Department of
Education

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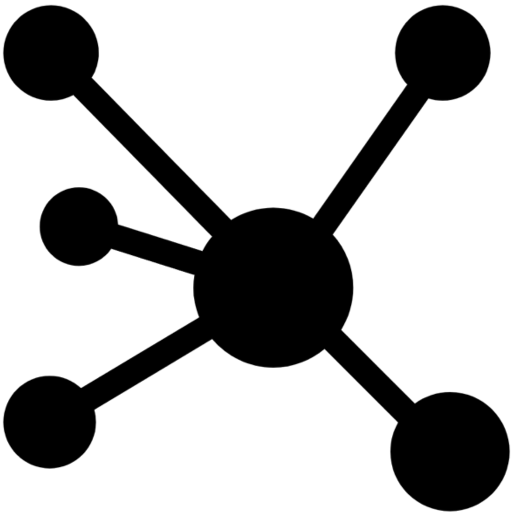
Year 12 ATAR Physics

Unit 3: Equilibrium, Torque and Stability



Gravity and motion

Physics ATAR Year 12 Unit 3

Equilibrium, Torque and Stability

Syllabus Points Covered

When an object experiences a net force at a distance from a pivot and at an angle to the lever arm, it will experience a torque or moment about that point.

This includes applying the relationship

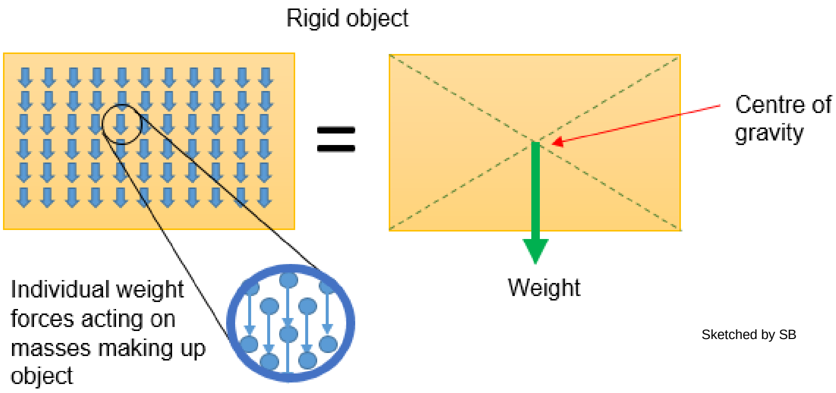
$$\tau = r\,F\sin\theta$$

Centre of mass

The **centre of mass** (COM) is a point in or near an object where the average mass is and can therefore represent a point where all of the mass can be thought of to be.

In practice it must be the balance point of the object. If all of the mass were concentrated at this centre of mass, then the object would behave in the same way.

The **centre of gravity** (COG) is a point on the object where the average weight acts.



Sketched by SB

Once the centre of gravity is known, then all of the weight of the object can be thought of as acting at or from this one point which greatly simplifies calculations.

For all calculations, assume that COM and COG are the same point.

Note that the COG can be outside the body of an object.

Image from SIDE

Stability

An object is stable if it will not topple over. When COM is over the base it is stable. If disturbed, the COM is lifted but it returns to its original position.

If the COM passes outside the base of an object, it topples.

The closer the centre of mass of an object is to the ground and the larger the base, the more stable the object is.

Greatest stability comes from low centre of mass and large area (base) over which the centre of mass acts. A large force is then required to move the centre of mass beyond the base.

The Formula One racing car has a low COG and a wide base which means it is very stable when going around corners.

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Types of stability

Stable: Stable position with centre of mass over the base. Slight movement has no overall effect - centre of mass may be raised but object returns to its original position

Unstable: Centre of mass is over the pivot. Slight movement causes object to move resulting in a new, often lower, centre of mass.

Neutral: Centre of mass is over the pivot. Slight movement causes object to move to a new position although the centre of mass usually remains at the same height.

In the space below draw an image to show the different types of stability.

Stable	Unstable	Neutral

Torque (or moment of a force)

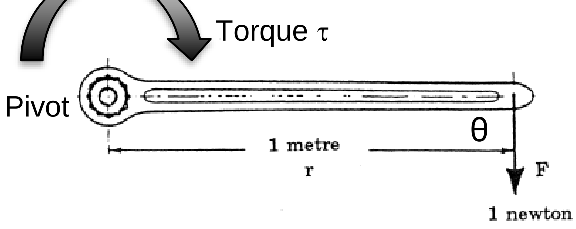
A *torque* or *moment of a force* is a turning effect created when a linear force is applied to a lever arm with a pivot and the object rotates.

$\tau = F \times r \times \sin\theta$ where τ = torque or moment in Nm

F = applied force at 90^0 in N

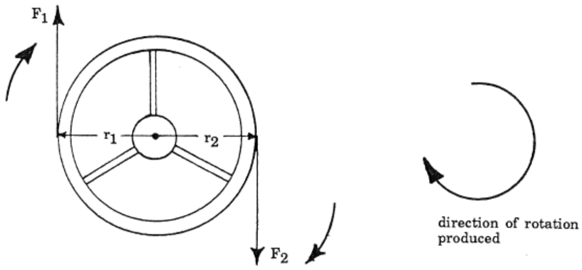
r = distance of point of application of force from pivot point

θ = angle between lever and force.



When F and r are perpendicular, maximum torque occurs. $\tau = F \times r$

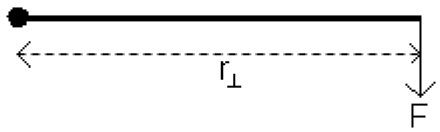
Applying a torque to a steering wheel will cause it to turn. Two equal forces applied at different points cause the rotation about a pivot point. This is referred to as a <u>couple</u>.



Example: You pick up a hammer that has a mass of 1.25 kg. You are holding it 15.0 cm from the centre of mass. What is the value of the torque or moment of force you are applying to keep the hammer horizontal?

F = mg
= 1.25 x 9.8
= 12.25 N
r = 0.150 m

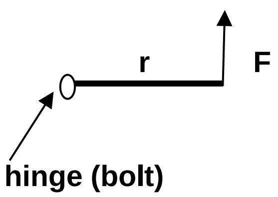
Always draw a diagram



$\tau = Fr$
= 12.25 x 0.150
= 1.8375
<u>τ = 1.84 Nm</u>

When calculating a torque, you need to multiply the force by the **perpendicular** distance.

This is straight forward if the force and lever at right angles.



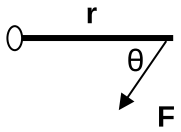
$\tau = F \times r$

Note: in the above situation the spanner is applying an anticlockwise torque to shift the nut. The nut is resisting this torque and applies a clockwise torque. If the anticlockwise torque is large enough, the nut will shift.

Diagrams on this page are from the School of Isolated and Distance Education

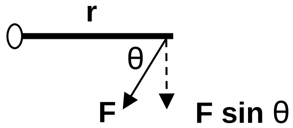
However, the force and the lever are not always at right angle to each other. If this is the case, there are a number of ways to calculate the torque.

- Measure the angle between the force and the lever.



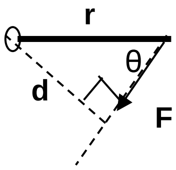
τ = F x r x sin θ

- Find a component of the force that is perpendicular to the lever.



τ = F x r
τ = F sin θ x r
τ = F x r x sin θ

- Extend the force line until a perpendicular line can be drawn to the hinge.



τ = F x d **d = r sin θ**
τ = F x r x sin θ

Exercise 1: Torques

Below are some questions to assist your understanding. Complete Exercise 1 and check your answers at the end of the book.

1. Jasmine, who has a mass of 35.0 kg, is sitting on a uniform seesaw which is 5.00 m long and pivoted in the middle. She is sitting 0.545 m from one end. What torque does she produce?
2. A 24.0 kg pelican is sitting at the end of a uniform horizontal support pole for a street lamp. The support pole is 0.750 m long and has a mass of 10.0 kg. Calculate the torque on the pivot point of the street lamp.
3. A mechanic applied a force to the end of a wrench in an attempt to undo the nut. The effective length of the wrench is 17 cm and he applies a force at an angle of 25° to the wrench. If the nut requires a torque of 18 Nm to move it, how much force will the mechanic need to apply?
4. How can the task of shifting the nut be made easier?

For a rigid body to be in equilibrium, the sum of the forces and the sum of the moments must be zero

This includes applying the relationships

$\sum F = 0$, $\tau = r F \sin\theta$, $\sum \tau = 0$

Diagrams on this page are from the School of Isolated and Distance Education

Conditions for Equilibrium

For an object to be in equilibrium, it must either be at rest or moving at constant velocity.

This means that all of the forces acting on it must be balanced.

All the forces acting downwards on the object must equal all of the forces acting upwards.
Similarly, all the forces acting left must equal all the forces acting right.
The sum of all the forces add to zero – $\Sigma F = 0$.
Net acceleration $a_{net} = 0$.

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The bike in the diagram above is in equilibrium.

The girl provides a downward force due to her weight.
The bike provides an upward force on the girl.

The girl and the bike provide a downward force on the ground.
The ground provides a force on the bike to support the weight.

$\Sigma F_{down} = \Sigma F_{up}$

If a man rides a bike at constant velocity, $\Sigma F = 0$ and $a_{net} = 0$.

The weight of the man and the bike must equal the reaction force from the ground.
Also, force provided by the man to move the bike forward must equal the forces of friction (from the ground, the air and the bike parts moving).

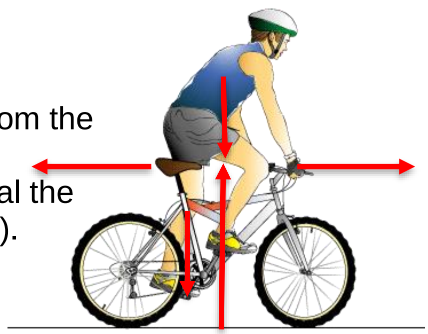


Image from SIDE

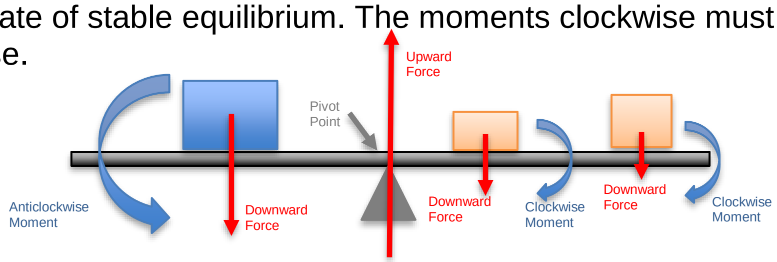
$\Sigma F_{down} = \Sigma F_{up}$
$\Sigma F_{left} = \Sigma F_{right}$

This is also true for objects that could rotate.

Consider the turning situation is a seesaw. The point about which the turning occurs is called the **fulcrum** or **pivot**.

The magnitude of the weights (**force**) placed either side of the fulcrum as well as where the weights are placed (**perpendicular distance of force**) can cause an imbalance.

When a turning forces are in balance e.g. in a seesaw balanced horizontally with masses on either side, it is in a state of stable equilibrium. The moments clockwise must equal the moments anticlockwise.



Sketched by SB

At this point, two conditions exist. The sum of the forces equals zero and the sum of the moments equals zero.

$$\Sigma M = 0 \quad \text{or } \Sigma\tau = 0$$

sum of the clockwise moments = sum of anti-clockwise moments

$$\Sigma CWM = \Sigma ACWM$$

$$\Sigma F = 0$$

sum of forces up = sum of forces down

$$\Sigma F_{up} = \Sigma F_{down}$$

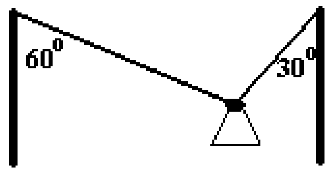
sum of forces left = sum of forces right

$$\Sigma F_{left} = \Sigma F_{right}$$

Equilibrium of Vectors – Resolving Vector Forces

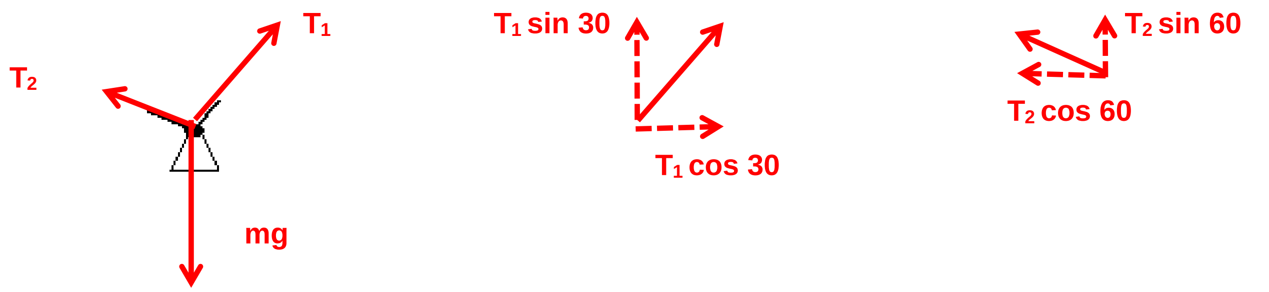
Some equilibrium problems can be solved by having an understanding of resolving vector forces.

Example: A street lamp weighing 50 kg, hangs from two poles as shown below. Calculate the tension in each wire.



The object is not moving. So, all the force up = all the forces down AND all the forces left = all the forces right. It is in **EQUILIBRIUM**.

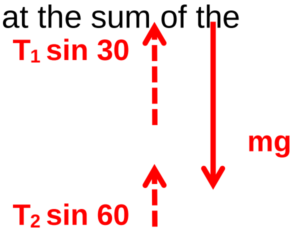
1. Resolve the forces into horizontal and vertical components.



2. Compare the horizontal forces. As the lamp is not moving, we know that the sum of all the horizontal forces = 0 ($\Sigma F_X = 0$). or forces left = forces right

$$T1\ \cos 30^0 = T2\ \cos 60^0$$
$$0.866\ T1 = 0.5\ T2$$
therefore $$T1 = 0.5774\ T2 \rightarrow \textit{equation 1}$$

3. Compare the vertical forces. As the lamp is not moving, we know that the sum of the vertical forces = 0 ($\Sigma F_Y = 0$) or forces up = forces down



$$T1\ \sin 30^0 + T2\ \sin 60^0 = 490$$
$$0.5\ T1 + 0.866\ T2 = 490 \rightarrow \textit{equation 2}$$

substitute equation (1) into equation (2)

$$0.5\ (0.5774\ T2) + 0.866\ T2 = 490$$
$$0.2887\ T2 + 0.866\ T2 = 490$$
$$1.1547\ T2 = 490$$
$$T2 = 424\ \text{N}$$

Diagrams on this page are from the School of Isolated and Distance Education

4. Substitute for T2 back into equation (1)

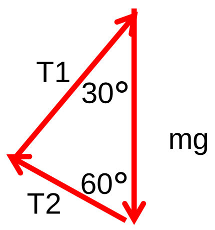
$$\begin{aligned} T1 &= 0.5774\ T2 \\ &= 0.5774 \times 424 \\ &= 245\ \text{N} \end{aligned}$$

5. Write down answer: <u>T2 = 424 N</u>

<u>T1 = 245 N</u>

Another method is to draw a <u>triangle of forces</u>.
The three forces create a closed triangle.
This indicates that all the forces are balanced and the net force is zero.

6. Use cos rule or sin rule to find T1 and T2.



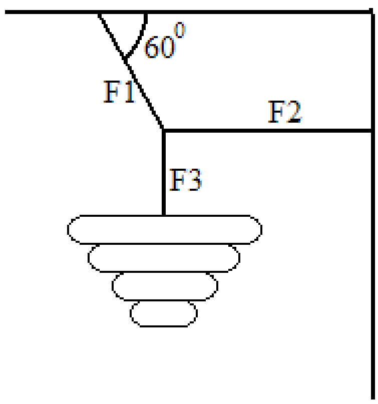
$$\frac{T1}{\sin 60} = \frac{mg}{\sin 90} \qquad T2 = 424\ \text{N}$$

$$\frac{T2}{\sin 30} = \frac{mg}{\sin 90} \qquad T1 = 245\ \text{N}$$

Exercise 2: Equilibrium of forces

Below are some questions to assist your understanding. Complete Exercise 2 and check your answers at the end of the book.

1. A chandelier is hanging from a ceiling as shown. The mass of the chandelier is 200 kg. Find the forces F1, F2 and F3.



2. A lamp of mass 4.80kg is suspended over the centre of a 15.0m wide road by means of two wires running to the tops of two 7.00m high poles on either side of the road. If the lamp is suspended 6.50m above the road, calculate the tensions in the wires.

Diagrams on this page are from the School of Isolated and Distance Education

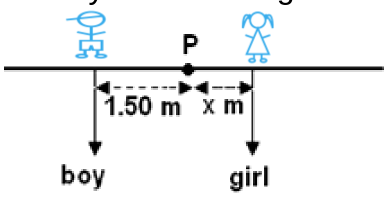
Equilibriums involving torques

When problems involve moments, for equilibrium to be established, the moments clockwise must balance the moments anticlockwise.

In addition, the forces must be balanced.

Example: A boy of mass 45.0 kg sits 1.50 m from the pivot point of a seesaw on the right hand side. Where must a girl of mass 40.0 kg sit in order for the seesaw to balance?

Always draw a diagram



BOY $F = ma$
$F = 45 \times 9.8$
$F = 441$ N
$r_\perp = 1.5$ m
GIRL $F = 40 \times 9.8$
$F = 392$N
$r_\perp = ?$

take moments about P
for balance $\Sigma M = 0$
therefore $\Sigma CM = \Sigma ACM$
$Fr_\perp$ (boy) $= Fr_\perp$ (girl)
$441 \times 1.5 = 392 \times r$ (girl)
r (girl) $= \dfrac{441 \times 1.5}{392}$
r (girl) $= 1.69$ m

Girl is sitting 1.69 m from the left of the centre (fulcrum)

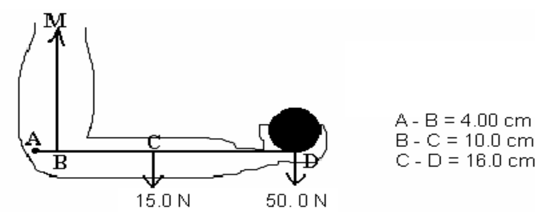
Always state where you are taking the moments from.

Exercise 3: Equilibrium of moments

Below are some questions to assist your understanding. Complete Exercise 3 and check your answers at the end of the book.

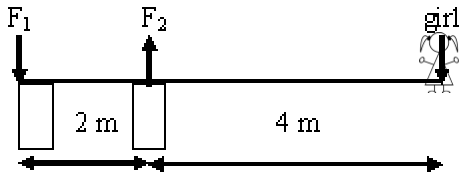
1. A man weighing 840 N sits on a seesaw 1.50 m from the pivot. His son and daughter sit on the other side and balance it. The girl weighs 4.20×10^2 N and is 0.900 m from the pivot. What is the weight of the boy who sits 1.80 m from the pivot?

2. A man holds a 50.0 N ball in his arm as shown below. The pivot point is the elbow and the forearm weighs 15.0 N. Calculate the force the muscles (M) must apply for the arm to hold the ball horizontal.



Diagrams on this page are from the School of Isolated and Distance Education with the exception of the male and female figures in example. Images sourced from Clker-Free-Vector-Images from Pixabay

3. What are the forces F_1 and F_2 that the supports exert on the diving board shown in the diagram below, when a 45.0 kg girl stands at the end of the board, if the mass of the board is 25.0 kg?



◈ *Forces are vector quantities so you should include a direction.*

4. A uniform metre rule of mass 0.800 kg has a mass of 0.200 kg suspended at the 90.0 cm mark. At what point must the rule be supported in order to balance?

A suggested approach to problem solving

Marks are awarded for working and if you go through this process you can often get more than 50% of the marks, even if you cannot solve the equations, perhaps even if you get the equations wrong. You <u>must always show all working</u>.

1. Read question carefully and form a mental picture of the situation then identify the object that is in equilibrium including its pivot point.
2. Draw a large clear fully labelled diagram. Use a ruler and pencil.
3. Identify the forces acting on the object that is in equilibrium and put them in the diagram in correct position.
4. Identify the unknown forces and angles required by the question and mark them in the diagram. Give them a symbol.
5. Check that you have all the forces correctly shown. Show only forces acting on the object in question.
6. Take moments about the point of action of the force with unknown direction (if applicable).
7. Equate the components of the forces horizontally and vertically to zero. $\Sigma F_H = 0$ $\Sigma F_V = 0$
8. Solve for the unknowns. Be careful with the mathematics.
9. Give direction of vector. Check you have the correct number of significant figures & the correct units.

Equilibrium involving moments and forces

For a body to be in equilibrium (not moving at all),

Σ**M** = **0** (sum of moments equals zero) and Σ**F** = **0** (sum of forces equals zero).

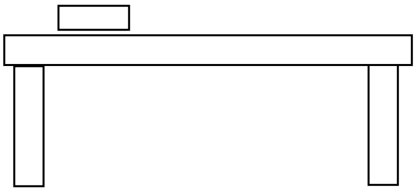
These are known as the <u>conditions for equilibrium</u>.

Diagrams on this page are from the School of Isolated and Distance Education

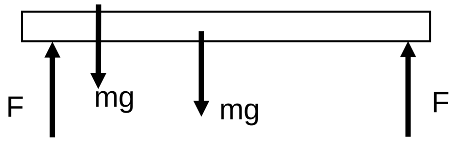
FORCES AT RIGHT ANGLES

Bridges

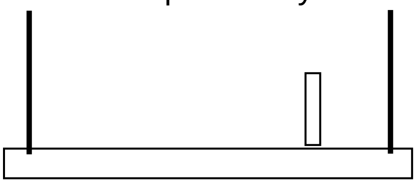
Bridge problems are those that have a beam supported by two pillars at each end. Masses are supported by the beam. To solve, use $\Sigma F_{up} = \Sigma F_{down}$.

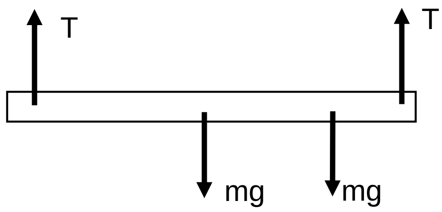


- Free body diagram



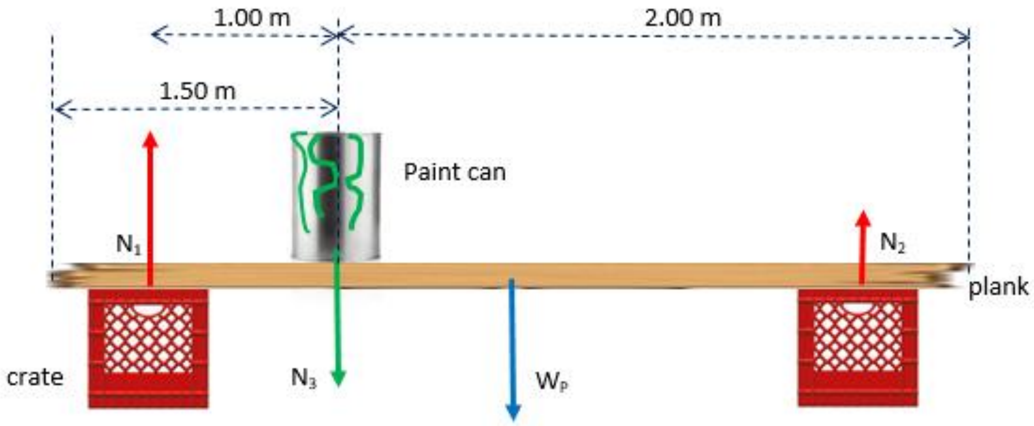
Platforms suspended by cables are similar.





Diagrams are from the School of Isolated and Distance Education

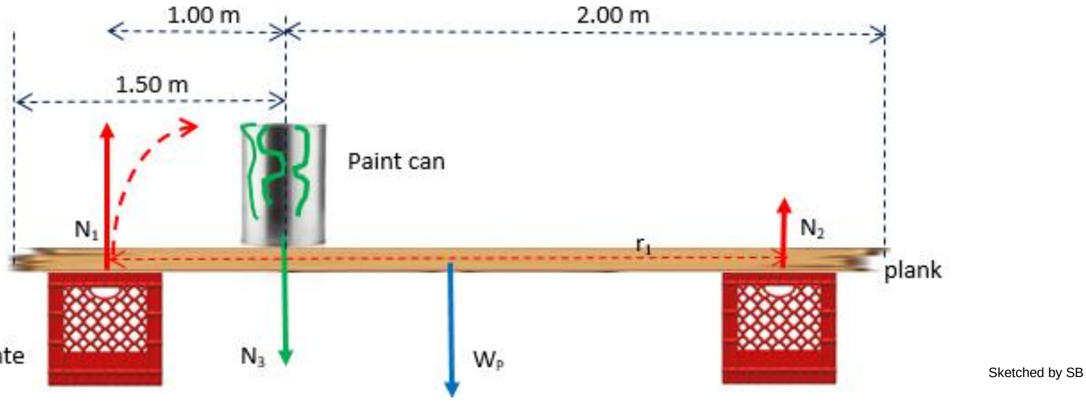
Example: A 4.00 m long uniform plank of wood (mass 25.0 kg) is placed on top of two milk crates which act as support on some level ground. A 15.0 kg can of paint is placed onto the plank, 1.50 m from one end. If the centres of the two crates are 1.00 m and 2.00 m from the centre of the paint can, calculate the normal force N_1 due to the crate acting on the plank.



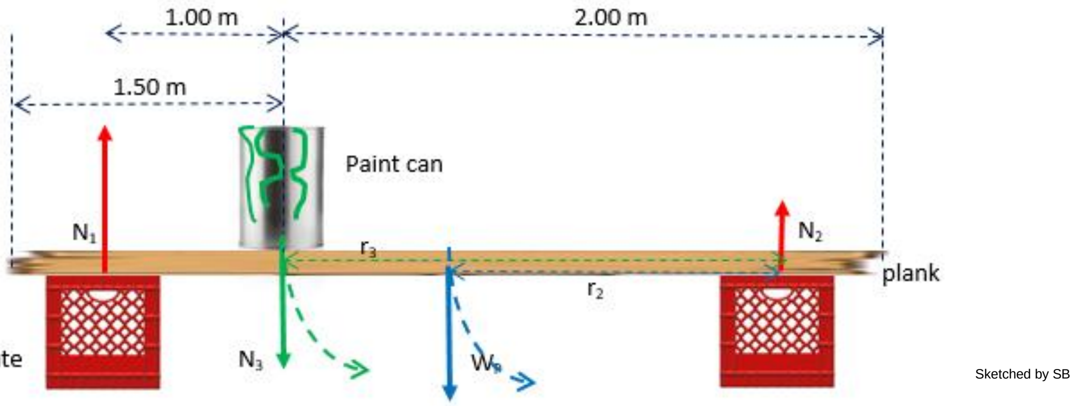
Sketched by SB

The plank is in equilibrium. $\Sigma M = 0$ so $\Sigma CWM = \Sigma ACWM$.
$\Sigma F = 0$ so $\Sigma F_{up} = \Sigma F_{down}$ and $\Sigma F_{left} = \Sigma F_{right}$

Take moment or torques about the point where the force N_2 acts. This choice eliminates an unknown variable, N_2.
The clockwise torque = $N_1 \times r_1$



The anticlockwise torque = $N_3 \times r_3 + W_p \times r_2$



$\Sigma CWM = \Sigma ACWM$

$N_1 \times r_1 = N_3 \times r_3 + W_p \times r_2$

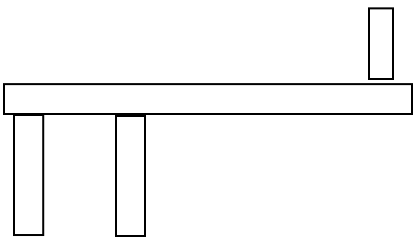
$$N_1 = \frac{N_3 \times r_3 + W_p \times r_2}{r_1}$$

$$= \frac{15.0 \times 9.80 \times 2.00 + 25 \times 9.80 \times 1.50}{3.00}$$

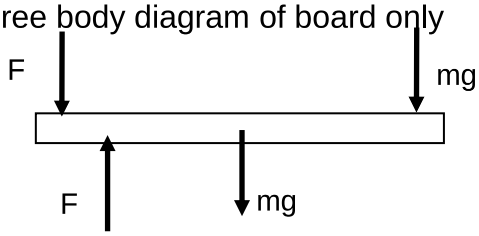
$= 220.5\ N \qquad = 2.21 \times 10^2\ N$

Diving boards

These problems usually involve a board supported at two point, supporting a weight that is beyond one of the support points. To solve, use $\Sigma F_{up} = \Sigma F_{down}$ and $\Sigma CWM = \Sigma ACWM$



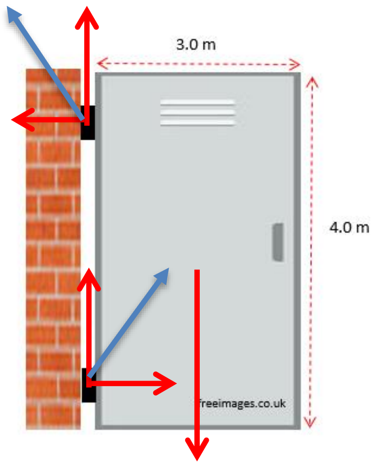
Free body diagram of board only



Diagrams are from the School of Isolated and Distance Education

Doors

A door is normally supported by two hinges that each supply the blue forces shown.



Sketched by SB

Since $\Sigma F_{left} = \Sigma F_{right}$, the horizontal components of the hinge forces must be equal in magnitude.

Since $\Sigma F_{up} = \Sigma F_{down}$ the vertical components added together must equal the weight of the door.

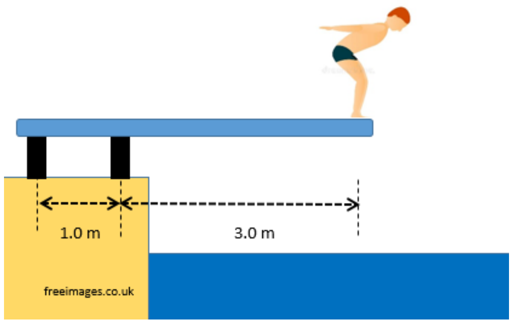
Also, ΣCWM = ΣACWM by taking moments about either A or B.

Exercise 4: Equilibrium of forces and moments

Below are some questions to assist your understanding. Complete Exercise 4 and check your answers at the end of the book.

1. A 20.0 m long bridge is supported at each end by pillars. The bridge has a mass of 3.52 x 10^4 kg. On the bridge is a truck which has a mass of 2.60 x 10^3 kg and a car which has mass of 985 kg. The truck is 4.00 m from one end and the car is 5.00 m behind the truck. Calculate the force on each pillar.

2. Calculate the forces F_A and F_B that the supports exert on the diving board when a 60 kg person stands on its tip. The board has a mass of 15 kg



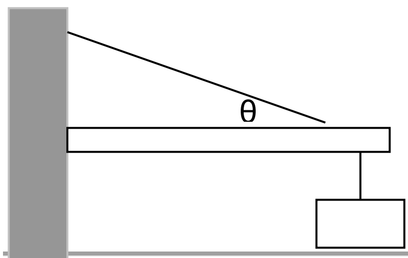
Sketched by SB

3. A door of mass 100 kg mass is supported by two hinges, A and B, on a wall of size 4 m x 3 m.
   Draw the forces on the diagram.
   The force on each hinge is the same. Find the force on each hinge.

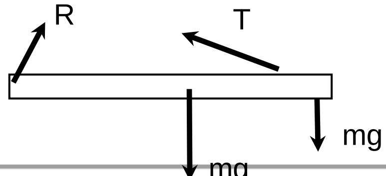
FORCES NOT AT RIGHT ANGLES

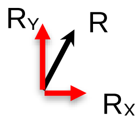
Signs

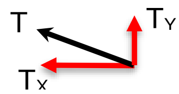
Sign problems are those that have a beam supported by a cable attached to a wall. Masses are supported by the beam. To solve, find a component of the tension in the cable, then use ΣCWM = ΣACWM and $\Sigma F_{up} = \Sigma F_{down}$.



Free body diagram of beam only







Diagrams are from the School of Isolated and Distance Education

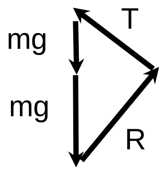
In order to find the reaction force from the wall, R, there are a number of ways to do this.

$R_Y + T_Y = mg + mg$ and $R_X = T_X$

By taking moments about R, this will eliminate R.

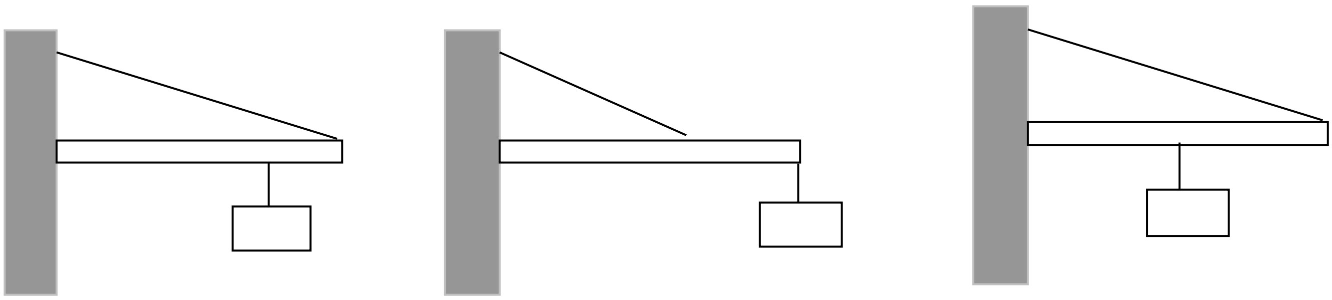
The clockwise moments will equal mgr + mgr.

The anticlockwise moment will equal $T_Y.r$, which is the same as $T\sin\theta.r$



Or, a triangle of forces could be drawn to solve for R.

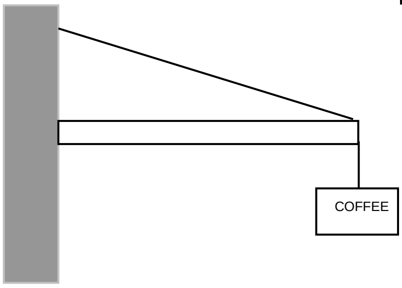
There are a number of ways that sign problems can be given.



Exercise 5: Equilibrium of signs

Below are some questions to assist your understanding. Complete Exercise 5 and check your answers at the end of the book.

1. The diagram below shows a sign hanging in front of a shop. If the sign weighs 2.40 x 10^2 N and the angle the rope makes with the bar is 55^0 (angle near the coffee sign), how much tension is in the rope? The bar has a mass of 5.00 kg and is 0.800 m long.



When dealing with angles in moments, students sometimes forget to multiply by the distance. For example, in the example above, the anticlockwise moment is (T sin 55 x 0.80)

Diagrams on this page are from the School of Isolated and Distance Education

2. A street lamp of mass 2.25 kg is suspended above the roadway from one end of a horizontal rod, 1.80m long, which is fastened at its other end to a vertical post. The mass of the rod is 3.00 kg. It and the lamp are partly supported by a light cable, attached 50.0 cm from the lamp and making an angle of 60.0^0 with the vertical post to which the other end is fastened.

   Calculate the tension in the cable and the direction and magnitude of the force the post exerts on the rod.

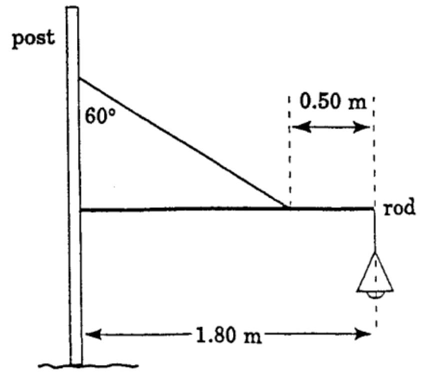


Diagram from the
School of Isolated and Distance Education

Ladders

The ladder is fixed into the ground so it cannot slide.

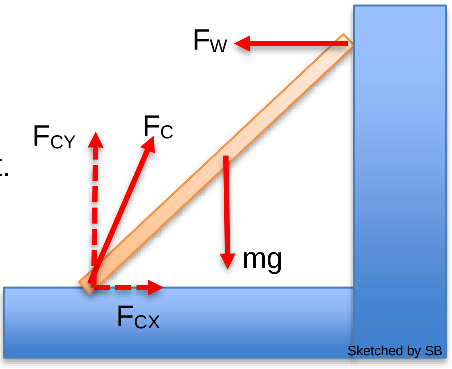
F_c in the diagram is the force provided to the foot of the ladder.
It does not necessarily go at the same angle as the ladder.

The wall is usually smooth.
This means that F_w from the wall only has a horizontal component.

The ladder has a mass, usually in the centre of the ladder.
There are sometimes people standing on the ladder.

In this example, $F_w = F_{cx}$ and $F_{cy} = mg$.

Taking moments about F_c can be used to solve the forces.



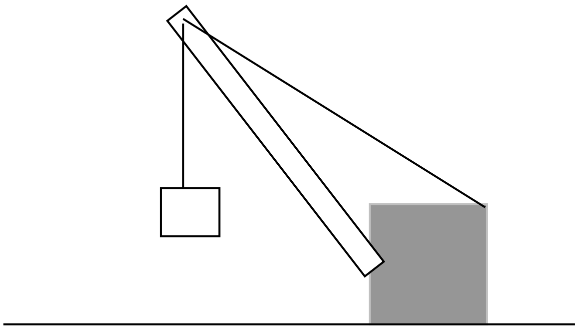
Exercise 6: Ladder

Below are some questions to assist your understanding. Complete Exercise 6 and check your answers at the end of the book.

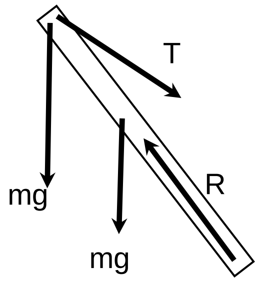
A 5.0 m long ladder leans against a wall at a point 4.0 m above a cement floor. The ladder is uniform and has a mass of 12.0 kg. Assuming the wall is frictionless (but the floor is not), determine the forces exerted on the ladder by the wall and the floor.

Boom crane

Here, the cable is under tension (it pulls inward) and the boom is under compression (it pushes outward). At the top of the boom, forces here must be at equilibrium. To solve, find all the components of the forces, then use $\Sigma F_{up} = \Sigma F_{down}$.



Free body diagram of boom only



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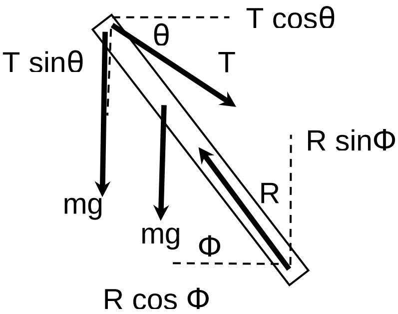
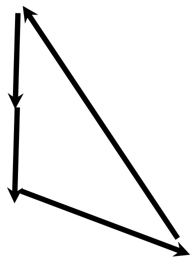


$$\Sigma F_{up} = \Sigma F_{down}$$
$$T \sin\theta + mg + mg = R \sin\phi$$

$$\Sigma F_{left} = \Sigma F_{right}$$
$$R \cos\phi = T \cos\theta$$

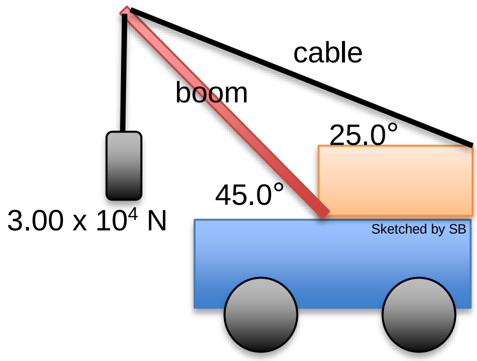
or



Diagrams are from the School of Isolated and Distance Education

Exercise 7: Crane

Find the tension in the cable and the compression in the boom. Assume the mass of the boom is negligible compare to the load carried.

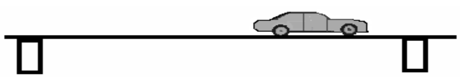


Exercise 8: Miscellaneous

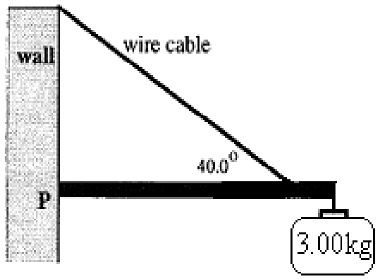
Below are some questions to assist your understanding. Complete Exercise 8 and check your answers at the end of the book.

1. If you stand with your left side and left foot against the wall you will fall over if you lift your right foot. Explain, using the concept of torque then estimate the amount of torque causing you to fall over in the situation described above.

2. A long bridge of mass 1.00×10^4 kg is supported by two large columns as shown. The centre of mass of each column is 24.0 m apart. If a 9.00×10^2 kg car is two-thirds of the way across the bridge, what is the force each column is supporting?



3. The diagram shows a wall bracket used to hang a sign from a wall. The dimensions of the wall bracket are as follows:
   Mass beam = 1.50 kg
   Length beam = 1.00 m
   Wire attached 0.750 m along beam
   Find:
   a. The tension in the wire cable.
   b. The compression in the beam.
   c. The reaction at the wall, point P.



4. A lawn roller of mass 2.00×10^2 kg and radius 40.0 cm is being pulled over a step as shown. The step is 8.00 cm high and the handle is horizontal.
   a. What force is required to cause the roller to begin moving
   b. At what angle to the horizontal will the least force be needed to pull the roller over the step. Justify your answer with appropriate calculations.

Diagrams are from the School of Isolated and Distance Education

Below are some additional questions to assist your understanding. Complete the questions and check your answers at the end of the book.

Equilibrium

1. A person holds a rope which hangs vertically and supports a mass of 6.00 kg.
   (a) What is the tension in the rope?
   (b) What force is exerted by the man on the rope?

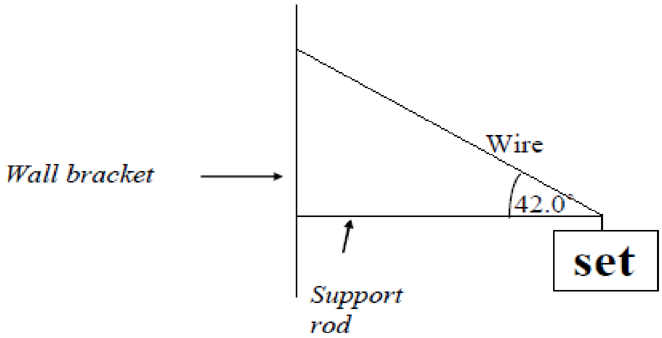
2. A weight is suspended by two strings, each making an angle of 30.0˚ with the vertical. If the tension in each string is 3.90 N, what was the weight?

3. Find the force that must be applied upwards parallel to the plane in order to hold a mass of 142 g at rest on a smooth plane inclined at 24.0˚ to the horizontal.

4. A 2.4×10^2 N vehicle is to be pulled up a 30.0˚ incline at constant speed. How great a force parallel to the incline is needed if friction effects can be neglected?

5. Find the horizontal force required to keep a small wooden block of mass 280 g at rest on a smooth plane inclined at 30.0˚ to the horizontal.

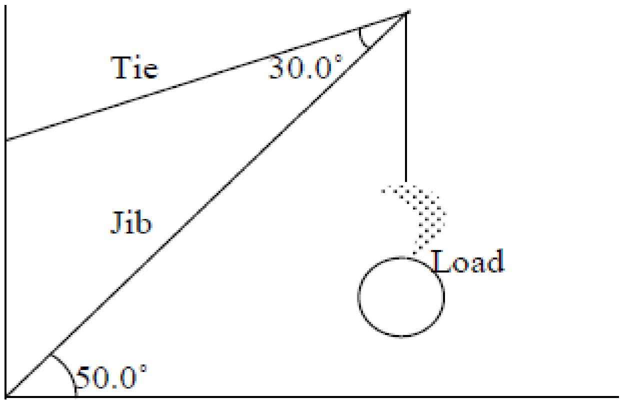
6. A sign weighing 425 N is supported by a wire at an angle of 42.0˚ as shown in the diagram. Determine the magnitudes and the directions of the forces exerted by the wire and by the bracket. Assume that the wall provides a perpendicular reaction force on the rod.



7. A large vertical pole has three light cables connected to it running in the south, east and north-west directions. The pole is kept vertical only by these three cables, which are horizontal. The tension in the north-west cable is 2.800×10^3 N.
   (a) Find the tensions in the other two cables.
   (b) If the south cable breaks, in what direction will the pole begin to fall?

8. A uniform drawbridge of mass 2.4×10^3 kg is 7.2 m long. It is pivoted at one end at the bottom of a wall, and it can be raised by a chain which is connected to the drawbridge 4.8 m from the pivot and to the wall 4.8 m above the pivot. When the tension in the chain is just sufficient to start raising the drawbridge from the horizontal position, calculate:
   (a) the tension in the chain
   (b) the total force exerted on the pivot.

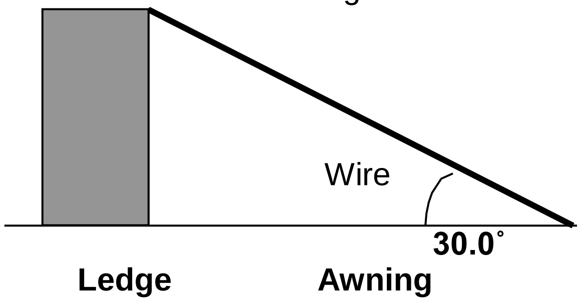
9. The diagram represents a jib crane. What are the forces in the jib and tie when the load is 8.00×10^2 kg? (let the length of the Jib be L)



Diagrams from the School of Isolated and Distance Education

10. A uniform veranda awning weighing 1.00×10^2 N is supported as shown in the diagram. Find:

    (a) the tension in the wire
    (b) the reaction at the ledge corner



11. A uniform ladder of length 5.66 m and mass 40 kg is resting against a smooth wall at an angle of 45.0˚. Find the magnitude and direction of the reaction of the ground on the ladder, assuming the ground is rough enough to prevent the ladder slipping.

12. A uniform ladder 1.00×10^1 m long and having a mass of 24.0 kg rests against a smooth vertical wall with its base on rough level ground. If the base of the ladder is 8.0 m from the wall, find the reaction forces exerted on the ladder by

    (a) the wall, and
    (b) the ground

13. Engineers design racing cars with a low centre of mass and widely spaced wheels to improve their high speed cornering ability. Explain, using moments, the physics on which they base this design.

14. By inverting your body to a handstand position your centre of mass moves much closer to the ground. A lower centre of mass should give you more stability. Why is it therefore more difficult to remain in a handstand position than to stand upright as normal?

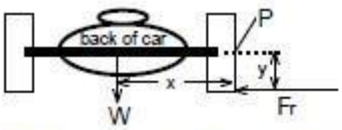
15. Explain why you cannot avoid swaying from side to side when you try to walk along a straight line with your arms folded

Diagram from the School of Isolated and Distance Education

Answers:

1. (a) 58.8 N (b) 58.8 N
2. 6.75 N
3. 0.57 N
4. 1.2×10^2 N
5. 1.6 N
6. 635 N, 472 N
7. (a) 1.980×10^3N (b) north
8. (a) 2.5×10^4 N
   (b) 1.9×10^4 N at 18.4˚ above the horizontal (to the bridge)
9. Jib 1.47×10^4 N, Tie 1.01×10^4 N
10. (a) 1.00×10^2 N
    (b) 1.00×10^2 N in direction 30.0˚ above the horizontal
11. 196 N at 63.5˚ to ground
12. (a) 1.6×10^2 N (b) 2.8×10^2 N at 56.2˚ above horizontal
13.

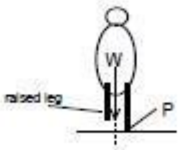
Car is moving away from viewer



When the car is turning to the left there is a large frictional force applied at the base of the right hand wheel. Fr creates a clockwise torque about P tending to roll the car. The weight force creates a stabilising anticlockwise torque about P. While cornering we want the anticlockwise torque to remain larger than the clockwise torque. This is achieved by reducing the moment arm y and increasing the moment arm x. This is done by lowering the centre of mass of the car and increasing the wheel base.

14. Besides your arms not having the muscle strength of your legs, your hands on the ground have limited ability to counteract unbalanced torques about the end of your forearm. When standing your weight force on your foot acts forward of your heel and the tension in the calcanean tendon adjusts to counteract unbalanced torques produced as your centre of mass moves slightly forwards or backwards.

15.



When standing still your weight is distributed evenly over both of your feet as your weight force acts down through a vertical line somewhere between each foot. As you take a step and you raise your leg to move it forward your weight force creates an unbalanced turning moment about your other foot at P. To stop this effect we lean to one side and transfer our weight momentarily over just one foot. When we do this the line of action of our weight force acts through our grounded foot and does not produce an unbalanced moment. The reverse happens as we take the next step causing us to sway to the other side.

Diagrams on this page are from the School of Isolated and Distance Education

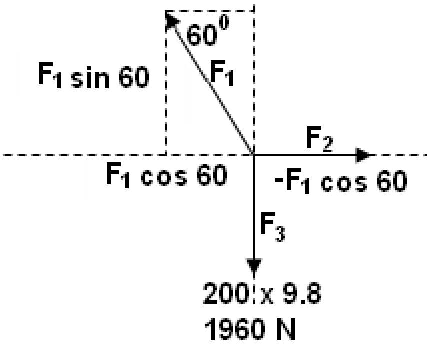
Answers to exercises

Exercise 1: Torques

1. **F = mg** **τ = Fr**
   = 35 x 9.8 **= 343 x 1.955**
   = 343 N **τ = 670.565 Nm** **<u>τ = 671 Nm</u>**
2. **here the pole has two forces, bird and weight of pole.**
   $\tau = (Fr_\perp)_{\text{bird}} + (Fr_\perp)_{\text{pole}}$
   = (24 x 9.8 x 0.75) + (10 x 9.8 x 0.375)
   = 176.4 + 36.75
   = 213.15 **<u>τ = 213 Nm</u>**
3. $\tau = F\,r\sin\theta$
   $F = \dfrac{\tau}{r\sin\theta} = \dfrac{18}{0.17 \times \sin 25} = 251\ \text{N}$
4. **Apply more force**
   Use a longer handle – attach a piece of pipe.
   Lower the friction on the nut (CRC) – lower the CW torque.

Exercise 2: Equilibrium of forces

1.



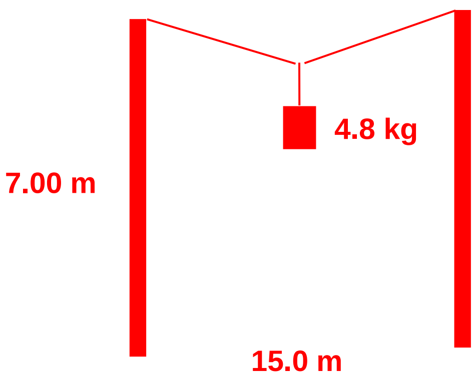
Vertical forces $\Sigma F_Y = 0$

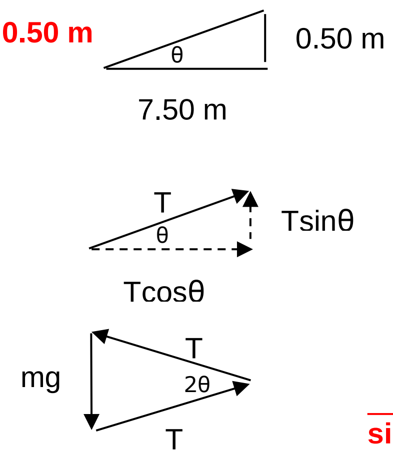
F_1 sin 60 = 1960 9.8 **F_3 = weight = 200 x**
F_1 = 1960 ÷ sin 60 **<u>F_3 = 1.96 x 10³ N down</u>**
F_1 = 2263 N
<u>F_1 = 2.26 x 10³ N 60⁰ to horizontal</u>

Horizontal forces $\Sigma F_X = 0$

F_2 = F_1 cos 60
= 2263 x 0.5
= 1132 N **<u>F_2 = 1.13 x 10³ N right</u>**

2.





$\tan\theta = \dfrac{0.50}{7.50}$ $\theta = 3.80^\circ$

$\Sigma F_Y = 0$
$2T\sin\theta = 4.8 \times 9.8$
$T = 355\ \text{N}$

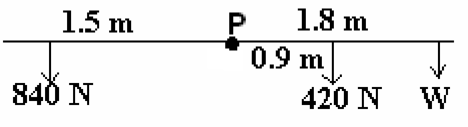
$\dfrac{T}{\sin(90 - \theta)} = \dfrac{mg}{\sin 2\theta}$

$T = \dfrac{4.80 \times 9.8 \times \sin 86.2}{\sin 7.6} = 355\ \text{N}$

Diagrams on this page are from the School of Isolated and Distance Education

Exercise 3: Equilibrium of moments

1.

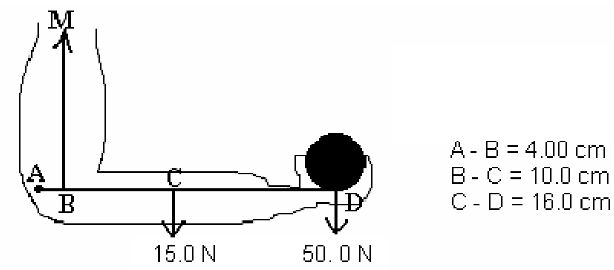


take moments about P

$$\Sigma M = 0$$
$$\Sigma CM = \Sigma ACM$$
$$(420 \times 0.9) + (W \times 1.8) = (840 \times 1.5)$$
$$378 + 1.8W = 1260$$
$$1.8\,W = 882$$
$$W = 490\text{ N}$$

<u>the weight of the boy is 490 N</u>

2.



Additional information:

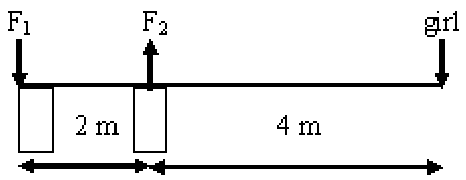
A-B = 0.04 m
B-C = 0.10 m
C-D = 0.16 m
A-C = 0.14 m
A-D = 0.30 m

Taking moments about A, $\Sigma M = 0$

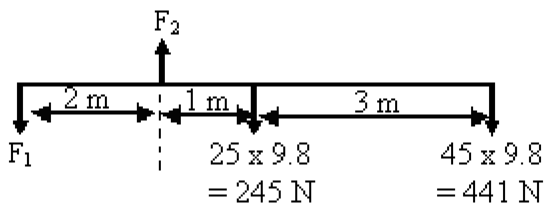
$$\Sigma CWM = \Sigma ACWM$$
$$(15 \times 0.14) + (50 \times 0.3) = (m \times 0.04)$$
$$2.1 + 15 = 0.04\,m$$
$$17.1 = 0.04\,m$$

m = 427.5 **<u>force of muscle = 428 N</u>**

3.



Redraw diagram.



take moments about F_2

$$\Sigma M = 0$$
$$\Sigma CWM = \Sigma ACWM$$
$$(245 \times 1.0) + (441 \times 4) = F_1 \times 2$$
$$245 + 1764 = 2F_1$$
$$2009 = 2F_1$$
$$F_1 = 1004.5\text{ N down}$$

Now at equilibrium, the sum of the forces is

$$F_2 = F_1 + F_w + F_{girl}$$
$$= 1004.5 + 245 + 441$$
$$F_2 = 1690.5\text{ N up}$$

so <u>$F_1 = 1.00 \times 10^3$ N down</u>

<u>$F_2 = 1.69 \times 10^3$ N up</u>

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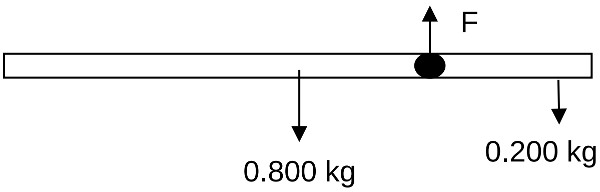
4.



$\Sigma F_Y = 0$ **F** = 0.800 **x** 9.8 + 0.200 **x** 9.8 = 9.80 **N**

take moments about end

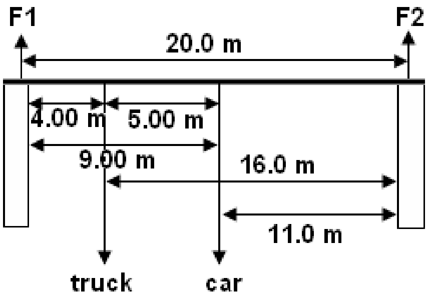
0.800 **x** 9.8 **x** 0.5 + 0.200 **x** 0.900 **x** 9.8 = **F x X**

X = 0.58 **m or** 58 **cm from end**

8.0 **cm from middle**

Exercise 4: Equilibrium of forces and moments

1.



F car = mg = 985 x 9.8 = 9653 N
F truck = 2600 x 9.8 = 25480 N
F bridge = 3.52 x 10^4 x 9.8 = 318500 N

take moments about F1
ΣCWM = ΣACWM
(25480 x 4) + (9653 x 9.00) + (344960 x 10) = F2 x 20
101920 + 86877 + 344960 = 20F2
3638397 = 20F2
F2 = 181919 N **<u>F2 = 1.82 x 10^5 N up</u>**

F up = F down
F1 + F2 = truck + car + bridge
F1 + 181919 = 9653 + 25480 + 318500
F1 + 181919 = 380093
F1 = 198174 **<u>F1 = 1.98 x 10^5 N up</u>**

2. **take moments about A**

$Fr_{girl} + Fr_{board} = F_B r$

$F_B = \dfrac{60 \times 9.8 \times 4 + 15 \times 9.8 \times 2.0}{1.0}$

$F_B = 2.6 \times 10^3$ N up

$\Sigma F_{up} = \Sigma F_{down}$

$F_B = mg + mg + F_A$

$F_A = 2646 - (60 \times 9.8 + 15 \times 9.8)$

$F_A = 1.9 \times 10^3$ N

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3.

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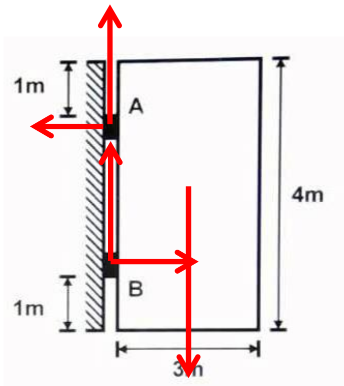
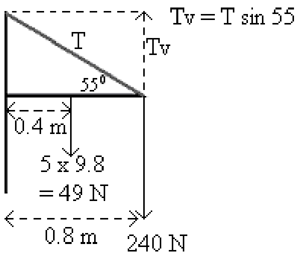


Take moments about A

$$\Sigma\ \mathrm{CWM} = \Sigma\ \mathrm{ACWM}$$
$$\mathrm{Fr} = \mathrm{Fr}$$
$$\mathrm{mgr} = F_{Bx}\ r$$
$$100 \times 9.8 \times 1.5 = F_{Bx} \times 2$$
$$F_{Bx} = 735\ \mathrm{N} = F_{Ax}$$
$$F_{up} = F_{down}$$
$$F_{Ay} + F_{By} = 100 \times 9.8$$
$$F_{Ay} = F_{By} = 490\ \mathrm{N}$$
$$F_A{}^2 = 735^2 + 490^2 \qquad F_A = F_B = 883\ \mathrm{N} \qquad \theta = 33.7°$$

Exercise 5: Equilibrium of signs

1.



take moments about pivot
ΣCWM = ΣACWM
(49 x 0.4) + (240 x 0.8) = T sin 55 x 0.8
19.6 + 192 = 0.6553T
211.6 = 0.6553T

T = 3.23 x 10² N

2.

-**take moments about fastener**
Tsin 30 **x** 1.3 = 2.25 **x** 9.8 **x** 1.8 + 3.00 **x** 9.8 **x** 0.90
T = 101 **N**
$R_x = T_x = 88.1$ **N**
$R_Y + T_Y = 3.00 \times 9.8 + 2.25 \times 9.8$
$R_Y = 0.565$ **N**
$R = \sqrt{(R_x)^2 + (R_Y)^2} = 88.1$ **N at** 0.37°

Exerise 6: Ladder

$$\Sigma F_Y = 0 \qquad mg - F_{cy} = 0 \qquad F_{cy} = mg = 12.0 \times 9.80 = 117.6\ \mathrm{N}$$

Find the angle $\sin\theta = \frac{4.0}{5.0} \qquad \theta = 53.13°$

Take moments about Fc pivot point

$$\Sigma\mathrm{CWM} = \Sigma\mathrm{ACWM}$$
$$mg \cos\theta \times r = F_w \sin\theta \times r$$
$$12.0 \times 9.8 \cos 53.13 \times 2.5 = F_w \sin 53.13 \times 5.0$$

F_w = 44.1 N

F_w = F_{cx} = 44.1 N

F_c = √(117.6² + 44.1²) = 126 N $\tan\theta = \frac{117.6}{44.1} = 69.4°$

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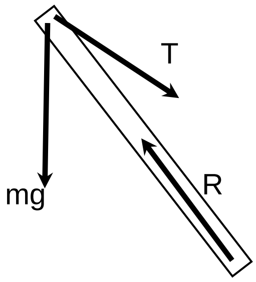
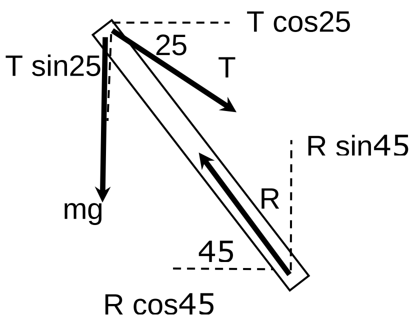
Exercise 7: Crane

Free body diagram of boom only

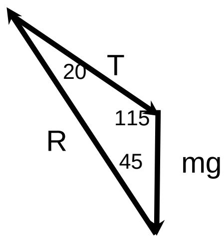




$$\Sigma F_{up} = \Sigma F_{down}$$
$$R\sin 45 = T\sin 25 + mg$$
$$R = \frac{T\sin 25 + 30000}{\sin 45}$$
$$\frac{T\sin 25 + 30000}{\sin 45} = \frac{T\cos 25}{\cos 45}$$
$$0.42262T + 30000 = 0.90631T$$
$$T = \frac{30000}{0.48369} = 62.0 \text{ kN}$$

$$\Sigma F_{left} = \Sigma F_{right}$$
$$T\cos 25 = R\cos 45$$
$$R = \frac{T\cos 25}{\cos 45}$$
$$R = 79.5 \text{ kN}$$

OR

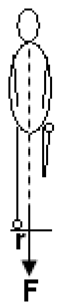


$$\frac{T}{\sin 45} = \frac{30000}{\sin 20}$$
$$T = 62.0 \text{ kN}$$

$$\frac{R}{\sin 115} = \frac{30000}{\sin 20}$$
$$R = 79.5 \text{ kN}$$

Exercise 8: Miscellaneous

1. **Stability comes when the centre of mass is within the base of the object. The centre of mass of a person is between their feet when standing upright so no torque. When you lift your right foot, your centre of mass is outside your base (your left foot) and therefore a torque is provided by your weight and you will fall over or more correctly, rotate.**



$r = 0.1$ m
$F = 60 \times 9.8$
$= 588$ N

$\tau = Fr$
$= 588 \times 0.1$
$= 58.8$

$\underline{\tau = 60 \text{ Nm}}$

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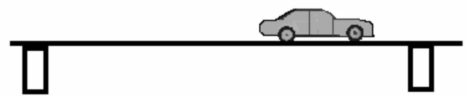
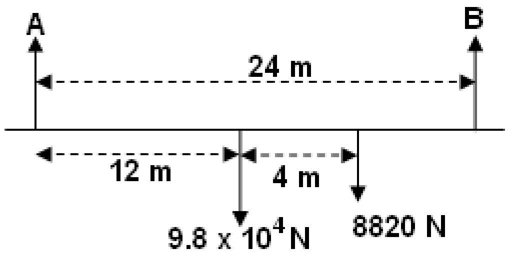
2.



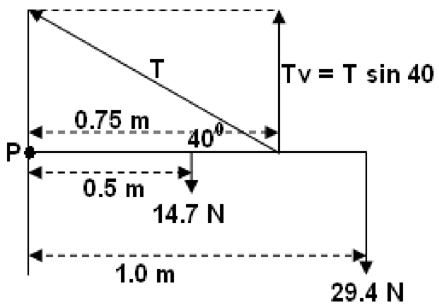


take moments about A

ΣCM = ΣACM
(9.8 x 10^4 x 12) + 8820 x 16) = B x 24
1.176 x 10^6 + 14112 = 24B
1317120 = 24B
<u>B = 5.49 x 10^4 N up</u>

F up = F down
A + 54880 = 9.8 x 10^4 + 8820
A = 9.8 x 10^4 + 8820 – 54880
<u>A = 5.19 x 10^4 N</u>

3.



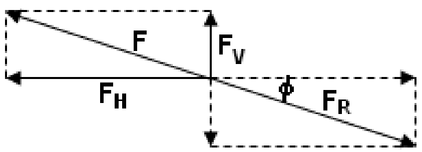
a.
Take moments about P
ΣCM = ΣACM
(14.7 x 0.5) + (29.4 x 1.0) = (T sin 40 x 0.75)
7.35 + 29.4 = 0.482T
36.75 = 0.482T
<u>T = 76.2 N</u>

b. Compression in beam is the horizontal component of tension force.

T_H = T cos 40
= 76.24 x cos 40
= 58.4 N **<u>compression = 58.4 N to the left</u>**

c. Reaction force is sum of horizontal and vertical components.

Horizontal is (b) above = 58.4 N left
Vertical: (14.7 + 29.4) down – (T sin 40) up
44.1 (down) – 48.98 (up)
4.88 N up



$F_R = \sqrt{(58.4^2 + 4.88^2)}$
= 58.6 N
$\phi = \tan^{-1}$ (4.88 ÷ 58.4)
= 4.78^0

<u>F_R = 58.6 N 4.78^0 below horizontal</u>

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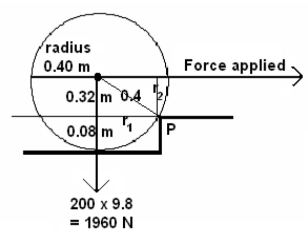
4.



$0.4^2 = r_1^2 + 0.32^2$

$r_1 = \sqrt{0.4^2 - 0.32^2)}$

$r_1 = 0.24$ m

$r_2 = 0.32$ m

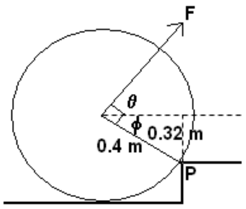
a. Take moments about P

$\Sigma CM = \Sigma ACM$

$F \times r_2 = F_w \times r_1$

$F \times 0.32 = 1960 \times 0.24$

$F = 1470$ **$\underline{F = 1.47 \times 10^3 \text{ N}}$**



the least amount of force will be when the applied force is perpendicular to the radius of the roller.

$\phi = \sin^{-1}(0.32 \div 0.40)$

$= 53.13^0$

$\theta = 90 - 53.13$

$= 36.9^0$ **<u>so minimum force is when force is 36.9⁰ above horizontal.</u>**

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