Stochastic dynamical modeling in biology

- Notes Week 2 -

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Notes 04-11-23

0.1 Power Spectral Density

Talking about power spectral density (PSD).

1. Important information to remember:
   - Lower bound of our time: $\frac{1}{M\tau}$
   - Upper bound of our time: $\frac{1}{2\tau}$
2. When we havd some sort of periodicity. We see it in the autocovariance function and within the PSD.
3. Real numbers have no phase. Complex numbers have a non-zero phase.
4. A phase that is at zero, when comparing two other components, the two elements oscillate in synchrony. However, if they were not at zero then they would possibly oscilliate not together. Ex. Preditor and Prey.
5. When applying a lot of normalization types like (Welch, Bartlett) method may produce a lot of noise within the phase. However, if we can reproduce at the point of the peak in the modulus, and the phase are the same. Then this is a possible true synchrony we see.

   Questions

0.2 Stochastic dynamical models

1. It won't tell us exactly what is going to happen. However, it will give us some idea about what will happen. i.e., it will tell us what the expected or probability of something happening will be.
2. Stochastic Iterative Map

$$X_k \sim g(K, X_{k-1}, X_{k-2}, ..., X_{k-P}) \tag{1}$$

   The random state of the system at time point k.

- If $X_k \in \mathbb{R}^n$ we call the SIM "numverical n-dimensional"
- If $P > 1$ we call the SIM delayed and p is the order of the SIM
- If $P = 1$ (immediately depends on the proceeding one) the SIM is called Markov
- If g does not explicitly depend on k,the SIM is called autonomous or time-homogeneious (no external factors change the dependency of the k)
- Autonomous Markov SIM with a discrete state space are also known as discrete-time Markov chains

Example:

- $N_k = rN_{k-1}$ Deterministic Model (Geometric growth)
- $N_k \sim \Pi(rN_{k-1})$ Stochastic Model -> Autonomous and markovian

Capital Pi (Π) to denote the poisson distribution. Only free parameter is the mean.

New Example:

- If we suppose that r depends on time. We now have multiple r's.
- $N_k \sim \Pi(r_k N_{k-1}) \rightarrow$ Still Markovian.
- However, it is no longer Autonomous as we have anther time related factor/outside factor that influences the growth.