

**Question 2****(8 marks)**

- (a) The twelfth row of Pascal's triangle begins with the numbers 1, 12, 66, 220, 495, 792, 924 and so on.

(i) State the value of  $\binom{12}{5}$ . (1 mark)

(ii) Deduce the value of  $\binom{13}{4}$ . (2 marks)

(iii) Calculate the sum of all the terms in the eighth row of Pascal's triangle. (1 mark)

- (b) The coefficient of the  $x^2$  term in  $x^3 + 7x^2 - 3x + 2$  is 7. Determine the coefficient of the  $x^2$  term in the expansion of

(i)  $(4x - 3)^2$ . (1 mark)

(ii)  $(2x + 1)^5$ . (3 marks)

**Question 2**
**(8 marks)**

- (a) The twelfth row of Pascal's triangle begins with the numbers 1, 12, 66, 220, 495, 792, 924 and so on.

- (i) State the value of  $\binom{12}{5}$ .

Solution
$\binom{12}{5} = 792$
Specific behaviours
✓ correct value

**(1 mark)**

- (ii) Deduce the value of  $\binom{13}{4}$

Solution
$\binom{13}{4} = \binom{12}{3} + \binom{12}{4} = 220 + 495 = 715$
Specific behaviours
✓ indicates use of terms in previous row
✓ correct value

**(2 marks)**

- (iii) Calculate the sum of all the terms in the eighth row of Pascal's triangle. **(1 mark)**

Solution
Sum = $2^8$ = 256
Specific behaviours
✓ evaluates

- (b) The coefficient of the  $x^2$  term in  $x^3 + 7x^2 - 3x + 2$  is 7. Determine the coefficient of the  $x^2$  term in the expansion of

- (i)  $(4x - 3)^2$ .

Solution
$16x^2 \dots$ Coefficient is 16
Specific behaviours
✓ correct value

**(1 mark)**

- (ii)  $(2x + 1)^5$ .

**(3 marks)**

Solution
Required term is $(2x)^2(1)^3 \times {}^5C_2 = 4x^2 \times 10 = 40x^2$
Coefficient is 40
Specific behaviours
✓ indicates elements of required term
✓ indicates use of ${}^5C_2$ and/or Pascals triangle
✓ correct value

**Question 20****(9 marks)**

An online grocery is offering new customers the opportunity to select 8 different products for just \$2 each. They can select from a range of 12 different canned items, 14 different snacks and 13 different drinks.

(a) Determine how many different selections can be made. (2 marks)

(b) Determine how many different selections can be made that just include drinks. (2 marks)

(c) Determine how many different selections can be made if it does not include a snack. (2 marks)

Fred knows that he is going to buy a can of baked beans, packet of chips and a bottle of water.

(d) Determine how many different selections can be made if he needs 8 different product in total. (3 marks)

**Question 20****(9 marks)**

An online grocery is offering new customers the opportunity to select 8 different products for just \$2 each. They can select from a range of 12 different canned items, 14 different snacks and 13 different drinks.

- (a) Determine how many different selections can be made.

**(2 marks)**

<b>Solution</b>
$12 + 14 + 13 = 39$
${}^{39}C_8 = 61\,523\,748$
<b>Specific behaviours</b>
✓ indicates choosing 8 from 37
✓ correct number

- (b) Determine how many different selections can be made that just include drinks.

**(2 marks)**

<b>Solution</b>
${}^{13}C_8 = 1\,287$
<b>Specific behaviours</b>
✓ indicates choosing 8 from 13
✓ correct number

- (c) Determine how many different selections can be made if it does not include a snack.

**(2 marks)**

<b>Solution</b>
${}^{25}C_8 = 1\,081\,575$
<b>Specific behaviours</b>
✓ indicates choosing 8 from 25
✓ correct number

Fred knows that he is going to buy a can of baked beans, packet of chips and a bottle of water.

- (d) Determine how many different selections can be made if he needs 8 different product in total.

**(3 marks)**

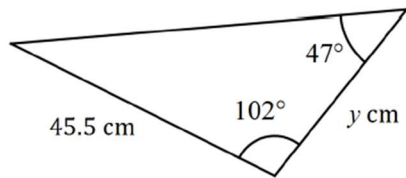
<b>Solution</b>
${}^{36}C_5 = 376\,992$
<b>Specific behaviours</b>
✓✓ indicates choosing 5 from 36
✓ correct number

**Question 9**

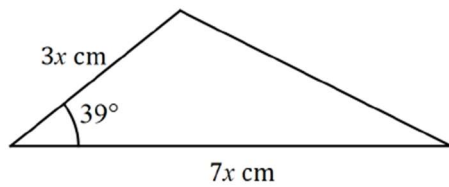
**(9 marks)**

- (a) Determine the size, to the nearest degree, of the largest angle in a triangle with sides of lengths 23 cm, 28 cm and 31 cm. (3 marks)

- (b) Determine the value of  $y$  in the diagram below. (3 marks)



- (c) The area of the triangle shown below is  $280 \text{ cm}^2$ . Determine the value of  $x$ . (3 marks)



**Question 9**

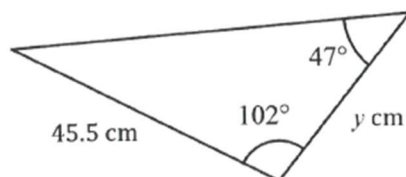
**(9 marks)**

- (a) Determine the size, to the nearest degree, of the largest angle in a triangle with sides of lengths 23 cm, 28 cm and 31 cm. (3 marks)

Solution
$\theta = \cos^{-1} \left( \frac{23^2 + 28^2 - 31^2}{2 \times 23 \times 28} \right)$ $\theta \approx 74^\circ$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ shows use of cosine rule</li> <li>✓ substitutes correctly</li> <li>✓ determines angle</li> </ul>

- (b) Determine the value of  $y$  in the diagram below.

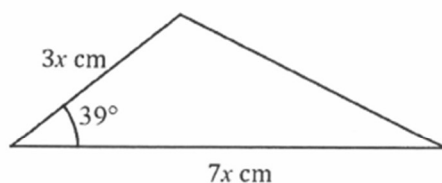
**(3 marks)**



Solution
$180 - 102 - 47 = 31^\circ$ $\frac{y}{\sin 31} = \frac{45.5}{\sin 47}$ $y = 32.04$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ determines angle</li> <li>✓ shows use of sin rule</li> <li>✓ determines value</li> </ul>

- (c) The area of the triangle shown below is  $280 \text{ cm}^2$ . Determine the value of  $x$ .

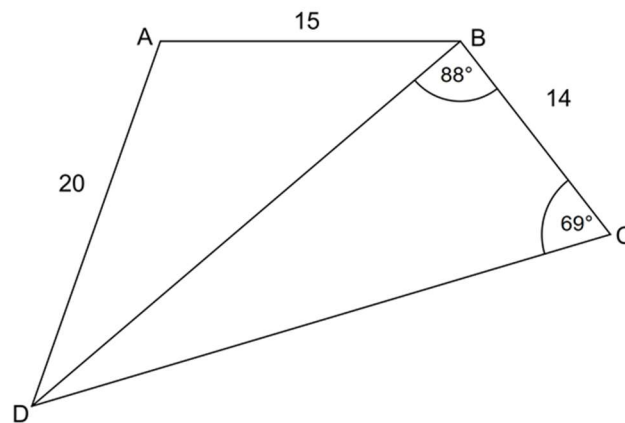
**(3 marks)**



Solution
$0.5 \times 3x \times 7x \times \sin 39 = 280$ $x^2 = 42.37$ $x = 6.51$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses area formula</li> <li>✓ substitutes correctly</li> <li>✓ solves for value</li> </ul>

**Question 20** (7 marks)

A vacant block in the shape of the quadrilateral ABCD, is shown below, with measurements in metres.



(a) Determine, with working:

(i) the length of DB, correct to the nearest metre. (2 marks)

(ii) the size of  $\angle ABD$ , correct to two decimal places. (2 marks)

Hence, or otherwise,

(b) state the area of the block. Show your working. (3 marks)

**Question 19 (10 marks)**

A pencil case contains ten pens; three red pens, five blue pens and two black pens. Ian was asked to select five pens from the pencil case.

(a) How many different selections can Ian make with regard to colour that contain:

(i) exactly three blue pens? (2 marks)

(ii) more black pens than red pens? (2 marks)

(b) Ian was asked to make a selection of five pens and the solution was  $\begin{pmatrix} 5 \\ 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ .

What selection was Ian asked to make?

(2 marks)

(c) Find the probability that Ian chooses:

(i) at least one red pen. (2 marks)

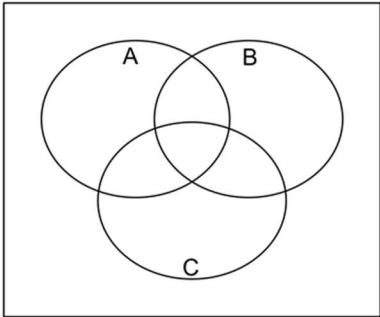
(ii) at most two red pens, given he chooses at least one red pen. (2 marks)



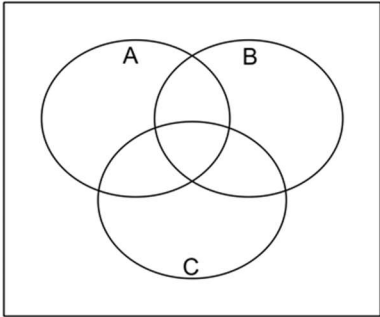
Question 21 (6 marks)

(a) Use the Venn diagrams below to shade the indicated regions.

(i)  $A \cup B'$

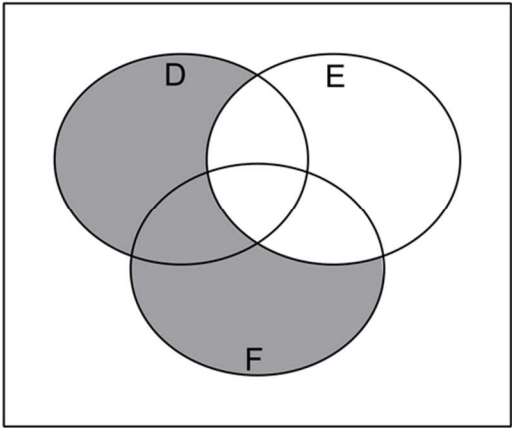


(ii)  $B \cap (C \cup A)'$  (4 marks)



(b) Use set notation to describe the region shaded. (2 marks)

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**Question 22 (8 marks)**

Graham travels to work each day of the working week by car.

He passes through three sets of lights in his journey.

The probability that he stops at the first set of lights is 0.7, while the probability that he doesn't stop at the second set of lights is 0.65.

The probability that he stops at the second set of lights given he stops at the first set of lights is 0.35

The probability that he stops at the third set of lights is 0.4, which is independent of stopping at the earlier sets of lights.

- (a) Are the events "stops at the first set of lights" and "stops at the second set of lights" independent events? Explain clearly using mathematical notation. (2 marks)

- (b) Determine the probability that Graham:

- (i) stops at every set of lights. (2 marks)

- (i) stops at either the first set or second set of lights or both. (2 marks)

- (iii) doesn't stop at any set of lights, given he didn't stop at the first set of lights. (2 marks)

**Question 23 (5 marks)**

A survey was conducted at the local high school concerning health food in their canteen. The question posed was: “Should the canteen introduce more health food?” The results are shown in the table below.

	Yes	No
Year 7 & 8	85	15
Year 11 & 12	40	60
Total	125	75

A student is selected at random from the two hundred students surveyed.

(a) Determine the probability that the student:

(i) voted Yes or was from Year 11 or 12. (1 mark)

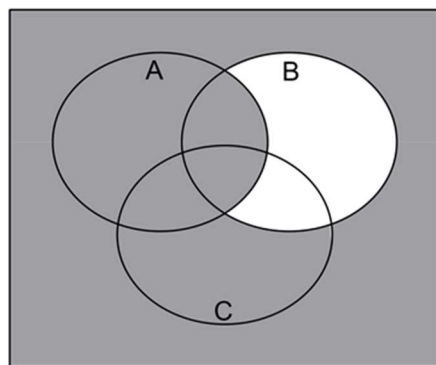
(ii) was from Year 7 or 8, given that the student voted Yes. (2 marks)

(b) The events “Introduce more health food” and “School Year” are not independent events according to this survey. Explain, using entries from the table why this is the case.

(2 marks)

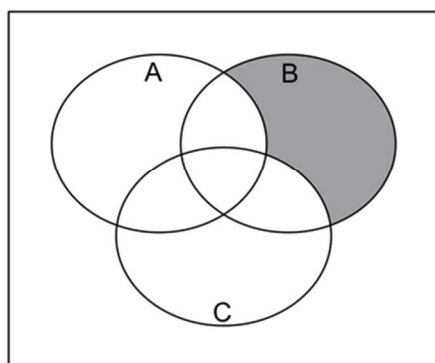
19. (a) (i)  $\binom{5}{3}\binom{5}{2} = 10 \times 10 = 100$  ✓✓
- (ii)  $\binom{2}{2}\binom{3}{1}\binom{5}{2} + \binom{2}{1}\binom{3}{0}\binom{5}{4} + \binom{2}{2}\binom{3}{0}\binom{5}{3}$  ✓  
 $= 30 + 10 + 10 = 50$  ✓
- (b) How many different selections containing four blue pens and a black pen are possible? ✓
- (c) (i)  $P(\text{at least one}) = 1 - P(0) = 1 - \frac{\binom{3}{0}\binom{7}{5}}{\binom{10}{5}}$  ✓
- $= 1 - \frac{21}{252} = \frac{231}{252}$  ✓
- (ii)  $P(\text{at most two red} \mid \text{at least one red})$
- $= \frac{\binom{3}{1}\binom{7}{4} + \binom{3}{2}\binom{7}{3}}{231}$  ✓
- $= \frac{105 + 105}{231} = \frac{210}{231}$  ✓ [10]
20. (a) (i)  $\frac{DB}{\sin 69^\circ} = \frac{14}{\sin 23^\circ}$  ✓  
 $\therefore DB = 33 \text{ m}$  ✓
- (ii)  $20^2 = 15^2 + DB^2 - 2(15)(DB)\cos(\text{ABD})$  ✓  
 $\therefore \angle \text{ABD} = 19.84^\circ$  ✓
- (b)  $\text{Area}(\triangle \text{ABD}) = \frac{1}{2} \times 15 \times DB \times \sin(\text{ABD})$  ✓  
 $= 85.16 \text{ m}^2$  ✓
- $\text{Area}(\triangle \text{BCD}) = \frac{1}{2} \times 14 \times DB \times \sin 88^\circ$  ✓  
 $= 234.01 \text{ m}^2$  ✓  
 $\therefore \text{Total area} = 319.17 \text{ m}^2$  ✓ [7]

21. (a) (i)



✓✓

(ii)



✓✓

(b)  $E' \cap (D \cup F)$

✓✓

[6]

22. (a)  $P(L1) = 0.7$ ,  $P(L2) = 0.35$ ,  $P(L2|L1) = 0.35$  and  $P(L3) = 0.4$   
Since  $P(L2) = P(L2|L1) = 0.35$   
then independent events.

✓

✓

(b) (i)  $P(\text{all three}) = P(L1) \times P(L2) \times P(L3)$   
 $= 0.7 \times 0.35 \times 0.4 = 0.098$

✓✓

(ii)  $P(L1 \cup L2) = P(L1) + P(L2) - P(L1 \cap L2)$   
 $= 0.7 + 0.35 - 0.7 \times 0.35 = 0.805$

✓✓

(iii)  $P((L1 \cup L2 \cup L3)' | L1')$   
 $= P(L2' \cup L3')$   
 $= 0.65 + 0.6 - 0.65 \times 0.6 = 0.86$

✓

✓

[8]

23. (a) (i)  $\frac{185}{200}$

✓

(ii)  $\frac{85}{125}$

✓✓

(b)  $P(Y) = \frac{125}{200}$  and  $P(7\&8) = \frac{100}{200}$

$P(Y \cap 7\&8) = \frac{85}{200}$

✓

Since  $P(Y) \times P(7\&8) \neq P(Y \cap 7\&8)$  then not independent.

✓

[5]

**Question 8 (6 marks)**

- (a)** (i) Complete the following table of Pascal's triangle where  $n = 5$  and  $n = 6$ . (2 marks)

n	$b^0$	$b^1$	$b^2$	$b^3$	$b^4$	$b^5$	$b^6$
3	1	3	3	1			
4	1	4	6	4	1		
5							
6							

- (ii) Hence, expand  $(p - q)^6$  fully. (2 marks)

- (b)** Given that the 11<sup>th</sup> term in the expansion of  $(p - 2q)^{25}$  is  $\binom{25}{a} (p)^b (-2q)^c$ , state the value of  $a$ ,  $b$  and  $c$ . (2 marks)

**Question 11 (10 marks)**

A survey of the 200 properties on a housing estate was undertaken. Part of the data collected related to the number of bedrooms and the number of toilets in each property as shown in the table below:

		Number of toilets			
		1	2	3 or more	Total
Number of bedrooms	1	35			35
	2		5	0	55
	3	5	65		
	4 or more	0	0		25
	Total				

(a) Complete the table. (2 marks)

(b) A property on the estate is selected at random. Find the probability that the property has:

(i) exactly 3 bedrooms. (1 mark)

(ii) at least two toilets. (1 mark)

(iii) exactly 3 bedrooms and at least 2 toilets. (2 marks)

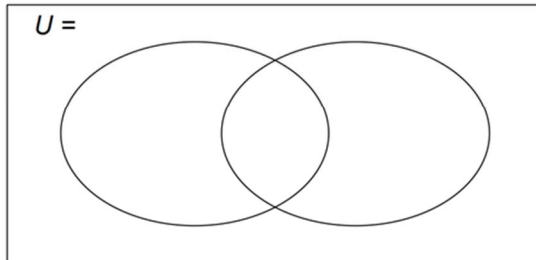
(iv) at most 2 bedrooms, given that it has exactly 2 toilets. (2 marks)

(c) Three properties are selected at random from those that have exactly 3 bedrooms. Calculate the probability that one property has 1 toilet, another has 2 toilets and the third property has at least 3 toilets. (2 marks)

**Question 16 (10 marks)**

In a group of Year 11 students, 60% are girls and 40% are boys. 35% of the students are taller than 172 cm. The probability that a boy is taller than 172 cm is 0.206.

- (a) Complete the Venn diagram below. (3 marks)



- (b) A Year 11 student is selected at random. Find the probability that:
- (i) the student is a girl taller than 172 cm. (1 mark)
  - (ii) the student is a boy less than 172 cm. (1 mark)
  - (iii) a student is taller than 172 cm, given that the student is a girl. (2 marks)
- (c) There are 265 students in Year 11.  
State the ratio of the number of girls over 172 cm to the number of boys over 172  
in the form 1:  $k$  where  $k$  is rounded to 3 significant figures. (3 marks)

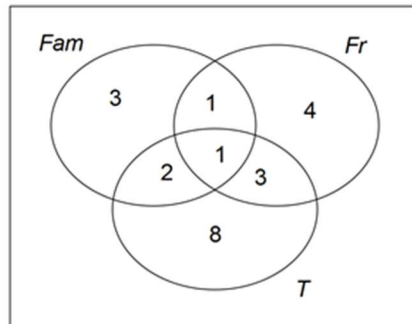


**Question 18 (9 marks)**

Jerome saves his favourite digital images on his tablet in three separate folders, namely "Family", "Friends" and "Travel". His Family folder contains 3 images, his Friends folder contains 4 images and his Travel folder contains 8 images. All the images are different.

- (a) (i) How many ways can Jerome arrange the 15 images in a row across the computer screen before they are put into the folders? (1 mark)
- (ii) Jerome chooses 2 images from each folder to use as background images. How many different selections of background images are there? (2 marks)
- (iii) Calculate the probability that if Jerome chooses 6 images at random, there are two from each folder. (2 marks)

Jerome chooses a further 7 favourite images and draws the following Venn diagram:



- (b) (i) Shade in the area  $Fam \cap Fr \cap T$  and explain its significance in this context. (2 marks)
- (ii) Given that an image was taken when travelling, what is the probability that it had friends in it? (2 marks)

**Question 20 (6 marks)**

The probability that Hannah goes swimming on any given day is 0.2. On a day that she goes swimming the probability that she has a burger for dinner is 0.75. On a day when she does not go swimming, the probability that she has a burger for dinner is  $x$ . The probability that Hannah has a burger for dinner on any day is 0.39.

- (a) Determine the value for  $x$ . (2 marks)
- (b) Given that Hannah had a burger for dinner, find the probability that she went swimming that day. (2 marks)
- (c) Are the two events "having a burger" and "goes swimming" independent? Justify your answer mathematically. (2 marks)

8. (a) (i)

5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1

(ii)  $p^6 - 6p^5q + 15p^4q^2 - 20p^3q^3 + 15p^2q^4 - 6pq^5 + q^6$  ✓✓  
 (b)  $a = 10 \quad b = 15 \quad c = 10$  ✓✓

[6]

11. (a)

		Number of toilets			Total
		1	2	3 or more	
Number of bedrooms	1	35	0	0	35
	2	50	5	0	55
	3	5	65	15	85
	4 or more	0	0	25	25
Total		90	70	40	200

(b) (i)  $\frac{85}{200}$  ✓✓  
 (ii)  $\frac{110}{200}$  ✓

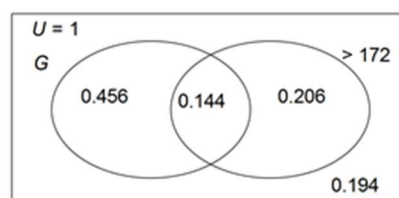
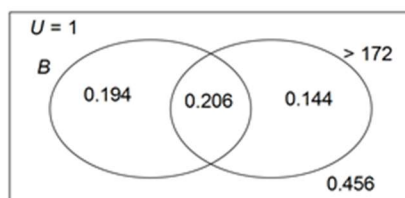
(iii)  $\frac{65}{200} + \frac{15}{200} = \frac{80}{200}$  ✓✓

(iv)  $\frac{5}{70}$  ✓✓

(c)  $\frac{{}^5C_1 \times {}^{65}C_1 \times {}^{15}C_1}{{}^{85}C_3} = \frac{4875}{98770} = 0.04936$  ✓✓

[10]

16. (a)



(b) (i) 0.144 ✓✓✓  
 (ii) 0.194 ✓  
 (iii)  $\frac{0.144}{0.6} = 0.24$  ✓✓

(c) Girls over 172 cm = 38 ✓  
 Boys over 172 cm = 55 ✓  
 38:55 = 1:1.45 ✓

[10]

18. (a) (i)  $15! = 1\,307\,674\,368\,000$

✓

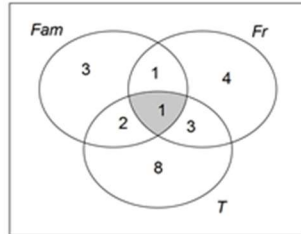
(ii)  ${}^3C_2 \times {}^4C_2 \times {}^8C_2 = 504$

✓✓

(iii)  $\frac{{}^3C_2 \times {}^4C_2 \times {}^8C_2}{{}^{15}C_6} = \frac{72}{715} = 0.100699$

✓✓

(b) (i)



✓

This image has family and friends and travel in it.

✓

(ii)  $\frac{4}{14}$

✓✓

[9]

20. (a)  $(0.2 \times 0.75) + 0.8x = 0.39$

✓

$x = 0.3$

✓

(b)  $\frac{0.15}{0.39} = \frac{5}{13}$

✓✓

(c) The events are not independent.

✓

$P(S|B) \neq P(S)$

$\frac{15}{39} \neq 0.2$

✓

[6]