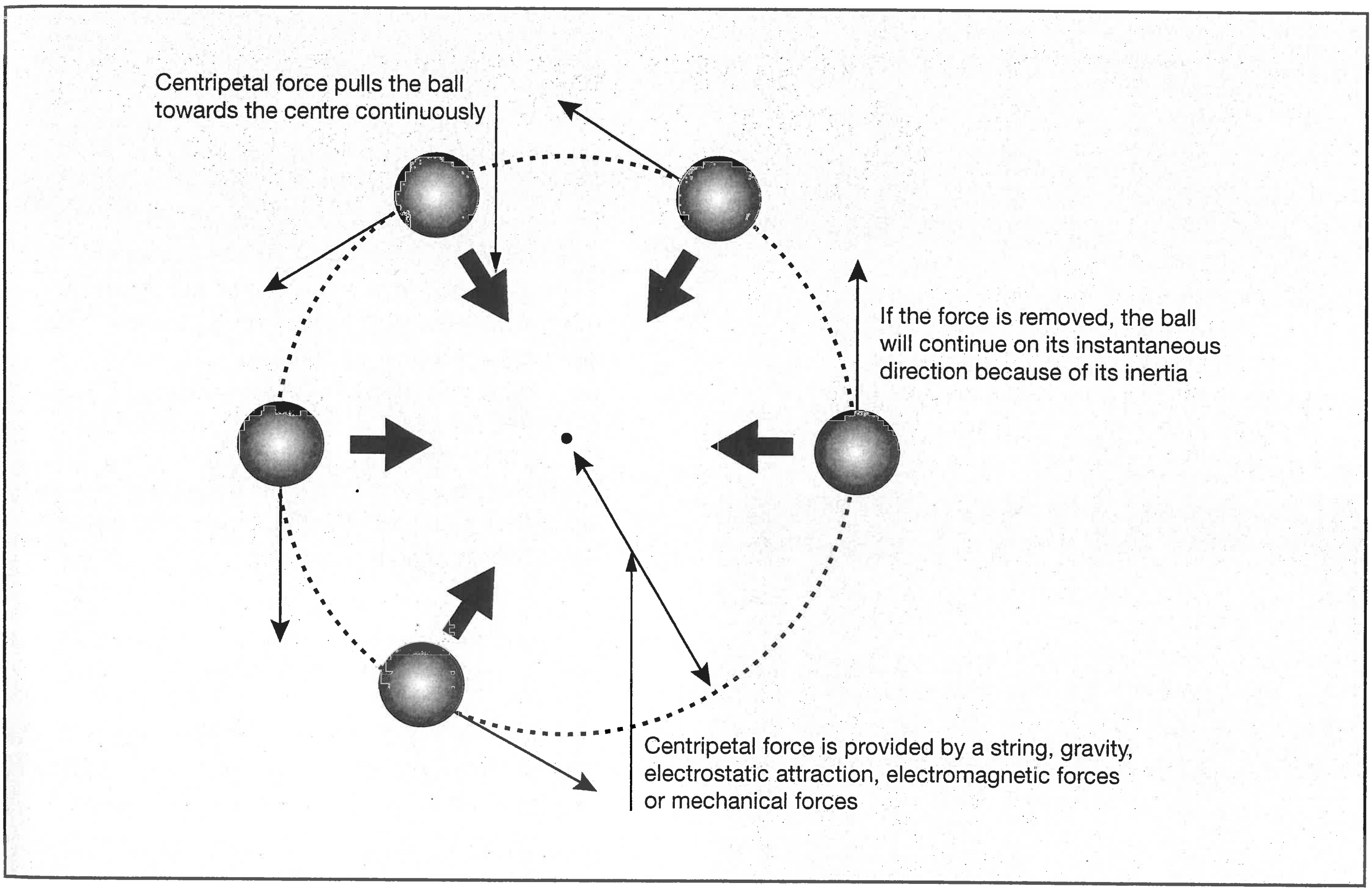
16 Circular Motion

When a spacecraft is in a stable orbit around the Earth, it is actually falling towards Earth all the time. The result is that it follows a curved path – the orbital path. Because its direction of travel is always changing, a force must be acting on it. The force involved in causing any object to move in a circular path is called a **centripetal force**. Since the only force acting on it is the gravitational force, then the **centripetal force is the gravitational force**.

Without a centripetal force, objects travelling in a circular path would move in a straight path. Centripetal forces act in all sorts of common situations, but they are always directed towards the centre of the circle of motion.



The equation we use to calculate the centripetal force acting on an object is: $$F_c = \frac{mv^2}{r}$$

Where F_c = centripetal force

m = mass of object in orbit

v = orbital speed of object

r = radius of orbit

From this we get the equation for the centripetal acceleration of an object in orbit: $$a_c = \frac{v^2}{r}$$

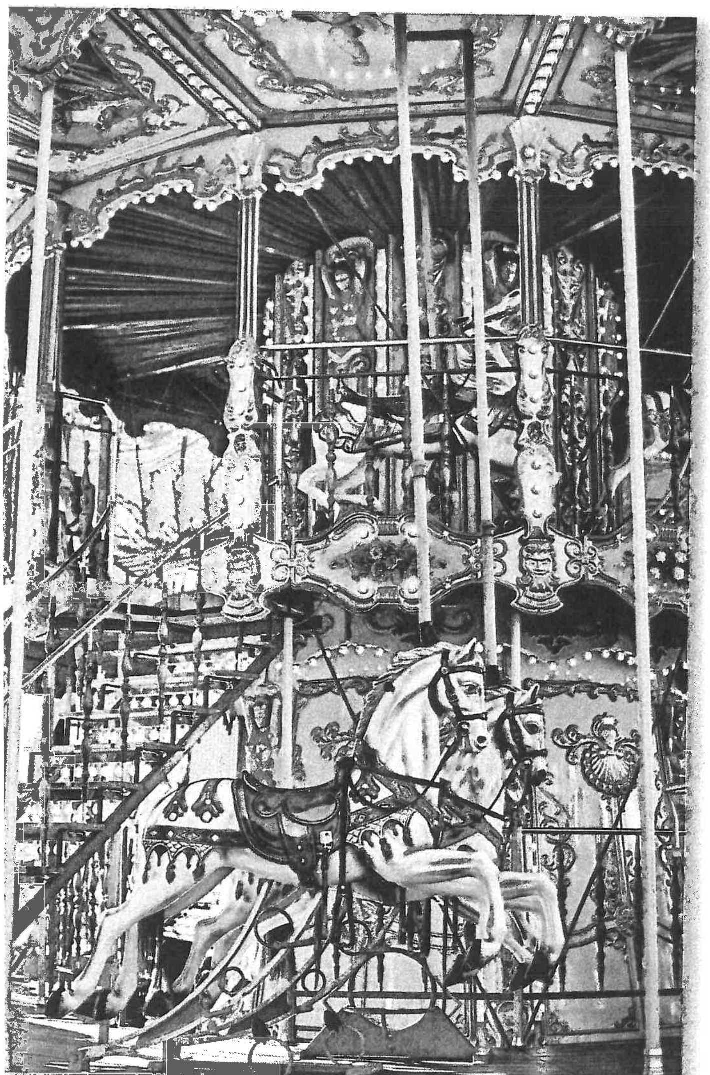
Where v = orbital speed of object

r = radius of orbit

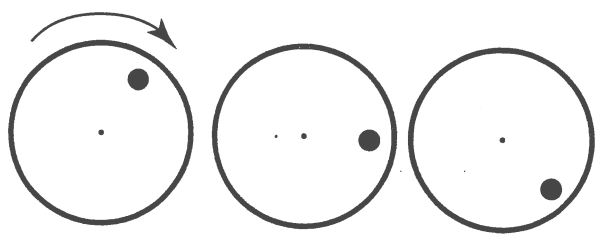
a_c = centripetal acceleration

QUESTIONS

1. A 200 kg satellite is orbiting the Earth at an altitude of 250 km. Its orbital speed is 27 800 km h^{-1}. Given the mass of the Earth as 6×10^{24} kg and its diameter as 12 760 km, find the:
   (a) Centripetal force acting on the satellite.
   (b) Centripetal acceleration of the satellite.
2. A 2 tonne truck goes around a curve of radius 150 m at 12 m s^{-1}.
   (a) Calculate the sideways friction between the road and the truck.
   (b) Calculate its centripetal acceleration.
3. A rock of mass 1.5 kg is tied to a piece of thread 1.2 m long and swung around in a horizontal circle. It moves through three rotations each second. Find:
   (a) Its linear speed.
   (b) Its centripetal acceleration.
   (c) The centripetal force acting on the rock.
4. P, Q and R are three 30 kg children on a merry-go-round 1.0 m, 2.0 m and 3.0 m from its centre. It is rotating four times each minute. Calculate the centripetal force on each child.

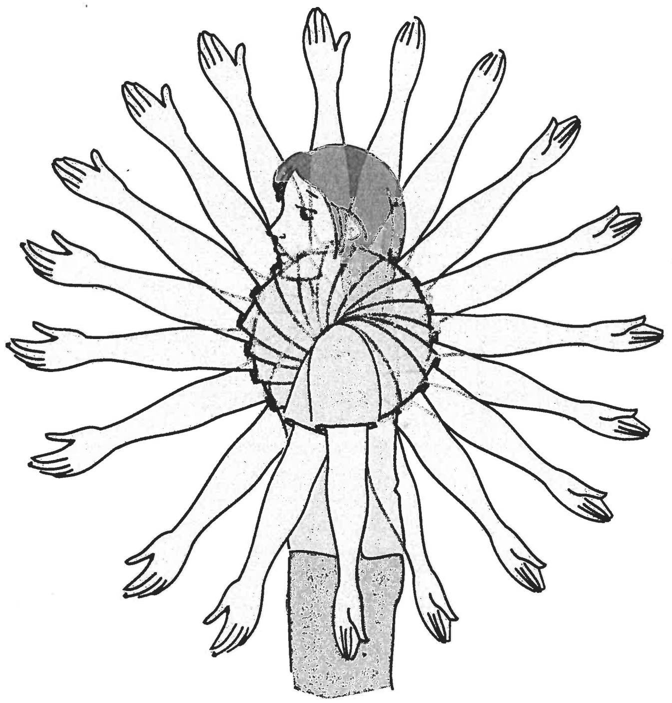


5. Three identical planets, X, Y and Z orbit a star at a radius of R, $2R$ and $3R$ respectively. Each planet rotates once about the star in time T. The linear speed of planet X is v. Find the ratio of:
   (a) The orbital speeds of X, Y and Z.
   (b) Their centripetal accelerations.
   (c) The centripetal forces acting on them.
6. Three other identical planets, U, V and W, also at a radius R, $2R$ and $3R$ respectively rotate about their sun with the same linear velocity, v.
   Find the ratio of:
   (a) The periods of U, V and W.
   (b) Their centripetal accelerations.
   (c) The centripetal forces acting on them.
7. The Russian Mir space station had a mass of 130 tonnes and orbited Earth at an altitude of 480 km with an orbital speed of 7621.4 m s^{-1}. The diameter of Earth is 12 760 km.
   (a) What centripetal force was acting on it?
   (b) Find the value of the acceleration due to gravity acting on an astronaut in Mir.
8. A disc is rotated at 5.0 Hz. The diagram shows three successive positions of a 25 g coin on the disc as taken by a stroboscopic camera.



   Calculate the centripetal force acting on the coin if it is 1.25 m from the centre of the disc.
9. Find the centripetal force acting on a satellite of mass 200 kg orbiting at an altitude of 1000 km above planet X. Its orbital speed is 18 000 km h^{-1} and X has a diameter of 8000 km.
   (A) 720 N
   (B) 1.0×10^6 N
   (C) 5.0×10^6 N
   (D) 1.3×10^7 N
10. How would the force acting on the object in Question 9 change if the mass of planet X was doubled and all other variables were the same?
11. Which statement about a satellite in a stable orbit is correct?
   (A) The gravitational force acting on the satellite is cancelled by the centripetal force.
   (B) There is zero net force acting on the satellite.
   (C) There is no centripetal force acting on the satellite, only the gravitational force.
   (D) The centripetal force acting on the satellite is the gravitational force.

12. The diagram below represents a stroboscopic picture of a girl's arm in almost uniform circular motion with a linear speed of 10 m s^{-1}. The diagram has not been drawn to scale. The frequency of the stroboscope was 40 hertz.



(a) Calculate the period of rotation of the arm.
(b) Calculate its frequency of rotation.
(c) Calculate the approximate length of the girl's arm.
(d) Calculate the centripetal acceleration of her fingertips.

13. An astronaut is in a 5000 kg spaceship which is orbiting planet Z at an altitude of 36 000 km. The acceleration due to gravity at that altitude is 4.0 m s^{-2}. How fast is the spaceship moving?
(A) 0.01 m s^{-1}
(B) 9.6×10^{-5} m s^{-1}
(C) 3.0×10^{-6} m s^{-1}
(D) Unable to be calculated without additional data.

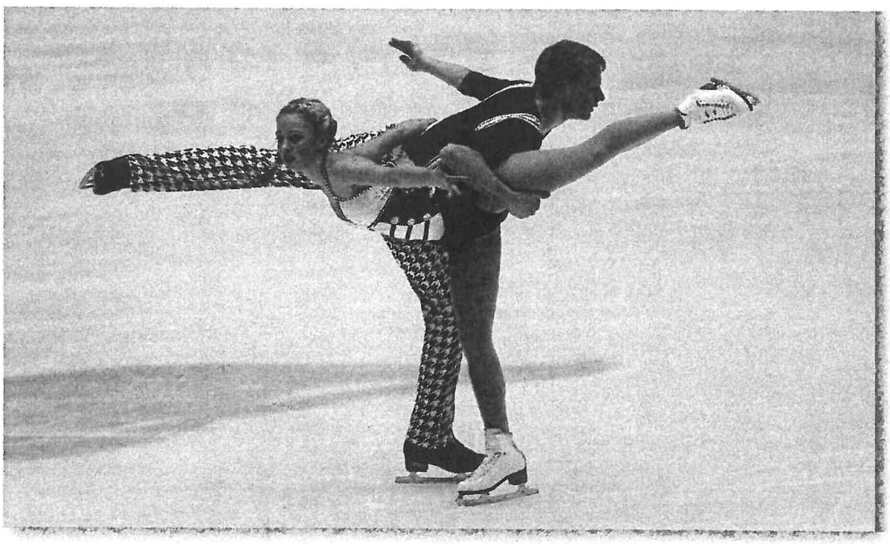
14. During the spin cycle of a washing machine, the clothes stick to the outer wall of the barrel as it spins at a rate as high as 1800 revolutions per minute. The radius of the barrel is 25 cm.
(a) Determine the speed of the clothes which are located on the wall of the spin barrel.
(b) Determine the acceleration of the clothes.

15. A manufacturer of CD-ROM drives claims that the player can spin a disc as frequently as 1200 revolutions per minute.
(a) If spinning at this rate, what is the speed of the outer row of data on the disc if this row is located 5.6 cm from the centre of the disc?
(b) What is the acceleration of the outer row of data?

16. George and Jenny are partners in pair figure skating. Last weekend, they perfected the death spiral element for inclusion in their upcoming competition. During this manoeuvre, George holds Jenny by the hand and swings her in a circle while she maintains her blades on the ice, stretched out in a nearly horizontal orientation.
Determine the net force which must be applied to Jenny, mass 50 kg, if her centre of mass rotates in a circle with a radius of 65 cm once every 1.8 seconds.

17. The merry-go-round at Physics Park takes riders on a spin at 8.5 m s^{-1}. The radius of the circle the outside riders move is about 7.5 m.
(a) Determine the time for outside riders to make one complete circle.
(b) Determine the acceleration of the riders.

18. In the display window of the toy store at the local mall, a battery-powered plane is suspended from a string and flying in a horizontal circle. The 650 gram plane makes a complete circle every 2.25 seconds. The radius of the circle is 0.9 m.
(a) Determine the orbital speed of the plane.
(b) Calculate its acceleration.
(c) What is the tension in the string as it flies in the horizontal circle?



15 Projectile Motion Problems 4

1. (a) From the grid, vertical displacement = about 7.8 squares = 0.78 m
   (b) From $\Delta y = u_y t_{fall} + \frac{1}{2}gt^2$
   $0.78 = 0 + 4.9t^2$
   Therefore $t = 0.4$ s (rounded)
   (c) Time between flashes = time/number of time intervals
   Number of time spaces (do not count dots because the first dot is at time zero – count spaces between dots = 14
   Therefore, time between flashes = time/14 = 0.0286 s
   (d) Frequency = (time between flashes = period)$^{-1}$ = $(0.0286)^{-1}$ = 35 Hz
   (e) Initial velocity = u_x (remember $u_y = 0$) = range (count the squares again)/time taken = about 0.98/0.4 = 2.45 m s^{-1}
2. (a) Measuring from the diagram, about 50° to the horizontal
   (b) By measuring the horizontal and vertical displacements as in the diagram, then applying the scale for the horizontal displacemen displacement = about 1.92 m
   (c) Again, by measuring and applying the scale = 1.44 m
   (d) From $(v_y)_{top}{}^2 = u_y + 2g\Delta y$, we get $u_y = 5.3$ m s^{-1}
   (e) From $v_y = u_y + gt_{rise}$, $t_{rise} = 0.54$ s
   (f) From the diagram. It takes 7 time intervals to reach maximum height = 0.54 s
   Therefore each time interval = period of the stroboscope = 0.077 s
   (g) Frequency of stroboscope = (period)$^{-1}$ = $(0.077)^{-1}$ = 12.9 Hz
   (h) Total time of flight = number of time intervals × 0.077 = 19 × 0.077 = 1.463 s
   Therefore, horizontal velocity = range/total time = 6.2/1.463 = 4.24 m s^{-1}
   (i) From $u_x = u \cos \theta$, $4.24 = u \cos 50°$
   Therefore $u = 6.6$ m s^{-1} at 50° to the horizontal
3. (a) Using a protractor and appropriately drawn reference lines on the diagram, $\theta = 35°$
   (b) By scale, knowing the height of the tower = 30 m, height above launch = 27 m
   (c) From $(v_{top})^2 = 0 = u_y{}^2 + 2g\Delta y = u_y{}^2 + 2 \times 9.8 \times 27$
   Therefore, $u_y = 23$ m s^{-1}
   (d) From $u_y = u \sin \theta$, $u = u_y/\sin 35° = 40.01$ m s^{-1}
   (e) From $v_{top} = 0 = u_y + at_{rise}$, $t_{rise} = 23/9.8 = 2.35$ s
   (f) From the diagram, 2.35 s = 8 time intervals
   Therefore the time interval between flashes = period of the stroboscope = 2.35/8 = 0.293 s
   (g) Frequency = 0.293^{-1} = 3.4 Hz
   (h) From a scale calculation = 72 m, or
   From the diagram, time to reach level with the side of the tower horizontally from the launch position = 8 time intervals = 8 × 0.293 =
   And $u_x = u \cos 35° = 40 \times \cos 35° = 32.7$ m s^{-1}
   So, distance between angry bird launch and tower = $u_x \times t = 32.7 \times 2.35 = 77$ m (note scale errors apply)
   (i) Launch velocity = vector sum of u_x and $u_y = \sqrt{(40^2 + 23^2)} = 46.1$ m s^{-1} at 30° to the horizontal (compared to 40.1 m s^{-1} in part (d) note estimating and rounding off errors apply here)
4. (a) By counting vertical grid = 16 squares = 160 cm = 1.6 m
   (b) Horizontal distance between plot points = 8 grid lines × 8 intervals = 64 grid lines = 6.4 m
   (c) From $(v_y)_{top}{}^2 = 0 = u_y{}^2 + 2g\Delta y$
   $u_y{}^2 = 2 \times 9.8 \times 1.6$
   Therefore $u_y = 5.6$ m s^{-1}
   (d) From $(v_y)_{top} = 0 = u_y + gt$
   We get $t_{rise} = 0.57$ s
   Therefore $t_{flight} = 2 \times 0.57 = 1.14$ s
   (e) There are 4 time intervals to rise, therefore 1 time interval = 0.57/4 = 0.143 s
   (f) From range = $u_x \times t_{flight}$
   $6.4 = u_x \times 1.14$
   Therefore $u_x = 5.6$ m s^{-1}
   (g) Since magnitude of $u_x = u_y$, angle of launch = 45°
   Therefore from $u_x = u \cos 45° = 5.6$
   We get $u = 7.92$ m s^{-1} at 45° to the horizontal

16 Circular Motion

1. (a) 1799 N towards centre of the Earth
   (b) 9.0 m s^{-2} towards centre of Earth
2. (a) 1920 N to the centre
   (b) 0.96 m s^{-2} to centre
3. (a) 22.6 m s^{-1}
   (b) 426.4 m s^{-2} to centre
   (c) 640 N to centre
4. P = 5.27 N to centre
   Q = 10.5 N to centre
   R = 15.8 N to centre
5. (a) 1 : 2 : 3
   (b) 1 : 2 : 3
   (c) 1 : 2 : 3

6. (a) 1 : 2 : 3
   (b) 6 : 3 : 2
   (c) 6 : 3 : 2
7. (a) 36 N to centre
   (b) 72 m s^{-2} to centre
8. 30.84 N
9. B
10. It would not change – the centripetal force is independent of the mass of the planet.
11. D
12. (a) 0.5 s (This assumes 20 images of her arm – one practically invisible lying perpendicularly parallel to her leg. If you calculated on the basis of 19 images, your answers will be slightly different as shown in brackets for each answer) (0.475 s)
    (b) 2.0 Hz (2.1 Hz)
    (c) About 0.8 m (0.76 m)
    (d) About 125 to 126 m s^{-2} to centre (132 m s^{-2})
13. D
14. (a) From $v = s/t = (1800 \times 2\pi \times 0.25)/60 = 47.1$ m s^{-1}
    (b) From $a_c = v^2/r = 8890$ m s^{-2} towards centre of rotation
15. (a) From $v = s/t = (1200 \times 2\pi \times 0.056)/60 = 3.52$ m s^{-1}
    (b) From $a_c = v^2/r = 221.3$ m s^{-2} towards centre of rotation
16. From $v = s/t = (2\pi \times 0.65)/1.8 = 2.27$ m s^{-1}
    Then, from $F_c = mv^2/r = 396$ 3 N
17. (a) From $v = s/t$, $t = s/v = 2\pi t/t = 2\pi \times 7.5/8.5 = 5.55$ s
    (b) From $a_c = v^2/r = 8.5^2/7.5 = 9.63$ m s^{-2}
18. (a) From $v = 2\pi r/t = 2\pi \times 0.9/2.25 = 2.57$ m s^{-1}
    (b) From $a_c = v^2/r = 7.35$ m s^{-2} towards centre of rotation
    (c) From $F_c = ma_c = 0.65 \times 7.35 = 4.78$ N (acting both ways in the string)

17 A Practical Analysis

1. A Readings 1, 5, 6 and 7, to examine the relationship between the mass of Y and the period of rotation.
   B Readings 1, 8, 9 and 10, to examine the relationship between mass of X and the period of rotation.
   C Readings 1, 2, 3 and 4 look at the relationship between the period of rotation and the radius.
2. A Period of rotation and mass of Y.
   B Period of rotation and mass of X.
   C Period of rotation and radius of rotation.
3. A Mass of X and radius.
   B Mass of Y and radius.
   C Masses of X and Y.
4. 1. 1 s
   2. 1.3 s
   3. 1.4 s
   4. 1.7 s
   5. 0.8 s
   6. 0.7 s
   7. 0.5 s
   8. 1.6 s
   9. 2.2 s
   10. 2.7 s
5. 1, 8, 9, 10
6. (a) A plot of mass X vs. T^2 will be a straight line.
   (b) A plot of (mass Y)$^{-1}$ vs. T^2 will be a straight line.
   (c) A plot of radius vs. T^2 will be a straight line.
7. A T^2 proportional to mass X
   B T^2 proportional to (mass Y)$^{-1}$
   C T^2 proportional to radius
8. $T = \sqrt{\dfrac{\text{radius} \times \text{mass X}}{\text{mass Y}}}$
9. A = 2.0 s
   B = 0.55 s
   C = about 190 g
   D = about 25 g
   E = about 0.1 m