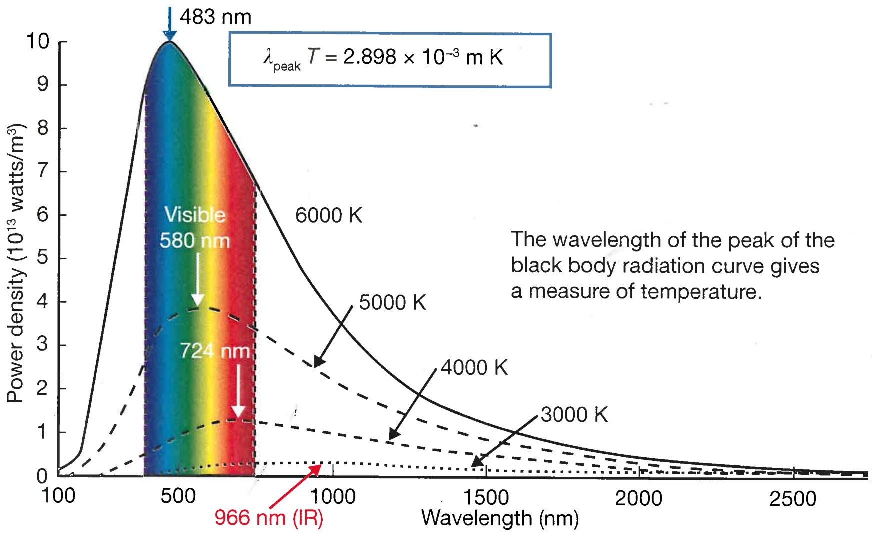
27 Wien's Displacement Law



Wien's displacement law states that the wavelength distribution of thermal radiation from a black body at any temperature has essentially the same shape as the distribution at any other temperature, except that each wavelength is displaced on the graph.

From this general law, it follows that there is an inverse relationship between the wavelength of the peak of the emission of a black body and its temperature when expressed as a function of wavelength. Planck's experimental results, supported this idea, and in later years astronomers found it crucial in determining the surface temperatures of stars.

$$\boxed{\lambda_{max} T = b}$$

Where λ_{max} is the peak wavelength

T is the absolute temperature of the black body, and

b is a constant of proportionality called *Wien's displacement constant*, equal to 2.898×10^{-3} m K

For wavelengths near the visible spectrum, it is often more convenient to use the nanometre in place of the metre as the unit of measure. In this case, $b = 2\ 897\ 768.5(51)$ nm K.

QUESTIONS

1. (a) The surface temperature of the Sun is 5778 K. What will be the peak wavelength of the light it emits in its black body radiation curve?
   (b) Using other data sources, what colour in the visible spectrum should this light be?
   (c) What colour do we perceive the Sun to be?
   (c) Suggest a reason for this (you may have to research this).
2. What is the peak radiation given out by a typical wood fire at a temperature of 1500 K?
3. (a) Taking our body temperatures at an average 37°C, calculate our peak radiation wavelength and frequency.
   (b) Using other data sources, where is this in the electromagnetic spectrum?
   (c) What device has been developed to take advantage of this fact?
4. The wavelength of the radiation in space resulting from the Big Bang is about 1.063 mm. What temperature does this make space?

Wilhelm Wien (1864-1928) was awarded the 1911 Nobel Prize in Physics for work on thermal properties of matter.

(i) $c = f\lambda$; $\lambda = c/f = (3 \times 10^8)/(9 \times 10^{21}) = 3.33 \times 10^{-14}$ m
$E = hf = (6.626 \times 10^{-34}) \times (9 \times 10^{21}) = 5.96 \times 10^{-12}$ J
(j) $E = hc/\lambda = (6.626 \times 10^{-34}) \times (3 \times 10^8)/(2 \times 10^{-7}) = 9.94 \times 10^{-19}$ J
$f = c/\lambda = (3 \times 10^8)/(2 \times 10^{-7}) = 1.5 \times 10^{15}$ Hz
(k) $E = hc/\lambda = (6.626 \times 10^{-34}) \times (3 \times 10^8)/(4 \times 10^{-11}) = 4.97 \times 10^{-15}$ J
$f = c/\lambda = (3 \times 10^8)/(4 \times 10^{-11}) = 7.5 \times 10^{18}$ Hz
(l) $E = hc/\lambda = (6.626 \times 10^{-34}) \times (3 \times 10^8)/(5.0) = 3.98 \times 10^{-26}$ J
$f = c/\lambda = (3 \times 10^8)/(5.0) = 6.0 \times 10^7$ Hz

9. (a) $E = hf = (6.626 \times 10^{-34}) \times (5 \times 10^6) = 3.31 \times 10^{-27}$ J
eV $= E/q_e = (3.31 \times 10^{-27})/(1.6 \times 10^{-19}) = 2.07 \times 10^{-8}$ eV
(b) $E = hf = (6.626 \times 10^{-34}) \times (2 \times 10^{10}) = 1.33 \times 10^{-23}$ J
eV $= E/q_e = (1.33 \times 10^{-23})/(1.6 \times 10^{-19}) = 8.28 \times 10^{-5}$ eV
(c) $E = hf = (6.626 \times 10^{-34}) \times (4.5 \times 10^{14}) = 2.98 \times 10^{-19}$ J
eV $= E/q_e = (2.98 \times 10^{-19})/(1.6 \times 10^{-19}) = 1.86$ eV
(d) $E = hc/\lambda = (6.626 \times 10^{-34}) \times (3 \times 10^8)/(500 \times 10^{-9}) = 3.98 \times 10^{-19}$ J
eV $= E/q_e = (3.98 \times 10^{-19})/(1.6 \times 10^{-19}) = 2.48$ eV
(e) $E = hc/\lambda = (6.626 \times 10^{-34}) \times (3 \times 10^8)/(450 \times 10^{-9}) = 4.42 \times 10^{-19}$ J
eV $= E/q_e = (4.42 \times 10^{-19})/(1.6 \times 10^{-19}) = 2.76$ eV
(f) $E = hc/\lambda = (6.626 \times 10^{-34}) \times (3 \times 10^8)/(400 \times 10^{-9}) = 4.97 \times 10^{-19}$ J
eV $= E/q_e = (4.97 \times 10^{-19})/(1.6 \times 10^{-19}) = 3.11$ eV
(g) $E = hc/\lambda = (6.626 \times 10^{-34}) \times (3 \times 10^8)/(250 \times 10^{-9}) = 7.95 \times 10^{-19}$ J
eV $= E/q_e = (7.95 \times 10^{-19})/(1.6 \times 10^{-19}) = 4.97$ eV
(h) $E = hf = (6.626 \times 10^{-34}) \times (2 \times 10^{19}) = 1.33 \times 10^{-14}$ J
eV $= E/q_e = (1.33 \times 10^{-14})/(1.6 \times 10^{-19}) = 8.28 \times 10^4$ eV
(i) $E = hf = (6.626 \times 10^{-34}) \times (3 \times 10^{21}) = 1.99 \times 10^{-12}$ J
eV $= E/q_e = (1.99 \times 10^{-12})/(1.6 \times 10^{-19}) = 1.24 \times 10^7$ eV

10. (a) C
(b) $E = hf$

11. B

12. D

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1. (a) From $\lambda_{max} \times T = 2.898 \times 10^{-3}$, we get $\lambda_{max} = (2.898 \times 10^{-3}) \div 5778 = 5.02 \times 10^{-7}$ m = 502 nm
(b) This is in the blue-green region of the visible spectrum.
(c) Yellow.
(d) Our eyes perceive colour differently to scientific instruments which measure wavelengths and frequencies accurately.
2. From $\lambda_{max} \times T = 2.898 \times 10^{-3}$, we get $\lambda_{max} = (2.898 \times 10^{-3}) \div 1500 = 1.932 \times 10^{-6}$ m = 1932 nm
3. (a) $T = 37 + 273 = 310$ K
From $\lambda_{max} \times T = 2.898 \times 10^{-3}$, we get $\lambda_{max} = (2.898 \times 10^{-3}) \div 310 = 9.348 \times 10^{-6}$ m = 9.35 μm
(b) Infra-red.
(c) Night vision glasses.
4. From $\lambda_{max} \times T = 2.898 \times 10^{-3}$, we get $T = (2.898 \times 10^{-3}) \div (1.063 \times 10^{-3}) = 2.73$ K

28 Energy from the Sun

1. L = Surface area × energy per square metre $= 4 \times \pi \times (6.9 \times 10^8)^2 \times 6.41 \times 10^7 = 3.837 \times 10^{26}$ W
2. $E = \sigma T^4$; $T^4 = E/s = (6.41 \times 10^7)/(5.67 \times 10^{-8}) = 1.1305 \times 10^{15}$; $T = 5798.5$ K
3. The 11 year sunspot cycle.
4. $I_\odot = L/4\pi D^2 = (3.837 \times 10^{26})/(4 \times \pi \times (1.5 \times 10^{11})^2) = 1356.5$ W m^{-2}
5. $I_1/I_2 = d_2^2/d_1^2$; $I_1 = I_2 d_2^2/d_1^2 = (6.41 \times 10^7) \times (6.9 \times 10^8)^2/(1.5 \times 10^{11})^2 = 1356.4$ W m^{-2}
6. Mercury = 9393 W m^{-2}
Venus = 2616 W m^{-2}
Mars = 592 W m^{-2}
Jupiter = 50.4 W m^{-2}
Saturn = 15 W m^{-2}
Uranus = 3.71 W m^{-2}
Neptune = 1.51 W m^{-2}
Pluto = 0.91 W m^{-2}
7. (a) $E = mc^2$; $m = E/c^2 = (3.837 \times 10^{26})/(3 \times 10^8)^2 = 4.26 \times 10^9$ kg
(b) $m = (4.26 \times 10^9) \times (365 \times 24 \times 60 \times 60) = 1.34 \times 10^{17}$ kg $= 1.34 \times 10^{14}$ tonnes
(c) $t = (6 \times 10^{24})/(4.26 \times 10^9) = 1.41 \times 10^{15}$ s $= 4.47 \times 10^7$ years!
(d) $m = (1.34 \times 10^{17}) \times (4.5 \times 10^9) = 6.03 \times 10^{26}$ kg
(e) $n = (6.03 \times 10^{26})/(6 \times 10^{24}) = 100.5$ Earth masses
(f) $\% = (6.03 \times 10^{26}) \times 100/(2 \times 10^{30}) = 0.03\%$
8. (a) $A = \pi r^2 = \pi \times (6.38 \times 10^6)^2 = 1.28 \times 10^{14}$ m^2
(b) $E = 1356.4 \times 1.28 \times 10^{14} = 1.74 \times 10^{17}$ W
(c) $A = 4\pi r^2/2 = 2 \times \pi \times (6.38 \times 10^6)^2 = 2.56 \times 10^{14}$ m^2
(d) E = total energy from Sun/area $= (1.74 \times 10^{17})/(2.56 \times 10^{14}) = 679.7$ W m^{-2}