\begin{bmatrix}-12\\5\end{bmatrix}$ and $\vec{w}=\begin{bmatrix}1\\2\end{bmatrix}$. Find: **(a)** $2\vec{v}-3(\vec{u}+\vec{w})$ **(b)** $\|\vec{v}\|$

(c) a unit vector in the direction of $\vec{v}$. **(d)** a unit vector perpendicular to $\vec{v}$.

20. Let $\vec{u}=\begin{bmatrix}2\\3\end{bmatrix}$, $\vec{v}=\begin{bmatrix}-8\\6\end{bmatrix}$ and $\vec{w}=\begin{bmatrix}3\\-5\end{bmatrix}$. Find: **(a)** $2\vec{v}-3(\vec{u}+\vec{w})$ **(b)** $\|\vec{v}\|$

(c) a unit vector in the direction of $\vec{v}$. **(d)** a unit vector perpendicular to $\vec{v}$.

21. (a) Find a unit vector in the direction of $\vec{u}=\begin{bmatrix}-5\\12\end{bmatrix}$.

(b) Find a unit vector in the direction of $\vec{v}=\begin{bmatrix}3\\-4\end{bmatrix}$.

(c) Find a unit vector in the direction of $\vec{w}=\begin{bmatrix}3\\-4\\12\end{bmatrix}$.

(d) Find two unit vectors perpendicular to $\vec{a}=\begin{bmatrix}1\\2\\3\end{bmatrix}$.

22. (a) Find a unit vector in the direction of $\vec{u}=\begin{bmatrix}-24\\7\end{bmatrix}$.

(b) Find a unit vector in the direction of $\vec{v}=\begin{bmatrix}30\\-24\end{bmatrix}$.

(c) Find a unit vector in the direction of $\vec{w}=\begin{bmatrix}3\\-4\\7\end{bmatrix}$.

(d) Find two unit vectors perpendicular to $\vec{a}=\begin{bmatrix}4\\-2\\3\end{bmatrix}$.

23. **(a)** Find a unit vector in the direction of $\vec{u} = \begin{bmatrix} -8 \\ 15 \end{bmatrix}$.

(b) Find a unit vector in the direction of $\vec{v} = \begin{bmatrix} 20 \\ -21 \end{bmatrix}$.

(c) Find a unit vector in the direction of $\vec{w} = \begin{bmatrix} 12 \\ -21 \\ 28 \end{bmatrix}$.

(d) Find two unit vectors perpendicular to $\vec{a} = \begin{bmatrix} 21 \\ -12 \\ 16 \end{bmatrix}$.

24. **(a)** Find a unit vector in the direction of $\vec{u} = \begin{bmatrix} -12 \\ 16 \end{bmatrix}$.

(b) Find a unit vector in the direction of $\vec{v} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$.

(c) Find a unit vector in the direction of $\vec{w} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$.

(d) Find two unit vectors perpendicular to $\vec{a} = \begin{bmatrix} 3 \\ 5 \\ -7 \end{bmatrix}$.

25. Let $P = (2, 1)$, $Q = (5, 1)$ and $R = (3, 4)$.

(a) Find the midpoints M_{PQ} and M_{PR} of segments $\overline{PQ}$ and $\overline{PR}$.

(b) Find the midpoint M of the segment $\overline{M_{PQ} M_{PR}}$.

(c) Express $\overrightarrow{PM}$ in terms of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$: $a \cdot \overrightarrow{PQ} + b \cdot \overrightarrow{PR} = \overrightarrow{PM}$
[Illustrate this with a picture]

26. Let $P = (1, 3)$, $Q = (5, 1)$ and $R = (4, 6)$.

(a) Find the midpoints M_{PQ} and M_{PR} of segments $\overline{PQ}$ and $\overline{PR}$.

(b) Find the midpoint M of the segment $\overline{M_{PQ} R}$.

(c) Express $\overrightarrow{PM}$ in terms of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$: $a \cdot \overrightarrow{PQ} + b \cdot \overrightarrow{PR} = \overrightarrow{PM}$
[Illustrate this with a picture]

27. Let $P = (2, 5)$, $Q = (4, -1)$ and $R = (5, 2)$.

(a) Find the midpoints M_{PQ} and M_{PR} of segments $\overline{PQ}$ and $\overline{PR}$.

(b) Find the midpoint M of the segment $\overline{M_{PR}\,Q}$.

(c) Express $\overrightarrow{PM}$ in terms of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$: $a \cdot \overrightarrow{PQ} + b \cdot \overrightarrow{PR} = \overrightarrow{PM}$
[Illustrate this with a picture]

28. Let $P = (4, 6)$, $Q = (0, -2)$ and $R = (-4, 2)$.

(d) Find the midpoints M_{PQ} and M_{PR} of segments $\overline{PQ}$ and $\overline{PR}$.

(e) Find the midpoint M of the segment $\overline{M_{PQ}\,R}$.

(f) Express $\overrightarrow{PM}$ in terms of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$: $a \cdot \overrightarrow{PQ} + b \cdot \overrightarrow{PR} = \overrightarrow{PM}$
[Illustrate this with a picture]

29. Let $\vec{u} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$. Compute the following quantities:

(a) $2\vec{u} + 3\vec{v}$ **(b)** $3\vec{u} - 2\vec{v} + \vec{w}$ **(c)** $\vec{u} \cdot \vec{v}$ **(d)** $\vec{v} \cdot \vec{w}$ **(e)** $(2\vec{u} + 3\vec{v}) \cdot \vec{w}$

30. Let $\vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$. Compute the following quantities:

(a) $2\vec{u} + 3\vec{v}$ **(b)** $3\vec{u} - 2\vec{v} + \vec{w}$ **(c)** $\vec{u} \cdot \vec{v}$ **(d)** $\vec{u} \cdot \vec{w}$ **(e)** $(2\vec{w} + 3\vec{v}) \cdot \vec{u}$

31. Let $\vec{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Compute the following quantities:

(a) $2\vec{u} + 3\vec{v}$ **(b)** $3\vec{u} - 2\vec{v} + \vec{w}$ **(c)** $\vec{u} \cdot \vec{v}$ **(d)** $\vec{u} \cdot \vec{w}$ **(e)** $(2\vec{w} + 3\vec{v}) \cdot \vec{u}$

32. Let $\vec{u} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Compute the following quantities:

(a) $2\vec{u} + 3\vec{v}$ **(b)** $3\vec{u} - 2\vec{v} + \vec{w}$ **(c)** $\vec{u} \cdot \vec{v}$ **(d)** $\vec{u} \cdot \vec{w}$ **(e)** $(2\vec{w} + 3\vec{v}) \cdot \vec{u}$

33. Find the angles between the following pairs of vectors:

(a) $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ **(c)** $\begin{bmatrix} 9 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -12 \\ 4 \end{bmatrix}$ **(e)** $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

34. Find the angles between the following pairs of vectors:

(a) $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 5 \end{bmatrix}$ **(b)** $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} -8 \\ 12 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 9 \end{bmatrix}$ **(e)** $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

35. Find the angles between the following pairs of vectors:

(a) $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ **(b)** $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 12 \\ -6 \end{bmatrix}$ **(c)** $\begin{bmatrix} -3 \\ 11 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 1 \end{bmatrix}$ **(e)** $\begin{bmatrix} 6 \\ -10 \end{bmatrix}$ and $\begin{bmatrix} 15 \\ 9 \end{bmatrix}$

36. Find the angles between the following pairs of vectors:

(a) $\begin{bmatrix} 5 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} -10 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ **(c)** $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ -10 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ **(e)** $\begin{bmatrix} -1 \\ -5 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$

37. Let $P = (2, 3)$, $Q = (-2, 4)$ and $R = (-3, -2)$.

(a) Find the area of ΔPQR **(b)** Find the angles of ΔPQR

(c) Find the lengths of the sides of ΔPQR

(d) Find the base of the altitude from Q, using a projection vector.

38. Let $P = (4, 1)$, $Q = (-3, 5)$ and $R = (-6, -2)$.

(a) Find the area of ΔPQR **(b)** Find the angles of ΔPQR

(c) Find the lengths of the sides of ΔPQR

(d) Find the base of the altitude from Q, using a projection vector.

39. Let $P = (2, 3)$, $Q = (8, 7)$ and $R = (3, 8)$.

(a) Find the area of ΔPQR **(b)** Find the angles of ΔPQR

(c) Find the lengths of the sides of ΔPQR

(d) Find the base of the altitude from Q, using a projection vector.

40. Let $P = (\ -1, 7\ )$, $Q = (\ 2, 1\ )$ and $R = (\ 7,\ 4\ )$.

(a) Find the area of ΔPQR **(b)** Find the angles of ΔPQR

(c) Find the lengths of the sides of ΔPQR

(d) Find the base of the altitude from Q, using a projection vector.

41. Use the law of cosines to find the angle between the lines $x + 4y = 7$ and $2x - 3y = 3$ [Hint: First find the point of intersection of the two lines, call it P. Then check that the points $Q = (\ 7, 0\ )$ and $R = (\ 0,\ -1)$ are each on one of the lines.

Use these three points, i.e. ΔPQR, and the law of cosines to compute the angle.]

42. Use the law of cosines to find the angle between the lines $2x - 3y = -5$ and $5x - y = 7$ [Hint: First find the point of intersection of the two lines, call it P. Then check that the points $Q = (\ 8, 7\ )$ and $R = (\ 3, 8\ )$ are each on one of the lines.

Use these three points, i.e. ΔPQR, and the law of cosines to compute the angle.]

43. Use the law of cosines to find the angle between the lines $2x + y = 5$ and $3x - 5y = 1$ [Hint: First find the point of intersection of the two lines, call it Q. Then check that the points $P = (\ -1, 7\ )$ and $R = (\ 7, 4\ )$ are each on one of the lines.

Use these three points, i.e. ΔPQR, and the law of cosines to compute the angle.]

44. Use the law of cosines to find the angle between the lines $5x + y = 7$ and $3x + 2y = 7$ [Hint: First find the point of intersection of the two lines, call it P. Then check that the points $Q = (\ 0, 7\ )$ and $R = (\ -1, 5\ )$ are each on one of the lines.

Use these three points, i.e. ΔPQR, and the law of cosines to compute the angle.]

45. (a) Graph in one picture the points $P = (\ 2, 3\ )$, $Q = (-1, 5)$ and $R = (\ 2,\ -3\ )$.

(b) Compute the distances: dist(P, Q), dist(P, R) and dist(Q, R).

(c) Compute the angles of the triangle $\triangle PQR$, using the law of cosines and the lengths you computed in part (b).

46. (a) Graph in one picture the points $P = (\ 2, 1\ )$, $Q = (\ -1, 7\ )$ and $R = (\ 7, 4\ )$.

(b) Compute the distances: dist(P, Q), dist(P, R) and dist(Q, R).

(c) Compute the angles of the triangle $\triangle PQR$, using the law of cosines and the lengths you computed in part (b).

47. (a) Graph in one picture the points $P = (3, 2)$, $Q = (5, 0)$ and $R = (2, -1)$.

(b) Compute the distances: $\text{dist}(P, Q)$, $\text{dist}(P, R)$ and $\text{dist}(Q, R)$.

(c) Compute the angles of the triangle $\triangle PQR$, using the law of cosines and the lengths you computed in part (b).

48. (a) Graph in one picture the points $P = (3, 2)$, $Q = (1, 5)$ and $R = (2, -3)$.

(b) Compute the distances: $\text{dist}(P, Q)$, $\text{dist}(P, R)$ and $\text{dist}(Q, R)$.

(c) Compute the angles of the triangle $\triangle PQR$, using the law of cosines and the lengths you computed in part (b).

49. Let $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

(a) Find the coordinates of the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $2\vec{w}$ and $\vec{v}-\vec{u}+2\vec{w}$.

(b) Graph in one picture the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $2\vec{w}$ and $\vec{v}-\vec{u}+2\vec{w}$.

(c) Compute the dot products $\vec{u}\cdot\vec{v}$, $(\vec{u}+\vec{v})\cdot(2\vec{w})$ and $(\vec{u}+\vec{v})\cdot(\vec{v}-\vec{u}+2\vec{w})$.

(d) Compute the angle between the vectors $\vec{u}$ and $\vec{v}$.

(e) Using the dot product find the angle between the vectors $\vec{v}-\vec{u}$ and $\vec{w}-\vec{u}$. How does this compare to the angle $\angle QPR$ in $\triangle PQR$ from problem 45. Explain.

50. Let $\vec{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$.

(a) Find the coordinates of the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $2\vec{w}$ and $\vec{v}-\vec{u}+2\vec{w}$.

(b) Graph in one picture the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $2\vec{w}$ and $\vec{v}-\vec{u}+2\vec{w}$.

(c) Compute the dot products $\vec{u}\cdot\vec{v}$, $(\vec{u}+\vec{v})\cdot(2\vec{w})$ and $(\vec{u}+\vec{v})\cdot(\vec{v}-\vec{u}+2\vec{w})$.

(d) Compute the angle between the vectors $\vec{u}$ and $\vec{v}$.

(e) Using the dot product find the angle between the vectors $\vec{v}-\vec{u}$ and $\vec{w}-\vec{u}$. How does this compare to the angle $\angle QPR$ in $\triangle PQR$ from problem 46. Explain.

51. Let $\vec{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

(a) Find the coordinates of the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $-3\vec{w}$ and $\vec{u}+\vec{v}-3\vec{w}$.

(b) Graph in **one** picture the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $2\vec{w}$ and $\vec{v}-\vec{u}+2\vec{w}$.

(c) Compute the dot products $\vec{u}\cdot\vec{w}$, $(\vec{u}+\vec{v})\cdot(-3\vec{w})$ and $(\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}-3\vec{w})$.

(d) Compute the angle between the vectors $\vec{u}$ and $\vec{v}$.

(e) Using the dot product find the angle between the vectors $\vec{v}-\vec{u}$ and $\vec{w}-\vec{u}$. How does this compare to the angle $\angle QPR$ in $\triangle PQR$ from problem 48. Explain.

52. Let $\vec{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

(a) Find the coordinates of the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $-3\vec{w}$ and $\vec{u}+\vec{v}-3\vec{w}$.

(b) Graph in **one** picture the vectors $\vec{u}+\vec{v}$, $\vec{v}-\vec{u}$, $2\vec{w}$ and $\vec{v}-\vec{u}+2\vec{w}$.

(c) Compute the dot products $\vec{u}\cdot\vec{w}$, $(\vec{u}+\vec{v})\cdot(-3\vec{w})$ and $(\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}-3\vec{w})$.

(d) Compute the angle between the vectors $\vec{u}$ and $\vec{v}$.

(e) Using the dot product find the angle between the vectors $\vec{v}-\vec{u}$ and $\vec{w}-\vec{u}$. How does this compare to the angle $\angle QPR$ in $\triangle PQR$ from problem 47. Explain.

53. (a) Find two vectors that are perpendicular to $\vec{u} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

(b) Give a geometric interpretation of the set of all vectors perpendicular to $\vec{u} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

(c) Find four vectors that are perpendicular to $\vec{v} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$.

(d) Give a geometric interpretation of the set of all vectors perpendicular to $\vec{u} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$.

54. (a) Find two vectors that are perpendicular to $\vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

(b) Give a geometric interpretation of the set of all vectors perpendicular to $\vec{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

(c) Find four vectors that are perpendicular to $\vec{v} = \begin{bmatrix} 3 \\ 4 \\ 12 \end{bmatrix}$.

(d) Give a geometric interpretation of the set of all vectors perpendicular to $\vec{u} = \begin{bmatrix} 3 \\ 4 \\ 12 \end{bmatrix}$.

55. Find two vectors that are perpendicular to $\vec{u} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$.

(b) Give a geometric interpretation of the set of all vectors perpendicular to $\vec{u} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$.

(c) Find four vectors that are perpendicular to $\vec{v} = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}$.

(d) Give a geometric interpretation of the set of all vectors perpendicular to $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}$.

56. A parallelogram with sides of equal length is called a **rhombus**. Show that the diagonals of a rhombus are perpendicular.

(*Hint*: Use two vectors $\vec{u}$ and $\vec{v}$ as adjacent sides and build the rhombus.
Then, find the diagonals in terms of those vectors and use dot products)

57. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

58. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$.

59. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

60. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

61. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} 3 \\ 4 \\ 12 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$.

62. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$

63. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

64. Find the orthogonal projection of $\vec{u} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

65. Show that for any three vectors $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$, and $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

a) $\left(\vec{v} + \vec{w}\right) \cdot \vec{u} = \vec{v} \cdot \vec{u} + \vec{w} \cdot \vec{u}$

b) $\vec{v} \cdot \left(t\,\vec{w}\right) = t\left(\vec{v} \cdot \vec{w}\right)$ [where t is a scalar.]

c) Is $(\vec{v} - \vec{w}) \cdot (\vec{v} + \vec{w}) = \vec{v}^{\,2} - \vec{w}^{\,2}$?

66. Show that for the vectors $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$.

a) $\text{proj}_{\vec{v}}(\vec{w}) \parallel \vec{v}$

b) $\left(\vec{w} - \text{proj}_{\vec{v}}(\vec{w})\right) \perp \vec{v}$

67. Show that for the vectors $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$.

a) $\text{proj}_{\vec{v}}(\vec{w}) \parallel \vec{v}$

b) $\left(\vec{w} - \text{proj}_{\vec{v}}(\vec{w})\right) \perp \vec{v}$

68. In general for $\vec{v}, \vec{w} \in \mathbb{R}^n$ show that

a) $\text{proj}_{\vec{v}}(\vec{w}) \parallel \vec{v}$

b) $\left(\vec{w} - \text{proj}_{\vec{v}}(\vec{w})\right) \perp \vec{v}$