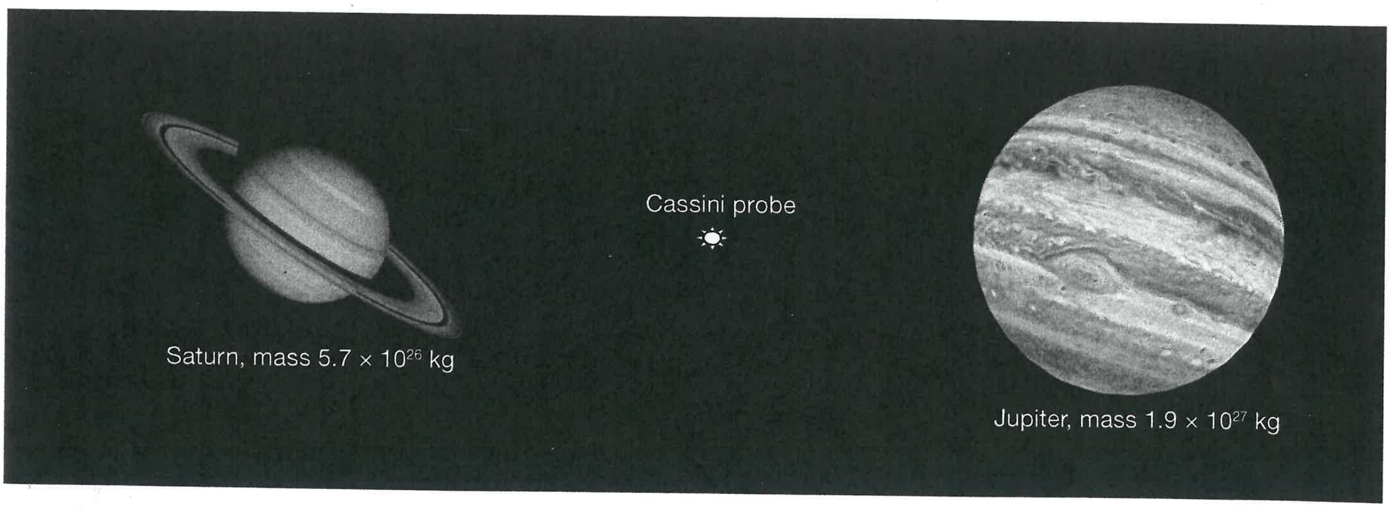
4

## QUESTIONS

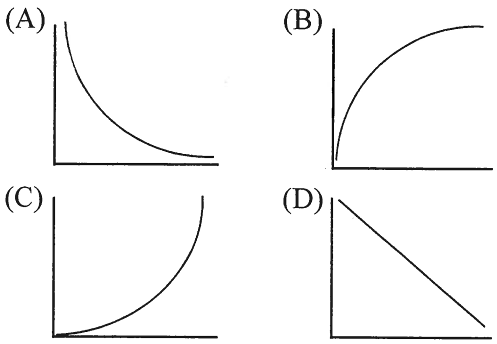
1. The radius of the Earth is 6378 km and its mass is $5.974 \times 10^{24}$ kg. Calculate the acceleration due to gravity on top of Mount Everest, 8800 m above sea level.
2. You have a mass of 60 kg. Predict your 'loss' in weight at the top of Sydney Tower, 305 m above sea level.
3. Calculate the gravitational force between you and another person (both mass 60 kg), 5 m apart.
4. The Moon's mass is $7.35 \times 10^{22}$ kg. Its diameter is 3467 km. Compare its gravitational acceleration to that of Pluto, mass $1.27 \times 10^{22}$ kg, diameter 2320 km.
5. The mass of Ganymede is $1.5 \times 10^{23}$ kg and its diameter is about 5270 km.
   (a) Calculate the acceleration due to gravity on the surface of Ganymede.
   (b) How does this compare with Earth's gravity?
   (c) Calculate the weight of a 10 kg mass on Ganymede's surface.
6. Two moons have masses $M$ and $2M$ and radii $R$ and $2R$ respectively. Compare their accelerations due to gravity.
7. Predict the effect on the gravitational force between two objects of:
   (a) Doubling the distance between them.
   (b) Doubling both masses.
   (c) Halving one mass and the distance between them.
8. Compare the gravitational force of attraction between masses X and Y, 10 kg and 20 kg respectively, 5 m apart, on the Moon ($g = 1.6$ m s$^{-2}$) and on Earth.
9. A space shuttle is orbiting Earth in a gravitational field of strength 8.9 N kg$^{-1}$. The mass of the Earth is $6 \times 10^{24}$ kg and its radius is 6380 km.
   (a) What is the altitude of the shuttle?
   (b) What is the weight of a 65 kg astronaut in this shuttle?
10. At some point between the Moon and the Earth will be a zero gravity point. Given the mass of the Moon and Earth as $7.35 \times 10^{22}$ kg and $6 \times 10^{24}$ kg respectively and the distance between their centres as 385 000 km, calculate how far this point is from Earth.
11. A satellite in orbit around a planet at a distance of $8.0 \times 10^{7}$ km has $3.0 \times 10^{10}$ J of kinetic energy. What is the weight of this satellite?
12. An astronaut weighs $W$ on Earth. What would the astronaut weigh on a planet which had twice the mass of Earth and half its radius?
13. An object of mass 12 kg weighs 156 N on planet X. What is the magnitude of the acceleration due to gravity on planet X?
14. An object has a mass of 9.0 kg on Earth and a weight of 101.43 N on Saturn. According to this data, what is the value of the acceleration due to gravity on Saturn?
15. An object weighs 147 N on Earth and 84 N on planet X. What is the acceleration due to gravity on planet X?
16. In 2002 the space probe *Cassini*, mass 2200 kg, was directly between Saturn and Jupiter as shown in the diagram (not to scale).




Find the net force on Cassini when it is $4.2 \times 10^{11}$ km from Jupiter and $3.9 \times 10^{11}$ km from Saturn.

17. Which of the following statements does *not* relate to Newton's laws of gravitation?
(A) A gravitational force exists between all masses.
(B) This gravitational force depends on the masses of the objects.
(C) The gravitational force depends inversely on the square of the distance between the objects.
(D) The gravitational force depends on the universal gravitational constant.

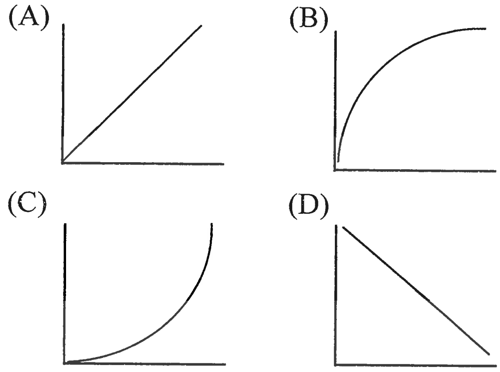
18. Which graph best shows the relationship between gravitational force and distance from the centre of a planet?




19. (a) What is the gravitational force between two objects, each of mass 40 kg when they are 6 m apart?
(A) $1.78 \times 10^{-8}$ N
(B) $2.96 \times 10^{-9}$ N
(C) $4.45 \times 10^{-10}$ N
(D) $7.41 \times 10^{-11}$ N
(b) What would the new force be if both masses and the distance between them were doubled?

20. Two planets, X and Y have masses $4M$ and $9M$ and diameters $8R$ and $18R$ respectively. What is the ratio of their gravitational forces on their surfaces?
(A) 2 : 9 (B) 4 : 9 (C) 9 : 4 (D) 16 : 81

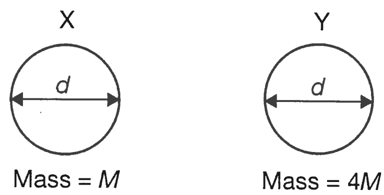
21. Which graph best shows the relationship between gravitational force and the mass of a planet?



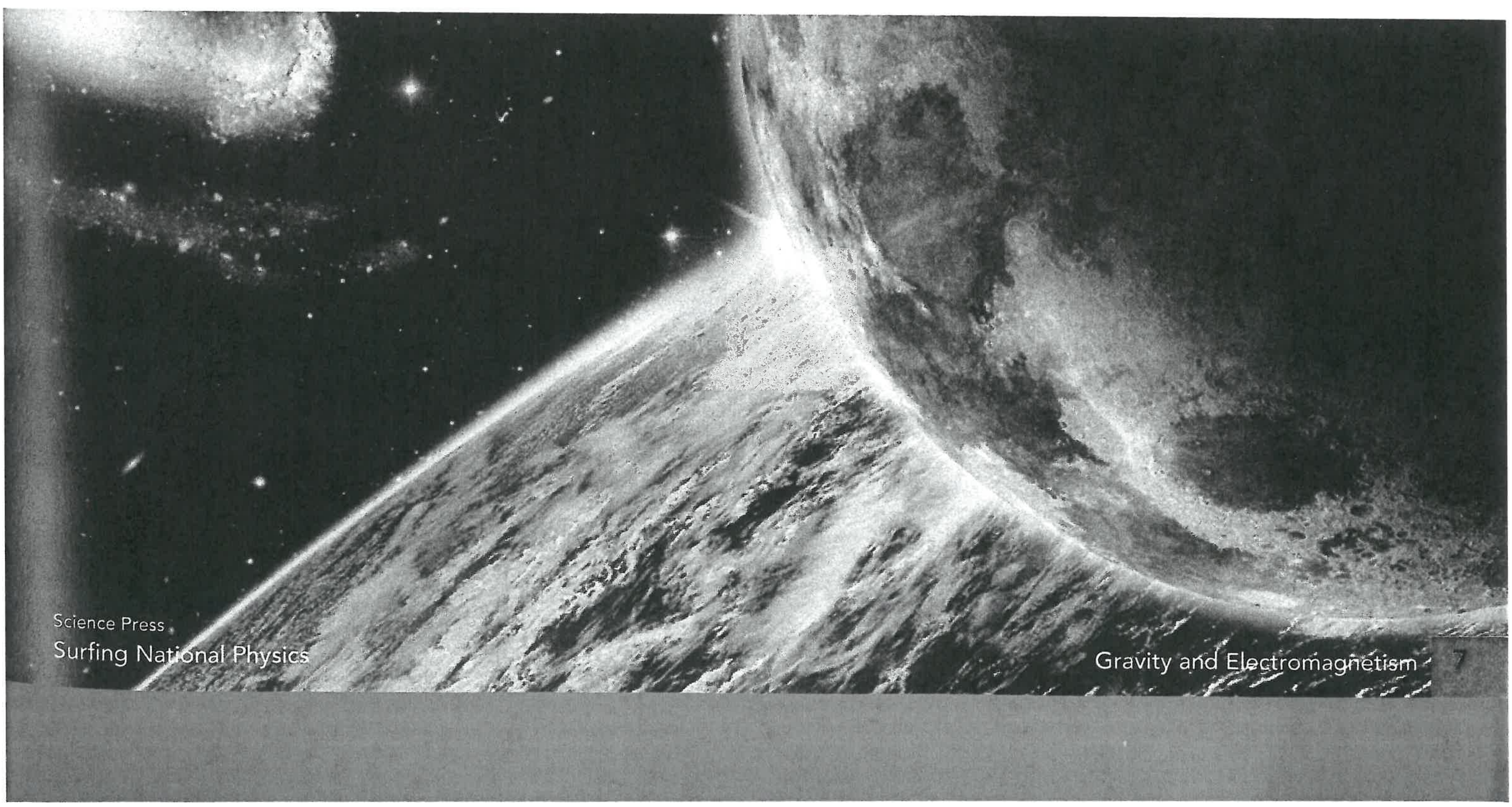

22. An astronaut has weight $W$ on planet X. What would be his weight on planet Y which has half the mass and half the diameter of X?
(A) $0.25W$ (B) $0.5W$ (C) $2W$ (D) $4W$

23. An astronaut in orbit 6000 km above the Earth experiences a gravitational force of $F$. His spaceship moves to a 12 000 km orbit. What is the new gravitational force the astronaut experiences?
(A) $0.25F$ (B) $0.44F$ (C) $0.5F$ (D) $2.25F$

24. Consider the two planets, X and Y shown below.




The gravitational force on X due to Y is $F$. What is the gravitational force on Y due to X?
(A) $0.25F$ (B) $0.5F$ (C) $F$ (D) $4.0F$



# 8 Changes in Gravitational Potential Energy

- If an object is moved from one position in a gravitational field, then the work done on it will be equal to its change in gravitational potential energy.
- If the work is done by the field to move it to a lower altitude above a planet, then this will also be equal to the increase in the kinetic energy of the object. The gravitational potential energy will decrease. Mathematically it will become a larger negative value.
- If the work is done by an external force (moving the mass to a higher altitude), then this will appear as an increase in gravitational potential energy. Mathematically it will become a smaller negative value.

Imagine moving an object from higher to lower altitude. From the diagram we can see that the initial gravitational potential energy of the object is:

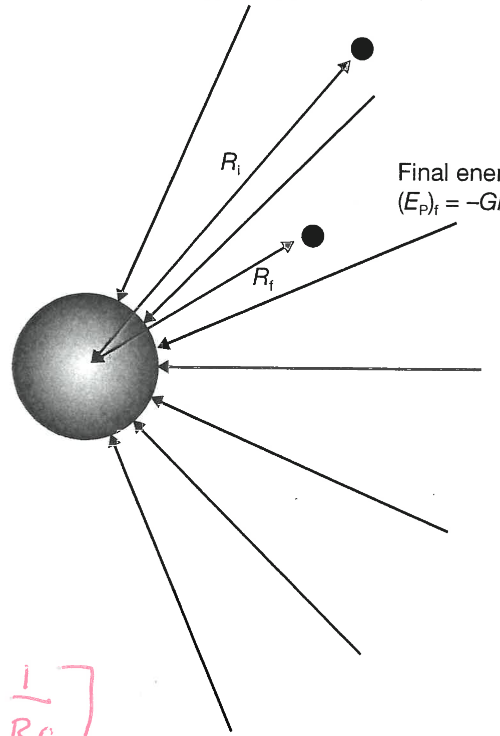
$$(E_P)_i = \frac{-GmM}{R_i}$$

And its final gravitational potential energy is:

$$(E_P)_f = \frac{-GmM}{R_f}$$

Therefore the change in gravitational potential energy, $\Delta E_P$ is given by:

$$\Delta E_P = (E_P)_f - (E_P)_i$$
$$= \frac{-GmM}{R_f} - \frac{-GmM}{R_i}$$




That is: $\Delta E_P = \frac{GmM}{R_i} - \frac{GmM}{R_f}$

Or $\Delta E_P = \cancel{GmM\left[\frac{1}{R_i - R_f}\right]}$ $= GMM\left[\frac{1}{R_i} - \frac{1}{R_f}\right]$

## QUESTIONS

1. (a) Calculate the change in the gravitational potential energy of a 90 kg satellite if its orbit is changed from 620 km above the surface of the Earth to 420 km altitude. The mass of the Earth is $6 \times 10^{24}$ kg and its radius is $6.38 \times 10^6$ m.
   (b) If the satellite's initial orbital speed was 7561 m $s^{-1}$, and gravitational potential energy was all changed to kinetic energy, by how much would the speed of the satellite change?
2. A satellite is in orbit at a distance $d$ from the centre of a planet. It takes $W$ joules of work to raise the orbit to distance $2d$. How much additional work is required to place it into an orbit at a distance of $3d$?
3. If it requires $E$ joules of energy to raise a mass from the surface of the Earth to an altitude of 6000 km above the Earth, how much energy is required to raise it from the surface to an altitude of 12 000 km?
4. A satellite of mass 500 kg is boosted from an orbit of altitude 10 000 km to one of altitude 20 000 km. Given the diameter of Earth as 12 756 km, its mass as $6 \times 10^{24}$ kg, calculate the change in the gravitational potential energy of the satellite.

5. A satellite of mass 500 kg is raised from an orbital radius of $7.3 \times 10^6$ m to a geostationary orbit of orbital radius $4.2 \times 10^7$ m. Taking the mass of the Earth as $6 \times 10^{24}$ kg, what is the change in its gravitational potential energy?
(A) +3641.5 J
(B) –3641.5 J
(C) $+2.26 \times 10^{10}$ J
(D) $-2.26 \times 10^{10}$ J

6. The gravitational potential energy between two objects is $E$. The distance between the objects is halved. How much work was done in moving the objects closer together?
(A) $0.5E$ (B) $1.0E$ (C) $2.0E$ (D) $4.0E$

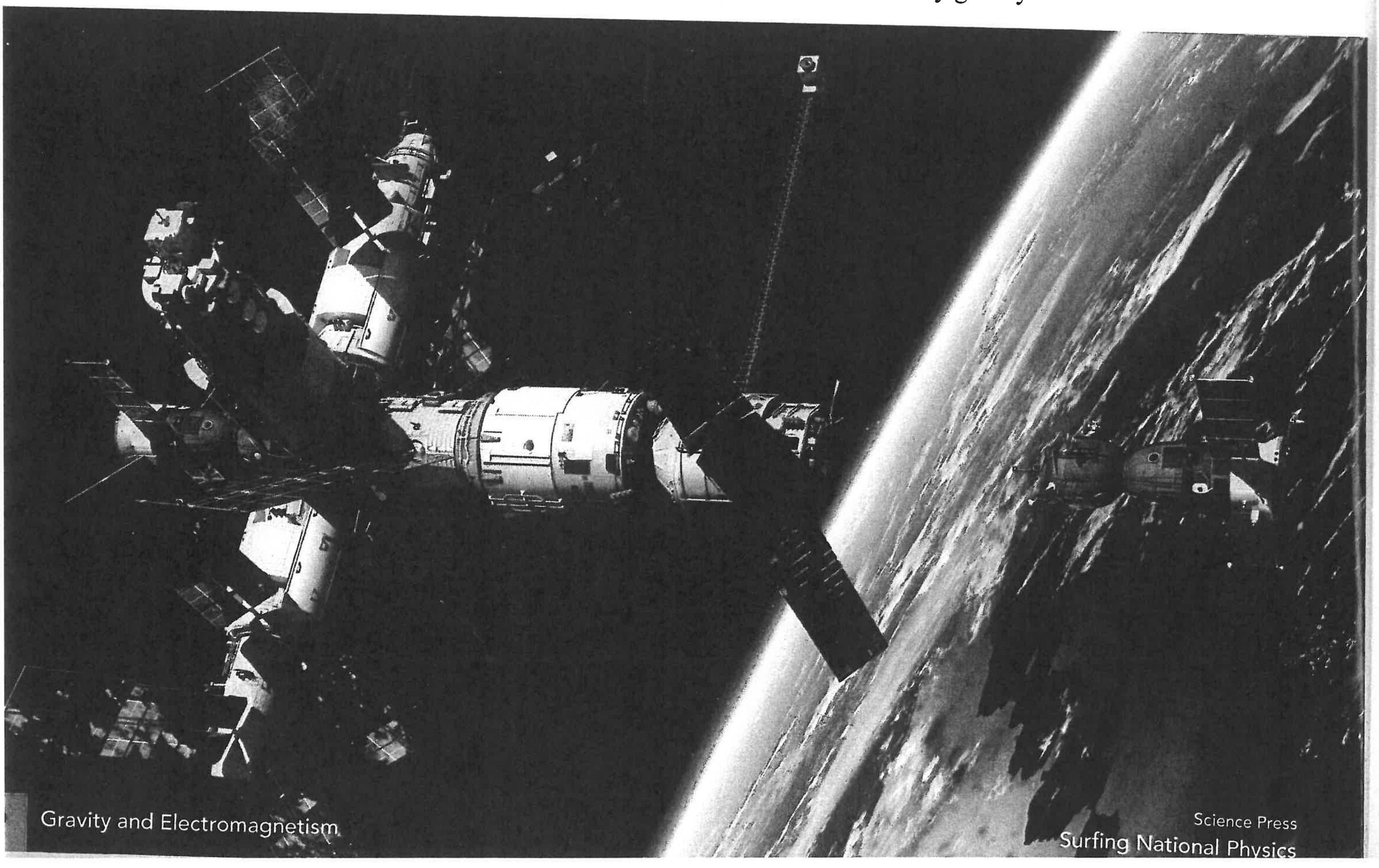
7. What is the relationship between the work done on an object and its gravitational potential energy?
(A) The work done on the object equals its gravitational potential energy.
(B) The work done on it equals the change in its gravitational potential energy.
(C) The work done on it equals the increase in its gravitational potential energy.
(D) The work done on it equals the decrease in its gravitational potential energy.

8. What work needs to be done against gravity to lift a 200 kg satellite to an altitude of 3000 km?
(A) $+1.05 \times 10^3$ J
(B) $-4.00 \times 10^9$ J
(C) $+4.00 \times 10^9$ J
(D) $+8.48 \times 10^{12}$ J

9. A satellite is moved to a higher altitude orbit. Which statement about this satellite is correct?
(A) Work is done by the satellite engines and its $E_P$ increases.
(B) Work is done by the satellite engines and its $E_P$ decreases.
(C) Work is done by gravity and its $E_P$ increases.
(D) Work is done by gravity and its $E_P$ decreases.

10. A rocket is in orbit distance $R$ from the centre of the Earth. At this height it has gravitational potential energy equal to $E$ joules. The rocket is then boosted to a higher orbit, orbital radius $3R$. How has its gravitational potential energy changed?
(A) Increased by one third.
(B) Decreased by one third.
(C) Increased by two-thirds.
(D) Decreased by two-thirds.

11. A rocket is in orbit distance $R$ from the centre of the Earth. At this height it has gravitational potential energy ($E_P$) equal to $E$ joules. The rocket is then boosted to an orbit where its $E_P$ is $3E$. Which statement about this rocket is correct?
(A) It is in a higher orbit and $2E$ work has been done on it by its engines.
(B) It is in a higher orbit and $2E$ work has been done on it by gravity.
(C) It is in a lower orbit and $2E$ work has been done on it by its engines.
(D) It is in a lower orbit and $2E$ work has been done on it by gravity.



We now have two equations to describe the force that attracts an orbiting satellite to Earth – Newton's gravitation equation and the equation for centripetal force. Equating these we get the following.

$$F_c = \frac{m_{satellite} v^2}{r} = F_g = \frac{G m_{satellite} m_{Earth}}{r^2}$$

Rearranging: $v = \sqrt{\frac{G m_{Earth}}{r}}$

Where $r$ = radius of Earth + altitude
$= 6.38 \times 10^6$ m + altitude
$m_{Earth} = 5.97 \times 10^{24}$ kg
$G = 6.67 \times 10^{-11}$ N m$^2$ kg$^{-1}$

**Notice that the orbital speed does not depend on the mass of the satellite.** From this equation we can see that the orbital speed of a satellite is inversely proportional to the square root of its distance from the centre of the Earth (or appropriate planet) – that is, the lower the orbit, the faster the satellite needs to go to stay in a stable orbit.

**Example:** A satellite has a mass of 500 kg and orbits at an altitude of 200 km. Find its orbital speed.

*Solution:* Data:

$r = 6\,380\,000 + 200\,000$
$= 6\,580\,000$ m
$G = 6.67 \times 10^{-11}$ N m$^2$ kg$^{-1}$
$M_{Earth} = 5.974 \times 10^{24}$ kg

Calculation:

From $v = \sqrt{\frac{G m_{Earth}}{r}}$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.974 \times 10^{24}}{6\ 580\ 000}}$$

$= 7782$ m s$^{-1}$

## QUESTIONS

1. Three moons around planet X have masses $M$, $4M$ and $9M$.
   (a) If the moons are all the same distance from the planet's centre, calculate the ratio of their orbital speeds.
   (b) If the distances of these moons from the planet centre are $R$, $4R$ and $9R$ respectively, calculate the ratio of their orbital speeds.
2. Three identical moons are in orbit around planets of mass $M$, $4M$ and $9M$. The planets have the same radii.
   (a) If they have the same orbital speed, find the ratio of their distances from the planet centres.
   (b) If these distances are the same, find the ratio of their orbital speeds.
3. Calculate the orbital speed needed to keep a satellite in a stable orbit at an altitude of 1500 km above the Earth.
4. An Earth satellite has an orbital speed of 28 000 km h$^{-1}$. Calculate its altitude.
5. The mass of Mars is $6.42 \times 10^{23}$ kg and its radius is 6794 km. Calculate the orbital speed needed to keep a Mars probe in a 200 km orbit.
6. Jupiter has a mass of $1.9 \times 10^{27}$ kg. Its diameter is 142 984 km. Calculate the orbital speed needed to keep a satellite in orbit at 200 km.
7. Calculate the altitude that a satellite would orbit Jupiter if its orbital speed is $3 \times 10^4$ m s$^{-1}$.
8. The mass of the Sun is $1.99 \times 10^{30}$ kg. Its diameter is 1 392 530 km. The diameter of Earth is 12 756 km. The distance between the Sun and the Earth is 150 000 000 km. Calculate the orbital speed of the Earth.
9. If the distance between the centre of the Earth and the Sun is 150 000 000 km, calculate the orbital speed of the Earth.

# 22 Kepler's Laws of Planetary Motion

At the time when he developed them (1609), Kepler's laws were radical claims. The belief at the time was that orbits should be based on perfect circles, and Kepler's mathematics actually supported the Copernican view of the Universe. A circle is a form of ellipse, and most of the planets follow orbits which can be closely approximated as circles, so it is only after careful observation and analysis that the orbits of the planets are found to be elliptical.

## Assumption: Orbits circular

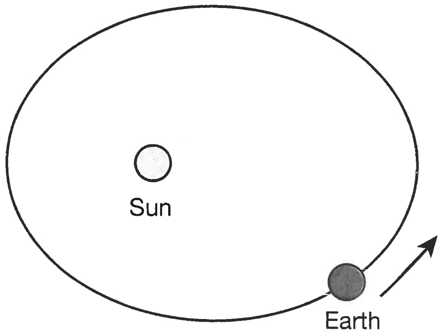
If the eccentricity (the degree to which the orbit deviates from being circular) of a planetary orbit is zero, then Kepler's laws state:

- The planetary orbit is a circle.
- The Sun is in the centre.
- The speed of the planet in the orbit is constant.
- The square of the sidereal period is proportionate to the cube of the distance from the Sun.

## Kepler's laws: Orbits elliptical

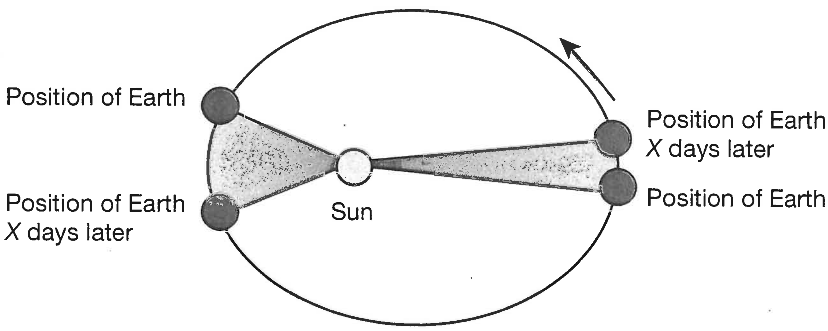
1. Kepler's first law states the following.

> All planets move in elliptical orbits with the Sun at one focus and the other focus empty.




2. Kepler's second law states the following.

> The line joining the planet to the Sun sweeps out equal areas in equal times. Obviously, to do this, the orbital speed of the planet will change depending on its position relative to the Sun.




3. Kepler's third law states the following.

> The squares of orbital periods of the planets are directly proportional to the cubes of the mean distances from the Sun.

From this it follows that the farther a satellite is from Earth, the longer it will take to complete and orbit, the greater the distance it will travel to complete an orbit, and the slower its average speed will be.

Note that when Kepler first derived this law he assumed that the orbits of the planets were essentially circular. We use it by substituting the average distance of the planet from the Sun, otherwise the mathematics is too difficult for this level.

From our equation for finding the average velocity of a moving object, and Newton's equation for the orbital speed o object, we get the velocity of an object in Earth orbit as follows.

$$v = \frac{s}{t} = \frac{\text{orbit circumference}}{\text{orbital period}} = \frac{2\pi r}{T} = \sqrt{\frac{Gm_{\text{Earth}}}{r}}$$

Squaring: $$\boxed{\frac{4\pi^2 r^2}{T^2} = \frac{Gm_{\text{Earth}}}{r}}$$

On rearranging: $$\boxed{\frac{r^3}{T^2} = \frac{GM}{4\pi^2} = \text{constant}}$$

This equation, often referred to as **Kepler's law of periods**, can be generalised to find the orbital period, $T$, of any orbiting mass around any planet, mass $M$ and radius $r$, to:

$$\boxed{\frac{(r_{\text{Earth}})^3}{(T_{\text{Earth}})^2} = \frac{(r_{\text{Mars}})^3}{(T_{\text{Mars}})^2} = \frac{(r_{\text{Jupiter}})^3}{(T_{\text{Jupiter}})^2} = \text{etc}}$$

Another way to use Kepler's law of periods is to equate the ratio for different orbiting bodies. Note that $r$ = distance between the centres of the two objects involved.

## QUESTIONS

1. Calculate the orbital period of the Moon. Its average distance from the Earth is 406 676 km and its diameter is 3467 km.
2. The mass of the Sun is $1.99 \times 10^{30}$ kg. Its diameter is 1 392 530 km and that of Earth is 12 756 km. The distance between the Sun and Earth is $1.5 \times 10^8$ km. Calculate the period of Earth around the Sun.
3. Calculate the altitude of an Earth satellite with a period of 24 hours.
4. Given the radius of the Earth's orbit around the Sun as 1 astronomical unit (AU), and its period as 1 year, and the radius of Mars' orbit as 1.5 AU, calculate Mars' period.
5. Calculate the period of an Earth satellite at an altitude of 600 km.
6. Use Kepler's law of periods to find the missing information in the table.

| Planet | Radius of orbit | Orbital period |
|---|---|---|
| Mercury | A | 59 Earth days |
| Venus | 0.72 AU | B |
| Saturn | C | 29.46 Earth years |
| Pluto | 39.3 AU | 248 Earth years |

7. Use Kepler's law of periods to calculate the missing quantities in the table below.

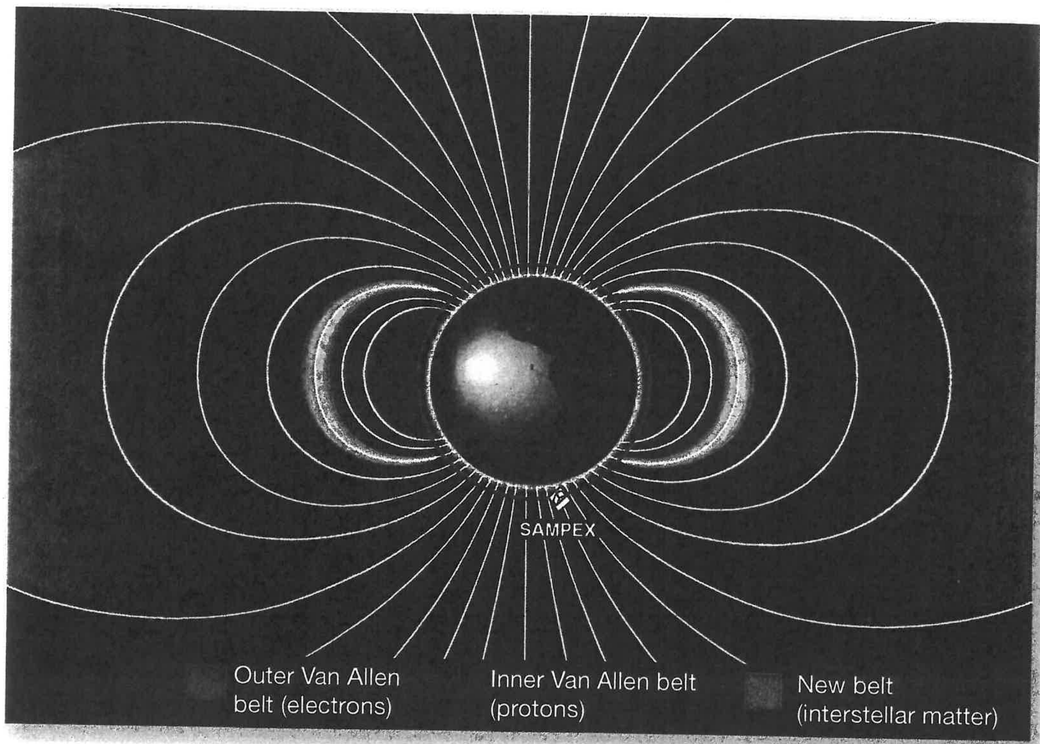
| Heavenly body | Period of rotation | Diameter (km) | Mass (kg) | Altitude of satellite (km) |
|---|---|---|---|---|
| Earth | 23 hr 56 min | A | $5.97 \times 10^{24}$ | 35 946.6 |
| Moon | 27 days 7 hr | 1737 | $7.35 \times 10^{22}$ | B |
| Mars | 24 hr 37 min | 6794 | $6.42 \times 10^{23}$ | C |
| Jupiter | D | 142 984 | $1.9 \times 10^{27}$ | 87 620.7 |
| Uranus | 17 hr 18 min | 51 800 | E | 56 956.7 |

# 24 Types of Orbits

The Van Allen belts of ionisation limit the region of space above the Earth that satellites can use because of the threat the ionisation poses to humans and electronic equipment.

The Van Allen belts are regions of high concentration of charged particles from the solar wind trapped in the Earth's magnetic field.

The outer belt (at geostationary and geosynchronous orbit altitude) is composed mainly of electrons, the middle belt (1000 km) mainly protons, and the inner belt (300 km) contains larger ions.



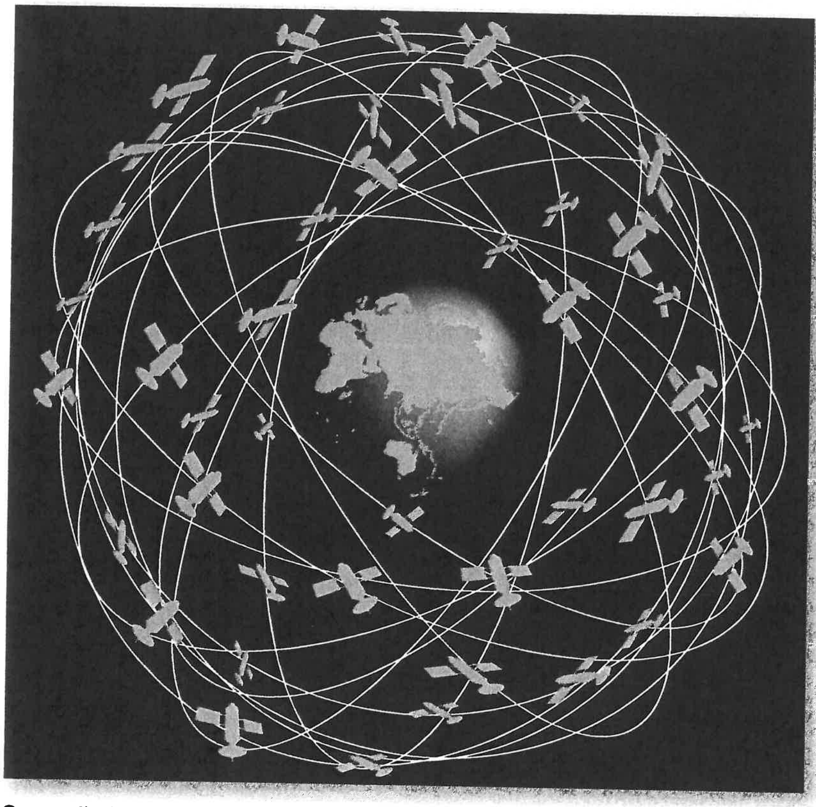
Van Allen belts of ionisation.

The Van Allen belts present shielding problems for the electronics and people in satellites placed in orbit between 300 and 36 000 km (which is where most of our satellites orbit).

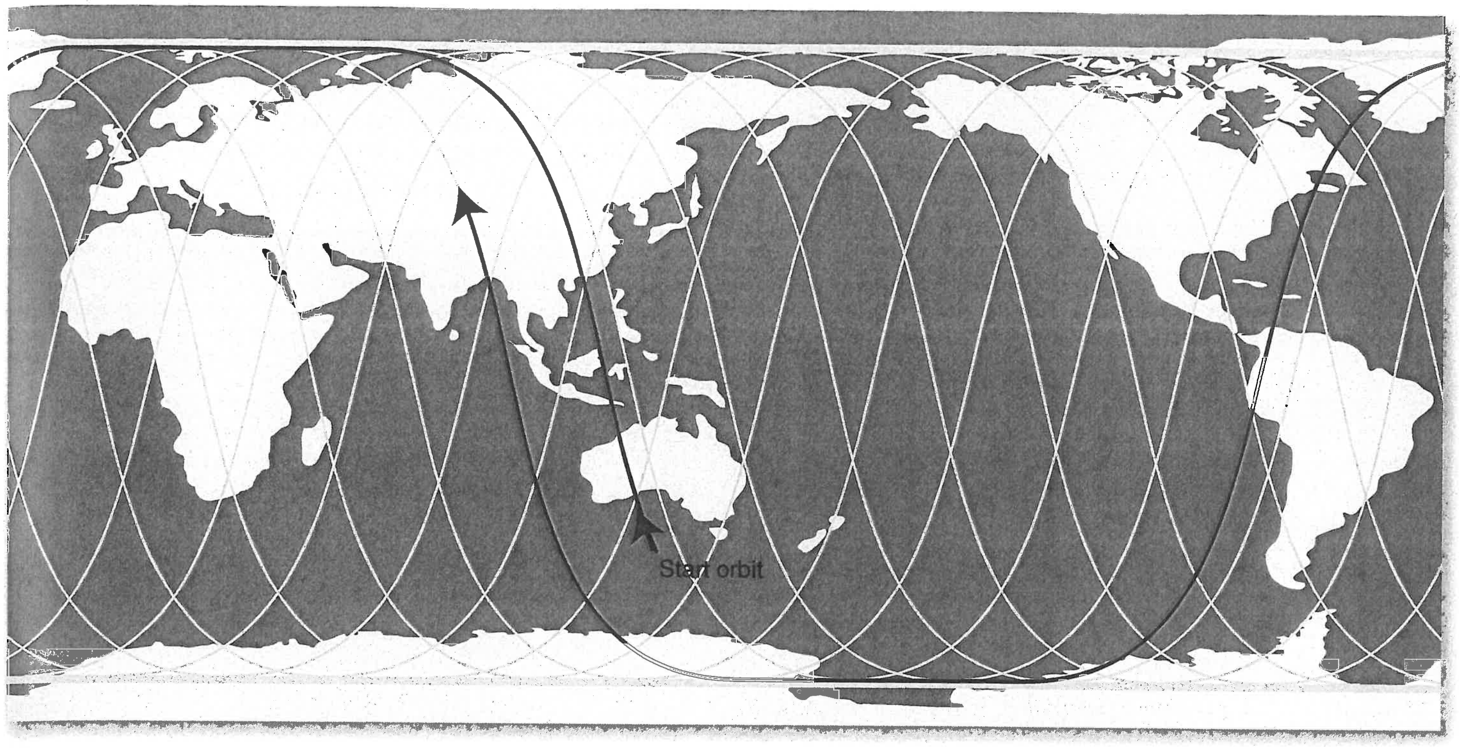
The minimum altitude required to avoid significant atmospheric friction is 250 km, the region between this and 1000 km can be used quite safely. This is known as the **low Earth orbit** region.

Manned spacecraft usually orbit between 250 km and 400 km, as do any satellites that may need occasional repairs by spacecraft crew. The Hubble telescope orbits a little higher, at 600 km. Spy satellites also orbit in this region, although they usually occupy a north-south orbital so they can cover the entire surface of the Earth every 24 hours.

**Geostationary** satellites orbit the Earth over the equator with a period of 23 hours, 56 minutes and 4 seconds – 86 164 seconds – one **sidereal day**. This is the time for the Earth to rotate once on its axis. A geostationary satellite will occupy the same position in space above the Earth. Geostationary orbits are useful for communications and weather satellites.


Constellation of GPS satellites orbiting Earth.

A **geosynchronous** orbit is a 23 hour, 56 minute and 4 second orbit which is not equatorial. Over the period of the orbit the satellite therefore traces a complicated pattern across the Earth as it and the Earth rotate in different alignments. Unless the orbit is a long way from the equatorial plane, receiving dishes can still track its signals, or stationary receivers can have built-in amplifiers to compensate for varying signal strengths.

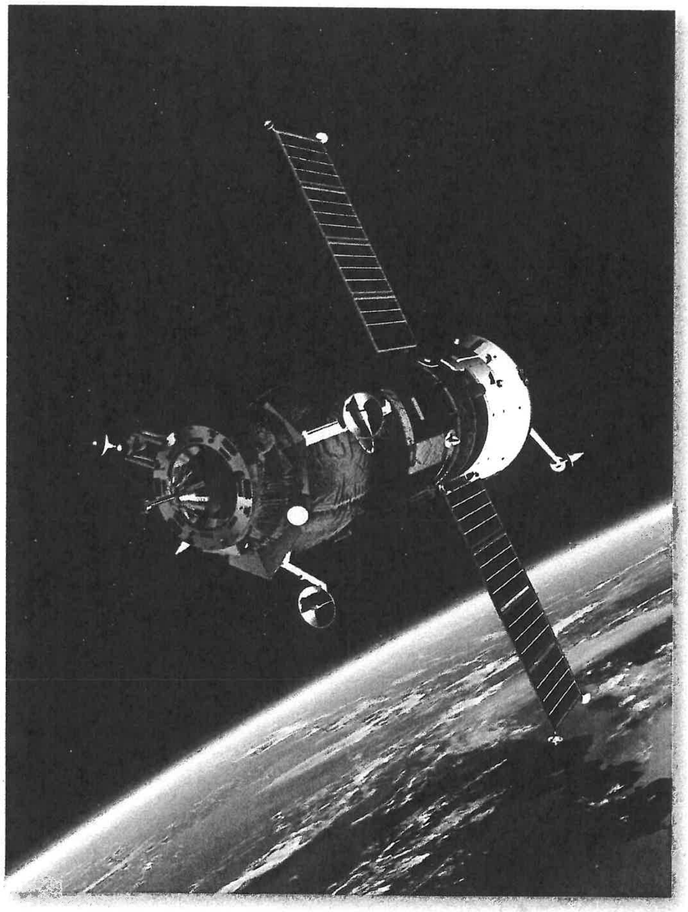



**Polar orbits** are used mostly by spy satellites because they cover the whole surface of the Earth at least once per day.

## QUESTIONS

1. Explain why many satellites are limited to altitudes between 250 and 1000 km.
2. Explain why it is useful to place satellites that may need maintenance in low Earth orbits.
3. Explain why a north-south orbital is more appropriate for a spy satellite than an equatorial orbit.
4. Calculate the altitude of a geostationary satellite.
5. Explain why geostationary orbits are useful for communications satellites.
6. Calculate the missing values (A to I) in the table.

| Satellite | Low Earth lower limit | Low Earth upper limit | Geostationary |
|---|---|---|---|
| Altitude (km) | A | B | C |
| Period (h) | D | E | F |
| Orbital speed (m $s^{-1}$) | G | H | I |



7. Explain why geostationary satellites orbit in 23 hours, 56 minutes and 4 seconds, and not 24 hours?
8. Geosynchronous orbits are not as useful for communications satellites as geostationary orbits. Explain this.
9. Compare geostationary and geosynchronous satellites.

14. (a) 3000 N
(b) 306.1 kg
(c) Not very possible for an average mother (or father!).
(d) Seatbelt is essential for increased safety.
(e) Newton's first law of motion (inertia).
15. (a) 5.0 m $s^{-2}$
(b) P = 2.5 m
Q = 4.0 m
R = 4.5 m
S = 7 m
T = 0.77 s
U = 1.10 s
V = 1.79 s
(c) Mass = 80 kg
Weight = 400 N

## 3 Acceleration Due to Gravity – Another Experiment

1. Diagram should show simple pendulum.
2. To determine the relationship between the length of a pendulum and its period of oscillation, *and* to use this relationship to calculate a value for the acceleration due to gravity.
3. The longer the pendulum the longer the period of oscillation.
4. Mass of pendulum bob (unnecessary as swing is independent of this), type of string, size of swing (again unnecessary), stopwatch used.
5. Length of the string.
6. Graph should curve upwards.
7. From an extrapolation of your graph, you should get 0.25 m.
8. Oscillation time increases with pendulum length but not direct relationship.
9.

| Length of pendulum string (m) | Average time for one swing (s) | (Time of swing)$^2$($s^2$) |
|---|---|---|
| 0.5 | 1.41 | 1.99 |
| 1.0 | 2.00 | 4.00 |
| 1.5 | 2.45 | 6.00 |
| 2.0 | 2.83 | 8.00 |
| 2.5 | 3.16 | 9.99 |
| 3.0 | 3.46 | 11.97 |
| 3.5 | 3.74 | 13.99 |
| 4.0 | 4.00 | 16.00 |

10. Graph should be a straight line.
11. 0.25 m
12. Answer to Question 11 should be more accurate as extrapolating a straight line graph is more accurate than extrapolating a curve.
13. Period of oscillation squared is directly proportional to length of the pendulum.
14. One oscillation.
15. Periodic motion.
16. The length of the pendulum is directly proportional to the square of its period of oscillation.
17. Calculating for each experimental value and taking an average should give a value of 9.8/9.9 m $s^{-2}$.

**A Xenos swinging time**

1. 6.49 m $s^{-2}$
2. No change – mass is not a variable affecting the period of swing of a pendulum.
3. 49.33 s
4. Time for 20 swings would be greater on the Moon as the Moon's gravity would be smaller.
5. Gravity on X will be greater than that on Y, so time for 20 swings will be less.

## 4 Newton's Laws of Gravitation

1. 9.77 m $s^{-2}$
2. 0.332 N if weight on Earth calculated using $w = mg$ ($g = 9.8$). If calculated using Newton's formula, $g$ works out to be 9.796, so difference is 0.03 N.
3. $9.6 \times 10^{-9}$ N
4. 2.6 times greater.
5. (a) 1.44 m $s^{-2}$
(b) 0.15 times
(c) 14.4 N
6. 2 : 1
7. (a) Force will be $\frac{1}{4}$ original value.
(b) Force will be 4 times larger.
(c) Force will be doubled.

8. 1 : 1. Gravitational force between X and Y depends only on the mass of X and Y, and the distance between them – this will be the same anywhere in the Universe.
9. (a) 325.7 km
   (b) 578.5 N
10. Approximately $3.47 \times 10^5$ km from the centre of Earth. (*Hint*: Take distance of zero field point as $x$ km from Earth and $(385\ 000 - x)$ km from Moon. Use $g = GM_E/x^2 = GM_m/(3\ 895\ 000 - x)^2$ and take square roots of both sides before doing the maths).
11. From $W = mg = mv^2/r = 2E_K/r = 750$ N
12. $8W$
13. 13
14. 11.27 m $s^{-2}$
15. 5.6 m $s^{-2}$
16. $1.03 \times 10^{-3}$ N towards Jupiter
17. D
18. A
19. B
20. C
21. A
22. C
23. B
24. C

## 5 Gravitational Field

1.

| Factor (position on Earth) | Value of $g$ | Explanation |
|---|---|---|
| Sea level, middle of Australia | > 9.8 | Thick continental crust underneath with probably higher than average density. |
| Sea level, on boat in middle of ocean | < 9.8 | Deep water underneath and thinner crust means lower than average density. |
| Sea level at the equator | < 9.8 | Largest Earth radius underneath, so $g$ less than average, largest decrease due to rotation of Earth. |
| Sea level, London | ≈ 9.8 | Possibly average thickness of crust under, and on edge of continental shelf. |
| Sea level, North Pole | > 9.8 | Smallest radius and zero effect from rotation. |
| Equator, top of high mountain | < 9.8 | Largest radius, greatest effect from rotation, more effect from higher altitude. |
| Equator, deep down mine shaft | < 9.8 | Largest radius, greatest effect from rotation, mine depth insignificant compared to radius. |
| In plane, flying over ocean | < 9.8 | Deep water underneath and thinner crust plus additional effect of higher altitude. |

2. 9.789 m $s^{-2}$ (9.8 m $s^{-2}$)
3. (a) 1.2 m $s^{-2}$ (N $kg^{-1}$)
   (b) 3.75 m $s^{-2}$
4. The closer we are to a mass, the stronger will be the gravitational field, and since the spacing between the vector lines used to represent gravitational field represents the relative strength of the field, the lines will be closer together closer to the mass.
5. The field will be uniform in strength and in a constant direction.
6. B
7. D
8. Zero – Gravitational force is a vector quantity and therefore has direction. A mass placed at this midpoint would experience identical forces in opposite directions (towards each of the planets). The net force on the object would be zero.
9. D (The relative increases in both the mass and radius are required to work out the strength of the gravitational field from $g = GM/R^2$ before an answer can be worked out here).
10. (a) B
    (b) (A) The acceleration due to gravity at the surface of *X is greater than that of Y.*
    (B) *Correct choice.*
    (C) The gravitational force acting on an object above the surface of X will be *greater* than the gravitational force acting on the same object the same distance above the surface of Y.
    (D) The gravitational potential energy of an object above the surface of X will be *greater* than the gravitational potential energy of the same object the same distance above the surface of Y.
11. D
12. D
13. (a) 0.25 g m $s^{-2}$
    (b) 4 g m $s^{-2}$
    (c) 2 g m $s^{-2}$
    (d) 0.5 g m $s^{-2}$
14. (a) $3.34 \times 10^{-10}$ m $s^{-2}$ directed away from $M_1$ through P (bearing 000)
    (b) $3.66 \times 10^{-10}$ m $s^{-2}$ directed away from $M_2$ through P (bearing 318°)
    (c) $6.54 \times 10^{-10}$ m $s^{-2}$ bearing 338°
15. (a) A
    (b) E
    (c) F
    (d) E

## 6 Gravitational Potential Energy 1

1. We choose the surface of the Earth as the zero $E_P$ position when we are considering masses close to the surface (rather than astronomical distances). This is more convenient, and usually we are really wanting to know how much work needs to be done to change the position of a mass within Earth's gravitational field.
2. A = 122.5 J
   B = 4.5 m
   C = 5.5 kg
   D = 9.8 m $s^{-2}$
   E = 87.36 J
   F = 12.5 m
   G = 2.5 kg
   H = 10 m $s^{-2}$
   I = 432 J
   J = 6.5 m
   K = 2.0 kg
   L = 29.4 m $s^{-2}$
3. (a) 1470 J $kg^{-1}$ (Make sure your units are correct.)
   (b) $1.47 \times 10^6$ J (1 cubic metre of water is 1000 kg.)

## 7 Gravitational Potential Energy 2

1. *GPE* is measured relative to a zero value at infinity, whereas we usually consider the energy *change* relative to the Earth's surface. This has more meaning to us than the absolute value compared to infinity.
2. The value of *g* is affected by altitude, so if altitude is significant, then this equation will give an incorrect answer.
3. The absolute value of *GPE* is measured compared to a value of zero at infinity. Moving away from infinity, objects are considered to be falling into a gravity well – their *KE* increases, so their *GPE* decreases. To decrease from zero means to go into negative values.
4. The mass at 200 m is closer to infinity, so it has more *GPE or* more work has to be done to lift the mass to a height of 200 m so it has more *GPE* (relative to the surface) – i.e. a *less negative* value.
5. The work done to lift it to 2*d* (relative to the surface) will be twice that to lift it to *d*, so *GPE* relative to surface will be double *but* absolute *GPE* (relative to infinity) will only be double if 2*d* is halfway to infinity.
6. (a) 193 720 km
   (b) 93 670 km
7. (a) $2.88 \times 10^8$ J
   (b) 4803.6 m $s^{-1}$
8. $-5.5 \times 10^9$ J
9. (a) A
   (b) C
10. (a) Position X (Its $E_P$ is less negative, so it is greater.)
    (b) Position X
11. A
12. $1.15 \times 10^{10}$ J
13. $-1.14 \times 10^{10}$ J
14. (a) A
    (b) E
    (c) E
    (d) F
    (e) E

## 8 Changes in Gravitational Potential Energy

1. (a) $-1.51 \times 10^8$ J
   (b) Speed would increase by 1833 m $s^{-1}$
2. *W*/3 joules
3. 0.35*E* joules (if you have *E*/3 it is close enough at the moment)
4. $4.6 \times 10^9$ J
5. D
6. B
7. B
8. C
9. A
10. C
11. A

## 9 Resolution of Vectors

1. (a) Horizontal = 21.6 N
   Vertical = 13.5 N
   (b) Horizontal = 20.35 N
   Vertical = 11.75 N
   (c) Horizontal = 27.9 N
   Vertical = 6.96 N

## 18 Circular Motion on a Banked Curve

1. (a) From $v^2 = gr \tan \theta$
   $v^2 = 9.8 \times 240 \times \tan 7° = 288.8$
   Therefore $v = 1$ m s$^{-1}$
   (b) Since mass is not a variable in the equation, $v = 17$ m s$^{-1}$
2. (a) From $v^2 = gr \tan \theta$
   $20^2 = 9.8 \times 250 \times \tan \theta$
   From which $\theta = 9.3°$
   (b) From $F_C = mv^2/r = 750 \times 20^2/250$
   $= 1200$ N towards the centre of the circle of motion
3. (a) From $v^2 = gr \tan \theta$
   $(60/3.6)^2 = 9.8 \times 80 \times \tan \theta$
   From which $\theta = 19.5°$
   (b) From $F_c = mv^2/r = 2000 \times 16.67^2/80 = 6947$ N towards the centre of the circle of motion
4. From $v^2 = gr \tan \theta$
   $v^2 = 9.8 \times 150 \times \tan 10° = 259.2$
   Therefore $v = 16.1$ m s$^{-1}$
5. From $v^2 = gr \tan \theta$
   $(500/3.6)^2 = 9.8 \times r \times \tan 40°$
   Therefore $r = 2346$ m
6. From $v^2 = gr \tan \theta$
   $v^2 = 9.8 \times 315 \times \tan 30° = 1782.3$
   Therefore $v = 42.2$ m s$^{-1}$ = 152 km h$^{-1}$
7. (a) From $v^2 = gr \tan \theta$
   $v^2 = 9.8 \times 350 \times \tan 15°$
   From which $v = 30.32$ m s$^{-1}$
   (b) From $F_c = mv^2/r$
   $m = F_c r/v^2 = 3939 \times 30.32^2/350 = 1500$ kg
8. (a) From $v^2 = gr \tan \theta$
   $900 = 1.67 \times 15 \times \tan \theta$
   From which $\theta = 88.4°$
   (b) From $v^2 = gr \tan \theta$
   $900 = 9.8 \times 15 \times \tan \theta$
   From which $\theta = 80.7°$
9. From $v^2 = gr \tan \theta$
   $(50)^2 = 9.8 \times r \times \tan 80°$
   Therefore $r = 45$ m
10. (a) From $v^2 = gr \tan \theta$ (for curve A)
   $v^2 = 9.8 \times 75 \times \tan 56°$
   From which $v = 33$ m s$^{-1}$
   From $v^2 = gr \tan \theta$ (for curve B)
   $v^2 = 9.8 \times 125 \times \tan 56°$
   From which $v = 42.6$ m s$^{-1}$
   Therefore ratio of speeds A : B = 1 : 1.3
   (b) From $v^2 = gr \tan \theta$
   $(33)^2 = 9.8 \times 125 \times \tan \theta$
   From which $\theta = 41.6°$
   (c) From $v^2 = gr \tan \theta$
   $(42.6)^2 = 9.8 \times 75 \times \tan \theta$
   From which $\theta = 68°$

## 19 Newton, Circular Motion and Orbital Speed

1. (a) 1 : 1 : 1 (Orbital speed is independent of the mass of the moons.)
   (b) 36 : 9 : 4
2. (a) 1 : 4 : 9
   (b) 1 : 4 : 9
3. 7108.6 m s$^{-1}$ = 25 591 km h$^{-1}$
4. 203.6 km
5. 2474.4 m s$^{-1}$ = 8908 km h$^{-1}$
6. 131 317 m s$^{-1}$ = 472 740 km h$^{-1}$
7. 133 662 km
8. 29 671 m s$^{-1}$ = 106 818 km h$^{-1}$
9. 29 747 m s$^{-1}$ = 107 089 km h$^{-1}$

## 20 The Geocentric Model of the Solar System

1. Geocentric means 'Earth centred' – refers to models of the Solar System which have the Sun and planets circling the Earth at the centre.
2. Retrograde motion is the apparent backwards motion of the planets sometimes as observed from Earth.
3. Epicycles were small circular orbits of planets within their main orbit around the Earth. They were proposed to explain retrograde motion.
4. Because they were travelling in a separate circular orbit around some imaginary point in space as well as around the Earth, then the planets would sometimes be going backwards in this circle relative to Earth.
5. Because existing models of the Universe did not explain the epicycle, they had to be changed to explain the epicycles.
6. The Universe referred to all they could see – the Sun, the Earth, the known planets and the few known Moons and the stars, comets and meteors.
7. Without experimental evidence there was really no reason to believe anyone rather than anyone else, but in addition, the fame and renown of the ancient astronomers was so great that it was almost a sacrilege to question their beliefs. Later, it became sacrilegious to do so as the Church was the owner of all knowledge and power.
8. Technology was limited to naked eye observations, so the data early astronomers worked with was limited. However, their models were not put to the 'test' by making predictions which, if accurate, would have supported their models. In simpler terms 'experiments' were not done to test their models.
9. Early astronomers had no idea of the size or scope of the Universe. They did not distinguish the Sun, Earth and planets as a tiny part of a much larger whole. To them, everything they could see was part of one system that was centred on the Earth.
10. (a) The diagram is showing how epicycles work using Jupiter as the example. The lines connecting the Earth and Jupiter are extended to the fixed far distant circle of stars so that we can visualise the relative position of the Earth and Jupiter against that fixed background.
    (b) Answers will vary, for example: Motion of Jupiter against the background of distant stars relative to Earth.
11. Many ancient civilisations have contributed to astronomy.
    Astronomical observations commenced very early in the timeline of civilised people.

## 21 The Heliocentric Model of the Solar System

1. Heliocentric means 'Sun centred' – refers to models of the Solar System which have the planets circling the Sun at the centre.
2. Initially, the 'scientific greatness' of men like Aristotle and Ptolemy was so great that one had to be very brave to question their ideas. Later, when the Church took these ideas as its own, it became a crime to question them – to the point of being put to death if you questioned the beliefs of the Church. It took courageous men like Galileo, whose own 'greatness' dissuaded the Church from making a martyr of him, to put the old ideas to the test. For his troubles, Galileo spent the last years of his life under house arrest.
3. Naked eye observations 'see' everything in the heavens as orbiting the Earth – the Sun rises in the east, travels across the sky and sets in the west – as does the Moon, the stars and planets (except for the retrograde motion of some which caused complications).
4. The most significant was Galileo's observation of the phases of Venus, but it was also supported by Kepler's mathematics. Galileo's observation that the Moons of Jupiter orbited Jupiter and not the Earth contradicted existing geocentric models, but by itself, was not necessarily evidence for a heliocentric model.
5. All models were limited to naked eye observations until Galileo's use of a telescope.
6. The main technological development was the invention of the telescope. This of course required technological developments in the making of lenses and in the idea of using lens combinations to enhance magnification.
7. It was partly the power of the Church that limited the model's success as well as the fact that, because of the limited technology available at the time, the model was no better at predicting the motion of heavenly bodies than that of Ptolemy.
   The biggest problem with a heliocentric model was that, if the Earth is rotating about the Sun, then there should be some relative movement of the stars during the year (parallax movement). Technology at the time was not good enough to detect this.
8. (a) It is in this orientation when Mars undergoes retrograde motion relative to Earth and so the geocentric model would say that Mars was closer to the Earth because of its epicycle.
   (b) The heliocentric model would say that it was in this orientation that Earth and Mars were actually at their closest point together.
9. (a) He is referring to the work done by scientists who preceded him such as Brahe, Kepler and Galileo.
   (b) Brahe, Kepler and Galileo.
10. Uranus did not follow the orbital path predicted by Kepler and Newton's mathematics, so it was hypothesised that another planet may exist and which was influencing the gravitational fields and altering this path. Further searching with higher powered telescopes resulted in the discovery of Neptune.

## 22 Kepler's Laws of Planetary Motion

1. 2 659 027 s = 30.8 days
2. 31 905 953 s = 369.3 days
3. 35 856 km
4. 1.84 years
5. 5804 s = 1.61 hours = 96.7 minutes
6. A = 0.3 AU
   B = 224.5 days
   C = 9.5 AU
7. A = 12 756 km
   B = 87 517 km
   C = 17 026 km
   D = 9 hr 50.4 min
   E = $8.68 \times 10^{25}$ kg

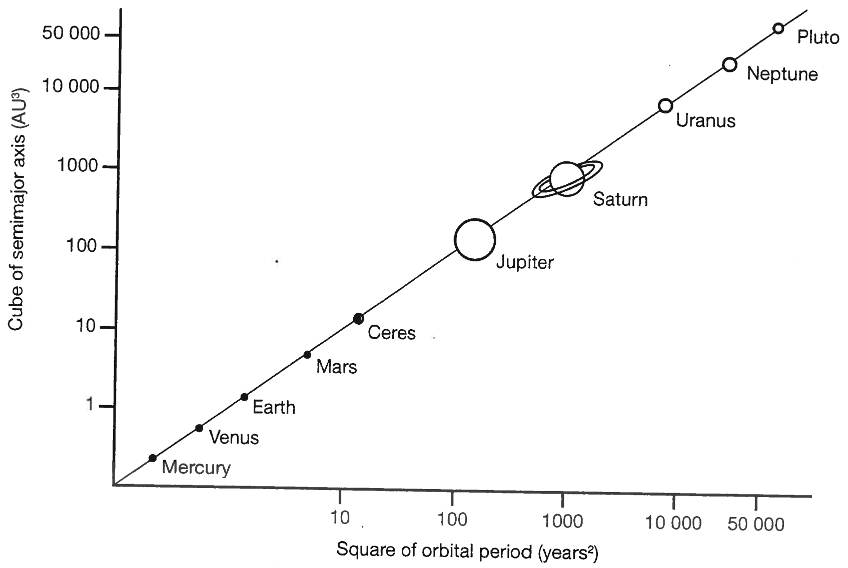
## 23 Kepler's Third Law

1. (a) The astronomical unit is a distance measure, being the distance from the Sun to the Earth.
   (b) It is used because it is a more understandable and sensible unit to use than, say, km (1 AU = 150 000 000 km).
   (c) For distances beyond the Solar System, the astronomical unit becomes too small to be sensible, so we tend to use larger distance measures.
   (d) Light years, parsec.
2.

| Planet/asteroid | Distance³ from Sun (AU)³ | Period² (Earth years)² |
|---|---|---|
| Mercury | 0.06 | 0.06 |
| Venus | 0.37 | 0.38 |
| Earth | 1.00 | 1.00 |
| Mars | 3.51 | 3.53 |
| Ceres | 21.95 | 22.09 |

| Planet | Distance³ from Sun (AU)³ | Period² (Earth years)² |
|---|---|---|
| Jupiter | 140.6 | 140.9 |
| Saturn | 857.4 | 867.9 |
| Uranus | 7100 | 7069 |
| Neptune | 27 027 | 27 166 |
| Pluto | 61 770 | 61 767 |

3. (a) The line of best fit will be a straight line. The figures clearly show a 1 : 1 ratio for each plot point. In addition, given that the data reflects Kepler's law of periods, then this law states that $R^3/T^2$ is constant, which is the condition for a straight line graph.
   (b)




   (c) The graph shows a direct relationship between the variables, it therefore reflects Kepler's law of periods (third law).
   (d) $R^3/T^2 = \text{constant} = GM/4\pi^2$
   (e) A linear scale from 0 to 60 000+ would either make the graph ridiculously large, or the plot points up to at least Saturn so close together that it would be difficult to distinguish between them.
   (f) No. This is a legitimate strategy to use for data which spreads across such a large range.
   (g) Yes. Regardless of the scaling on the axes, there is still a linear relationship between the data.
   (h) Any heavenly object orbiting the Sun will lie somewhere on this graph. Kepler's third law holds for all objects which are orbiting the same primary.
   (i) No. The Moon orbits the Earth not the Sun, and therefore will have a different constant value if Kepler's third law is applied to it.
   (j) Yes. Halley's comet orbits the Sun and is therefore part of the same orbital system.
   (k) Kepler's laws are based on the orbits of the planets being approximately circular. Halley's comet traces a very elliptical orbit, so its distance from the Sun varies quite considerably.

## 24 Types of Orbits

1. This places them below the lower Van Allen belt and so they are not subject to damaging ionisation from the belt.
2. Easier and cheaper to get to.
3. North-south orbital will cover all of the Earth's surface at least once every day (depending on speed/altitude – remember spy satellites are LEO satellites and have a period of about 90 minutes). Equatorial orbit only covers Earth's surface around the equator.
4. 35 779.4 km
5. They remain above the same position of the Earth so transmitters and receivers do not have to 'track' the satellite – they remain in a fixed position.

6. A = 250
B = 1000
C = 35 799.4
D = 1.49
E = 1.75
F = 23.56.04
G = 7752.4
H = 7348
I = 3056
7. 23 hr, 56 min and 4 s is the actual period of rotation of the Earth.
8. Geosynchronous orbits are not equatorial so the satellite is not stationary with respect to the Earth's surface – transmitters and receivers would need to 'track' its movement, or have built-in circuitry to compensate for a variable signal.
9. Geostationary orbit is equatorial and satellite position is fixed relative to Earth's surface, while a geosynchronous orbit is not equatorial and its position is not fixed relative to surface.

## 25 Space Debris

1. Space debris, also known as orbital debris, space junk, and space waste, is the collection of defunct objects in orbit around Earth.
2. Orbiting satellites can be damaged or totally destroyed by collisions with space debris. Their solar power collecting cells can be damaged and they can stop operating because of loss of power. Collisions with small particles of space debris may not cause significant damage but could slow them down and accelerate their orbital decay, thus shortening their operational lifetime.
3. The use of Whipple shield, orbiting backwards so that the tail area and used engines protect the cabin and cargo bay from collisions, parking behind the Space Station and using it as a protective barrier.
4. Answers will vary, for example: The strategy seems extremely costly and inefficient in that, as proposed, it will eliminate only a small number of pieces of space debris (ten) per day. Given an estimated 19 000 pieces larger than 5 cm in diameter and the number growing each day as further mishaps occur, it would seem to be a 'drop in the ocean' strategy.
5. Much of the debris at low altitudes will undergo orbital decay and burn up in the Earth's atmosphere within a short period of time because the atmosphere is significantly denser at low altitudes. Debris at high altitudes may still be there in a hundred years time.
6. $(KE)_{debris} = \frac{1}{2} \times 0.025 \times 7500^2 = 703\ 125$ J
$(KE)_{car} = \frac{1}{2} \times 1500 \times 30^2 = 675\ 000$ J, which is approximately the same value, therefore will have the same effect during collision.
7. (a) As shown in Question 6, the energy of even small pieces of space debris moving fast is huge, enabling them to do huge amounts of damage or even destroy satellites and other spacecraft.
(b) $(KE)_{debris} = \frac{1}{2} \times 1.0 \times 7500^2 = 28\ 125\ 000\ \text{J} = = \frac{1}{2} \times m \times 30^2$
Giving $m = 62\ 500$ kg!
8. Answer will vary, but could include some of the following.
The dumping of unused fuel into space once orbital altitudes have been released will reduce the risk of explosion of that fuel at a later date.
A 'one-up/one-down' launch license policy for Earth orbits. Launch vehicle operators would have to build into their launch vehicles the ability to rendezvous with, capture and deorbit an existing derelict satellite from approximately the same orbital plane, as well as send their own vehicle into an unstable orbit at the end of its working lifetime.
Robotic refuelling of satellites to extend their operational lifetime rather than replacing them with new satellites.
Rocket stages or satellites that retain enough propellant can power themselves into a decaying orbit thereby reducing their orbital time from several hundred years to maybe 10 or 15 years.
Instead of using rockets (and therefore storing fuel which is an explosion risk) to accelerate orbital decay when the working life of a spacecraft is ended, an electrodynamic tether can be attached to the spacecraft on launch. At the end of its lifetime it is rolled out and slows down the spacecraft through increased atmospheric drag. Tethers of up to 30 km have been successfully deployed in orbit to date.
Use a remotely controlled vehicle to rendezvous with debris, capture it, and return to a central station or push it into a lower orbit where orbital decay will be accelerated.
Dedicated 'interceptor satellites' to match orbits with and capture large pieces of space debris, and either ejecting them into a lower orbit or storing them and deorbiting with them when the storage bay is full.
A Space Sweeper with Sling-Sat (4S) grappling satellite mission that sequentially captures and ejects debris into lower orbits where orbital decay occurs more quickly. The momentum from these interactions provides a free impulse to the craft to move it to additional targets.

## 26 Forensic Science and Projectiles

1. Spinning the football increases its angular momentum and inertia and increases the chance it will follow a straight path to the receiver.
2. The longer a force acts on an object the more it will accelerate. The more it accelerates, the larger its velocity and momentum. So, the longer the length of a rifle barrel, the greater the impulse applied to the bullet and the faster it will leave the barrel. The faster it is going, the longer its range will be.
3. Full metal jacket refers to the entire body of a bullet being coated with a thin covering of a metal such as copper. This makes it harder and increases its penetrating power. However, a partially covered bullet may do more damage as it may deform or break up when it hits its target.
4. The greater the inertia of a ballistic missile, the truer its flight path will be as it will be more difficult to be moved off course by differences in air density, or breezes and the like. Spinning the bullet as it moves down the barrel of the gun or rifle firing it increases its angular momentum and inertia and makes its flight path truer.
5. Any projectile will be affected by gravity the moment it leaves the barrel. A projectile aimed directly at a target will hit the ground before it gets there, or the target below the point aimed at. It must be aimed above the desired impact point to counteract gravitational fall.
6. The denser the air, the more air resistance offered to a projectile. Wet weather or high humidity air will slow a ballistic projectile more than dry, hot weather. Windy weather will drive a projectile off course.
7. There are so many guns, so many bullet types, so much information that it would be impossible to process evidence and compare it for possible matches without computers.