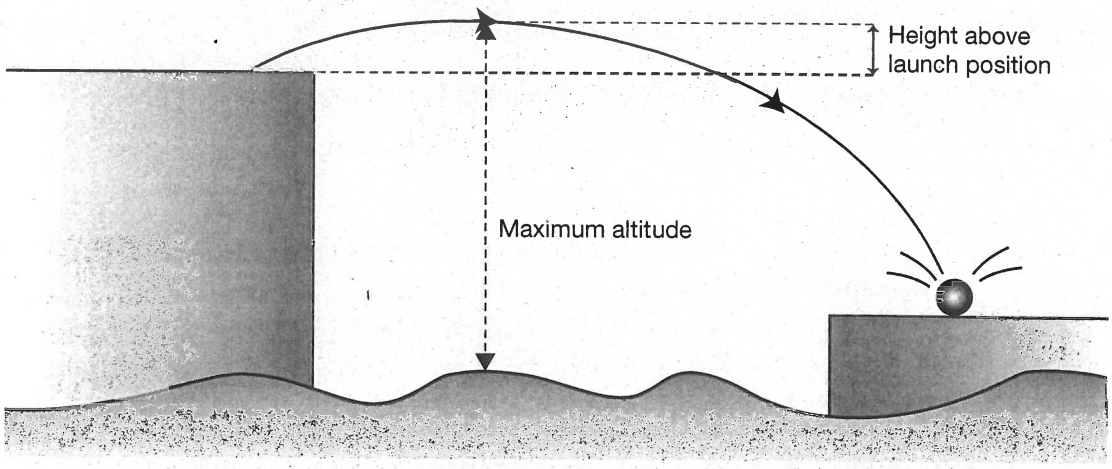
14 Projectile Motion Problems 3

Object landing at a different level

Special considerations:

- Total vertical displacement = difference in height between the two levels.
- If target lower, then vertical displacement is negative (assume upwards direction positive).
- If target higher, vertical displacement positive.
- Vertical velocity at top of flight = 0.
- Time to rise *does not* equal time to fall.
- Time to rise *is not* half the time of flight.
- Speed at launch *does not* equal speed at landing.
- Angle of launch *does not* equal angle of landing.
- Two halves of flight are *not* symmetrical.
- Maximum height occurs when vertical velocity = 0.



QUESTIONS

For each of the problems below, consider a projectile fired at an angle and landing on a surface either below or higher than that from which it is fired, as shown in the diagram.

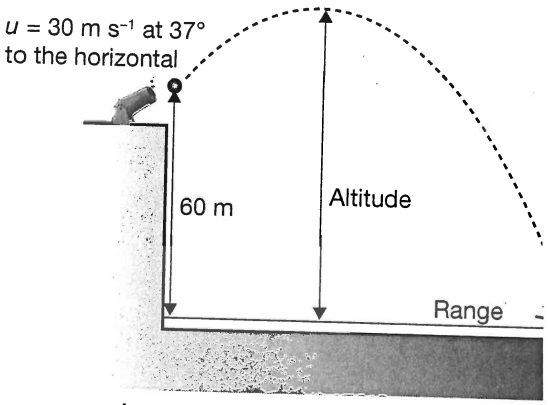
For each problem find any of these quantities which are not in the given data (not necessarily in the order given).

(a) The initial velocity of the projectile.
(b) Its initial horizontal velocity.
(c) Its initial vertical velocity.
(d) Its range.
(e) Its maximum height above the launch position.
(f) The time it takes to reach maximum height.
(g) Its time of flight.

1. A projectile is fired at 30° to the horizontal from the top of a cliff 200 m high. Its initial speed is 49 m s^{-1}.
2. A projectile is fired from the top of a 120 m high cliff at 25 m s^{-1}. It lands on the ground 6.4 s after firing.
3. A cannon is at the top of a 60 m high cliff firing at a castle on top of an adjacent cliff 110 m high. The cannon and castle are 200 m apart horizontally. The cannonballs hit 20 s after firing.
4. A cannonball is fired at 50 m s^{-1} from the top of a 200 m high cliff so that maximum range is achieved.
5. A boy throws a rock at 15 m s^{-1} from the top of a 75 m high cliff. The rock lands in the water at the bottom of the cliff 4.0 s later.
6. A projectile is thrown up from the top of a 60 m high cliff. It rises to a maximum height of 44.1 m above the cliff top. It hits the ground 76 m out from the base of the cliff.
7. A ball is thrown out from the edge of a 40 m high cliff with a velocity of 35.1 m s^{-1} at 30° to the horizontal.
8. A cannonball is fired at 40 m s^{-1} at 40° to the horizontal from the top of a 218.7 m cliff and hits a target 300 m from the base of the cliff.
9. A projectile fired up into the air from the top of a 75 m high cliff hits the ground 500 m out from the base 10 s later.
10. A cannon fires from the top of a 150 m high cliff at a castle 300 m from the base of the cliff. The ball hits the castle 15 s later.

Now answer these questions

11. A cannonball is fired at 40 m s^{-1} at an angle of 35° to the horizontal. Calculate the height at which the ball hits a vertical cliff 50 m away.
12. A player kicked a football from 30 m in front goalposts. The ball just cleared the crossbar was 3.0 m above the ground 2.0 s later. Calcu
    (a) The angle at which the ball was kicked.
    (b) The ball's initial speed.
13. A projectile is fired from the top of a cliff and rea height of 147 m above its launch point. It lands 9 from the base of the cliff 20 s after launch. Calcu
    (a) The height of the cliff.
    (b) The velocity at which the projectile was lau
14. A rock is dropped into a well and the distance travelled is $5.0t^2$ m, where t is the time it took fall. If the water splash is heard 2.75 seconds the rock was dropped, and the speed of sound 340 m s^{-1}, approximate the height of the well.
15. A mass of 5.0 kg is thrown vertically upwards the top of a building at 22 m s^{-1}. It hits the gro 4.8 seconds later. What is the height of the bui
16. Consider the projectile shown in the diagram.



Find:
(a) Its time of flight.
(b) Its maximum height above the ground.
(c) Its range.

13 Projectile Motion Problems 2

All answers in order of information requested in the chapter (answers may differ slightly due to rounding errors).

1. 20 m s^{-1} at 30° to horizontal, 17.3 m s^{-1}, 10 m s^{-1}, 35.3 m, 5.1 m, 1.02 s, 2.04 s
2. 30 m s^{-1} at 60° to horizontal, 15 m s^{-1}, 26 m s^{-1}, 79.5 m, 34.5 m, 2.65 s, 5.3 s
3. 35.6 m s^{-1} at 55.8° to horizontal, 20 m s^{-1}, 29.4 m s^{-1}, 120 m, 44.1 m, 3.0 s, 6.0 s
4. 57 m s^{-1} at 37° to horizontal, 45.4 m s^{-1}, 34.3 m s^{-1}, 318.2 m, 60.0 m, 3.5 s, 7.0 s
5. 258.5 m s^{-1} at 50° to horizontal, 166.1 m s^{-1}, 198 m s^{-1}, 6712 m, 2000 m, 20.2 s, 40.4 s
6. 296.9 m s^{-1} at 81.9° to horizontal, 41.7 m s^{-1}, 294 m s^{-1}, 2500 m, 4410 m, 30 s, 60 s
7. 200 m s^{-1} at 45° to horizontal, 141.4 m s^{-1}, 141.4 m s^{-1}, 4080 m, 1020.1 m, 14.43 s, 28.86 s
8. 150 m s^{-1}at 40° to horizontal, 115 m s^{-1}, 96.4 m s^{-1}, 2262 m, 474 m, 9.84 s, 19.67 s
9. 46.2 m s^{-1} at 60° to horizontal, 23.1 m s^{-1}, 40 m s^{-1}, 188.4 m, 81.6 m, 4.08 s, 8.16 s
10. 152.3 m s^{-1} at 74.8° to horizontal, 40 m s^{-1}, 147 m s^{-1}, 1200 m, 1102.5 m, 15 s, 30 s
11. 64 m s^{-1} at 50° to horizontal, 40 m s^{-1}, 50 m s^{-1}, 408.2 m, 127.5 m, 5.1 s, 10.20 s
12. 286.9 m s^{-1} at 25° to horizontal, 260 m s^{-1}, 121.2 m s^{-1}, 6433 m, 750 m, 12.37 s, 24.74 s
13. 400 m s^{-1} at 30° to horizontal, 346.4 m s^{-1}, 200 m s^{-1}, 14133 m, 2041 m, 20.4 s, 40.8 s
14. 176.1 m s^{-1} at 36° to horizontal, 143 m s^{-1}, 103 m s^{-1}, 3000 m, 543.3 m, 10.5 s, 21 s
15. 173.2 m s^{-1} at 58° to horizontal, 91.60 m s^{-1}, 147 m s^{-1}, 2748 m, 1102.5 m, 15 s, 30 s
16. (a) 78.4 m
    (b) 145.4 m s^{-1} at 15.5° to horizontal
17. (a) 6.93 s
    (b) 113.4 m s^{-1} at 17.4° to horizontal
18. (a) 91.8 m
    (b) 4.33 s
    (c) 22.9 m

14 Projectile Motion Problems 3

All answers in order of information requested in the chapter (answers may differ slightly due to rounding errors).

1. 49 m s^{-1} at 30° to horizontal, 42.4 m s^{-1}, 24.5 m s^{-1}, 396.9 m, 30.6 m, 2.5 s, 9.36 s
2. 25 m s^{-1} at 30.3° to horizontal, 21.6 m s^{-1}, 12.6 m s^{-1}, 138 m, 8.1 m, 1.3 s , 6.4 s
3. 101 m s^{-1} at 84.3° to horizontal, 10 m s^{-1}, 100.5 m s^{-1}, 200 m, 515 m, 10.3 s, 20 s
4. 50 m s^{-1} at 45° to horizontal, 35.4 m s^{-1}, 35.4 m s^{-1}, 387.2 m, 63.9 m, 3.6 s, 10.94 s
5. 15 m s^{-1} at 3.2° to horizontal, 14.97 m s^{-1}, 0.85 m s^{-1}, 59.9 m, 0.037 m, 0.087 s, 4.0 s
6. 31 m s^{-1} at 71.2° to horizontal, 10 m s^{-1}, 29.4 m s^{-1}, 76 m, 44.1 m, 3.0 s, 7.6 s
7. 35.1 m s^{-1} at 30° to horizontal, 30.4 m s^{-1}, 17.6 m s^{-1}, 158.1 m, 15.8 m, 1.8 s, 5.2 s
8. 40 m s^{-1} at 40° to horizontal, 30.64 m s^{-1}, 25.7 m s^{-1}, 300 m, 33.6 m, 2.62 s, 9.8 s
9. 65 m s^{-1} at 39.7° to horizontal, 50 m s^{-1}, 41.5 m s^{-1}, 500 m, 87.9 m, 4.2 s, 10 s
10. 66.6 m s^{-1} at 72.5° to horizontal, 20 m s^{-1}, 63.5 m s^{-1}, 300 m, 205.7 m, 6.5 s, 15 s
11. 23.6 m
12. (a) 37°
    (b) 18.8 m s^{-1}
13. (a) 886.4 m
    (b) 72.7 m s^{-1} at 47.6° to horizontal
14. Let T_1 be the time it takes the rock to reach the bottom of the well. If Δy is the depth of the well, then
    From $\Delta y = u_y t + \frac{1}{2}gt^2$ ($u_y = 0$), then $\Delta y = 5T_1^2$
    If T_2 be the time it takes soundwave from the splash to reach the top of the well, then $\Delta y = 340T_2$
    Therefore $5T_1^2 = 340T_2$
    We are given that $T_1 + T_2 = 2.75$
    Therefore $T_2 = 2.75 - T_1$
    So, $5T_1^2 = 340\ (2.75 - T_1)$
    So, $5T_1^2 + 340T_1 - 935 = 0$
    Solving, we get $T_1 = 2.65$ s
    From which, the depth of the well = $T_1^2 = 5(2.65)^2 = 35.1$ m
15. From $\Delta y = u_y t + \frac{1}{2}gt^2$ (Δy = vertical distance fallen)
    $\Delta y = 22 \times 4.8 - \frac{1}{2} \times 9.8 \times 4.8^2 = 7.3$ m
16. (a) Vertical component of its velocity = $u \sin 37° = 18.05$ m s^{-1}
    From $\Delta y = u_y t + \frac{1}{2}gt^2$, we get
    $-60 = 18.05t - 4.9t^2$
    From which, $t = 5.8$ s
    (b) Vertical velocity at top of flight = 0
    Therefore, from $v_y^2 = u_y^2 + 2g\Delta y$ (this time, Δy = maximum height above launch position)
    So, $0 = 18.05^2 - 19.6\Delta y$
    From which $\Delta y = 16.62$ m
    Therefore, maximum altitude above the ground = 16.62 + 60 = 76.62 m
    (c) Horizontal component of the velocity, $u_x = 30 \cos 37° = 23.96$ m s^{-1}
    Range = u_x × time of flight = 23.96 × 5.8 = 138.96 m = 139 m