MSP_7_cviko

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Assume that $X_1, ..., X_{n_1}$ are IID binomially distributed variables, and $Y_1, ..., Y_{n_2}$ are IID geometrically distributed variables.

- find a mle estimate for probability parameter from just binomial part
- find a MLE estimate for probability parameter from joint sample $X_1, ...X_{n_1}, Y_1, ..., Y_{n_2}$
- let the parameter n for binomial distribution be $n = 100$, you observed $n_1 = 100$ binomial trials with $\sum_{i=1}^{n_1} x_i = 1008$, then you observed $n_2 = 100$ geometric trials with $\sum_{j=1}^{n_2} y_j = 1247$. Use likelihood ratio test to decide whether the value of a probability parameter changed. Use $\alpha = 5\%$

$X_i \sim Bi(n; \pi_B)$ $\qquad \pi_B = \pi_G = \pi \rightarrow$

$Y_i \sim Ge(\pi_G)$ $\qquad L(\pi) = \prod_{i=1}^{n_1} \binom{n}{x_i} \pi^{x_i} (1-\pi)^{n-x_i} \cdot \prod_{j=1}^{n_2} \pi (1-\pi)^{y_j - 1}$

$$\ell(\pi) = \underbrace{\ln\left(\prod_{i=1}^{n_1} \binom{x_i}{n}\right) + \sum_{i=1}^{n_1} x_i \cdot \ln(\pi) + \sum_{i=1}^{n_1} (n - x_i) \cdot \ln(1-\pi)} + n_2 \ln(\pi) + \sum_{j=1}^{n_2} (y_j - 1) \ln(1-\pi)$$

$$\frac{\partial \ell}{\partial \pi} = \frac{\sum_{i=1}^{n} x_i}{\pi} - \frac{n_1 n}{1-\pi} + \frac{\sum_{i=1}^{n} x_i}{1-\pi} + \frac{n_2}{\pi} - \frac{\sum_{j=1}^{n_2} y_j - n_2}{1-\pi} = 0$$

$$(1-\pi) \sum_{i=1}^{n_1} x_i - \pi n_1 n + \pi \sum_{i=1}^{n_1} x_i + (1-\pi) n_2 - \pi \sum_{j=1}^{n_2} y_j + \pi n_2 = 0$$

$$\sum_{i=1}^{n_1} x_i - \pi n_1 n + n_2 - \pi \sum_{j=1}^{n_2} y_j = 0 \rightarrow \hat{\pi} = \frac{\sum_{i=1}^{n_1} x_i + n_2}{n_1 n + \sum_{j=1}^{n_2} y_j} \doteq 0{,}0985$$

$$\frac{\partial \ell}{\partial \pi_B} = \frac{\sum x_i}{\pi_B} - \frac{n_1 n}{1-\pi_B} + \frac{\sum x_i}{1-\pi_B} = 0 \rightarrow \sum x_i - \pi n_1 n = 0$$

$$\hat{\pi}_B = \frac{\sum x_i}{n_1 n} = 0{,}1008$$

$$\hat{\pi}_G = \frac{1}{\bar{Y}} \doteq 0{,}0802$$

$$LR = 2\Big[\Big(\ln\Big(\prod_{i=1}^{n_1} \binom{x_i}{n}\Big) + \sum_{i=1}^{n_1} x_i \ln(\hat{\pi}_B) + \Big(n_1 n - \sum_{i=1}^{n_1} x_i\Big) \ln(1-\hat{\pi}_B) + n_2 \ln(\hat{\pi}_G) + \sum_{j=1}^{n_2} y_j \ln(1-\hat{\pi}_G) - n_2 \ln(1-\hat{\pi}_G)\Big)$$
$$\Big(\ln\Big(\prod_{i=1}^{n_1} \binom{x_i}{n}\Big) + \sum_{i=1}^{n_1} x_i \cdot \ln(\hat{\pi}) + \Big(n_1 n - \sum_{i=1}^{n_1} x_i\Big) \ln(1-\hat{\pi}) + n_2 \ln(\hat{\pi}) + \sum_{j=1}^{n_2} y_j \ln(1-\hat{\pi}) - n_2 \ln(1-\hat{\pi})\Big)\Big] =$$

$$2\Big[\sum x_i \ln\Big(\frac{\hat{\pi}_B}{\hat{\pi}}\Big) + \Big(n_1 n - \sum_{i=1}^{n_1} x_i\Big) \ln\Big(\frac{1-\hat{\pi}_B}{1-\hat{\pi}}\Big) + n_2 \ln\Big(\frac{\hat{\pi}_G}{\hat{\pi}}\Big) + \Big(\sum_{j=1}^{n_2} y_j - n_2\Big) \ln\Big(\frac{1-\hat{\pi}_G}{1-\hat{\pi}}\Big)\Big] =$$

$\doteq 5{,}586$ $\qquad \overline{W}_{0{,}05} = \langle 0; \chi^2_{0{,}95}(1)) = \langle 0; 3{,}841)$

$LR \notin \overline{W}_{0{,}05} \Rightarrow$ ZAMÍTÁM

Let $X_1, ..., X_9$ be IID distributed Bernoulli trials.

- Identify a distribution of $\sum_{i=1}^{9} x_i$
- If $\sum_{i=1}^{9} x_i = 3$ test a hypothesis $H_0 : \pi = 1/2$ against $H_1 : \pi < 1/2$ for $\alpha = 0,15$
- Estimate β and power, if the critical value of π to detect is $\pi = 1/4$

$\sum X_i \sim Bi(9; \pi)$

$x_i = 3 \rightarrow P(X \leq 3 \mid \pi = \frac{1}{2}) \doteq 0{,}254 > \alpha \rightarrow$ NEZ H_0

$W_\alpha = \{0; 1; 2\}$ - protože $p(0) + p(1) + p(2) < \alpha$

$\beta \rightarrow$ NEZ $\wedge$ NEPLATÍ $\rightarrow P(X \in \overline{W}_\alpha \mid H_1) = P(X \geq 3 \mid \pi = \frac{1}{4}) \doteq 0{,}4$

Assume that the observations 11, 5; 6, 7; 7, 8; 7, 1; 10, 3; 12, 2; 11, 4; 12, 6; 9, 8; 10, 9 follow a normal distribution with unknown parameters μ, σ.

- Compute 95% CI's for both μ and σ.
- Using $\alpha = 5\%$ test a hypothesis $H_0 : \mu = 11$ against $H_1 : \mu \neq 11$.
- Derive a test for $H_0 : \sigma^2 = \sigma_0^2$.
- Test $H_0 : \sigma^2 = 2$ against $H_1 : \sigma^2 \neq 2$ using $\alpha = 1\%$

$\bar{X} = 10{,}03$ $\qquad s^2(x) = 4{,}542$

$s(x) = \sqrt{\frac{\wedge}{n-1} \cdots} \doteq 2{,}131$

$\mu \in \langle 8{,}505; 11{,}555)$

$\sigma^2 \in \langle 2{,}149; 15{,}140)$

$\sigma \in \langle 1{,}466; 3{,}891)$

$t = -1{,}44$ $\qquad t_{0{,}975}(9) \doteq 2{,}262$

$t \in \overline{W}_\alpha$ $\qquad \overline{W}_\alpha = (-2{,}262; 2{,}262)$

$\Rightarrow$ NEZAMÍTÁM

1) Rozptyl nezávisí na μ

2) pro $N(\mu, 1)$ platí $\frac{(n-1) S^2(X)}{1} \sim \chi^2(n-1)$

3) pro $N(\mu; \sigma_0^2) \rightarrow \frac{(n-1) \cdot S^2(X)}{\sigma_0^2} \sim \chi^2(n-1)$

$\Rightarrow$ KDYŽ PLATÍ H_0 $\quad t = \frac{(n-1) S^2(X)}{\sigma_0^2} \sim \chi^2(n-1)$

$t = 20{,}44$ $\qquad \overline{W}_{0{,}01} = (1{,}735, 23{,}589)$

$t \in \overline{W} \Rightarrow$ NEZ.