

Appendix A: 连续时间的傅里叶变换对

傅里叶变换: $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$

傅里叶反变换: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega$

	Time Function	Fourier Transform
1	$x(t) = \begin{cases} 1, & t \leq T_1, \\ 0, & t > T_1. \end{cases}$	$\frac{2 \sin(\omega T_1)}{\omega}$
2	$\frac{\sin(Wt)}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \omega < W \\ 0, & \omega > W \end{cases}$
3	$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j\omega}$
4	$t \exp(-at)u(t), \quad a > 0$	$\frac{1}{(a + j\omega)^2}$
5	$\delta(t)$	1
6	$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
7	$x(t) = 1$	$2\pi\delta(\omega)$
8	$\delta(t - t_0)$	$\exp(-j\omega t_0)$
9	$\exp(j\omega_0 t)$	$2\pi\delta(\omega - \omega_0)$
10	$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
11	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
12	$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - \frac{2\pi k}{T})$
13	$x(t) = \begin{cases} 1, & t \leq T_1 \\ 0, & T_1 < t \leq \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin(k\omega_0 T_1)}{k} \delta(\omega - k\omega_0)$

14	$\frac{t^{n-1}}{(n-1)!} \exp(-at)u(t), \quad a > 0$	$\frac{1}{(a + j\omega)^n}$
15	$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$

Appendix B: 离散时间的傅里叶变换对

傅里叶变换: $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$

傅里叶反变换: $x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})e^{j\omega n} d\omega$

	Time Function	Fourier Transform
1	$x[n] = \begin{cases} 1, & n \leq N_1, \\ 0, & n > N_1. \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
2	$\frac{\sin[Wn]}{\pi n}, \quad 0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \leq \omega \leq W \\ 0, & W < \omega \leq \pi \end{cases}$ $X(\omega)$ periodic with period 2π
3	$a^n u[n], \quad a < 1$	$\frac{1}{1 - ae^{-j\omega}}$
4	$(n+1)a^n u[n], \quad a < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$
5	$\delta[n]$	1
6	$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$
7	$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$
8	$\delta[n - n_0]$	$\exp(-j\omega n_0)$
9	$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$
10	$\cos[\omega_0 n]$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$

11	$\sin[\omega_0 n]$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$
12	$\sum_{n=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(\omega - \frac{2\pi k}{N})$
13	$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & N_1 < n \leq \frac{N}{2} \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - \frac{2\pi k}{N})$
14	$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad a < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$
15	$\sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - \frac{2\pi k}{N})$

CT 傅里叶级数: $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \quad a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$

DT 傅里叶级数: $x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \quad a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n}$

$e^{jx} = \cos(x) + j \sin(x)$

$y(t) = x(t) * h(t) \equiv \int_{-\infty}^{+\infty} x(\tau) h(t - \tau) d\tau$

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n - k]$