Gather 6 observations of adult human weight (y) and height (x).
For $\alpha = 0,05$ and using these observations

- find estimates for β_1 and β_2 for regression line $y = \beta_1 + \beta_2 x$
- test $H_0 : \beta_2 = 0$ against $\beta_2 \neq 0$
- find a CI for β_1
- compute R-squared
- compute leverage for the height farthest from the mean height
- estimate the value of weight for height $x_0 = 180$ cm
- compute CI and PI for your predicted value

	x	y	x^2	y^2	xy
1	181	80	32 761	6400	14 480
2	175	70	30625	4900	12 250
3	190	100	36 100	10 000	19 000
4	191	105	36 481	11025	20055
5	185	84	34 225	7056	15 540
6	165	60	27 225	3600	9900
Σ	1087	499	197 417	42981	91 225

$\bar{x} = 181{,}1\bar{6}$

$\bar{y} = 83{,}1\bar{6}$

①

$$\det(X^T X) = n \cdot \Sigma x_i^2 - (\Sigma x_i)^2 = 2933$$

$$b_2 = \frac{n \Sigma x_i y_i}{\det(X^T X)} = 1{,}683$$

$$b_1 = \bar{y} - b_2 \bar{x} = -221{,}732$$

⑥

$$\hat{y} = -221{,}732 + 1{,}683x$$

$$\hat{y}(180) = -221{,}732 + 1{,}683 \cdot 180 = 81{,}208$$

$H_0: \beta_2 = 0$

$H_A: \beta_2 \neq 0$

$$t = \frac{b_2 - \beta_{20}}{S_{res}\sqrt{g^{22}}} \sim t(6-2)$$

$$t = 7{,}693$$

$$G = (X^T X)^{-1}$$

$$g_{11} = \frac{\Sigma x_i^2}{\det(X^T X)} = 67{,}389$$

$$g_{22} = \frac{n}{\det(X^T X)} = 0{,}002045$$

$$S_e = \Sigma y_i^2 - b_1 \Sigma y_i - b_2 \Sigma x_i y_i = 93{,}593$$

$$\Sigma(y_i - b_1 - b_2 x_i)^2$$

$$S_{res}^2 = \frac{S_e}{n-k} = \frac{S_e}{6-2} = 23{,}398$$

$$S_{res} = 4{,}837$$

② $\overline{W}_{0,05} = \langle -t_{0,975}(4); t_{0,975}(4) \rangle$

$\langle -2,776 ; 2,776$

$t \notin \overline{W} \Rightarrow H_0$ zamítáme

③

$\beta_1 \in \langle b_1 - t_{1-\frac{\alpha}{2}}(n-k) \cdot S_{res} \cdot \sqrt{g_{11}} ; b_1 + t_{1-\frac{\alpha}{2}}(n-k) \cdot S_{res} \sqrt{g_{11}} \rangle$

$\beta_1 \in \langle -331.96 ; -111.504 \rangle$

④ $R_2 = 1 - \frac{S_e}{\Sigma y_i^2 - n\bar{y}^2} = 0,937$

⑤ $h_{66} = \frac{1}{n} + \frac{(x_6 - \bar{x})^2}{\Sigma x_i^2 - n\bar{x}^2} = 0,685$ $x_0 = 180$ $(0,16934)$

⑦ CI $x_0 = 180$

$y(180) \langle \hat{y}(180) - t_{1-\frac{\alpha}{2}}(n-k) s_{res} \sqrt{d(x_0)} ; \hat{y}(180) + t_{1-\frac{\alpha}{2}}(n-k) s_{res} \sqrt{d(x_0)} \rangle$

$y(180) \in \langle 75,682 ; 86.734 \rangle$

$d(x_0) = \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma x_i^2 - n\bar{x}^2} = 0,16934$

PI

$x_0 = 180$

$y(180) \langle \hat{y}(180) - t_{1-\frac{\alpha}{2}}(n-k) s_{res} \sqrt{1+d(x_0)} ; \hat{y}(180) + t_{1-\frac{\alpha}{2}}(n-k) s_{res} \sqrt{1+d(x_0)} \rangle$

$y(180) \in \langle 66.688 ; 95.727 \rangle$