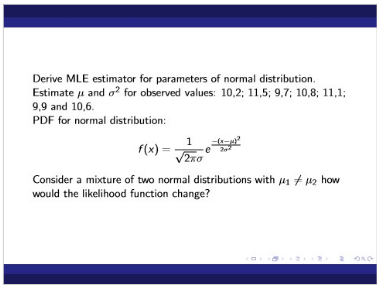
MSP_4_cviko

pondělí 7. října 2024 18:20



pro n pozorování

1) $L(\mu,\sigma^2)=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$

$l(\mu,\sigma^2)=\sum_{i=1}^{n}\ln\left(\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}\right)=$

$\sum_{i=1}^{n}\left[\ln\left(\frac{1}{\sqrt{2\pi}}\right)+\ln\left(\frac{1}{\sigma}\right)+\frac{-(x_i-\mu)^2}{2\sigma^2}\right]=$

$n\cdot\ln\frac{1}{\sqrt{2\pi}}+n\ln\left(\frac{1}{\sigma}\right)-\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$

$\frac{\partial l}{\partial\mu}=-\sum_{i=1}^{n}\frac{\cancel{2}(x_i-\mu)}{\cancel{2}\sigma^2}=\frac{n\mu}{\sigma^2}-\frac{\sum_{i=1}^{n}x_i}{\sigma^2}\;\Big|\;\frac{\partial l}{\partial\mu}=0\to\frac{n\mu}{\sigma^2}=\frac{\sum_{i=1}^{n}x_i}{\sigma^2}\to\hat{\mu}=\frac{\sum_{i=1}^{n}x_i}{n}=\bar{x}$

$\frac{\partial l}{\partial\sigma}=-\frac{n}{\sigma}+\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^3}\;\Big|\;\frac{\partial l}{\partial\sigma}=0\to\frac{n}{\sigma}=\frac{1}{\sigma^3}\cdot\sum_{i=1}^{n}(x_i-\bar{x})^2\to\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$

$\mu\to\hat{\mu}=\bar{x}$

$\hat{\mu}=10{,}5429$

$\hat{\sigma}^2=0{,}362449$

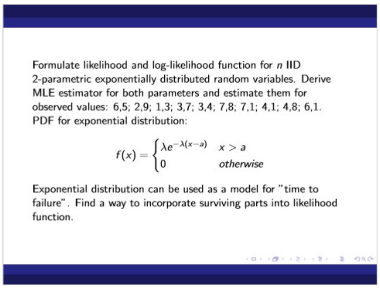
2) Směs A → 2 složky n_1 a n_2 pozorování ZNÁMÉ

$L(\mu_1;\mu_2;\sigma^2)=\prod_{i=1}^{n_1}\frac{1}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{(x_i-\mu_1)^2}{2\sigma^2}}\cdot\prod_{j=1}^{n_2}\frac{1}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{(x_j-\mu_2)^2}{2\sigma^2}}$

Směs B → n_1 a n_2 NEZNÁMÉ

$L(\mu_1;\mu_2;\sigma^2;p)=\prod_{i=1}^{N}\left[p\cdot\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{(x_i-\mu_1)^2}{2\sigma^2}}+(1-p)\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{(x_i-\mu_2)^2}{2\sigma^2}}\right]$

Problém → log moc nepomůže pro nalezení max



1) $L(a;\lambda)=\prod_{i=1}^{n}\lambda\cdot e^{-\lambda(x_i-a)}\to l(a;\lambda)=n\cdot\ln(\lambda)-\sum_{i=1}^{n}\lambda(x_i-a)$

$\frac{\partial l}{\partial\lambda}=\frac{n}{\lambda}-\sum_{i=1}^{n}(x_i-a)\to\hat{\lambda}=\frac{n}{\sum_{i=1}^{n}(x_i-a)}=\frac{1}{\bar{x}-\min(x_i)}$

$\frac{\partial l}{\partial a}=n\lambda\to\hat{a}=\min_i(x_i)$

$\bar{x}=4{,}77\to\hat{\lambda}=0{,}288$

$\min(x_i)=1{,}3$

2) Censorování ⇒ v čase x_c ještě fungoval

Necenz.: $L(\lambda;a)=\prod_{i=1}^{n_1}\lambda\cdot e^{-\lambda(x-a)}$

Cenz.: $L(\lambda;a)=\prod_{j=1}^{n}(1-F(x_j))=\prod_{j=1}^{n}\left[1-(1-e^{-\lambda(x_j-a)})\right]=\prod_{j=1}^{n_2}e^{-\lambda(x_j-a)}$

Celkem: $L(\lambda;a)=\prod_{i=1}^{n_1}\lambda e^{-\lambda(x_i-a)}\cdot\prod_{j=1}^{n_2}e^{-\lambda(x_j-a)}$

pokud $n_1=0\to l(\lambda,a)=\sum_{j=1}^{n_2}-\lambda(x_j-a)\to\frac{\partial l}{\partial\lambda}=-\sum_{j=1}^{n_2}(x_i-a)$

$\frac{\partial l}{\partial\lambda}$ není funkcí λ ⇒ λ nejde odhadnout

Amount of latecomers into MSP lessons can be viewed as a random variable with Poisson distribution (see PMF). Derive MLE for parameter λ and estimate lambda given following observations from last year: 1; 2; 4; 2; 1; 4; 1; 1; 3; 5.

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

Where $x \in \mathbb{N} \cup \{0\}$

1) $L(\lambda) = \prod_{i=1}^{n} \frac{\lambda^{x_i}}{x_i!} e^{-\lambda}$ $\quad \ell(\lambda) = \sum_{i=1}^{n} x_i \ln(\lambda) - \sum_{i=1}^{n} \ln(x_i!) - n\lambda$

$\frac{\partial \ell}{\partial \lambda} = \frac{\sum x_i}{\lambda} - n \to \bar{x} = \hat{\lambda}$

$\bar{x} = 2{,}4$

Bonus

Try to estimate population size of some species via capture-recapture scheme. Capture-recapture uses hypergeometric distribution vith PMF:

$$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

python code

KÓD V ODKAZU, LOGIN DO GOOGLE