Answers should be **exact** if possible (e.g. $\frac{3}{7}$ instead of $0.42857\cdots$, or $\sqrt{2}$ instead of $1.414\cdots$)

If exact answers are not possible, approximations should be given to **at least** 4 decimal places.

1. Convert the following angles from degrees to radians [Write answer in the form $\frac{m}{n}\pi$].

f. $48° = \frac{48°}{180°}\pi\,(\text{rad}) = \boxed{\frac{4}{15}\pi\,(\text{rad})}$ **h.** $198° = \frac{198°}{180°}\pi\,(\text{rad}) = \boxed{\frac{11}{10}\pi\,(\text{rad})}$

3. Convert the following angles from degrees to radians [Write answer in the form $\frac{m}{n}\pi$].

b. $240° = \frac{240°}{180°}\pi\,(\text{rad}) = \boxed{\frac{4}{3}\pi\,(\text{rad})}$ **h.** $528° = \frac{528°}{180°}\pi\,(\text{rad}) = \boxed{\frac{44}{15}\pi\,(\text{rad})}$

5. Convert the following angles from radians to degrees

e. $1.8\overline{6}\pi\,(\text{rad}) = (1.8666666\cdots)\pi\,(\text{rad}) = \frac{28}{15}\pi\,(\text{rad}) = \frac{28}{15}180° = 336°$

g. $0.1234\,(\text{rad}) = 0.1234\frac{180°}{\pi} = \boxed{7.0703°}$

7. Convert the following angles from radians to degrees

d. $\frac{24}{5}\pi\,(\text{rad}) = \frac{24}{5}180° = \boxed{864°}$

f. $6.625\pi\,(\text{rad}) = 6.625\cdot 180° = \boxed{1192.5°}$

11. Compute the following values using the given table

[Do not use decimal approximations]

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

a. $\sin(-315°) = \sin(360° - 315°) = \sin(45°) = \boxed{\frac{1}{\sqrt{2}}}$

b. $\cos(180°) = \cos(2\cdot 90°) = 2\cdot\cos^2(90°) - 1 = \boxed{-1}$ or alternatively

$\cos(180°) = -\cos(0°) = -1$

c. $\tan(240°) = \tan(240° - 180°) = \tan(60°) = \frac{\sin(60°)}{\cos(60°)} = \boxed{\sqrt{3}}$

d. $\csc\left(-\frac{19}{6}\pi\right) = \csc(-570°) = \csc(150°) = \frac{1}{\sin(150°)} = \frac{1}{\sin(30°)} = \boxed{2}$

e. $\sec\left(\frac{2}{3}\pi\right) = \sec(120°) = \frac{1}{\cos(120°)} = \frac{1}{-\cos(60°)} = \boxed{-2}$

f. $\cot\left(-\frac{5}{4}\pi\right) = \cot(-225°) = \cot(135°) = \frac{\cos(135°)}{\sin(135°)} = \frac{-\cos(45°)}{\sin(45°)} = \boxed{-1}$

13. Let $O = (0,0)$ and $Q = (1,0)$. The point P on the unit circle, in the **third quadrant**, has $x-$coordinate: $x_P = -0.8$. If $\angle POQ = \theta$ compute

a. $\sin(\theta) = -\sqrt{1-\cos^2(\theta)} = -\sqrt{1-(-0.8)^2} = \boxed{-0.6}$

b. $\cos(\theta) = \boxed{-0.8}$

c. $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{-0.6}{-0.8} = \boxed{\frac{3}{4} = 0.75}$

d. $\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{-0.6} = \boxed{-\frac{5}{3}}$

e. $\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{-0.8} = \boxed{-\frac{5}{4}}$

f. $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{-0.8}{-0.6} = \boxed{\frac{4}{3}}$

g. $\theta = 180° + \cos^{-1}(0.8) = \boxed{216.8699°}$

17. Find the angles θ (in degrees) in the following cases

a. $\sin(\theta) = -\frac{\sqrt{3}}{2}$ with $180° \le \theta \le 360°$ $\Rightarrow$ $\boxed{\theta = 240° \text{ or } \theta = 300°}$

b. $\cos(\theta) = -\frac{\sqrt{3}}{2}$ with $0° \le \theta \le 180°$ $\Rightarrow$ $\boxed{\theta = 150°}$

c. $\tan(\theta) = -1$ with $90° \le \theta \le 270°$ $\Rightarrow$ $\boxed{\theta = 135°}$

d. $\csc(\theta) = -2$ with $90° \le \theta \le 270°$ $\Rightarrow$ $\boxed{\theta = 210°}$

e. $\sec(\theta) = -2$ with $0° \le \theta \le 360°$ $\Rightarrow$ $\boxed{\theta = 120° \text{ or } \theta = 240°}$

f. $\cot(\theta) = -\sqrt{3}$ with $0° \le \theta \le 360°$ $\Rightarrow$ $\boxed{\theta = 120° \text{ or } \theta = 300°}$

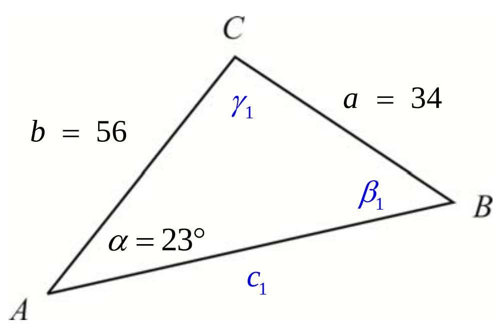
19. Find the angles θ (in degrees [exact values]) in the following cases

b. $\cos(\theta) = -\frac{1}{2}$ with $-90° \le \theta \le 90°$ $\boxed{\text{no such } \theta \text{ exist}}$

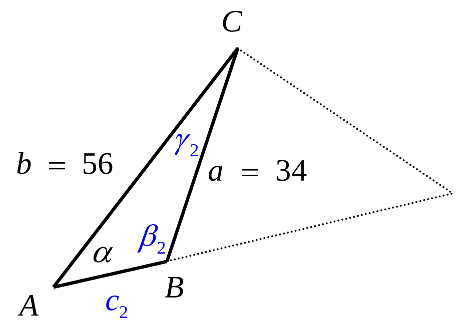
e. $\sec(\theta) = -\frac{2}{3}\sqrt{3}$ with $-720° \le \theta \le -90°$ $\Rightarrow$ $\boxed{\theta = -150° \text{ or } \theta = -210° \text{ or } \theta = -510° \text{ or } \theta = -570°}$

f. $\cot(\theta) = -1$ with $450° \le \theta \le 900°$ $\Rightarrow$ $\boxed{\theta = 495° \text{ or } \theta = 675° \text{ or } \theta = 855°}$

23. A triangle has angle $\alpha = 23°$, and sides $a = 34$ and $b = 56$



or



First compute β, note that there are two possibilities as this is an "ambiguous" case:

b. $\frac{\sin(\beta)}{56} = \frac{\sin(23°)}{34} \Rightarrow \beta_1 = \sin^{-1}\left(\frac{56\sin(23°)}{34}\right) = \boxed{40.0576°}$

and $\beta_2 = 180° - 40.0576° = \boxed{139.9424°}$

a. $\gamma_1 = 180° - 23° - \beta_1 = \boxed{116.9424°}$ and $\gamma_2 = 180° - 23° - \beta_2 = \boxed{17.0576°}$

c. $\frac{c_1}{\sin(\gamma_1)} = \frac{b}{\sin(\beta_1)} \Rightarrow c_1 = \frac{56\sin(\gamma_1)}{\sin(\beta_1)} = \boxed{77.5718}$

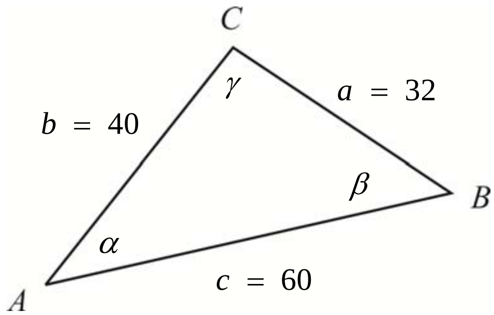
and $\frac{c_2}{\sin(\gamma_2)} = \frac{b}{\sin(\beta_2)} \Rightarrow c_2 = \frac{56\sin(\gamma_2)}{\sin(\beta_2)} = \boxed{25.5247}$

d. Area of $\Delta ABC = \frac{1}{2}ab\sin(\gamma_1) = \frac{1}{2}\cdot 34\cdot 56\sin(\gamma_1) = \boxed{848.6721}$ and

Area of $\Delta ABC = \frac{1}{2}ab\sin(\gamma_2) = \frac{1}{2}\cdot 34\cdot 56\sin(\gamma_2) = \boxed{279.2527}$

e. Distance of C to $\overleftrightarrow{AB} = 56\sin(23°) = \boxed{21.8809}$

25. A triangle has sides $a = 32$, $b = 40$ and $c = 60$



a. $\alpha = \cos^{-1}\left(\dfrac{b^2+c^2-a^2}{2bc}\right) = \boxed{29.5414°}$

b. $\beta = \cos^{-1}\left(\dfrac{a^2+c^2-b^2}{2ac}\right) = \boxed{38.0475°}$ **c.** $\gamma = 180° - \alpha - \beta = \boxed{112.4111°}$

d. Area of $\Delta ABC = \dfrac{1}{2}\cdot a\cdot b\cdot \sin(\gamma) = \boxed{591.6621}$

e. Distance of C to $\overleftrightarrow{AB} = b\sin(\alpha) = \boxed{19.7221}$

29. a. $\csc(\theta) = -3.125 \Rightarrow \sin(\theta) = \dfrac{1}{-3.125} = \boxed{-0.32}$

b. $\cos(\theta) = \sqrt{1-\sin^2(\theta)} = \boxed{0.9474}$

c. $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{-0.32}{0.9474} = \boxed{-0.3378}$

f. $\theta = \sin^{-1}(-0.32) = \boxed{-18.6629°}$

33. Use trig identities to exactly compute the following [e.g. $\sin(75°) = \sin(30° + 45°)$]

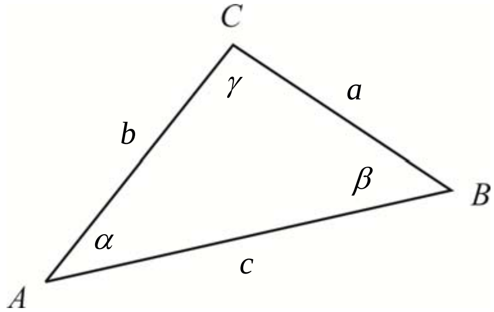
e. $\sec(165°) = \dfrac{1}{\cos(120°+45°)} = \dfrac{1}{\cos(120°)\cos(45°) - \sin(120°)\sin(45°)} = \boxed{\sqrt{2}-\sqrt{6}}$

f. $\cot(195°) = \dfrac{\cos(150°+45°)}{\sin(150°+45°)} = \dfrac{\cos(150°)\cos(45°) - \sin(150°)\sin(45°)}{\sin(150°)\cos(45°) + \cos(150°)\sin(45°)} = \boxed{2+\sqrt{3}}$

h. $\cos(2x) = 2\cos^2(x) - 1 \Rightarrow \cos(x) = \pm\sqrt{\dfrac{1+\cos(2x)}{2}}$

$$\cos(22.5°) = +\sqrt{\frac{1+\cos(45°)}{2}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

35. Find the areas of the following triangles



a. $a = 3$, $b = 4$ and $c = 5$ so that $\gamma = 90°$, and thus: Area of $\Delta ABC = \frac{1}{2} \cdot a \cdot b = \boxed{6}$

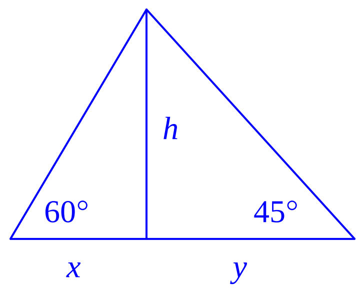
e. $\alpha = 30°$, $b = 5$ and $c = 6$: Area of $\Delta ABC = \frac{1}{2} \cdot b \cdot c \sin(30°) = \boxed{7.5}$

i. $\alpha = 60°$, $\beta = 45°$ and $c = 20$:

$$\frac{a}{\sin(\alpha)} = \frac{c}{\sin(\gamma)} \Rightarrow a = \frac{20\sin(60°)}{\sin(75°)} = 30\sqrt{2} - 10\sqrt{6} \quad \text{hence}$$

Area of $\Delta ABC = \frac{1}{2} \cdot a \cdot c \sin(\beta) = \frac{1}{2}\frac{20\sin(60°)}{\sin(75°)} 20\sin(45°) = \boxed{100\left(3 - \sqrt{3}\right)}$

Here is another method:



Let $c = 20 = x + y$ then for the height h we have simultaneously: $x\sqrt{3} = h = y$.

Hence $20 = x + x\sqrt{3}$ i.e. $x = \frac{20}{1+\sqrt{3}}$ and $y = \frac{20\sqrt{3}}{1+\sqrt{3}}$.

Therefore Area of $\Delta ABC = \frac{1}{2} \cdot 20 \cdot h = \frac{1}{2} \cdot 20 \cdot \frac{20\sqrt{3}}{1+\sqrt{3}} = \boxed{100\left(3 - \sqrt{3}\right)}$