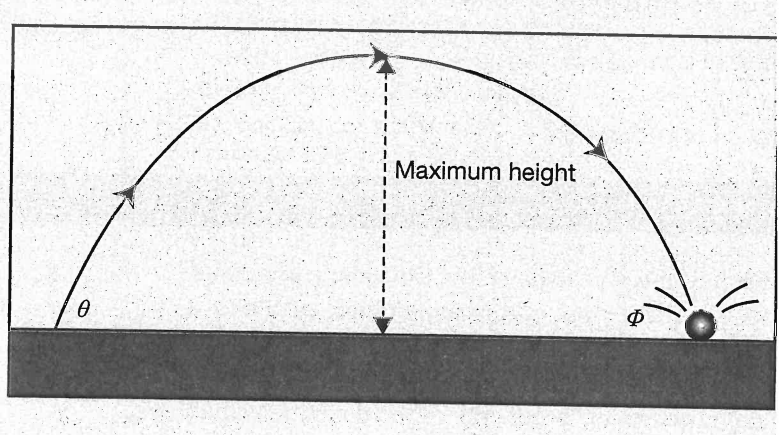
13 Projectile Motion Problems 2

Object thrown up and landing at same level

Special considerations:

- Total vertical displacement = 0.
- Vertical velocity at top of flight = 0.
- Time to rise = time to fall.
- Time to rise = half the time of flight.
- Speed at launch = speed at landing.
- Angle θ = angle Φ.
- Two halves of flight are symmetrical.
- Maximum height occurs when vertical velocity = 0.



QUESTIONS

For each of these problems, consider a projectile fired at an angle and landing on a surface at the same level as that from which it was fired, as shown in the diagram. For each problem find any of these quantities which are not in the given data (not necessarily in the order given).

(a) The initial velocity of the projectile.
(b) Its initial horizontal velocity.
(c) Its initial vertical velocity.
(d) Its range.
(e) Its maximum height.
(f) The time it takes to reach maximum height.
(g) Its time of flight.

1. Projectile fired at 20 m s^{-1} at 60° to the vertical.
2. Projectile is fired at 30 m s^{-1} and has a range of 79.5 m. It is in flight for 5.3 s.
3. Projectile is fired at 35.6 m s^{-1} at 55.8° to the horizontal and hits the ground 6 s later.
4. Projectile has a range of 318.2 m and rises to a maximum 60 m.
5. Projectile fired at 50° to the horizontal, rises 2.0 km.
6. Projectile's range is 2.5 km and rises for 30 s.
7. Projectile is fired at 200 m s^{-1} at 45°.
8. Projectile fired at 40° to the horizontal at 150 m s^{-1}.
9. Projectile rises to a height of 81.6 m after being fired at 60° to the horizontal.
10. Projectile hits a target 1.2 km away, 30 s after firing at 74.8°.
11. Projectile rises to a maximum height of 127.5 m ar has a range of 408.2 m.
12. Projectile rises to 0.75 km, having been fired at 286.9 m s^{-1}.
13. Projectile fired at 400 m s^{-1} and rises for 20.4 s.
14. Projectile fired with minimum velocity to hit a targ 3.0 km away 21 s after firing.
15. The vertical component of the velocity of a projectile is 147 m s^{-1} and the horizontal componer of its velocity is 91.6 m s^{-1}.

Now answer these questions

16. A projectile has a time of flight of 8.0 s and a range of 1120 m.
    (a) What maximum height does it reach?
    (b) At what velocity is it projected?
17. A projectile has a range of 750 m and reaches a maximum height of 58.8 m.
    (a) What is its time of flight?
    (b) What is its launch velocity?
18. A ball is hit at 30 m s^{-1} at 45° to the horizontal.
    (a) Calculate how far from the bat a fielder should stand to catch the ball at the same height as it was hit.
    (b) Calculate the time of flight if the fielder catches it.
    (c) Calculate how high the ball would rise after being hit.

13 Projectile Motion Problems 2

All answers in order of information requested in the chapter (answers may differ slightly due to rounding errors).

1. 20 m s^{-1} at 30° to horizontal, 17.3 m s^{-1}, 10 m s^{-1}, 35.3 m, 5.1 m, 1.02 s, 2.04 s
2. 30 m s^{-1} at 60° to horizontal, 15 m s^{-1}, 26 m s^{-1}, 79.5 m, 34.5 m, 2.65 s, 5.3 s
3. 35.6 m s^{-1} at 55.8° to horizontal, 20 m s^{-1}, 29.4 m s^{-1}, 120 m, 44.1 m, 3.0 s, 6.0 s
4. 57 m s^{-1} at 37° to horizontal, 45.4 m s^{-1}, 34.3 m s^{-1}, 318.2 m, 60.0 m, 3.5 s, 7.0 s
5. 258.5 m s^{-1} at 50° to horizontal, 166.1 m s^{-1}, 198 m s^{-1}, 6712 m, 2000 m, 20.2 s, 40.4 s
6. 296.9 m s^{-1} at 81.9° to horizontal, 41.7 m s^{-1}, 294 m s^{-1}, 2500 m, 4410 m, 30 s, 60 s
7. 200 m s^{-1} at 45° to horizontal, 141.4 m s^{-1}, 141.4 m s^{-1}, 4080 m, 1020.1 m, 14.43 s, 28.86 s
8. 150 m s^{-1}at 40° to horizontal, 115 m s^{-1}, 96.4 m s^{-1}, 2262 m, 474 m, 9.84 s, 19.67 s
9. 46.2 m s^{-1} at 60° to horizontal, 23.1 m s^{-1}, 40 m s^{-1}, 188.4 m, 81.6 m, 4.08 s, 8.16 s
10. 152.3 m s^{-1} at 74.8° to horizontal, 40 m s^{-1}, 147 m s^{-1}, 1200 m, 1102.5 m, 15 s, 30 s
11. 64 m s^{-1} at 50° to horizontal, 40 m s^{-1}, 50 m s^{-1}, 408.2 m, 127.5 m, 5.1 s, 10.20 s
12. 286.9 m s^{-1} at 25° to horizontal, 260 m s^{-1}, 121.2 m s^{-1}, 6433 m, 750 m, 12.37 s, 24.74 s
13. 400 m s^{-1} at 30° to horizontal, 346.4 m s^{-1}, 200 m s^{-1}, 14133 m, 2041 m, 20.4 s, 40.8 s
14. 176.1 m s^{-1} at 36° to horizontal, 143 m s^{-1}, 103 m s^{-1}, 3000 m, 543.3 m, 10.5 s, 21 s
15. 173.2 m s^{-1} at 58° to horizontal, 91.60 m s^{-1}, 147 m s^{-1}, 2748 m, 1102.5 m, 15 s, 30 s
16. (a) 78.4 m
    (b) 145.4 m s^{-1} at 15.5° to horizontal
17. (a) 6.93 s
    (b) 113.4 m s^{-1} at 17.4° to horizontal
18. (a) 91.8 m
    (b) 4.33 s
    (c) 22.9 m

14 Projectile Motion Problems 3

All answers in order of information requested in the chapter (answers may differ slightly due to rounding errors).

1. 49 m s^{-1} at 30° to horizontal, 42.4 m s^{-1}, 24.5 m s^{-1}, 396.9 m, 30.6 m, 2.5 s, 9.36 s
2. 25 m s^{-1} at 30.3° to horizontal, 21.6 m s^{-1}, 12.6 m s^{-1}, 138 m, 8.1 m, 1.3 s , 6.4 s
3. 101 m s^{-1} at 84.3° to horizontal, 10 m s^{-1}, 100.5 m s^{-1}, 200 m, 515 m, 10.3 s, 20 s
4. 50 m s^{-1} at 45° to horizontal, 35.4 m s^{-1}, 35.4 m s^{-1}, 387.2 m, 63.9 m, 3.6 s, 10.94 s
5. 15 m s^{-1} at 3.2° to horizontal, 14.97 m s^{-1}, 0.85 m s^{-1}, 59.9 m, 0.037 m, 0.087 s, 4.0 s
6. 31 m s^{-1} at 71.2° to horizontal, 10 m s^{-1}, 29.4 m s^{-1}, 76 m, 44.1 m, 3.0 s, 7.6 s
7. 35.1 m s^{-1} at 30° to horizontal, 30.4 m s^{-1}, 17.6 m s^{-1}, 158.1 m, 15.8 m, 1.8 s, 5.2 s
8. 40 m s^{-1} at 40° to horizontal, 30.64 m s^{-1}, 25.7 m s^{-1}, 300 m, 33.6 m, 2.62 s, 9.8 s
9. 65 m s^{-1} at 39.7° to horizontal, 50 m s^{-1}, 41.5 m s^{-1}, 500 m, 87.9 m, 4.2 s, 10 s
10. 66.6 m s^{-1} at 72.5° to horizontal, 20 m s^{-1}, 63.5 m s^{-1}, 300 m, 205.7 m, 6.5 s, 15 s
11. 23.6 m
12. (a) 37°
    (b) 18.8 m s^{-1}
13. (a) 886.4 m
    (b) 72.7 m s^{-1} at 47.6° to horizontal
14. Let T_1 be the time it takes the rock to reach the bottom of the well. If Δy is the depth of the well, then
    From $\Delta y = u_y t + \frac{1}{2}gt^2$ ($u_y = 0$), then $\Delta y = 5T_1^2$
    If T_2 be the time it takes soundwave from the splash to reach the top of the well, then $\Delta y = 340T_2$
    Therefore $5T_1^2 = 340T_2$
    We are given that $T_1 + T_2 = 2.75$
    Therefore $T_2 = 2.75 - T_1$
    So, $5T_1^2 = 340\ (2.75 - T_1)$
    So, $5T_1^2 + 340T_1 - 935 = 0$
    Solving, we get $T_1 = 2.65$ s
    From which, the depth of the well = $T_1^2 = 5(2.65)^2 = 35.1$ m
15. From $\Delta y = u_y t + \frac{1}{2}gt^2$ (Δy = vertical distance fallen)
    $\Delta y = 22 \times 4.8 - \frac{1}{2} \times 9.8 \times 4.8^2 = 7.3$ m
16. (a) Vertical component of its velocity = $u \sin 37° = 18.05$ m s^{-1}
    From $\Delta y = u_y t + \frac{1}{2}gt^2$, we get
    $-60 = 18.05t - 4.9t^2$
    From which, $t = 5.8$ s
    (b) Vertical velocity at top of flight = 0
    Therefore, from $v_y^2 = u_y^2 + 2g\Delta y$ (this time, Δy = maximum height above launch position)
    So, $0 = 18.05^2 - 19.6\Delta y$
    From which $\Delta y = 16.62$ m
    Therefore, maximum altitude above the ground = 16.62 + 60 = 76.62 m
    (c) Horizontal component of the velocity, $u_x = 30 \cos 37° = 23.96$ m s^{-1}
    Range = $u_x \times$ time of flight = 23.96 × 5.8 = 138.96 m = 139 m