Learning Objective

I will be able to solving problems related to right-angled triangle and non right-angled triangle problems, including bearings.

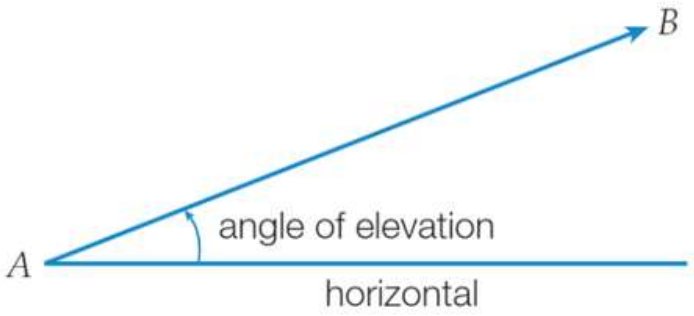
Success Criteria

At the end of the lesson, I will be able to:

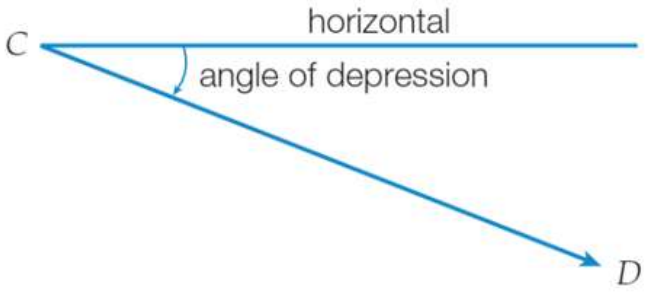
- **apply my knowledge of trigonometry to solving problems related to right-angled triangle and non right-angled triangle problems, including bearings.**

Concept Development

The angle of elevation from A to B is the angle between the line AB and the horizontal.
If you were positioned at A, you would need to raise your eyes from a horizontal line of sight to view an object at B.



The angle of depression from C to D is the angle between the line CD and the horizontal.
If you were positioned at C, you would need to lower your eyes from a horizontal line of sight to view an object at D.



Elevation: look up.
Depression: look down.

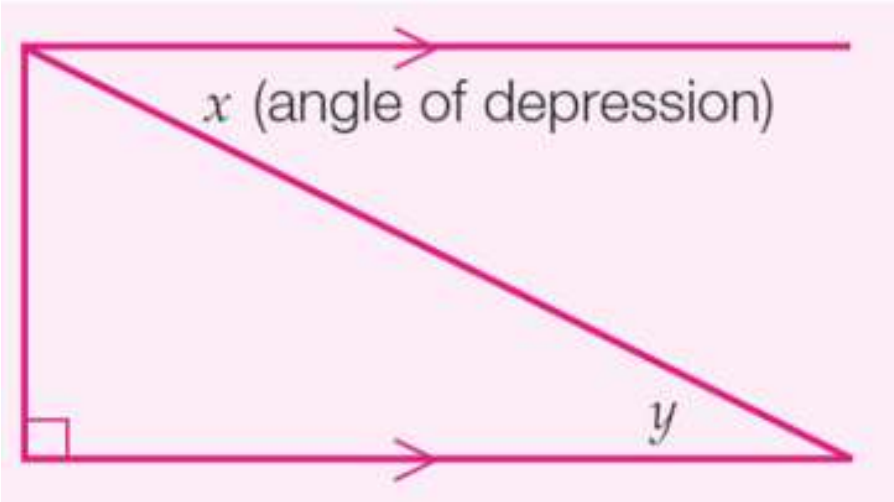
Concept Development

Elevation: look up.
Depression: look down.

The angle of depressions may not always be inside the triangle you draw.

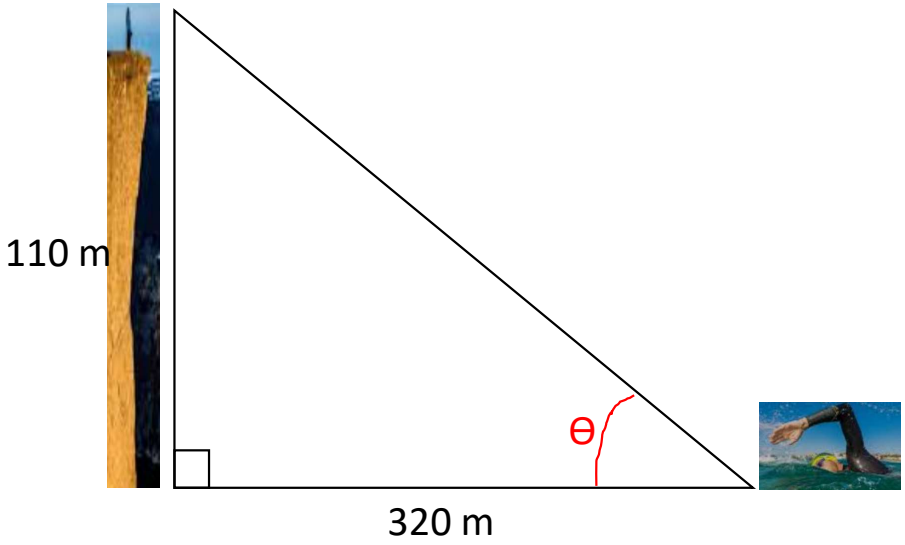
However, you can use alternate (Z) angles to find an angle inside the triangle.

x and y are alternate angles on parallel lines (same size).



Skill Development

A swimmer, 320 m from the base of a cliff, looks up and waves to her friend who is at the top of the 110 m cliff.
Find the angle of elevation from the swimmer to the top of the cliff. Give your answer to the nearest degree.



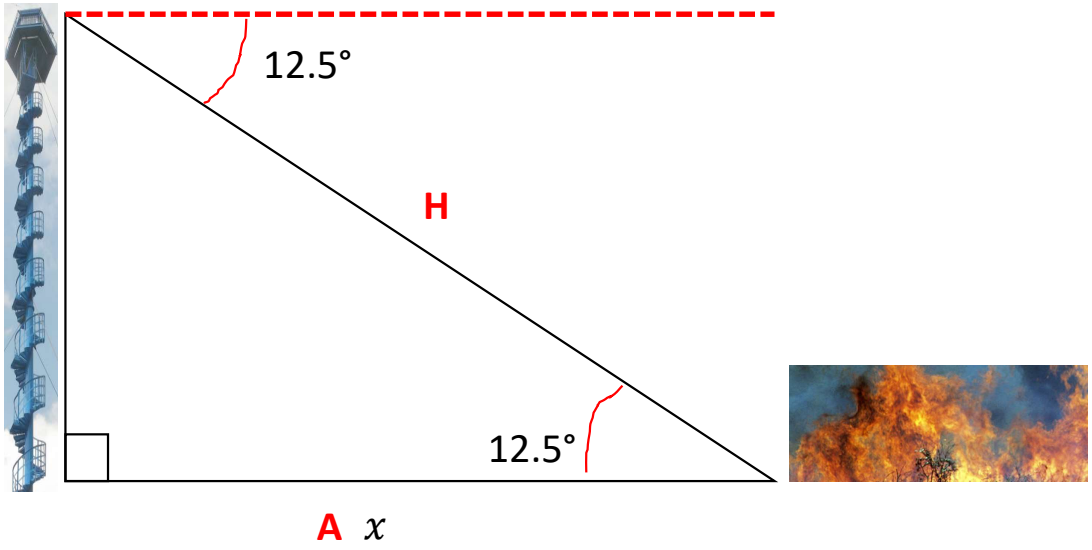
$$\tan\theta = \frac{110}{320}$$

$$\theta = 18.9704\ldots$$

The angle of elevation is 19°.

Guided Practice

A ranger at the top of a lookout spots a small fire at an angle of depression of 12.5°.
If the tower is 76 m high, what is the distance from the top of the lookout to the fire.
Give your answer to the nearest metre.



$$sin\ 12.5° = \frac{76}{x}$$

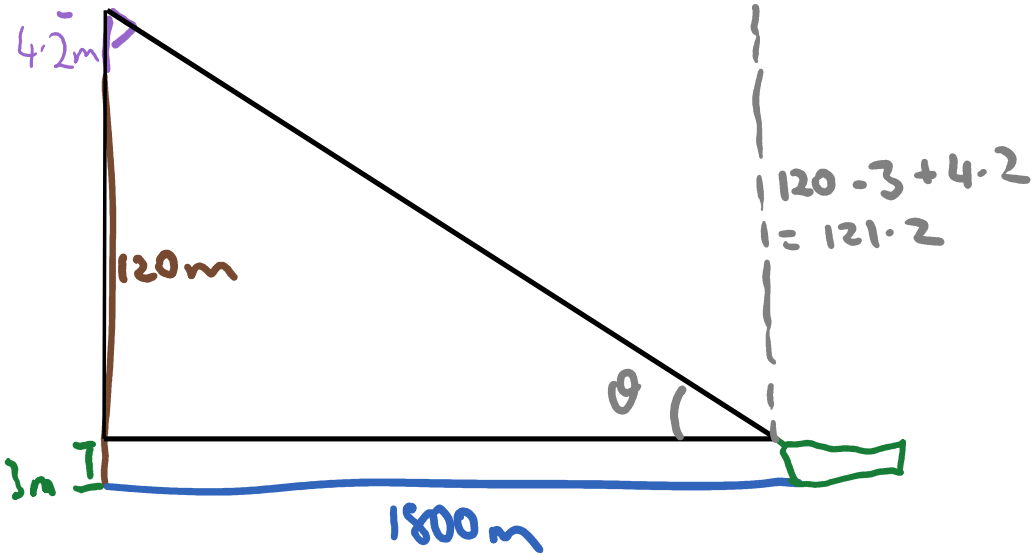
$$x = \frac{76}{sin\ 12.5°}$$

$$x = 351.1372\ \ldots$$

The distance is 351 m.

Guided Practice

A yacht is 1.8km away from the base of a 120 m cliff. The deck of the yacht is 3 m above sea level. What is the angle of elevation from the deck of the yacht to the top of a 4.2m flagpole on the edge of the clif? Give your answer to 2 decimal places.



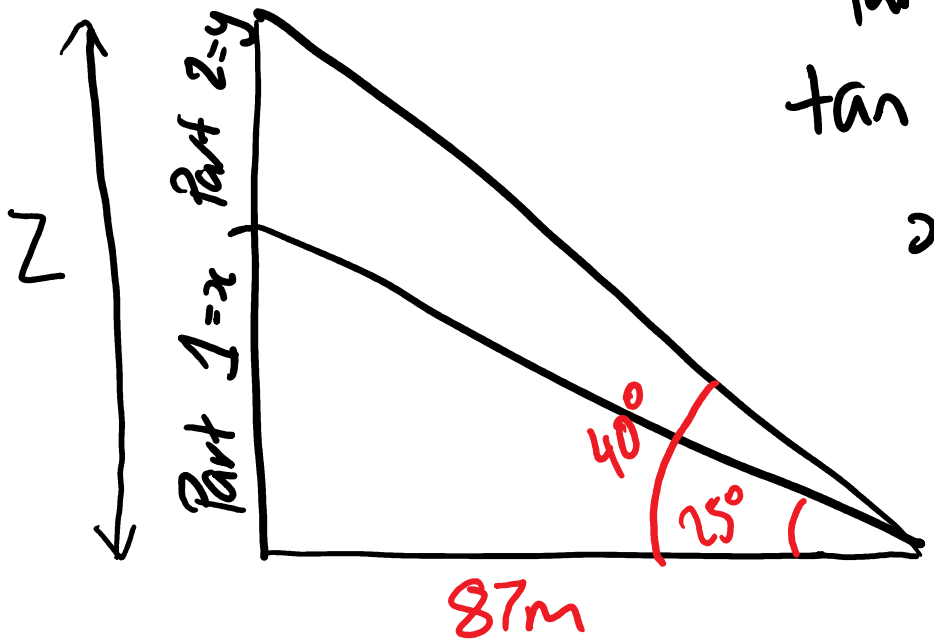
$$\tan\theta = \frac{121.2}{1800}$$

$$\theta° = \tan^{-1}\left(\frac{121.2}{1800}\right)$$

$$\theta = 3.85$$

Guided Practice

A radio station tower was built in two sections. From a point 87m from the base of the tower, the angle of elevation of the top of the first section is $25°$, and the angle of elevation of the top of the second section is $40°$. To the *nearest metre*, what is the height of the top section of the tower?



Part 1

$\tan 25 = \frac{x}{87}$

$x = 40.57\text{m}$

Part 2

$\tan 40 = \frac{z}{87}$

$z = 73.00\text{m}$

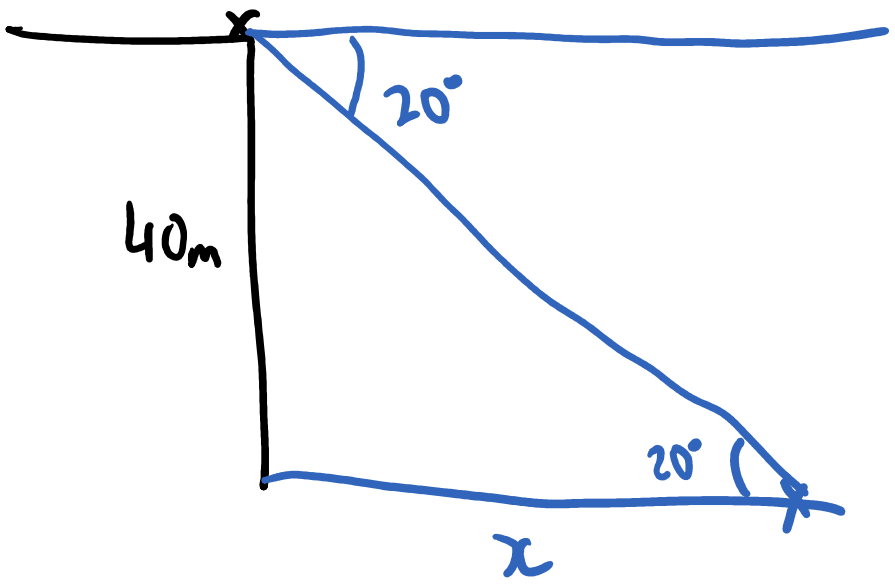
$y = 73 - 40.57$

$= 32.43$

The top section is 32m

Guided Practice

4 A person lying down on top of a cliff 40 m high observes the angle of depression to a buoy in the sea below to be 20°. If the person is in line with the buoy, find the distance between the buoy and the base of the cliff, which may be assumed to be vertical.



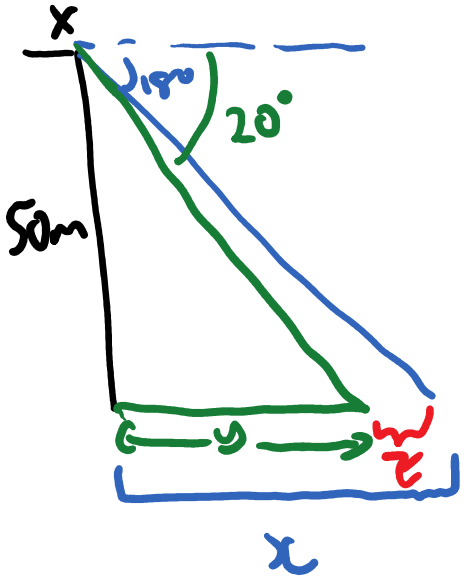
$$\tan(20) = \frac{40}{x}$$

$$x = \frac{40}{\tan(20)}$$

$$x = 109.90\text{ m}$$

Guided Practice

A person standing on top of a cliff 50 m high is in line with two buoys whose angles of depression are 18° and 20°. Calculate the distance between the buoys.



$$\tan(18°) = \frac{50}{x}$$

$$x = \frac{50}{\tan(18)}$$

$$\tan(20) = \frac{50}{y}$$

$$y = \frac{50}{\tan(18)}$$

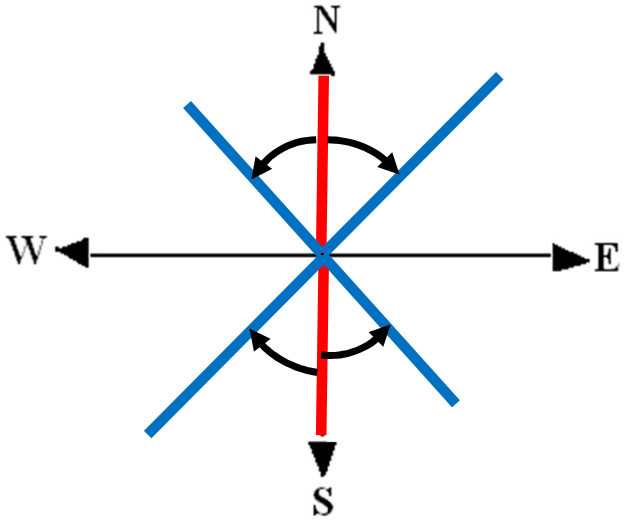
$$z = x - y$$

$$= 16.51\text{m} \ (2\text{dp})$$

Skill Development

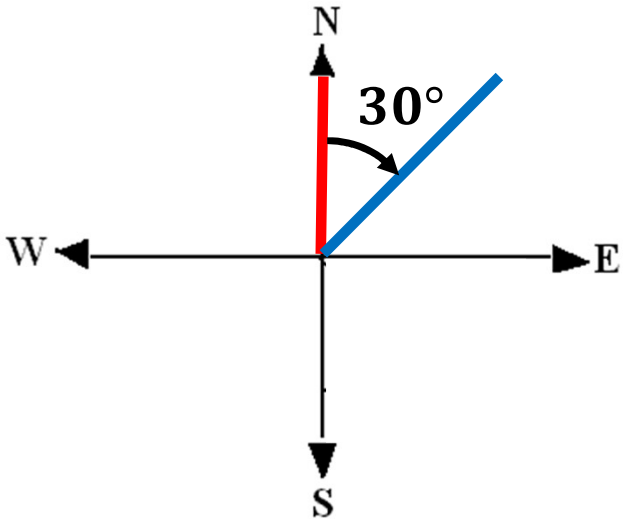
Compass Bearings give the angle:

- Given in terms of being north, south, east or west.
- Expressed as a certain number of degrees between 0^o and 90^o East or West of North or South.

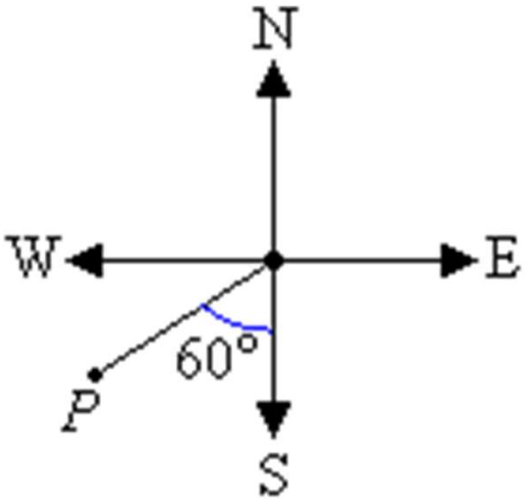


Skill Development

Express the following angles as a compass bearing.



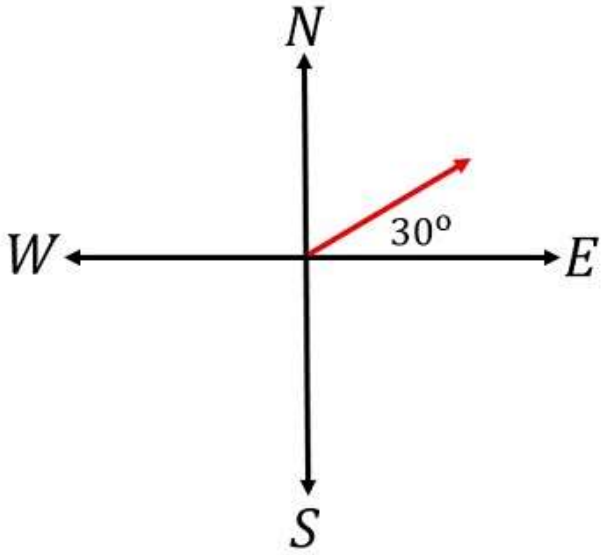
N30°E



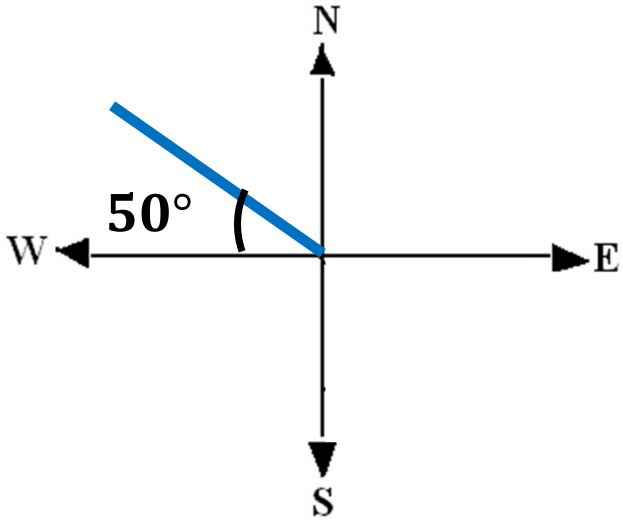
S60°W

Skill Development

Express the angle as a compass bearing.



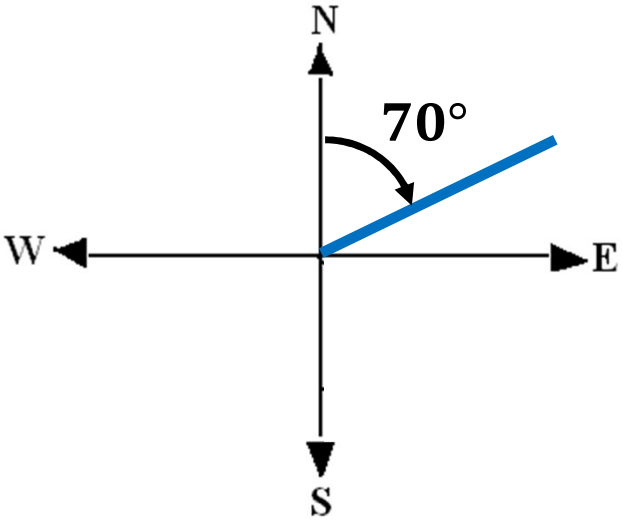
N60°E



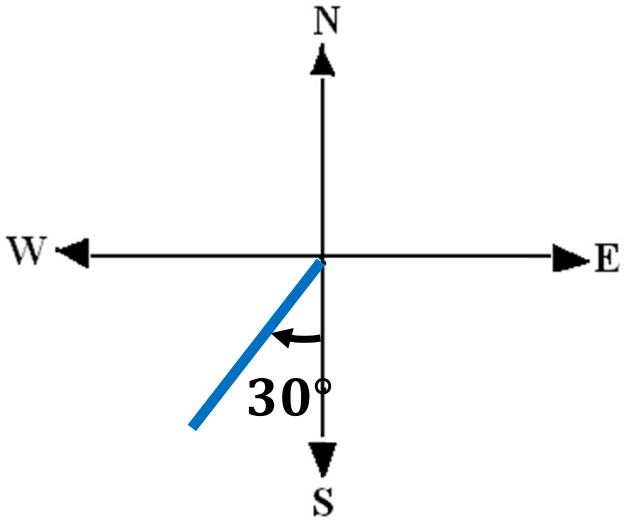
N40°W

Skill Development

Draw the compass bearing N70°E

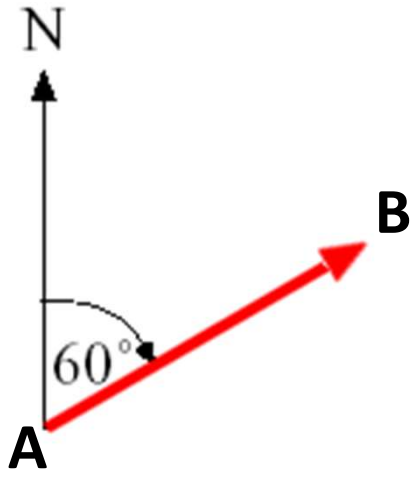


Draw the compass bearing S30°W



Skill Development

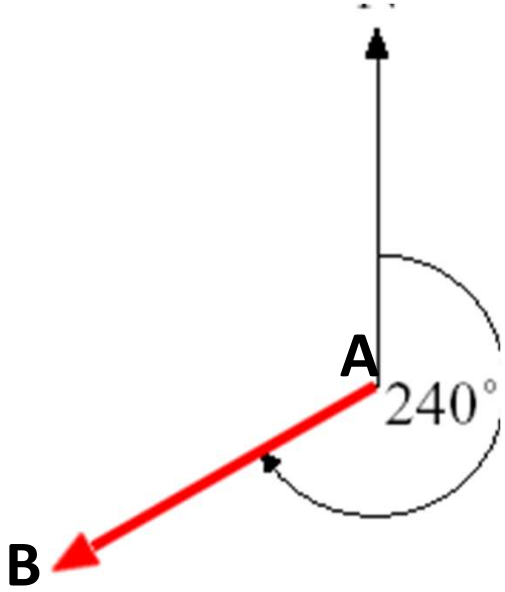
True Bearings give the angle in a clockwise direction from North



True Bearing is always written with three digits so is 060°

Skill Development

State the true bearing of B from A



The True Bearing of B from A is 240°

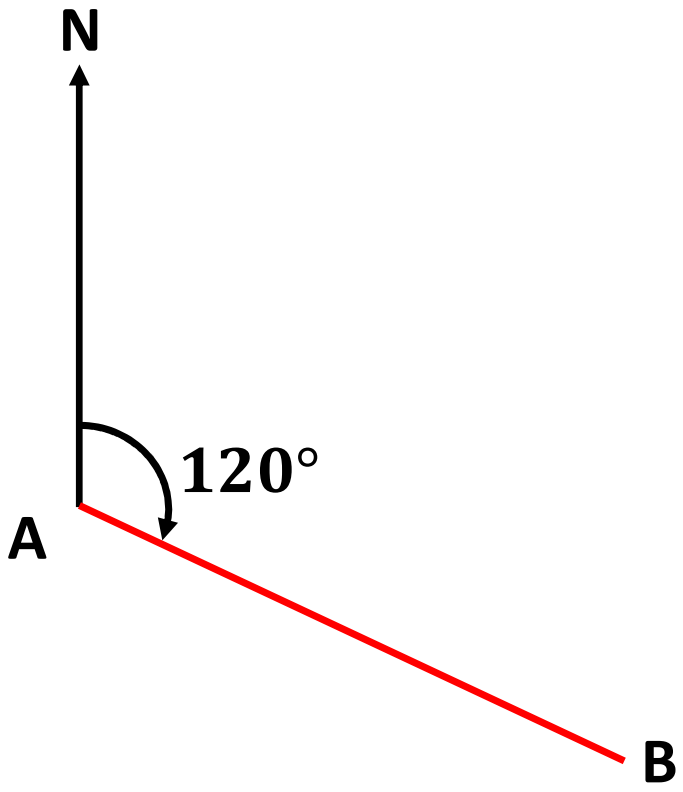
Skill Development

Draw the true bearing of B from A when it is $120°T$.

Step 1: Label A and draw a North arrow from A

Step 2: Estimate 120° from North Arrow

Step 3: Draw a straight line from A and Label B



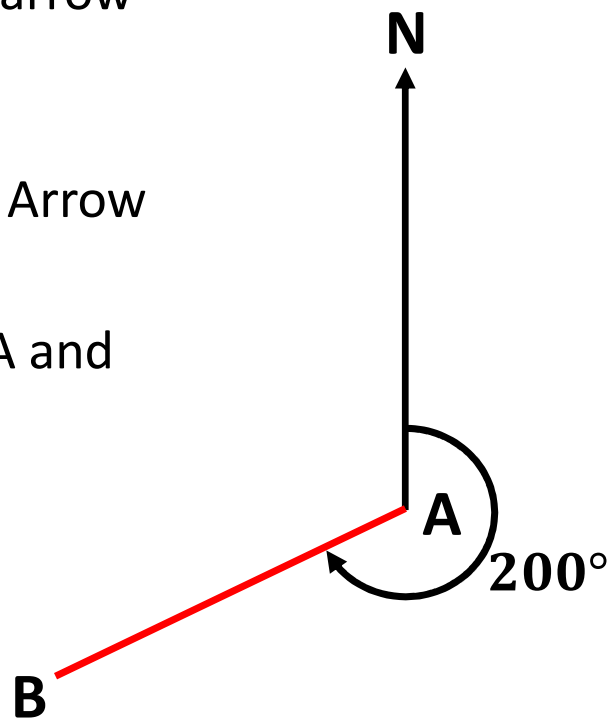
Skill Development

Draw the true bearing of B from A when it is $200°T$.

Step 1: Label A and draw a North arrow from A

Step 2: Estimate 200° from North Arrow

Step 3: Draw a straight line from A and Label B

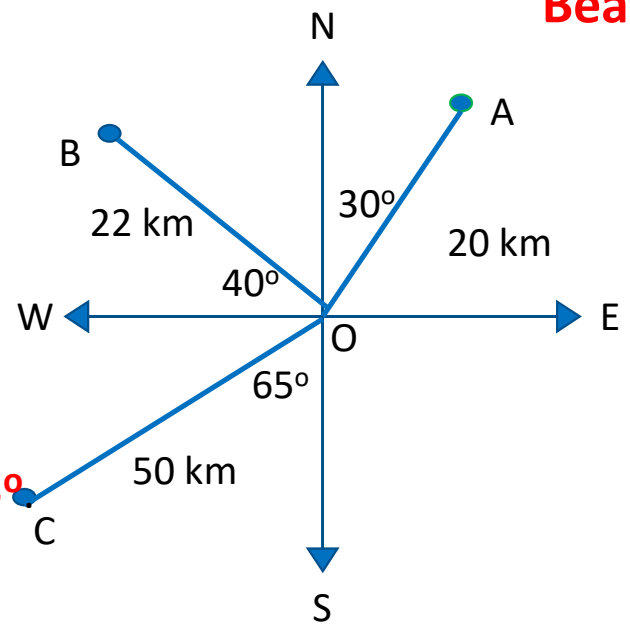


Concept Development

Determine the true bearing of A, B and C from O.

Bearing of B from O is 310°

Bearing of A from O is 030°.

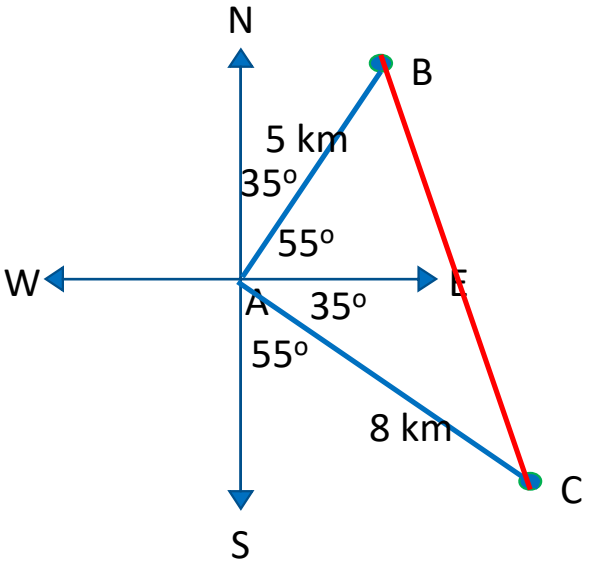


Bearing of C from O is 245°.

Guided Practice

From Port A, two ships B and C are observed in directions N35°E and S55°E respectively.

If Ship B is 5 km from A and Ship C is 8 km from A, what is the distance between the ships to the nearest 50m?



From the diagram,

$$\angle BAC = 55° + 35° = 90°$$

Using Pythagoras' Theorem

$$BC^2 = AB^2 + AC^2$$
$$= 25 + 64$$
$$= 89$$

Therefore $BC = 9.43398\ km$

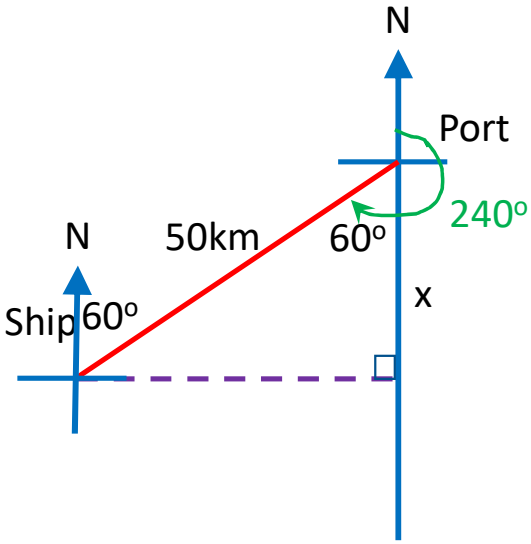
The distance between the ships is 9.45 km or 9450 m.

Guided Practice

A ship leaves a port and sails 50km on a bearing of 240°.

a) How far south of the port is the ship?

b) What is the bearing of the port from the ship?

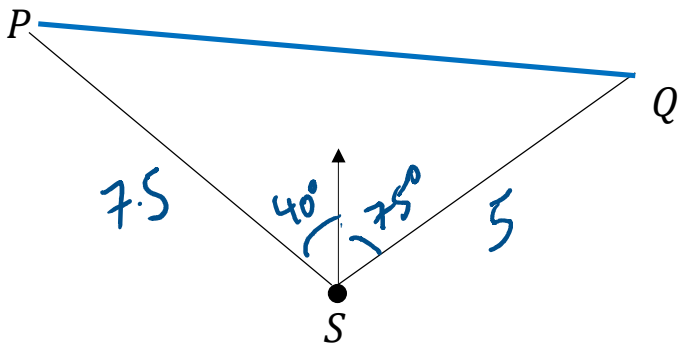


a) Extend a horizontal line from the ship to the North-South vertical line, forming a right-angled triangle.
Let x = the distance of the ship south of the port.
We know that the angle between the NS line and the hypotenuse of the angle is 240°-180°=60
Using trig ratios, $\cos 60° = \frac{x}{50}$
Therefore $x = 50 \cos 60° = 25$
The ship is 25km south of the port.

b) Place an axis on the ship.
Since the we know that there are two parallel vertical lines, the angle between the NS line and the red line is 60°.
Hence the bearing of the port from the ship is 060°.

Guided Practice

13 From a ship S, two other ships P and Q are on bearings 320° and 075° respectively. The distance PS is 7.5 km and the distance QS is 5 km. Find the distance PQ.



$$PQ^2 = 5^2 + 7.5^2 - 2(5)(7.5)\cos(115°)$$

$$PQ = 10.63\ km$$

Guided Practice

A yacht sails from point A on a bearing of 035° for 2000 m. It then alters course to a direction with a bearing of 320° and after sailing for 2500 m it reaches point B.

a Find the distance AB.

b Find the bearing of B from A.

$\theta = 180 - 35 - 40 = 105°$

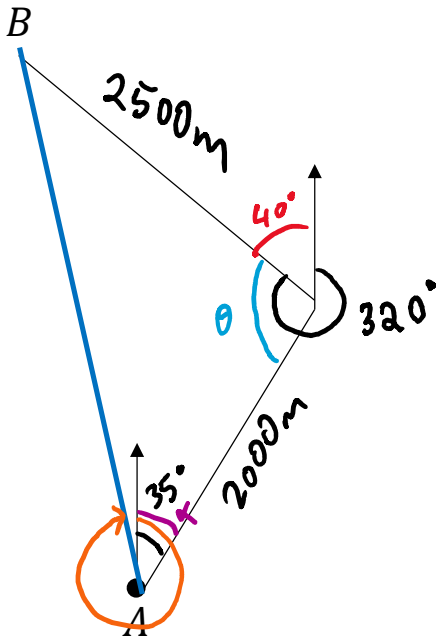
$AB^2 = 2000^2 + 2500^2 - 2(2000)(2500)\cos(105)$

$AB = 3583.04\ m$

$2500^2 = 2000^2 + AB^2 - 2(2000)(AB)\cos(\alpha)$

$\alpha = 42.37°$

$Bearing$
$= 360 - (\alpha - 35)$
$= 353°$

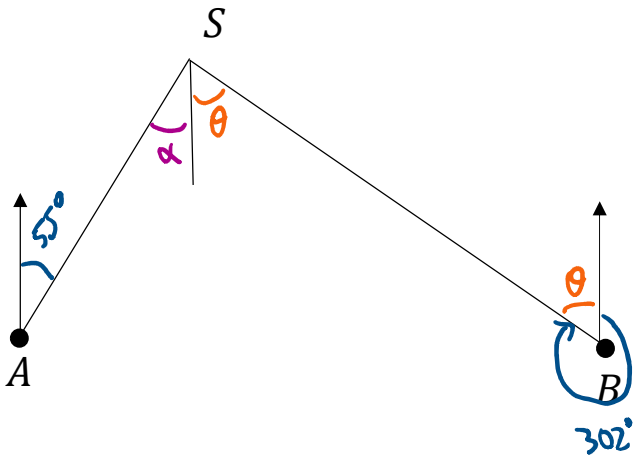


Guided Practice

10 The bearing of a ship S from a lighthouse A is 055°. A second lighthouse B is due east of A. The bearing of S from B is 302°. Find the magnitude of angle ASB.

$\theta = 360 - 302 = 58°$

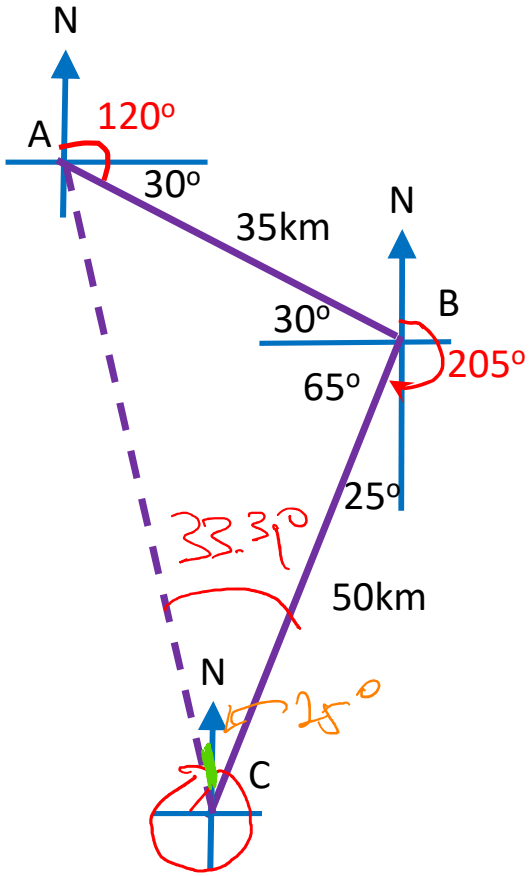
$\angle ASB = \alpha + \theta = 55 + 58 = 113°$



A ship leaves port A and sails for 35km on a bearing of 120° to port B. From there, the ship sails to port C which is 50km from B on a bearing of 205°.

a) Find the distance from port A to port C.

b) Find the bearing of port A from port C to the nearest degree.



a) Use the cosine rule.

From the diagram, angle ABC = 95°

$AC^2 = 35^2 + 50^2 - 2(35)(50)cos95°$

$AC \approx 63.4826$

Therefore the distance from port A to Port C is approximately 63.5km

b) Use the sine rule to calculate the angle ACB

$$\frac{35}{\sin \angle ACB} = \frac{AC}{\sin 95°}$$

$$\angle ACB = 33.31° \; or \; 146.69°$$

Reject 146.69° because $\angle ACB$ must be an acute angle.

Therefore the bearing is 360°-33.31°+25°=351.69°

The bearing of port A from port C is 352°

Independent Practice

Complete Cambridge Ex 13F