ANSWERS

Exercise 1A PAGE 8

1 36 **2** 8 **3** 16
4 1 **5** 1 **6** 2
7 1 **8** 2 **9** 1
10 8 **11** 1 **12** -1
13 -32 **14** 1 **15** -1
16 4 **17** 5 **18** 2
19 3 **20** 9 **21** $\frac{1}{9}$
22 $\frac{1}{4}$ **23** $\frac{1}{8}$ **24** $\frac{3}{4}$
25 $\frac{1}{6}$ **26** 7 **27** 5
28 $\frac{1}{5}$ **29** 1 **30** 1
31 1 **32** -2 **33** 125
34 $\frac{4}{3}$ **35** $\frac{2}{3}$ **36** 27
37 $\frac{1}{27}$ **38** $\frac{1}{16}$ **39** $\frac{3}{2}$
40 25 **41** 25 **42** $\frac{1}{16}$
43 $\frac{1}{512}$ **44** 2^{16} **45** 2^{13}
46 2^6 **47** 2^{14} **48** 2^8
49 2^0 **50** 2^{-1} **51** 2^{-3}
52 2^{-3} **53** 3^3 **54** 3^4
55 3^0 **56** 3^{-1} **57** $3^{0.5}$
58 $3^{0.25}$ **59** 3^{-3} **60** $3^{-0.5}$
61 3^6 **62** 10^2 **63** 10^{-1}
64 10^{-1} **65** 10^{-2} **66** 10^{-2}
67 10^0 **68** 10^6 **69** 10^6
70 10^6 **71** 10^9 **72** 10^{-3}
73 $10^{0.5}$ **74** 10^3 **75** $10^{-0.5}$
76 $10^{\frac{3}{2}}$ **77** 5 **78** 4
79 3 **80** 4 **81** 7
82 5 **83** 8 **84** 8
85 10 **86** 8 **87** 9
88 8 **89** 10 **90** 8
91 7 **92** 3 **93** 2
94 2 **95** 15 **96** 3
97 4 **98** a^7 **99** $\frac{1}{a^3}$
100 $\frac{1}{b^5}$ **101** b^6 **102** $\frac{1}{b^6}$
103 a^2 **104** $\frac{1}{a^6}$ **105** a^2
106 b^{10} **107** b^{18} **108** $-4a^5$
109 $16a^5$ **110** $\frac{1}{a^2}$ **111** $\frac{a^9}{b^3}$
112 $6a^3$ **113** $\frac{1}{a^3}$ **114** a^2
115 $2a$ **116** $8a^5$ **117** $32a^5$
118 $4a^2b^4$ **119** $\frac{10y^2}{a^5}$ **120** $\frac{3}{2a^4b^3}$
121 $\frac{y}{3x}$ **122** $-9a^8b$ **123** a^2b^5
124 $\frac{3a^4}{2b^2}$ **125** $\frac{1}{a^5}$ **126** $\frac{y^3}{x^4}$
127 $\frac{b^4}{a^8}$ **128** $a^4 + a$ **129** $a^2 + a^4$
130 $2a + 3$ **131** $\frac{4}{5}$ **132** $7(2^n)$
133 9

Exercise 1B PAGE 13

1 $x = 3$ **2** $x = 5$ **3** $x = 7$
4 $x = -3$ **5** $x = -5$ **6** $x = -7$
7 $a = 0.5$ **8** $a = -0.5$ **9** $y = -2$
10 $c = 3$ **11** $d = 3$ **12** $x = 3$
13 $x = 3$ **14** $x = 6$ **15** $x = 2$

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16 $x = 1$ **17** $x = 0.75$ **18** $x = -1$
19 $x = 3$ **20** $x = 2$ **21** $x = 5$
22 $k = 0.75$ **23** $p = -0.25$ **24** $x = 1.5$
25 $x = 1$ **26** $h = 6$ **27** $x = 0.5$
28 $x = 1$ **29** $x = -0.25$ **30** $n = 2$
31 $a = \pm 4$ **32** $p = \pm 10$ **33** $x = 2$
34 $x = 4$ **35** $x = 16$ **36** $x = 64$
37 $h = 0.25$ **38** $y = 0.5$ **39** $p = 3$
40 $x = 10\,000$ **41** $x = \pm 5$ **42** $x = \pm\frac{2}{3}$
43 $x = \pm 3$ **44** $x = 1000$ **45** $p = \pm 4$
46 $x = 4$ **47** $x = 81$ **48** $x = \frac{1}{16}$
49 $w = 4$ **50** $x = 2$ **51** $h = 24$
52 $x = -1$ or 7 **53** $w = \frac{9}{4}$, i.e., 2.25
54 $z = \frac{27}{8}$, i.e., 3.375
55 **a** The third solution is $x = 0$.
b $x = 0, x = -4, x = 4$
c **i** $x = 0, x = 4$ **ii** $x = 0, x = -5, x = 5$
iii $x = 0, x = 25$ **iv** $x = 0$

Exercise 1C PAGE 16

1 $x = 4.5$ **2** $x = 3.2$
3 $x = 2.4$ **4** $x = 2.8$
5 $x = 6.7$ **6** $x = 2.2$
7 $x = 2.46$ **8** $x = 2.55$
9 $x = 3.32$ **10** $x = 6.26$
11 $x = 3.86$ **12** $x = 3.53$
13 $x = 8.43$ **14** $x = -1.51$
15 $x = 4.35$ **16** $x = 3.34$
17 $x = 2.74$ **18** $x = -6.99, x = 3.38$

Miscellaneous exercise one PAGE 17

1 Completed table not given here. $y = -x^3 + 2x^2 - x + 7$
2 **a** Australia has an area of approximately $7\,682\,000$ km^2.
b Light travels at a speed of $300\,000\,000$ m/sec.
c A golf ball has a mass of approximately 0.045 kg.
d The Earth is approximately $150\,000\,000$ km from the sun.
e Gamma waves have a wave length less than $0.000\,000\,000\,01$ metres.
f The Earth orbits the sun at a speed of approximately $107\,000$ km/hr.
g In 1961 the first man in space, Yuri Gagarin, flew his spacecraft at a speed of $27\,400$ km/hr, i.e. approximately 7600 m/sec.
3 **a** At the beginning of this century China had a population of approximately 1.27×10^9 and India had a population of approximately 1.03×10^9.
b The egg cell, or ovum, with a radius of approximately 5×10^{-5} metres, i.e. 5×10^{-2} mm, is the largest single human cell.
c It is thought that approximately 1.1×10^6 people die each year of malaria.
d Some adult wasps of a particular species could weigh just 5×10^{-3} grams.
e Concorde, the first supersonic passenger airliner, had a cruising speed of 2.16×10^3 km/hr.
4 **a** $12\,000\,000$ **b** $46\,800$
c $305\,000\,000$ **d** 0.01
e 0.206 **f** 0.006
5 **a** 5^2 **b** 5^3 **c** $5^{0.5}$
d 5^{-1} **e** 5^{-2} **f** $5^{-0.5}$
g 5^7 **h** 5^7 **i** 5^6
j 5^{12} **k** 5^6 **l** 5^0
m 5^5 **n** 5^2 **o** 5^5
6 **a** $x = 5$ **b** $x = 4$
c $x = -2$ **d** $x = \frac{5}{2}$
e $x = \frac{5}{3}$ **f** $x = \frac{1}{3}$
g $x = -\frac{1}{3}$ **h** $x = -3$
i $x = -\frac{2}{5}$ **j** $x = \frac{1}{4}$
k $x = -\frac{3}{2}$ **l** $x = 8$
7 **a** $y = \pm\frac{1}{3}$ **b** $p = \frac{4}{9}$
c $x = \frac{3}{2}$ **d** $x = 8$
e $x = \pm 5$ **f** $t = \pm\frac{1}{5}$
g $t = \frac{27}{2}$ **h** $x = 9$
i $x = 0, x = 1$ **j** $x = 0, x = \pm\pm 1$
8 **a** $x = 1, x = 3$ **b** $x = 0, x = 2$
c $x = -1, x = 0$ **d** $x = 1, x = 3$

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Exercise 2A PAGE 25

1 Rule: $y = 3^x$

x	0	1	2	3	4	5
y	1	3	9	27	81	243

2 Rule: $y = 7^x$

x	0	1	2	3	4	5
y	1	7	49	343	2401	16 807

3 Rule: $y = 1.5 \times 2^x$

x	0	1	2	3	4	5
y	1.5	3	6	12	24	48

4 Rule: $y = 1.75 \times 8^x$

x	0	1	2	3	4	5
y	1.75	14	112	896	7168	57 344

5 Rule: $y = 2^{x+1}$

x	0	1	2	3	4	5
y	2	4	8	16	32	64

6 Rule: $y = 2.5 \times 4^{x+1}$

x	1	2	3	4	5	6
y	40	160	640	2560	10 240	40 960

7 **a** Quadratic **b** $y = x^2 + 1$

8 **a** Exponential **b** $y = 4^x$

9 **a** Linear **b** $y = 2x + 3$

10 **a** Quadratic **b** $y = 2x^2$

11 **a** Exponential **b** $y = 1.5(8)^x$

12 **a** Exponential **b** $y = 5^x$

13 **a** Quadratic **b** $y = x^2 + x$

14 **a** Exponential **b** $y = 6^x$

15 **a** Exponential **b** $y = 3(2)^x$

16 **a** Reciprocal **b** $y = \dfrac{60}{x}$

17 **a** Cubic **b** $y = x^3 + 1$

18 **a** Linear **b** $y = 20 - 3x$

19 **a** (0, 1)

b Discuss your answer with your teacher.

20–22 Discuss your answer with your teacher.

23 **a** $y = 2^x$ **b** $y = 3^x$

24 **a** 12 **b** 28 **c** 40

25 **a** $y = 3^{x-2}$ **b** $y = 2^x + 2$

c $y = 2^{x-2}$ **d** $y = 3^x - 2$

e $y = 3^{x+1} + 2$ **f** $y = 2^{x-2} - 2$

Exercise 2B PAGE 31

3 Approximately 61 million, assuming annual growth rate for the given years continues.

4 Approximately 6000.

5 **a** $A = 80$, $k = 1.08$

b 8%

c 1200

6 Approximately 1200.

7 **a** 68 **b** 29 **c** 0.84, 115

d 115 **e** 14

8 **a** $k \approx 80$, $a \approx 0.92$

b Approximately 27.

c Approximately 2021.

9 **a** 10 000 of A and 1000 of B.

b 4200 of A and 1300 of B.

c 6.2

10 **a** $k \approx 850$, a is between 0.9 and 0.91.

b Approximately 14 weeks.

Miscellaneous exercise two PAGE 35

1 **a** II **b** IV **c** III

d I **e** III **f** IV

g III **h** I

2 **a** $x = \pm 7$ **b** $x = \pm 10$ **c** $x = 10$

d $x = 2$ **e** $x = 4$ **f** $x = 0$

g $x = 3$ **h** $x = -1$ **i** $x = -2$

j $x = -3$ **k** $x = -1$ **l** $x = -2$

m $x = -3$ **n** $x = 0, \pm 5$ **o** $x = -0.125$

p $x = 0.5$ **q** $x = 0.25$ **r** $x = \pm 3$

3 **a** 12 000 **b** 12 610 000 **c** 0.000 26

d 6 **e** 12 630

4 **a** $y = 2^{x+3}$, $y = 8 \times 2^x$ **b** $y = 3^x - 2$

5 Check reasonableness of answers by evaluating $5^{1.6}$, $5^{2.4}$ and $5^{2.5}$ on a calculator.

6 –2 or 0

7 0 or 2

8 **a** $k \approx 18.9$, $a \approx 0.93$

b Approx. 6.22 a.m. (Remember, graph shows number of *half* hours.)

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Exercise 3A PAGE 42

1 18 **2** 26 **3** 44
4 38 **5** 42 **6** 36
7 72 **8** 44 **9** 42
10 46 **11** 324 **12** 2744
13 8 **14** 20 **15** 28
16 26 **17** 29 **18** 21
19 27 **20** 9 **21** 162
22 18 **23** 26 **24** 4374
25 27 **26** 216 **27** 343
28 91 **29** 4 **30** 7
31 49 **32** 94

Exercise 3B PAGE 48

1 $T_1 = 6$, $T_{n+1} = T_n + 4$.
2 $T_1 = 28$, $T_{n+1} = T_n - 2$.
3 $T_1 = 5$, $T_{n+1} = T_n + 10$.
4 $T_1 = 7.5$, $T_{n+1} = T_n + 2.5$.
5 $T_1 = 100$, $T_{n+1} = T_n - 11$.
6 $T_1 = 6$, $T_n = 2T_{n-1}$.
7 $T_1 = 0.375$, $T_n = 4T_{n-1}$.
8 $T_1 = 384$, $T_n = 0.25T_{n-1}$.
9 $T_1 = 50$, $T_n = 3T_{n-1}$.
10 $T_1 = 1000$, $T_n = 1.1T_{n-1}$.
11 Neither arithmetic nor geometric.
12 Geometric.
13 Arithmetic.
14 Arithmetic.
15 Neither arithmetic nor geometric.
16 Geometric.
17 Geometric.
18 Arithmetic.
19 Neither arithmetic nor geometric.
20 Neither arithmetic nor geometric.
21 Arithmetic.
22 Geometric.
23 $T_1 = 8$, $T_2 = 11$, $T_3 = 14$, $T_4 = 17$, $T_{n+1} = T_n + 3$.
24 $T_1 = 100$, $T_2 = 97$, $T_3 = 94$, $T_4 = 91$, $T_{n+1} = T_n - 3$.
25 $T_1 = 11$, $T_2 = 22$, $T_3 = 44$, $T_4 = 88$, $T_{n+1} = 2T_n$.
26 $T_1 = 2048$, $T_2 = 1024$, $T_3 = 512$, $T_4 = 256$, $T_{n+1} = 0.5T_n$.
27 **b** $N_{n+1} = N_n + 800$
28 **a** The sequence is a geometric progression.
b The next three terms after the first are 550, 605 and 665.5.
29 **a** The sequence is a geometric progression.
b The next three terms after the first are 1250, 1562.5 and 1953.125.
30 **a** The sequence is a geometric progression.
b The next three terms after the first are 21 600, 19 440 and 17 496.
31 **a** $T_1 = 3$, $T_{n+1} = T_n + 5$.
b The sequence is arithmetic.
32 **a** $T_1 = 1.5$, $T_{n+1} = T_n \times 2$.
b The sequence is geometric.
33 **a** $T_1 = 4$, $T_2 = 9$, $T_3 = 16$, $T_4 = 25$, $T_5 = 36$.
b Neither arithmetic nor geometric.
34 **a** After 1 year, 2 years, 3 years and 4 years the account is worth \$1296, \$1392, \$1488 and \$1584 respectively.
b The amounts are in arithmetic progression.
c $T_1 = \$1200$, $T_{n+1} = T_n + \$96$.
35 $T_1 = 4$, $T_{n+1} = T_n + 1$.
36 $T_1 = \$45\,000$, $T_{n+1} = T_n + \$1500$. The terms of the sequence progress arithmetically.
37 \$68 000 in 2014, \$71 400 in 2015, \$74 970 in 2016, \$78 718.50 in 2017.
$T_1 = \$68\,000$, $T_{n+1} = 1.05T_n$.
38 $T_1 = \$1500$, $T_{n+1} = 1.08T_n$.
39 $T_1 = \$36\,000$, $T_{n+1} = 0.85T_n$.

Exercise 3C PAGE 58

1 $T_{100} = 506$ **2** $T_{100} = 289$
3 $T_{100} = 815$ **4** $T_{100} = -120$
5 $T_{25} = 5 \times 2^{24}$ **6** $T_{25} = 1.5 \times 4^{24}$
7 $T_{25} = 8 \times 3^{24}$ **8** $T_{25} = 11 \times 2^{24}$
9 $T_{28} = 223$ **10** $T_{20} = 3\,495\,265$
11 $T_{19} = 774\,840\,977$ **12** $T_{45} = 6$
13 $T_1 = 48$, $T_{n+1} = T_n + 3$. Julie successfully completes 90 items on the 15th day.
14 Substituting y for T_n and x for n the rule $T_n = a + (n-1)d$ becomes $y = dx + (a - d)$.
This is the equation of a straight line with gradient d, cutting the y-axis at $(0, a - d)$.
15 Substituting y for T_n and x for n the rule
$T_n = ar^{n-1}$ becomes $y = ar^{x-1}$, i.e. $y = \left(\frac{a}{r}\right) r^x$.
An exponential function cutting the y-axis at $\left(0, \frac{a}{r}\right)$.

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16 As $n \to \infty$, the 'nd' term in the expression $a + (n-1)d$ will dominate.

Thus as $n \to \infty$, T_n will be increasingly large and positive if $d > 0$ and increasingly large and negative if $d < 0$.

17 As $n \to \infty$,the n in the expression ar^{n-1} will dominate.

Thus as $n \to \infty$,

if $r > 1$, T_n will become increasingly large, either positively or negatively dependent on the sign of the constant a.

if $r < -1$, T_n will become increasingly large, alternating between large negative and large positive.

if $-1 < r < 1$, T_n will become smaller and smaller, maintaining the sign of the constant a if r is positive and alternating between small positive and small negative if r is negative.

That is, if $-1 < r < 1$, as $n \to \infty$, $T_n \to 0$.

18 The first four terms are 8, 11, 14, 17. The 50th term is 155. The 100th term is 305.

19 The first four terms are 100, 97, 94, 91. The 50th term is –47. The 100th term is –197.

20 11, 22, 44, 88, 180224, 184549376

21 2048, 1024, 512, 256, 0.0625

22 **a** $T_n = 9 + (n-1) \times 6$, i.e. $T_n = 6n + 3$

b $T_n = 7 + (n-1) \times 1.5$, i.e. $T_n = 1.5n + 5.5$

23 **a** $3 \times 2^{n-1}$ **b** $100 \times 1.1^{n-1}$

24 **a** 856 **b** 3495

c The 142858th term.

25 **a** 126953.125 **b** The 14th term.

26 **a** 844700 **b** The 60th term.

27 1, 8, 27, 64, neither

28 **a** 64 **b** 7

29 **a** 1850 **b** 2000

30 **a** 7971615 **b** 5

31 **a** –1835008 **b** 7

32 The amount in the account at the end of ten years is $8635.70.

33 Just after the end of the 22nd year, i.e. early in the 23rd year.

34 $T_{n+1} = 1.08 \times T_n + \$200$, $T_{10} = \$11533.01$

35 $T_{n+1} = 1.08 \times T_n - \$200$, $T_{10} = \$5738.39$

Miscellaneous exercise three PAGE 61

1 **a** Quadratic **b** Exponential

c Linear **d** Quadratic

e Reciprocal **f** Linear

g Linear **h** Quadratic

i Quadratic **j** Reciprocal

k Linear **l** Exponential

2 **a** $x \approx 2.3$ **b** $x \approx 2.6$ **c** $x \approx 1.4$

3 **a** 3 **b** –3 **c** –1

d 0.5 **e** 0 **f** 1.5

g –6 **h** 1.5

4 **a** $T_n = 4 \times 1.5^{n-1}$ **b** $T_n = \frac{8}{3} \times 1.5^n$

5 243

6 **a** $x = 3$ **b** $x = -3$ **c** $x = -1$

d $x = 8$ **e** $x = \pm 8$ **f** $x = 11$

7 **a** 4 **b** 64 **c** 9

d 0.2 **e** 2

8 **a** –4, **b** –4, 4, 4 **c** Neither

9 **a** –2, **b** –4, 8, 16 **c** Neither

10 **a** $48 - 4k$ **b** 95.5

11 a^7 **12** $12x^3y^4$ **13** $\frac{3a^2}{2b^2}$

14 $72a^8b^3$ **15** $\frac{9}{8a^4b^3}$ **16** $\frac{48b}{a}$

17 $\frac{2a^5}{b^5}$ **18** $k^4 + 1$ **19** $p^3 - p^6$

20 125 **21** 25 **22** $\frac{8}{3}$

Exercise 4A PAGE 69

1 **a** 68 **b** 100 **c** 138

2 **a** 53 **b** 123 **c** 28

3 **a** –9 **b** 0 **c** 9

4 $T_1 = 6$, $T_2 = 11$, $T_3 = 16$, $T_4 = 21$.
$S_1 = 6$, $S_2 = 17$, $S_3 = 33$, $S_4 = 54$.

5 $T_1 = 11$, $T_2 = 14$, $T_3 = 17$, $T_4 = 20$.
$S_1 = 11$, $S_2 = 25$, $S_3 = 42$, $S_4 = 62$.

6 $T_1 = 22$, $T_2 = 19$, $T_3 = 16$, $T_4 = 13$.
$S_1 = 22$, $S_2 = 41$, $S_3 = 57$, $S_4 = 70$.

7 $T_1 = 25$, $T_2 = 32$, $T_3 = 39$, $T_4 = 46$. $T_5 = 53$. Yes

8 $T_1 = 1$, $T_2 = 4$, $T_3 = 9$, $T_4 = 16$. $T_5 = 25$. No

9 **a** 48 **b** 8780

10 **a** 174 **b** 60

11 5050

12 **b** 2088

13 **b** 14309

14 78 km, 1470 km **15** 285

16 $31500 **17** A: $762500 B: $734000

18 $6840

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Exercise 4B PAGE 73

1 $T_1 = 6$, $T_2 = 18$, $T_3 = 54$, $T_4 = 162$.
$S_1 = 6$, $S_2 = 24$, $S_3 = 78$, $S_4 = 240$.

2 $T_1 = 16$, $T_2 = 24$, $T_3 = 36$, $T_4 = 54$.
$S_1 = 16$, $S_2 = 40$, $S_3 = 76$, $S_4 = 130$.

3 $T_1 = 1$, $T_2 = 1$, $T_3 = 2$, $T_4 = 3$, $T_5 = 5$. No

4 $T_1 = 8$, $T_2 = 24$, $T_3 = 72$, $T_4 = 216$, $T_5 = 648$. Yes

5 32 767 **6** 40 940

7 650 871 **8** 104 139.36

9 3071.25, 12 287.25, 49 151.25

10 20 **11** 25

12 393 216, 524 287.5 **13** $1 015 000

14 **a** Approximately 5500 tonnes.
b Approximately 6050 tonnes.
c Approximately 6655 tonnes.
d Approximately 107 000 tonnes.

15 **a** Approximately $69 000.
b Approximately $79 350.
c Approximately $211 000.
d Approximately $1 218 000.

16 Entries in last line of table are:

1/1/18	$\$1200 \times 1.1^4$	$\$1200 \times 1.1^3$	$\$1200 \times 1.1^2$
$\$1200 \times 1.1$	$1200	$7326.12	

Immediately following the deposit of $1200 on 1/1/29 there will be $43 140 in the account, to the nearest dollar.

17 $14 784

18 **a** Approximately 2653.
b Approximately 3299.
c Approximately 3299.
d Approximately 43 200.
e Approximately 125 700.

19 **a** 17 years
b Approximately 79 500 tonnes.

20 **a** First term $\$P$, common ratio 1.095, number of terms 21.
b 829.7

Exercise 4C PAGE 79

1 GP A: **a** 0.4 **b** S_∞ exists and equals 40.
GP B: **a** 1.5 **b** S_∞ does not exist.
GP C: **a** 0.3 **b** S_∞ exists and equals 50.

2 **a** S_∞ exists and equals 200.
b S_∞ exists and equals 400.
c S_∞ does not exist.
d S_∞ exists and equals 450.
e S_∞ does not exist.
f S_∞ exists and equals 50.
g S_∞ exists and equals 0.9.
h S_∞ exists and equals 2048.

3 0.6

4 66

5 Table not shown here.
a 25 mg **b** 10 mg

6 250.

The idea that the athlete's performance might diminish according to some geometrical pattern is not unreasonable as he would tire as time went on. Hence the situation could feasibly be modelled by a geometrical sequence but we would be surprised if the numbers exactly fitted the model.

However, if the geometrical sequence were continued, by the 10th minute the athlete is completing the exercise approximately 7 times and by the 15th minute approximately 2 times so it could be argued that there is resting going on in these later minutes. If the athlete has to complete the exercise at least once each minute then counting would stop after about 18 minutes with a total of about 245 completions.

7 **a** 1.2 m **b** Approx. 9 cm. **c** 8 m

8 11.67 m

Miscellaneous exercise four PAGE 81

1 **a** 2^6 **b** 2^8 **c** 2^7
d 2^2 **e** 2^{10} **f** 2^2
g 2^{14} **h** 2^0 **i** 2^5

2 $\frac{1}{2}$ **3** 16 **4** $\frac{9}{4}$ (i.e., 2.25)

5 1 **6** 64 **7** $\frac{1}{25}$ (i.e., 0.04)

8 $\frac{1}{3}$ **9** 5 **10** $\frac{1}{7}$

11 Compare your reasoning with that of others in your class.

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12

	$T_1, T_2, T_3, T_4, T_5, \ldots$	Recursively defined
a	17, 22, 27, 32, 37, …	$T_n = T_{n-1} + 5, T_1 = 17$
b	100, 93, 86, 79, 72, …	$T_{n+1} = T_n - 7, T_1 = 100$
c	5, 15, 45, 135, 405, …	$T_n = 3T_{n-1}, T_1 = 5$
d	6, 10, 14, 18, 22, …	$T_{n+1} = T_n + 4, T_1 = 6$
e	2, 6, 18, 54, 162, …	$T_{n+1} = 3T_n, T_1 = 2$
f	17, 9, 1, –7, –15, …	$T_{n+1} = T_n - 8, T_1 = 17$

13 $a = 3, k = 2$, for **a** $T_{20} = 524\,288$, for **b** $T_{20} = 62$

14 **a** 15

15 **a** $x = 29, \quad T_{n+1} = T_n + 21, \quad T_1 = 8$

b $x = 20, \quad T_{n+1} = 2.5 \times T_n, \quad T_1 = 8$. Or:

$x = -20, \quad T_{n+1} = -2.5 \times T_n, \quad T_1 = 8$.

16 **a** When $t = 3.493$ (to three decimal places), i.e. in approximately 3.5 years.

b When $t = 6.986$ (to three decimal places), i.e. in approximately 7 years.

17 $T_1 = 30, T_{n+1} = T_n + 3$.

One day prior to the championships Rosalyn will practise for 90 minutes.

During the 20 days prior to the championships Rosalyn will practise for a total of 20 hours and 30 minutes.

18 After 20 years, account A will have a balance of \$3 207 135 (nearest dollar) compared to account B, which after 20 years will have a balance of \$1 949 636 (nearest dollar).

The organisers need to have \$607 906 available 'now'. (Rounded up to next dollar.)

Exercise 5A PAGE 88

1 **a** A → B, D → F

b B → D, F → H, H → I

c B, D, F, H

2 I: B, C, D, E, F, G — II: A

III: H — IV: A, D, G

V: A, D, H — VI: C, F

VII: A, E, G — VIII: A, B, F, G

IX: A, E, F, G — X: B, F, H

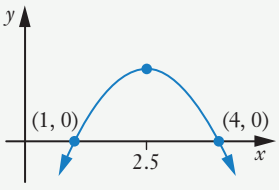
3 **a** C, E, H, K, M, O. **b** A, B, I, J, N, P.

c D, F, G, L.

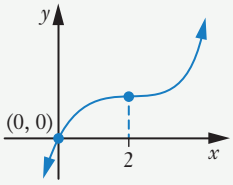
4 **a** 2 **b** 4 **c** 0 **d** –2

e –4 **h** 2 **g** 2

5



6



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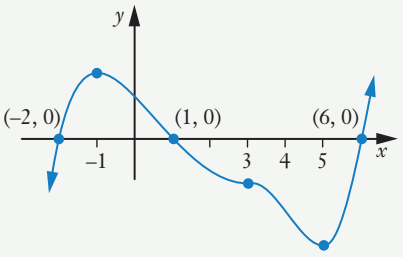

Exercise 5B PAGE 92

1

Point P	Point Q	Grad of chord PQ
(2, 4)	(4, 16)	6
(2, 4)	(3, 9)	5
(2, 4)	(2.5, 6.25)	4.5
(2, 4)	(2.1, 4.41)	4.1
(2, 4)	(2.01, 4.0401)	4.01
(2, 4)	(2.001, 4.004 001)	4.001
(2, 4)	(2.0001, 4.000 400 01)	4.0001

Thus the gradient of $y = x^2$ at $x = 2$ is 4.

2 $y = x^2$

x	0	1	2	3	4	5
grad	0	2	4	6	8	10

Compare your rule with those of others.

3 $y = 3x^2$

x	0	1	2	3	4	5
grad	0	6	12	18	24	30

Compare your rule with those of others.

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Exercise 5D PAGE 101

1 $2x$ **2** $3x^2$ **3** 1
4 $4x^3$ **5** 0 **6** $12x$
7 $24x^3$ **8** 7 **9** 16
10 $14x^6$ **11** $14x$ **12** 9
13 $\frac{x}{5}$ **14** $4x^5$ **15** $9x^5$
16 $2x^6$ **17** $8x$ **18** $20x^3$
19 $24x^2$ **20** 0 **21** $7x^6$
22 $24x^5$ **23** $18x$ **24** 5
25 0 **26** $18x^2$ **27** $32x^3$
28 $15x^4$ **29** $6x^5$ **30** $42x^6$
31 $16x^3$ **32** 10 **33** 12
34 12 **35** 12 **36** 80
37 7 **38** –20 **39** 2
40 0.8 **41** (1, 1) **42** (1, 1), (–1, –1)
43 (1.5, 6.75) **44** (0.5, 0.25), (–0.5, –0.25)
45 (1, 1) **46** (–1, 1)
47 $y = 6x - 4$ **48** $y = -6x - 3$
49 $y = 20x - 20$ **50** $y = -20x - 20$
51 $y = 16x - 24$ **52** $y = 18x - 72$
53 **a** 24 **b** –3 **c** $9x^2$ **d** 36
54 **a** 6 **b** 24 **c** $3x$ **d** 6
55 **a** y changes by 234 units (from 16 to 250) when x changes from $x = 2$ to $x = 5$.
b y changes at an average rate of 78 units per unit change in x when x changes from 2 to 5.
c When $x = 2$ the instantaneous change in y is 24 units per unit change in x.
d When $x = 5$ the instantaneous change in y is 150 units per unit change in x.
56 (–1, 8) gradient –16, (2, 32) gradient 32.
57 (–2, –8) gradient 12, (0, 0) gradient 0, (2, 8) gradient 12.
58 $\frac{1}{54}$, 1.5
59 $-\frac{1}{6}, \frac{1}{6}$.

Exercise 5E PAGE 104

1 $2x + 3$ **2** $3x^2 - 4$
3 $12x - 21x^2$ **4** $12x^3 + 6x^2 - 5$
5 $7 + 2x$ **6** $12x - 3$
7 $8x + 7$ **8** $15x^2 - 8x$
9 $20x^3 - 3$ **10** $4x + 7$
11 $-6x + 7$ **12** $1 + 2x + 3x^2 + 4x^3$
13 $-4 + 6x - 6x^2 + 4x^3$ **14** –3
15 24 **16** 21
17 11 **18** $y = 7x - 4$
19 $y = 13x - 50$ **20** $y = 8x - 34$
21 $y = 1$ **22** (–5, 76), (1, –2)
23 (–3, 0) gradient –8, (5, 0) gradient 8.
24 (5, –10) **25** (–3, 20), (1, –4)

Exercise 5F PAGE 107

1 $\frac{1}{2\sqrt{x}}$ **2** $-\frac{1}{x^2}$ **3** $-\frac{3}{x^2}$
4 $\frac{3}{\sqrt{x}}$ **5** $\frac{2}{x^{\frac{2}{3}}}$ **6** $\frac{3}{2}\sqrt{x}$
7 $\frac{2}{3x^{\frac{2}{3}}}$ **8** $-\frac{3}{x^4}$ **9** $-\frac{4}{x^5}$
10 $-\frac{6}{x^4}$ **11** $-\frac{20}{x^5}$ **12** $2x + \frac{1}{2\sqrt{x}}$
13 $6x - \frac{2}{\sqrt{x}}$ **14** $1 - \frac{1}{x^2}$ **15** $2x + \frac{2}{x^3}$
16 $\frac{1}{2\sqrt{x}} - \frac{3}{x^2}$ **17** $2x + 1 - \frac{1}{x^2} - \frac{2}{x^3}$
18 $-\frac{2}{x^2}$ **19** $-\frac{3}{2\sqrt{x^3}}$ **20** $-\frac{2}{x^{\frac{4}{3}}}$
21 $-\frac{1}{3x^{\frac{4}{3}}}$ **22** – 5 **23** 0.25
24 0.0625 **25** 11 **26** $\frac{8}{3}$
27 – 4 **28** –0.5
29 (2, 0.5), (–2, –0.5) **30** (0.25, 0.5)
31 (9, –243) **32** $y = 0.25x + 1$
33 $y = -x + 2$ **34** $y = -0.25x + 0.75$
35 $(-1\frac{1}{3}, -1\frac{11}{12}), (1\frac{1}{3}, 1\frac{11}{12})$
36 Answers not given here. Discuss with others in your class and your teacher.

Miscellaneous exercise five PAGE 109

1 **a** 4 **b** 16 **c** 3 **d** 7
e 9 **f** 4 **g** 4 **h** 7
i 3 **j** 3 **k** 0 **l** 4
m 2 **n** 4 **o** 9

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2 **a** 9 **b** 16

3 **a** 13 **b** –24

4 **a** 1024 **b** 1 073 741 824
c 2046 **d** 2 147 483 646

5 **a** $-3x^2$ **b** $10x - \frac{3}{\sqrt{x}}$ **c** $10x - \frac{2}{x^3}$

6 –4

7 **a** Reciprocal. $y = \frac{-6}{x}$
b Quadratic. $y = x^2 + 1$
c Linear. $y = 3x + 5$
d Exponential. $y = 5^x$
e Quadratic. $y = x(x + 1)$
f Exponential. $y = 10^x$
g Exponential. $y = 4 \times 2^x$, i.e. $y = 2^{x+2}$
h Reciprocal. $y = \frac{-24}{x}$
i Cubic. $y = 2x(x + 3)(x - 3)$

8 The other two angles of the triangle are of size 60° and 110°.

9 $T_1 = 0.8$, $T_{n+1} = 5 \times T_n$.

10 **a** 8×10^{11} **b** 8×10^{11} **c** 8×10^{21}
d 1.6×10^9 **e** 2×10^{-3} **f** 5×10^2

11 **a** Sequence 1: 5, 17, 53, 161, 485.
Sequence 2: 0.125, 0.25, 0.5, 1, 2.
Sequence 3: –5, 5, 15, 25, 35.
b Sequence 1: Neither.
Sequence 2: Geometric.
Sequence 3: Arithmetic.
c Sequence 1: 721
Sequence 2: 3.875
Sequence 3: 75.
d Sequence 1: 774 840 977
Sequence 2: 16 384
Sequence 3: 165.
e Sequence 1: 1 162 261 446
Sequence 2: 32 767.875
Sequence 3: 1440.

12 **a** $y = 5x - 1$
b $y = 23x - 29$ is tangent at (2, 17) and $y = 23x + 35$ is tangent at (–2, –11).

13 **a** The display tells us that from $x = 1$ to $x = 6$ the function has an average rate of change of 64.
b The display tells us that at $x = 5$ the function has an instantaneous rate of change of 105.
I.e., at $x = 5$, $\frac{df}{dx} = 105$.

14 C

15 **a** 3 **b** positive **c** negative

16 Required x-coordinates are 4 and 6. Required y-coordinate is 2.816.

Exercise 6A PAGE 117

1 $10r + 3$ **2** $3 + 6k - 18k^2$

3 $15r^2 - 2r + 15$ **4** $8p^3 + 9p^2 - 14$

5 $36t^2 + 18t - 8$

6 **a** 16 **b** 26 **c** 36

7 **a** 12 **b** 18 **c** –24

8 **a** 20π **b** 6π **c** 140

9 **a** 32π **b** 48π **c** 60π

10 **a** 4π **b** 36π **c** 400π

11 **a** $\frac{4\pi t^2}{25}$ m^2 **b** 0.64π m^2
c $\frac{8\pi t}{25}$ m^2/s **d** 0.96π m^2/s

12 **a** 120 **b** 3870
c 750 bct/h **d** $(500 + 30t^2)$ bct/h
e **i** 620 bct/h **ii** 1250 bct/h
iii 3500 bct/h

13 **a** 400 **b** 50 units/h **c** 8
d **i** 57 units/h **ii** 66 units/h
iii 69 units/h

14 **a** **i** 0.2 L **ii** 2088 L
b **i** 0.03 L/min **ii** 0.25 L/min
iii 2.89 L/min

15 **a** **i** 42 **ii** 44
iii 47 **iv** 70
b $(0.2t + 2)$ deer/yr
c **i** 3 deer/yr **ii** 4 deer/yr
iii 6 deer/yr

16 **a** 150 000 tonnes
b 48 000 tonnes
c Rate of decrease = $8000 + 840t - 60t^2$,
i.e. $\frac{dT}{dt} = (60t^2 - 840t - 8000)$ tonnes/year
d **i** 9440 tonnes/year **ii** 10 400 tonnes/year
iii 10 940 tonnes/year

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17 **a** 1000 cm^3 **b** 992.4 cm^3

c $(0.2t - 4)$ cm^3/s

d **i** -4 cm^3/s **ii** -3.4 cm^3/s

e 20 seconds **f** $a = 0, b = 20$.

Exercise 6B PAGE 126

(Sketches not given here – check using a graphic calculator).

1 **c** (–3, 61)

2 **c** (3, –50.5)

3 Local maximum point at (–3, 20).
Local minimum point at (1, –12).

4 Local maximum point at (1, 37).
Local minimum point at (5, 5).

5 Local (and global) maximum point at (2, 9).

6 Horizontal inflection at (0, 0).

7 Local (and global) minimum point at (0, 0).

8 Local minimum point at (0, 0).
Local maximum point at (2, 4).

9 Local (and global) minimum point at (1, 5).

10 Local (and global) min at (–2, –22).
Local max at (0, 10). Local min at (1, 5).

11 **a** (0, 0)

b (–3, 0), (0, 0)

c As $x \to +\infty, y \to +\infty$. As $x \to -\infty, y \to -\infty$.

d Local maximum point at (–3, 0).
Local minimum point at (–1, –4).

e Use a graphic calculator to check validity of your sketch.

f Minimum value is –20. Maximum value is 16.

12 Local maximum point at (0, 0).
Local minimum point at (1, –1).

a –1 **b** –5

Exercise 6C PAGE 130

Each solution should clearly show the use of calculus and justify a maximum (or minimum) value.

1 When $t = 8$, X has a local minimum value of 16.

2 When $p = 10$, A has a local maximum value of 300.

3 The maximum value of A is 20 and it occurs when $x = 10$ and $y = 2$.

4 The maximum value of A is 13.5 and it occurs when $x = 4.5$ and $y = 3$.

5 When $x = 35$ the maximum profit of \$725 is realised.

6 When $x = 120$ the maximum profit of \$9400 is realised.

7 **a** 25 m × 25 m

b 50 m × 25 m (with the existing wall forming one of the 50 m sides).

8 The manufacturer should spend \$30 000 on advertising to achieve the maximum profit of \$140 000.

9 For maximium capacity the dimensions need to be width 0.4m, length 0.6 m, height 0.5 m.

10 The maximum capacity is achieved when $x = 10$.

11 6 cm should be turned up along each edge to maximise the capacity of the gutter.

12 **a** $\$(2500 + 500x - 25x^2)$

b 10 **c** \$2

d 5000 **e** \$5000

13 Minimum N is 1100, to the nearest 100. (When $t = 16$).
Maximum N is 4600, to the nearest 100. (When $t = 24$).

14 **a** The body is 105 m from the origin after three seconds.

b $(t^2 - 12t + 50)$ m/s

c The initial velocity of the body is 50 m/s.

d When $t = 6$ the body is moving with minimum velocity and the body is then 156 metres from the origin.

15 81 mm, 880 cm

16 The owner should spend \$12 500 on security.

Exercise 6D PAGE 134

1 $6r - \frac{5}{r^2}$

2 **a** 100 **b** 40 **c** 20

3 Maximum at $(-\sqrt{2}, -2\sqrt{2})$, minimum at $(\sqrt{2}, 2\sqrt{2})$.

4 Minimum at (–2, 9), maximum at (2, 1).

5 Maximum at (–4, –18).

6 **a** $\frac{500}{x^2}$ **b** $\left(x^2 + \frac{2000}{x}\right)$ cm^2

c 10, 5, 300 cm^2

7 Correct to one decimal place, the base radius needs to be 4.4 cm and the height 8.8 cm.

8 When the base radius is 3.5 cm (correct to one decimal place) and the height is 14.0 cm (correct to one decimal place) the cost of material is minimised.

Miscellaneous exercise six PAGE 135

1 **a** 5^2 **b** 5^4 **c** 5^3 **d** 5^0

e 5^3 **f** 5^6 **g** 5^5 **h** 5^4

i 5^7 **j** 5^3 **k** 5^2 **l** 5^1

m 5^{10} **n** 5^4 **o** 5^{17} **p** 5^2

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q 5^6 **r** 5^3 **s** 5^5 **t** 5^6
u 5^7 **v** 5^8 **w** 5^2 **x** 5^3
y 5^2 **z** 5^2

2 **a** a^4 **b** $\frac{5a^7}{b^8}$ **c** $2 + 2^n$
d $x + 2x^4$ **e** 2^x **f** $\frac{3}{5}$

3 The first six terms are 97, 108, 119, 130, 141, 152.

4 The first five terms are 350, 70, 14, 2.8, 0.56.

5 **a** 6 **b** 3 **c** 9

6 **a** 60 **b** 24 **c** 105

7 **a** 15 **b** **i** 55 **ii** 120

8 **a** 14 535 **b** –442 860 **c** 12 582 906
d 500 **e** 5

9 From the graph, some of the points that the tangent at $x = 1$ seems to pass through are (–3, –11), (0, –2), (1, 1) and (3, 7). Thus the tangent at $x = 1$ has a gradient of 3. Thus the gradient of $f(x)$ at $x = 1$ is 3.

From the graph, some of the points that the tangent at $x = 2$ seems to pass through are (0, –16), (2, 8) and (3, 20). Thus the tangent at $x = 2$ has a gradient of 12. Thus the gradient of $f(x)$ at $x = 2$ is 12.

Finding the gradient of $y = x^3$ at $x = 1$ and at $x = 2$ using calculus confirms these values.

10 2

11 **a** 10 **b** 3 **c** 13

12 **a** D, H, K, P **b** B, F, I, K, N, O
c G, H, L, M **d** A, C, D, E, J, P

13 At the point (1, 2).

14 **a** (–1, –3), (1, 3) **b** (0.25, –1.25)

15 **a** 343 062, 1 698 992, 5 308 522
b 67 513, 223 973, 526 233

16 $y = 25x + 185$ (at the point (–5, 60)) and $y = 25x - 71$ (at the point (3, 4)).

17 **a** $a = -20, c = 260$ **b** $\$(260p - 20p^2)$
c $\$(400p - 20p^2 - 1820)$ **d** 10, 60, \$180

18 **a** 1.4×10^{24} **b** 7.2×10^{12}
c 6.8×10^{12} **d** 3.5×10^{1}
e 7×10^{24} **f** 2.45×10^{14}

19 **a** 200 **b** 1500 **c** 130
d **i** 30 organisms/h **ii** 105 organisms/h
iii 330 organisms/h

20 **a** The display tells us that from $x = 1$ to $x = 3$ the given function has an average rate of change of 41.
b The display tells us that at $x = 3$ the given function has an instantaneous rate of change of 109,
i.e., at $x = 3$, $\frac{df}{dx} = 109$.

21 **a** $\$(10 - 0.2x)$ per metre
b $(500 + 25x)$ metres
c $\$(5000 + 150x - 5x^2)$
d 15. Negative coefficient of x^2 in the quadratic, hence turning point is a maximum.

Exercise 7A PAGE 144

1 $\frac{1}{8}x^8 + c$ **2** $\frac{1}{6}x^6 + c$ **3** $\frac{1}{5}x^5 + c$
4 $\frac{1}{4}x^4 + c$ **5** $\frac{1}{3}x^3 + c$ **6** $\frac{1}{2}x^2 + c$
7 $x + c$ **8** $4x^3 + c$ **9** $2x^6 + c$
10 $2x^4 + c$ **11** $7x^2 + c$ **12** $6x + c$
13 $x^3 + 3x^2 + c$ **14** $2x^3 - x + c$
15 $7x + 3x^4 + c$ **16** $3x^2 - 3x^5 + c$
17 $7x - 4x^2 + c$ **18** $\frac{x^3}{3} + 3x + c$
19 $3x^6 + x + c$ **20** $2x^3 + \frac{1}{2}x^2 + c$
21 $4x^3 + 2x^4 + 2x + c$ **22** $x^3 - x^2 + \frac{1}{6}x^6 + c$
23 $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + c$ **24** $3x^4 + 3x^2 + 5x + c$
25 $x^3 + 5x^2 + 8x + c$ **26** $3x^3 + 4x^2 - x + c$
27 $\frac{1}{3}x^3 - 4x + c$ **28** $\frac{1}{3}x^3 - x^2 - 3x + c$
29 $2x^4 + x^3 + c$ **30** $x^4 + 4x^3 + 2x^2 + c$
31 $y = 2x^3 + 7$ **32** $y = \frac{3}{2}x^2 + 2x - 2$
33 $y = x^3 - x^2 + 6$ **34** $y = 2x^3 - 5x + 3$
35 $y = 3x + 2x^4 + 7$

36 **a** $f(x) = \frac{1}{2}x^3 + 2x^2 - x - 2$ **b** 8

37 **a** $\frac{3}{2}x^2 - 6x + 6$ **b** 24
c $a = -4$ or 8

38 $p = 27$ **39** (–4, 0), (0, 0), (4, 0)

40 $k = -1$

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Exercise 7B PAGE 146

1 $V = 3t^2 + 5t + 30$

2 **a** $x = t^2 - 6t + 7$ **b** 23 **c** 1 or 5

3 **a** $A = 2r^2 + 3r^4 + 2$ **b** 58

4 **a** $C = x^2 + 3x + 100$ **b** $C = x^3 + x^2 + 5000$

5 **a** $R = 50x$ **b** $R = 50x - 0.025x^2$

6 \$38 000 **7** $(7000 - 20t - 5t^2)$ cm^3

8 Increasing. $A = 100t + 10\,000$

9 $C = 40x + 1000$

10 $R = 200x - \frac{1}{20}x^2$, 150 000

11 **a** 29 **b** 43 **c** 176

Exercise 7C PAGE 150

1 $\frac{x^3}{3} + c$ **2** $\frac{x^2}{2} + c$ **3** $\frac{x^4}{4} + c$

4 $2x + c$ **5** $2x^5 + c$ **6** $2x^4 + c$

7 $2x^2 + x + c$ **8** $2x^3 - 5x + c$ **9** $4x^2 - 7x + c$

10 $\frac{x^2}{2} + 3x^3 + c$ **11** $\frac{x^2}{2} - x + c$

12 $2x^3 + \frac{11x^2}{2} + 3x + c$ **13** $2x^3 + 3x^2 + c$

14 $2x^4 - x^3 + c$ **15** $\frac{3x^4}{2} + 4x^3 + 3x^2 + c$

Miscellaneous exercise seven PAGE 150

1 10^4 **2** 10^{-1} **3** 10^6

4 10^8 **5** 10^2 **6** 10^6

7 $10^{0.5}$ **8** $10^{\frac{1}{3}}$ **9** $10^{1.5}$

10 0 **11** 5 **12** 5

13 4 **14** 5 **15** 4

16 5 **17** 8 **18** 2

19 $T_1 = 10$, $T_{n+1} = T_n + 6$.

The sum of the first fifteen terms exceeds the fifteenth term by 686 (i.e. by the sum of the first 14 terms).

20 **a** 0 **b** 5 **c** $10x + 5$

d $15x^2 + 10x + 5$ **e** $2x + \frac{1}{2\sqrt{x}}$ **f** $-\frac{3}{2x^4}$

21 **a** 29 **b** 9 **c** $8x - 3$ **d** 21

22 8 **23** 6

24 (–2, 3000), (16, 84)

25 **a** $y = 5^x + 1$ is $y = 5^x$ with 1 added to the right-hand side.

Thus the graph of $y = 5^x + 1$ is that of $y = 5^x$ translated up one unit.

b $y = 5^{x+1}$ is $y = 5^x$ with the x replaced by $x + 1$.

Thus the graph of $y = 5^{x+1}$ is that of $y = 5^x$ moved left 1 unit.

Alternatively we could write $y = 5^{x+1}$ as $y = 5 \times 5^x$ which is $y = 5^x$ with the right-hand side multiplied by 5. Thus the graph of $y = 5^{x+1}$ is also that of $y = 5^x$ dilated parallel to the y-axis, scale factor 5.

c $y = 5^{-x}$ is $y = 5^x$ with the x replaced by $-x$.

Thus the graph of $y = 5^{-x}$ is that of $y = 5^x$ reflected in the y-axis.

d Writing $y = \frac{1}{5^x}$ as $y = 5^{-x}$ we see that the answers to this part will be as for part **c**.

26 9

27 **a** (–1, –2) **b** (–1, –3), (1, 3)

28 **a** $a = 7, b = 3$ **b** (0, –21)

c Gradient is –10 at (–3, 0). Gradient is 10 at (7, 0).

d (5, –16) **e** $y = -4x - 21$

29 $a = 3, b = 4$. Gradient at P is –7. Gradient at Q is 7. Gradient at R is 1.

30 **a** $(2000 - 40x + 0.2x^2)$ dollars per unit

b \$500 per unit

c $P(x) = 500x - 20\,000 + 20x^2 - \frac{x^3}{15}$ dollars

d \$2000 per unit

31 From the display we can conclude that the graph of $y = 4x^3 + 9x^2 - 210x + 75$ has two stationary points, one is at (–5, 850) and the other is at (3.5, –378.25).

32 **a**

Width (cm)	Length (cm)	Height (cm)	Volume (cm^3)
10	20	120	24 000
20	40	90	72 000
30	60	60	108 000
40	80	30	96 000

A continuation of the table, for suitably chosen values for the width, leads to maximum volume achieved when, to the nearest cm, the width is 33 cm, the length is 66 cm and the height is 51 cm.

b Volume $= 300w^2 - 6w^3$.

Calculus, and consideration of the graph of $f(w) = 300w^2 - 6w^3$, confirms that volume is maximised for $w = \frac{100}{3}$ cm, i.e. 33 cm (nearest cm).

33 Base 4 m, height 8 m, area 32m^2.

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Exercise 8A PAGE 159

1 (Graph not shown here.)

a The car reaches C at 11.54 a.m. and the truck reaches town C at 12.15 p.m.

b From 8.30 a.m. to 9.30 a.m. the truck maintained a steady speed of 100 km/h.

c The average speed of the truck from A to B was 87 km/h (to the nearest km/h).

d The car passes the truck at 10.30 a.m. in town B, just as the truck is about to leave B.

2

A	2 m	2 m	6 m/s	6 m/s
B	6 m	6 m	0 m/s	0 m/s
C	8 m	8 m	2 m/s	–2 m/s
D	5 m	–5 m	5 m/s	5 m/s
E	9 m	–9 m	3 m/s	–3 m/s
F	1 m	–1 m	7 m/s	–7 m/s

3 **a** 4 m/s **b** 31 m/s

4 **a** 6 m **b** 5 m/s **c** 23 m/s

5 **a** 0 m **b** –3 m/s **c** 3 m/s

6 **a** 0 m **b** 1 m/s **c** 37 m/s

7 **a** 3 m **b** 6 m/s **c** 6 m/s

8 **a** –3 m **b** –20 m/s **c** 8 m/s

9 **a** 1 m **b** –6 m/s **c** 150 m/s

10 17 m, 14 m/s

11 2 m, 4 m/s

12 **a** 8 m **b** 8 m/s **c** 7

13 **a** 6 m **b** –1 m/s **c** 4

14 **a** 12 m **b** 8 m/s **c** 3

15 153 m

16 **a** 45 m **b** 105 m **c** 10
d 60 m **e** 15 m

17 **a** 8 m **b** 20 m **c** 4
d 20 m **e** 3 m

18 **a** 40 m **b** 52 m **c** 6
d 20 m **e** 7 m

19 **a** 94 m **b** 148 m **c** 5
d 162 m **e** 26 m

20 **a** 10, 0
b 120 m, 10 m/s upwards
c 0 m/s, 125 m

21 60 m/s

22 **a** 12 **b** 318 m
c *A* and *B* collide 'head-on'.

Exercise 8B PAGE 163

1 23 metres

2 34 metres

3 The body is at the origin when $t = 0.6$ and when $t = 4$.

4 –5 metres

5 At the origin the velocity of the body is –6 m/s, when $t = 2$, and 6 m/s, when $t = 4$.

6 **a** When $t = 5$. **b** –12.5 metres

7 19 metres

8 0.5 metres

Miscellaneous exercise eight PAGE 163

1 **a** (0, –6) **b** (0, 15) **c** (0, 6)
d (0, 3) **e** (0, –4) **f** (0, 12)

2 **a** (3, 0) **b** (–3, 0)
c (–3, 0), (3, 0) **d** (2, 0), (7, 0)
e (3, 0), (–2, 0), (3.5, 0)
f (1, 0), (–1, 0), (3, 0), (–4, 0)

3 2^{15} **4** 2^{-4} **5** 2^{-6}

6 2^{-1} **7** 2^{-2} **8** 2^{10}

9 2^{13} **10** 2^{6} **11** 2^{6}

12 2^{3} **13** 2^{0} **14** 2^{8}

15 $n = -5$ **16** $n = 5$ **17** $n = 13$

18 $n = 6$ **19** $n = 10$ **20** $n = \frac{5}{6}$

21 $n = 3$ **22** $n = 1$ **23** $n = 2$

24 15, 21, 27, 33, 39.

25 100, 93, 86, 79, 72.

26 4, 20, 100, 500, 2500.

27 6, 24, 96, 384, 1536.

28 213, 219, 225, 231, 237.

29 256, 384, 576, 864, 1296.

31 **a** $6x$ **b** $15x^2$ **c** $x + 3$
d $30x - 1$ **e** $6 + 18x$ **f** $-2x$

32 **a** $14x$ **b** $6x^2$
c $12x^3 + 3x^2 - 10x + 9$ **d** $9x^2 - 4x + 3$

33 7

34 **a** 10 **b** 6
c $6x + 2$ **d** 14

35 **a** (0.5, 1.25) **b** (–2, –40), (2, 40)
c (2, 10) **d** (–2, –20), (4, 16)

36 **a** –3 **b** –0.375

37 At the point (1, –4).

38 $y = -2x - 1$

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39 **a** From $t = 0$ to $t = 5$ the object travels 3 metres. The average speed is 0.6 m/sec.

b From $t = 5$ to $t = 8$ the object travels 12 metres. The average speed is 4 m/sec.

c When $t = 5$, the speed of the object is 1 m/sec.

d When $t = 8$, the speed of the object is 3 m/sec.

40 **a** –12 m/s **b** 12 m/s

c 2.5 **d** 1.5, 2.5

41 $a = 3, b = 5$. $(-1, -4)$

42 $y = 1.25x - 2$

43 $f(x) = 7x^3 - 15x^2, f(3) = 54, f'(2) = 24$

44 **a** 3399, 9744, 235 056

b 859, 1729, 14 401

45 **a** $60x + c$ **b** $30x^2 + c$

c $20x^3 + c$ **d** $15x^4 + c$

e $12x^5 + c$ **f** $10x^6 + c$

g $2x^4 - 5x^3 + 2x + c$

h $4x - \frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c$

i $\frac{1}{3}x^3 - 9x + c$ **j** $12x^4 - 8x^3 + c$

46 **a** $y = 2x^2 - 3x + 3$

b $y = 2x^3 - x^2 + 4x + 7$

c $y = 2x^4 - 4x^3 - 2x^2 + 11x - 10$

47 When $x = 2, y = 64$.

48 **a** 7500 cm^3

b 29 100 cm^3

c $(5400 - 900t + \frac{75t^2}{2})$ cm^3/s

d 3750 cm^3/s, 2400 cm^3/s, 150 cm^3/s.

49 **a** y changes by 320 units (from 1 to 321) when x changes from $x = 0$ to $x = 10$.

b y changes at an average rate of 32 units per unit change in x when x changes from 0 to 10.

c When $x = 0$, the instantaneous change in y is 2 units per unit change in x.

d When $x = 10$, the instantaneous change in y is 62 units per unit change in x.

50 D

51 \$18, 15 000

52 **a** 5 metres

b 57 metres

c From 5 metres to 57 metres in 4 seconds is an average rate of change of 13 m/sec.

d When $t = 2$, the instantaneous rate of change of x is 3 m/sec.

e When $t = 6$, the instantaneous rate of change of x is 27 m/sec.

53 **a** $250 - \pi r$ **b** $A = 500r - \pi r^2$

54 \$22 003.73

55 **a** $120 - x$ **b** $\$(360 + 3x - 0.05x^2)$

c \$4.50 **d** \$405

e $P(x) = \$(120 + 5x - 0.05x^2)$

f \$5.50 **g** \$245

56 **a** 25 m **b** 5 **c** 20 m

57 $a = -0.02, b = 4.8, c = 0.018, d = -6.6, e = 50, f = 30, g = 20, h = \frac{550}{3}$

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