18 Circular Motion on a Banked Curve

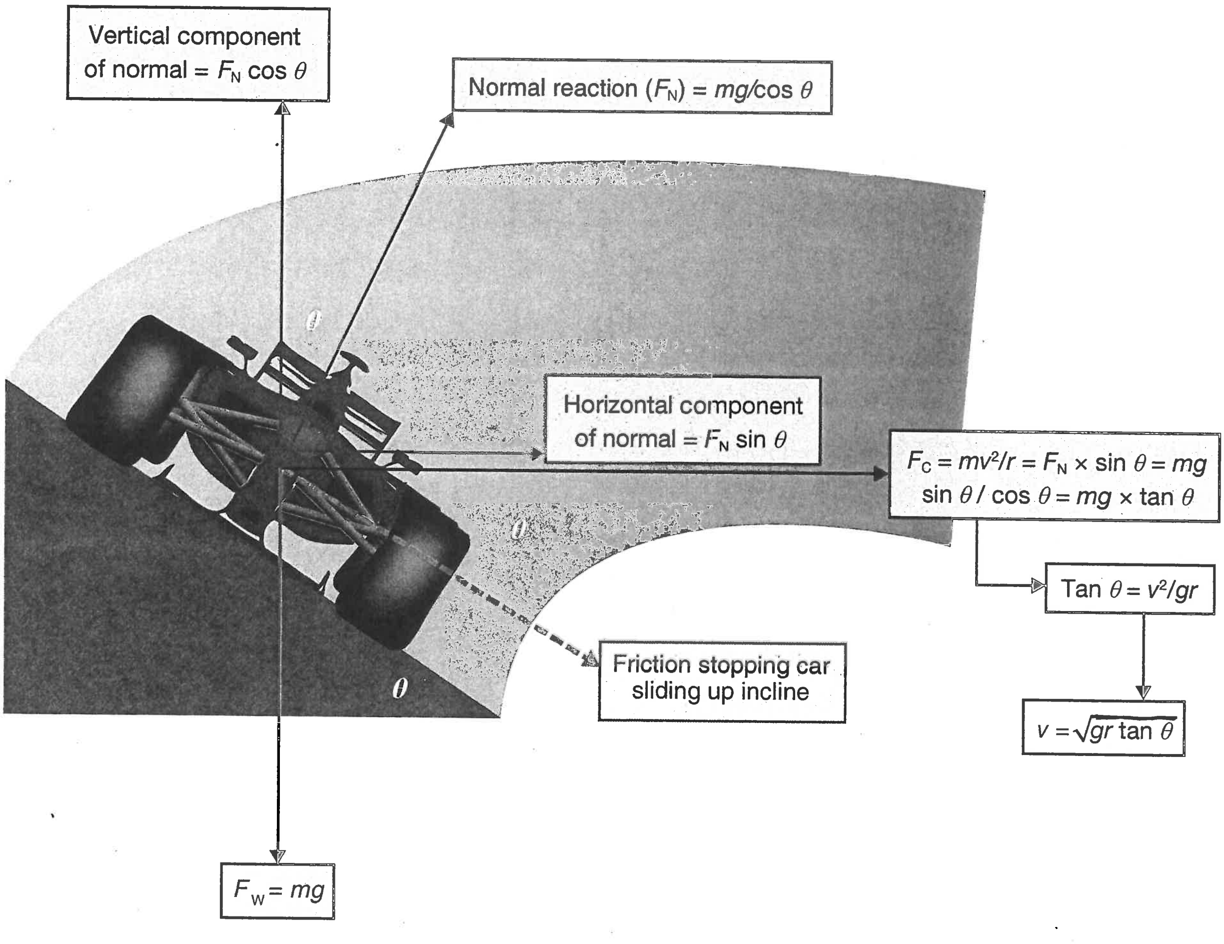
Consider a car driving around a curve banked at an angle of θ with respect to the horizontal. It is important for the mathematical analysis that you realise that the circle of motion has as its radius, a *horizontal* line drawn from the car to a point above the centre of the curve. This is the direction of the centripetal force acting on the car to cause the circular motion. That is, the radius of the circle is *not parallel* to the surface of the inclined.

Although we are not going to consider frictional forces in this treatment of motion on a banked curve, cars can take curves faster than the theoretically calculated speed if (and there always is) there is sideways friction acting between the tyres and the road surface.

Even though the car is moving, the car tyres are not slipping on the road surface, so the part of tyre in contact with the road is instantaneously at rest with respect to the road. If there was no friction, the car would slide towards the outside of the curve and eventually leave the road, so the friction must oppose this tendency and therefore must be directed down the incline.

In the examples below, we are assuming no friction, but the cars do not slide off the curves because of the banking.

Analysing the forces in the diagram we get the equations that connect the variables as shown. Note that we are ignoring frictional effects in the forwards direction of the motion.



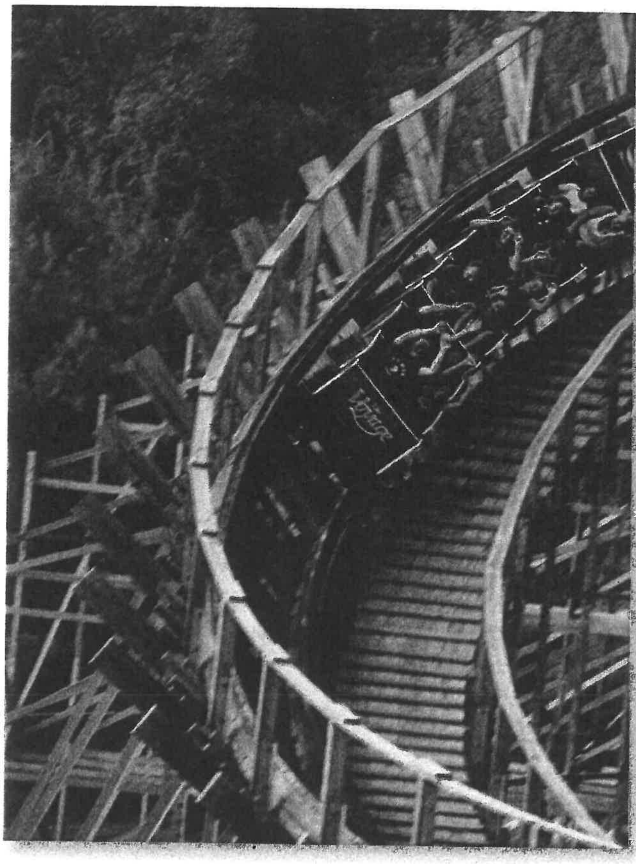
If we now analyse these forces further, then we get the following.

$$\text{Vertically, since at any instant } \Sigma F = 0, \text{ then } F_N \cos\theta = mg$$

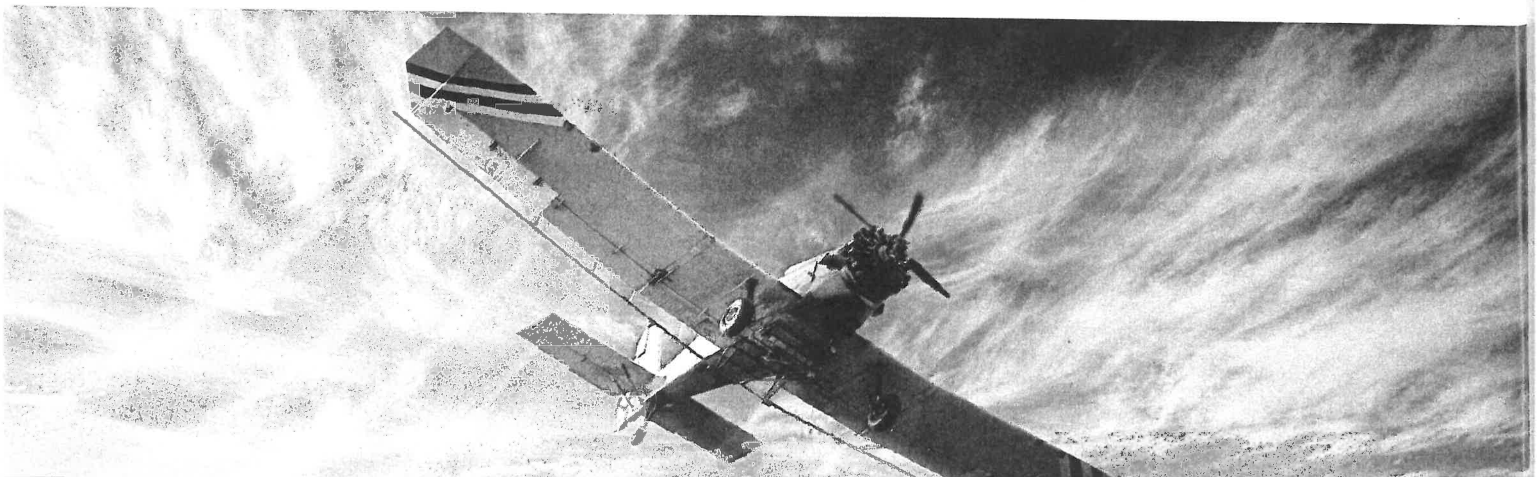
$$\text{Horizontally, net force is the centripetal force: } F_C = mv^2/r = F_N \sin\theta = mg \tan\theta$$

QUESTIONS

1. A 600 kg car is merging onto a major highway via a curve banked at 7° from the horizontal. The radius of the curve is 240 m.
   (a) What is the maximum safe speed for this car around the curve?
   (b) What would be the maximum safe speed for a car with mass 750 kg?
2. (a) Determine the minimum angle at which a road should be banked so that a 750 kg car travelling at 20.0 m s^{-1} can safely negotiate the curve if the radius of the curve is 250.0 m.
   (b) What will be the centripetal force acting on the car?
3. (a) If a curve with a radius of 80 m is properly banked for a car travelling 60 km h^{-1}, what must be the angle at which the curve is banked?
   (b) What would be the centripetal force acting on a 2 tonne truck taking the curve at 60 km h^{-1}?
4. A car is driven around a circle with a radius of 150 m, banked at an angle of 10°. Calculate the maximum velocity the car can travel.
5. An airplane is flying in a horizontal circle at a speed of 500 km h^{-1}. Its wings are tilted 40° to the horizontal, and force is provided by lift that is perpendicular to the wing surface. What is the radius of the circle?
6. At a car racing track, cars travel through a 315 m radius curve banked at 30°. What is their maximum safe speed? Give your answer in m s^{-1} and km h^{-1}.
7. A curve is banked at 15° and has a radius of 350 m. A car, with a centripetal force of 3939 N acting on it is negotiating the curve at its maximum safe speed.
   (a) What is the maximum safe speed for the curve?
   (b) What is the mass of the car?
8. (a) A roller coaster on the Moon has a curve with radius of 15.0 m. The coaster will go around this curve at 30 m s^{-1}. If the acceleration due to gravity is 1.67 m s^{-2} on the Moon, what is the optimum banking angle of the curve?
   (b) What would need to be the banking of the curve on Earth?
9. The curve in the roller coaster shown in the picture is banked at 80°. Assuming the track is frictionless, and that the coaster takes the curve at 50 m s^{-1} what is the radius of curvature of this bend?



10. Two frictionless curves are banked at 56°. Curve A has a radius of 75 m, and B a radius of 125 m.
    (a) What is the ratio of the maximum safe speed to take each curve?
    (b) At what angle should B have been banked so that cars could safely take both curves at the same speed as curve A?
    (c) At what angle should A have been banked so that cars could safely take both curves at the same speed as curve B?



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1. (a) From $v^2 = gr \tan\theta$
   $v^2 = 9.8 \times 240 \times \tan 7° = 288.8$
   Therefore v = ~~km s^{-1}~~ 17 M/S
   (b) Since mass is not a variable in the equation, $v = 17$ m s^{-1}
2. (a) From $v^2 = gr \tan\theta$
   $20^2 = 9.8 \times 250 \times \tan\theta$
   From which $\theta = 9.3°$
   (b) From $F_C = mv^2/r = 750 \times 20^2/250$
   = 1200 N towards the centre of the circle of motion
3. (a) From $v^2 = gr \tan\theta$
   $(60/3.6)^2 = 9.8 \times 80 \times \tan\theta$
   From which $\theta = 19.5°$
   (b) From $F_c = mv^2/r = 2000 \times 16.67^2/80 = 6947$ N towards the centre of the circle of motion
4. From $v^2 = gr \tan\theta$
   $v^2 = 9.8 \times 150 \times \tan 10° = 259.2$
   Therefore $v = 16.1$ m s^{-1}
5. From $v^2 = gr \tan\theta$
   $(500/3.6)^2 = 9.8 \times r \times \tan 40°$
   Therefore $r = 2346$ m
6. From $v^2 = gr \tan\theta$
   $v^2 = 9.8 \times 315 \times \tan 30° = 1782.3$
   Therefore $v = 42.2$ m s^{-1} = 152 km h^{-1}
7. (a) From $v^2 = gr \tan\theta$
   $v^2 = 9.8 \times 350 \times \tan 15°$
   From which $v = 30.32$ m s^{-1}
   (b) From $F_c = mv^2/r$
   $m = F_c r/v^2 = 3939 \times 30.32^2/350 = 1500$ kg
8. (a) From $v^2 = gr \tan\theta$
   $900 = 1.67 \times 15 \times \tan\theta$
   From which $\theta = 88.4°$
   (b) From $v^2 = gr \tan\theta$
   $900 = 9.8 \times 15 \times \tan\theta$
   From which $\theta = 80.7°$
9. From $v^2 = gr \tan\theta$
   $(50)^2 = 9.8 \times r \times \tan 80°$
   Therefore $r = 45$ m
10. (a) From $v^2 = gr \tan\theta$ (for curve A)
    $v^2 = 9.8 \times 75 \times \tan 56°$
    From which $v = 33$ m s^{-1}
    From $v^2 = gr \tan\theta$ (for curve B)
    $v^2 = 9.8 \times 125 \times \tan 56°$
    From which $v = 42.6$ m s^{-1}
    Therefore ratio of speeds A : B = 1 : 1.3
    (b) From $v^2 = gr \tan\theta$
    $(33)^2 = 9.8 \times 125 \times \tan\theta$
    From which $\theta = 41.6°$
    (c) From $v^2 = gr \tan\theta$
    $(42.6)^2 = 9.8 \times 75 \times \tan\theta$
    From which $\theta = 68°$

19 Newton, Circular Motion and Orbital Speed

1. (a) 1 : 1 : 1 (Orbital speed is independent of the mass of the moons.)
   (b) 36 : 9 : 4
2. (a) 1 : 4 : 9
   (b) 1 : 4 : 9
3. 7108.6 m s^{-1} = 25 591 km h^{-1}
4. 203.6 km
5. 2474.4 m s^{-1} = 8908 km h^{-1}
6. 131 317 m s^{-1} = 472 740 km h^{-1}
7. 133 662 km
8. 29 671 m s^{-1} = 106 818 km h^{-1}
9. 29 747 m s^{-1} = 107 089 km h^{-1}