

Derive MLE estimator for parameters of normal distribution. Estimate μ and σ^2 for observed values: 10.2; 11.5; 9.7; 10.8; 11.1; 9.9 and 10.6.

PDF for normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Consider a mixture of two normal distributions with $\mu_1 \neq \mu_2$ how would the likelihood function change?

$$\hat{\mu} = 10,5429$$

$$\hat{\sigma}^2 = 0,362449$$

$$1) L(\mu, \sigma^2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$l(\mu, \sigma^2) = \sum_{i=1}^n \ln \left(\frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \right) =$$

$$\sum_{i=1}^n \left[\ln \left(\frac{1}{\sqrt{2\pi}} \right) + \ln \left(\frac{1}{\sigma} \right) - \frac{(x_i-\mu)^2}{2\sigma^2} \right] =$$

$$n \cdot \ln \left(\frac{1}{\sqrt{2\pi}} \right) + n \ln \left(\frac{1}{\sigma} \right) - \sum_{i=1}^n \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial l}{\partial \mu} = - \sum_{i=1}^n \frac{2(x_i-\mu)}{2\sigma^2} = -\frac{n\mu}{\sigma^2} - \frac{\sum_{i=1}^n x_i}{\sigma^2} \quad \left| \frac{\partial l}{\partial \mu} = 0 \rightarrow \frac{n\mu}{\sigma^2} = -\frac{\sum_{i=1}^n x_i}{\sigma^2} \rightarrow \hat{\mu} = -\frac{\sum_{i=1}^n x_i}{n} = \bar{x} \right.$$

$$\frac{\partial l}{\partial \sigma} = -\frac{n}{\sigma} + \sum_{i=1}^n \frac{(x_i-\mu)^2}{\sigma^3} \quad \left| \frac{\partial l}{\partial \sigma} = 0 \rightarrow \frac{n}{\sigma} = \frac{1}{\sigma^3} \cdot \sum_{i=1}^n (x_i-\mu)^2 \rightarrow \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (x_i-\bar{x})^2 \right.$$

$$\mu \rightarrow \hat{\mu} = \bar{x}$$

2) Smis A \rightarrow 2 složby n_1 a n_2 pozorování ZNÁHE

$$L(\mu_1, \mu_2, \sigma^2) = \prod_{i=1}^{n_1} \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_i-\mu_1)^2}{2\sigma^2}} \cdot \prod_{j=1}^{n_2} \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_j-\mu_2)^2}{2\sigma^2}}$$

Smis B $\rightarrow n_1$ a n_2 NEZNÁHE

$$L(\mu_1, \mu_2, \sigma^2, p) = \prod_{i=1}^N \left[p \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_i-\mu_1)^2}{2\sigma^2}} + (1-p) \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_i-\mu_2)^2}{2\sigma^2}} \right]$$

Problém \rightarrow log moc nepomůže pro nalezení max

Formulate likelihood and log-likelihood function for n IID 2-parametric exponentially distributed random variables. Derive MLE estimator for both parameters and estimate them for observed values: 6.5; 2.9; 1.3; 3.7; 3.4; 7.8; 7.1; 4.1; 4.8; 6.1.

PDF for exponential distribution:

$$f(x) = \begin{cases} \lambda e^{-\lambda(x-a)} & x > a \\ 0 & \text{otherwise} \end{cases}$$

Exponential distribution can be used as a model for "time to failure". Find a way to incorporate surviving parts into likelihood function.

$$1) L(a, \lambda) = \prod_{i=1}^n \lambda \cdot e^{-\lambda(x_i-a)} \rightarrow l(a, \lambda) = n \cdot \ln(\lambda) - \sum_{i=1}^n \lambda(x_i-a)$$

$$\frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^n (x_i-a) \rightarrow \hat{\lambda} = \frac{n}{\sum_{i=1}^n (x_i-a)} = \frac{1}{\bar{x} - \min(x_i)}$$

$$\frac{\partial l}{\partial a} = n\lambda \rightarrow \hat{a} = \min(x_i)$$

2) Cenzurování \Rightarrow n case x_c ještě fungoval

$$\text{NeCense: } L(\lambda; a) = \prod_{i=1}^{n_1} \lambda \cdot e^{-\lambda(x_i-a)}$$

$$\text{Cense: } L(\lambda; a) = \prod_{j=1}^{n_1} [1 - F(x_j)] = \prod_{j=1}^{n_1} [1 - e^{-\lambda(x_j-a)}] = \prod_{j=1}^{n_2} e^{-\lambda(x_j-a)}$$

$$\text{Celkem: } L(\lambda; a) = \prod_{i=1}^{n_1} \lambda e^{-\lambda(x_i-a)} \cdot \prod_{j=1}^{n_2} e^{-\lambda(x_j-a)}$$

$$\text{pokud } n_1 = 0 \rightarrow l(\lambda, a) = \sum_{j=1}^{n_2} -\lambda(x_j-a) \rightarrow \frac{\partial l}{\partial \lambda} = -\sum_{j=1}^{n_2} (x_j-a)$$

$$\frac{\partial l}{\partial \lambda} \text{ není funkce } \lambda \Rightarrow \lambda \text{ nejde odhadnout}$$

$$\bar{x} = 4,77 \rightarrow \hat{\lambda} = 0,288$$

$$\min(x_i) = 1,3$$

Amount of latecomers into MSP lessons can be viewed as a random variable with Poisson distribution (see PMF). Derive MLE for parameter λ and estimate lambda given following observations from last year: 1; 2; 4; 2; 1; 4; 1; 1; 3; 5.

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

Where $x \in \mathbb{N} \cup \{0\}$

$$1) L(\lambda) = \prod_{i=1}^n \frac{\lambda^{x_i}}{x_i!} e^{-\lambda} \quad \ell(\lambda) = \sum_{i=1}^n x_i \ln(\lambda) - \sum_{i=1}^n \ln(x_i!) - n\lambda$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{\sum x_i}{\lambda} - n \rightarrow \bar{x} = \hat{\lambda}$$

$$\bar{x} = 2,4$$

Bonus

Try to estimate population size of some species via capture-recapture scheme. Capture-recapture uses hypergeometric distribution with PMF:

$$p(x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$$

python code

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