Bunching of Fractionally-Charged Quasiparticles Tunneling through High Potential Barriers

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Shot noise measurements were recently exploited to measure the charge of the quasiparticles in the Fractional Quantum Hall (FQH) regime. For fractional filling factors $\nu = 1/3$ and $2/5$ of the first Landau level, fractional charges $q = e/3$ and $e/5$, respectively, were measured [1–3]. We investigate here the interaction of $e/3$ quasiparticles with a strong backscatterer and find unexpected results. When a weak backscatterer is introduced in the path of an otherwise noiseless current of quasiparticles, stochastic *partitioning* of the quasiparticles takes place and shot noise proportional to their charge appears. Specifically, at $\nu = 1/3$, noise corresponding to $q = e/3$ appears. However, the measured charge increases monotonically as backscattering becomes stronger, approaching asymptotically $q = e$ [4,5]. In other words, only *electrons*, or alternatively, three *bunched* quasiparticles, tunnel through high potential barriers when impinged by a noiseless current of quasiparticles. Here we show that such bunching of quasiparticles by a strong backscatterer depends on the average occupation (*dilution*) of the *impinging* quasiparticle current. For a very dilute impinging current, bunching ceases altogether and the transferred charge approaches $q = e/3$.

These surprising results prove that a sparse beam of quasiparticles, each with charge $e/3$, tunnel through high potential barriers, originally thought to be opaque for them.

Already in 1918 W. Schottky determined the charge of the electron by measuring the average square of the current fluctuations, $S = \langle i^2 \rangle$, resulting from the stochastic emission of electrons in a vacuum tube - naming it *shot noise* [6] . His famous expression $S = 2qI$ is a result of independent random events obeying Poisson distribution. Here, S is the spectral density of the fluctuations (in units of A^2/Hz), q the charge of the particle and I the average current. Similar experiments were implemented [1–3] in the FQH regime [7], verifying the existence of fractionally charged quasiparticles [8]. A partially transmitting constriction, *Quantum Point Contact* (QPC), served in these experiments as an adjustable potential barrier in the path of the current, thus partitioning the transmitted charges. This random process is described by a binomial distribution, resulting with $S = 2qI(1-t)$ at zero temperature, with $t \in [0, 1]$ being the transmission coefficient of the QPC [4,9,10]. In the weak backscattering regime, the quasiparticles were found to traverse the QPC independent of each other and the measured charge was $q = e/3$ at filling factor $\nu = 1/3$ and $q = e/5$ for $\nu = 2/5$ [1–3]. As backscattering gets stronger, the tunneling of individual quasiparticles becomes correlated, and in the limit of a *pinched* QPC and $\nu = 1/3$ three quasiparticles were found to group together in order to tunnel through the barrier. Obviously, Schottky's formula is inapplicable for *correlated* (or *bunched*) *quasiparticles*, but it can still be used to characterize the system with the effect of bunching being incorporated into an effective charge $q(t)$. Hence, the noise for a pinched QPC becomes *electronic like* [4], namely, with an effective charge $q = e$. Moreover, a nearly universal behavior was found for the evolution of the effective charge $q(t)$ [5], starting at $q(\text{open QPC}) = e/3$ and monotonically increases toward $q(\text{pinched QPC}) = e$.

Here we explore the *bunching properties* of a pinched QPC when a *sparse* beam of $e/3$

quasiparticles impinges upon it. In other words, when there are not enough quasiparticles in close proximity to bunch into an 'electron', we may ask the following questions: (a) Will the barrier become opaque? (b) Will quasiparticles be 'borrowed' from lower-energy states to fill in for the missing ones in the beam? Or, (c) perhaps the bunching condition will be relaxed and individual quasiparticles will transverse the barrier? Theory does not provide yet answers to these questions.

Samples were fabricated in a high-mobility two-dimensional electron gas embedded in a GaAs-AlGaAs heterostructure. Measurements were done in a strong magnetic field $B \approx 13$ T and a fractional filling factor $\nu = 1/3$ in the FQH regime. Vanishing of the longitudinal resistivity ρ_{xx} assures that the (net) current is flowing chiraly along the edges of the sample in *edge states*. This allows measurements in *multi-terminal geometries* shown in Fig. 1. Two techniques are employed in order to partition the quasiparticle beam, hence making it *sparse* or *dilute*, before it impinges on the pinched QPC2. A straightforward scheme, shown in Fig. 1a, utilizes a noiseless current I_{inc} that impinges on a relatively *open* QPC1 and partially scatters toward QPC2 (although in this case the small *reflection coefficient* of QPC1 is responsible for the dilution of the current, for uniformity we stick to the notation of *transmission coefficient* $t_1 \to 0$). Most of the current continues toward $\boldsymbol{D_1}$ while the scattered part is a very dilute beam of quasiparticles with charge $q_1 = e/3$ [1,2,5] and dilution determined by t_1. Much of that dilute current is reflected back by QPC2 toward drain $\boldsymbol{D_2}$ and a small part t_2 is transmitted. A fraction $t_1 t_2$ of the incident current reaches the amplifier. This method cannot be applied, however, to achieve a moderately sparse beam of quasiparticles since a partly pinched QPC1 would lead to bunching of quasiparticles and to an *effective charge* $q_1 > e/3$ [5]. Hence, we employ also a geometry shown in Fig. 1b. Here, the incident current is being partitioned by transmission through a cascade of weakly backscattering QPCs (for each $t_i \to 1$), feeding QPC2 with current of quasiparticles with arbitrary dilution $t_1 = \prod_i t_i$. The fact that the partitioned charge by this method is $e/3$ is not obvious and a detailed study [11] was needed to prove that a beam of quasiparticles is indeed produced. We also tested, via detailed noise measurements [11], whether dilute quasiparticles suffer *intra-edge scattering* and subsequent equilibration during transport along the device edges. Equilibration establishes a new chemical potential and increases the occupation of each state below the chemical potential - hence, modifying the dilution of the beam. As shown in Ref. 11 such equilibration does not take place in our devices.

Using one of the two methods depicted in Fig. 1 we create a *noisy* beam of quasiparticles with charge $e/3$, which is being further partitioned by QPC2. The noise at $\boldsymbol{A}$ is measured with a spectrum analyzer after amplification by a cooled amplifier. The amplifier, being placed near the sample, has a very low current noise at its input, $\langle i^2_{amp}\rangle = 1.5 \times 10^{-28}$ A^2/Hz, when operating with bandwidth of 30 kHz and center frequency $f_0 \approx 1.5$ MHz. The value of f_0, chosen well above the cutoff of the ubiquitous $1/f$ noise, is determined by a resonance of LC circuit, with C determined by the capacitance of the coaxial cable connecting the sample and the amplifier and L by an added superconducting coil [1]. Reflected currents flow into the grounded terminals $\boldsymbol{D}$ and $\boldsymbol{T}$, leading to a constant *input* (at $\boldsymbol{S}$) and *output* (at $\boldsymbol{A}$) conductance $G = e^2/3h$ - independent of the transmission of the QPCs. This makes both the sample's equilibrium noise ($4k_BTG = 5 \times 10^{-29}$ A^2/Hz) and the sample-dependent amplifier's noise ($\langle i^2_{amp}\rangle/G^2$) independent of QPCs' transmission, allowing their subtraction from the measured noise (for comparison, the magnitude of the shot noise at $\boldsymbol{A}$ is typically in the 10^{-30} A^2/Hz range).

The configuration in Fig. 1a can be analyzed by means of *superposition* [12]. Consider first the 'injector' QPC1, characterized by transmission $t_1 \to 0$ toward QPC2 and partitioned charge $e/3$. Being a stochastic element, it generates (at zero temperature) noise $2(e/3)I_{inc}t_1(1-t_1)$, with $I_{inc}t_1$ the transmitted current, impinging on QPC2. This noise is attenuated with a factor t_2^2 by QPC2, resulting with a contribution of QPC1 to the total noise:

$$S_1 = 2(e/3)I_{inc}t_1(1 - t_1) \cdot t_2^2 \ . \tag{1}$$

Consider now QPC2, characterized by transmission t_2 and charge q_2 when impinged by a *noiseless* current I_{imp} of $e/3$ quasiparticles. It produces noise $2q_2I_{imp}t_2(1 - \widetilde{t_2})$, where $I_{imp}t_2 = I_2$ is the transmitted current and $\widetilde{t_2} = t_2\frac{e/3}{q_2}$ denotes the effective transmission for charge q_2 quasiparticles. This transmission $\widetilde{t_2}$ is determined self consistently with the charge q_2 in order to maintain the measured conductance of QPC2 [5]. We stress that even though the current $I_{imp} = I_{inc}t_1$ is noisy we still use the above expression to calculate the noise generated by QPC2, since the noise in I_{imp} was already taken into account in Eq. (1). The added contribution of QPC2 is therefore:

$$S_2 = 2q_2I_{inc}t_1t_2(1 - \widetilde{t_2}) \ . \tag{2}$$

The total noise in $\boldsymbol{A}$ is then $S_1{+}S_2$. The correctness of this analysis can be validated in the limit of a constant charge (say, $e/3$): $S_1{+}S_2 = 2(e/3)I_{inc}t_1t_2(1{-}t_1t_2)$, with $t_{tot} = t_1t_2$ being the total transmission from $\boldsymbol{S}$ to $\boldsymbol{A}$ - the standard expression for binomially distributed process. In the experiment we use the expression for $S_1 + S_2$ in order to determine the charge q_2 partitioned by QPC2. In the limit where both t_1 and t_2 are small S_1 and S_2 are of order of $O(t_1t_2^2)$ and $O(t_1t_2)$, respectively, with the first much smaller than the second. Hence, the measured noise is dominated by the contribution of the pinched QPC2:

$$S \approx 2q_2I_{inc}t_1t_2 = 2q_2I_2 \ . \tag{3}$$

We verify first that the noise produced by a pinched QPC, when fed with a quiet current, corresponds as expected to $q \approx e$. We find results similar to these in Ref. 5 with an example given in Fig. 3b. For $t_1 = 1$, hence feeding QPC2 with a noiseless current, and $t_2 \approx 0.1$, we measure indeed charge e. We then partition the incident current by setting $t_1 < 1$, hence impinging a noisy current of quasiparticles on QPC2 with $t_2 \approx 0.1$.

The noise seen in Fig. 2a corresponds to an average state-occupation $t_1 \approx 0.7$ and than in Fig. 2b to $t_1 \approx 0.2$. Calculating the expected noise [5] we take into account the finite temperature of the electrons ($T \approx 65$ mK) and the energy (or current) dependence of the total transmission $t_{tot} = t_1 t_2$ (current dependent transmissions are shown in the insets of Fig. 2). The average current is being varied over a large enough range, with the voltage V satisfying $q_2 V \gg k_B T$, to allow the noise to reach the linear regime. Nice agreement is found between the data and the independent particle model for $q_2 = 0.9e$ and $t_1 = 0.7$ (lightly diluted current) and for $q_2 = 0.55e$ and $t_1 = 0.2$ (highly diluted current).

A more striking example of the effect of beam dilution is demonstrated in Fig. 3, where the range of current I_2 is kept constant for different values of dilution. Obviously, a higher source voltage is required to obtain the same current I_2 when the current is more dilute. In comparison with the measured *electron charge* for a noiseless impinging current (Fig. 3b), a highly dilute current ($t_1 \approx 0.1$) impinging on the pinched QPC2 is found to produce a small charge $q_2 = 0.45e$ (Fig. 3a) - slightly above the quasiparticles charge.

Figure 4 summarizes the dependence of q_2 on the dilution t_1 of the impinging current on QPC2. Two examples, $t_2 = 0.1$ and $t_2 = 0.25$, are chosen, with corresponding charge e and $0.75e$, respectively, for a noiseless impinging current. The more dilute the impinging current is ($t_1 \to 0$), the smaller is the effective charge q_2 - approaching asymptotically $e/3$. The unavoidable conclusion is that *bunching of quasiparticles* is not an essential mechanism for quasiparticles transfer through high potential barriers! Bunching takes place **only** when the incoming states are highly occupied.

Before we conclude we may also ask how is the *transmission* of QPC2 affected by the dilution of the impinging quasiparticles. Present theory assumes only a noiseless current approaching a constriction within the framework of the Luttinger model. Also here we find counter intuitive results. In the linear regime, where the source voltage is small enough to keep the transmission almost energy independent (Fig. 5a) the transmission

t_2 is independent of dilution (although the source voltage for the dilute current is some ten times larger). This can be compared with the case where the same source voltage range is kept (Fig. 5b). Here, the transmission t_2 of the noiseless current strongly depends on voltage (approaching unity at $V > 50\ \mu$V). Since equilibration of quasiparticles had been ruled out [11], we conclude that the non-linearity of the pinched QPC depends strongly on the quasiparticle *current* and less on the quasiparticle *energy*. This rules out that the potential profile of the barrier in the QPC is responsible for non-linearity of the current. Moreover, the insensitivity of the transmission t_2 in the linear regime to the dilution of the impinging quasiparticle beam suggests equal probabilities of tunneling for a single quasiparticle and for bunched quasiparticles. In other words, noise and transmission measurements show that quasiparticles can transfer, with the same *ease*, either *one by one* or *bunched in groups*. Their bunching depends on the transparency of the barrier and on the preparation of the quasiparticle beam. It is now for theory to explain such a bizarre effect.

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Captions

Fig. 1. Schematic and actual representations of quasiparticles *injector* followed by a quasiparticle *filter*, both made of quantum point contacts, QPC1 and QPC2, respectively. **(a)** An injector made of a relatively open QPC1 partitions an incident noiseless (dc) current, injected from terminal $\boldsymbol{S}$. The scattered part (t_1), composed of a dilute beam of quasiparticles, impinges on a pinched QPC2. The resulting noise measured by a cooled, low-noise, amplifier at terminal $\boldsymbol{A}$ (see Ref. 1). The intermediate drain $\boldsymbol{D_2}$ prohibits multiple-reflections, and the grounded terminal $\boldsymbol{T}$ is used to fix the *output* impedance of the sample and make it independent of QPC settings. **(b)** An alternative scheme, suitable for producing a moderately dilute current, invokes a cascade of weakly backscattering QPCs *transmitting* a dilute quasiparticle beam (see Ref. 11). **(c)** A photograph of the actual device in the vicinity of the QPCs, formed by metallic gates (light gray regions) deposited on the surface of the GaAs-AlGaAs heterostructure embedding a two dimensional electron gas some 0.1 μm below the surface. Electron mobility is 2×10^6 cm^2/Vs

and areal carrier density is 1.1×10^{11} cm^{-2}, both measured at 4.2 K in the dark. The solid arrows correspond to the direction of current in configuration (a), while the other QPCs on the right (with current flow denoted by dotted arrows) are used when configuration (b) is employed. Ohmic contacts (serving as $\boldsymbol{S}$, $\boldsymbol{D}$, $\boldsymbol{T}$, $\boldsymbol{A}$) are outside the frame of the picture.

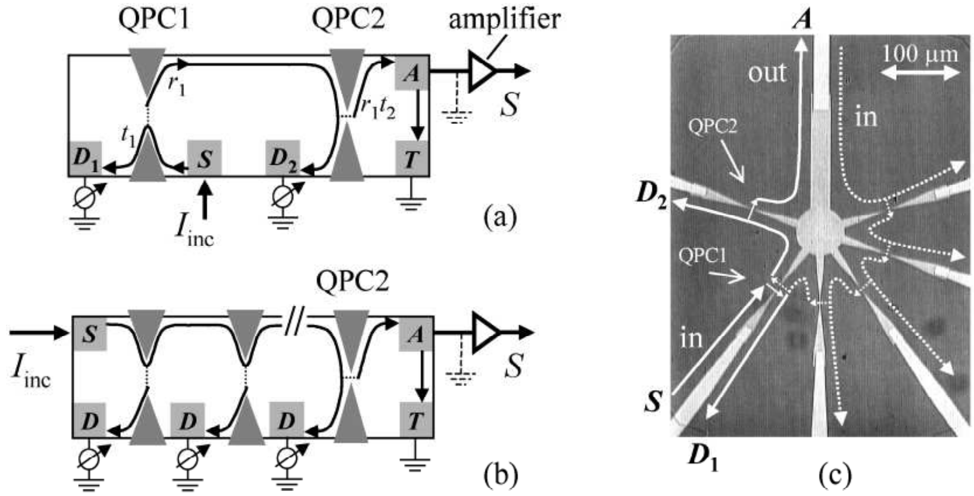
Fig. 2. Noise and transmission measurements of the pinched QPC2 (with transmission $t_2 \approx 0.1$ at zero bias) for two different values of dilution of the impinging current: $t_1 = 0.7$ **(a)** and 0.2 **(b)**. In the main graphs the measured noise is plotted against the transmitted current, together with the theoretical prediction of the independent particles model at a finite temperature. The intermediate curve in each graph represents the best fit to an arbitrary charge q_2. Various transmission coefficients, measured simultaneously with the noise, are shown in the insets against the incident (noiseless) current. Each inset shows the dilution level t_1 generated by the QPC1 *injector*, the transmission t_2 of the pinched QPC2 *in response to the dilute impinging current*, and the total transmission t_{tot}. Notice that the sensitivity of t_2 to the current depends on the dilution level of the impinging current.

Fig. 3. Comparison of the charge characterizing the pinched QPC2 for two extreme cases of the impinging current: not diluted (noiseless) and highly dilute, *keeping the same transmitted current.* **(a)** The noise produced by the pinched QPC2 when fed by a highly dilute impinging current, $t_1 \approx 0.1$, corresponds to a quasiparticle charge $q_2 \approx 0.45e$. The inset shows the current-dependent transmission t_1 (level of dilution) and the transmission t_2 (is fairly current independent for such a dilute beam). **(b)** The noise produced by the pinched QPC2 when fed by a noiseless current corresponds to almost an *electronic* charge. The inset verifies the charge $q_2 = e$ by measuring the noise over considerably wider range

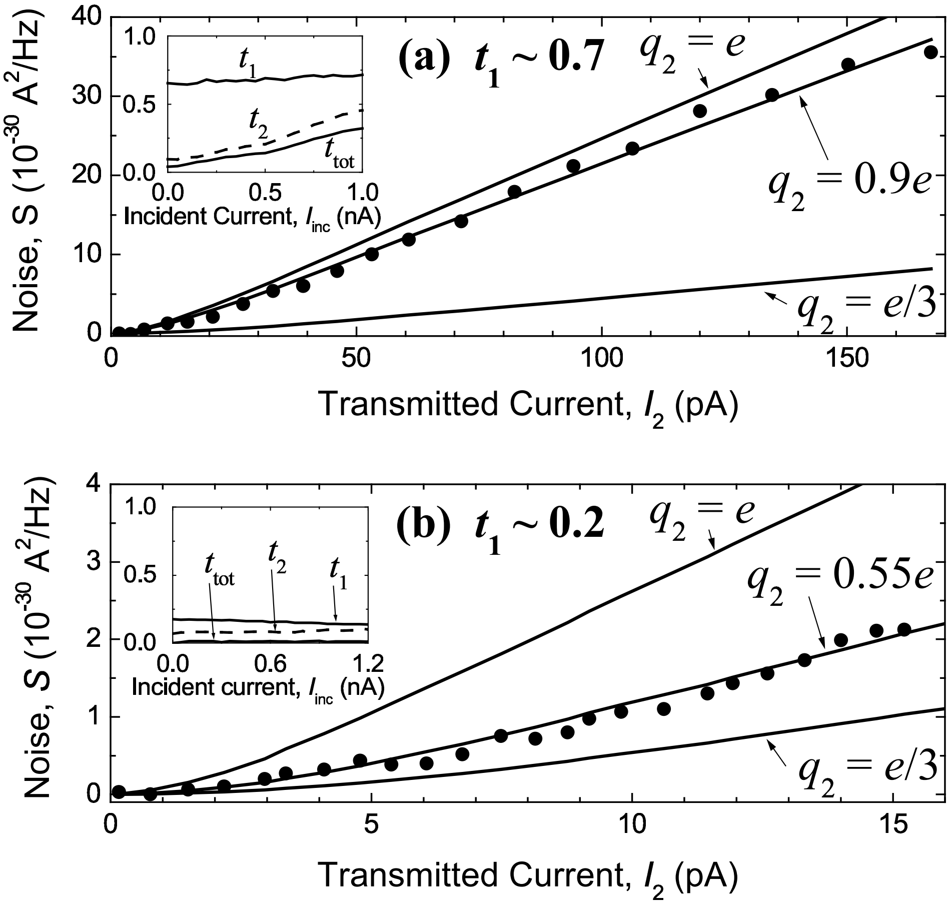
of transmitted current. Clearly, the charge characterizing QPC2 depends not only on the potential barrier height but also on the average occupation of the states (dilution) of the impinging current.

Fig. 4. Evolution of the effective charge q_2 that characterizes the pinched QPC2 in response to different values of dilution t_1 of the impinging current (extracted from curves similar to that in Fig. 3a). Results of three different measurements are shown - two complementary sets of data for $t_2 = 0.1$, with dilution produced by backscattering of a single QPC1 (Fig. 1a) and by transmission through five, relatively open, QPCs (Fig. 1b), and one set with $t_2 = 0.25$. In the case of $t_2 = 0.25$ only the extreme points are shown in order to simplify the graph. The dashed lines are only guide to the eye. Evidently, as the current impinging on the pinched QPC2 becomes more dilute, the charge drops from its original value toward $e/3$ in the limit of very high dilution. We conclude that individual, very sparse, quasiparticles tunnel through a pinched QPC - originally thought to be opaque for them.

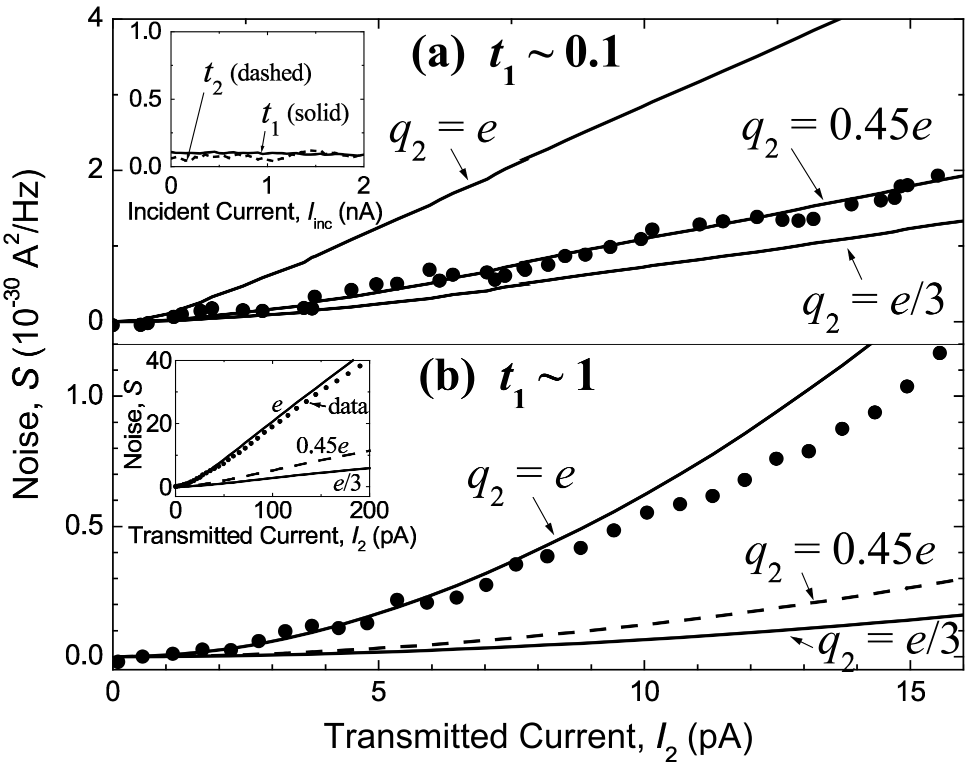
Fig. 5. Dependence of the transmission t_2 of the pinched QPC2 on the dilution t_1 of the impinging current. **(a)** Measurements in the linear regime. The transmission t_2 of a highly dilute impinging current ($t_1 \approx 0.1$, solid curve) and that of a noiseless current ($t_1 = 1$, dashed curve), with the same *transmitted current* range kept in both cases. The applied source voltage, on the other hand, reaches a maximal value of $170\ \mu$V for the noisy impinging current but only $16\ \mu$V in the noiseless case. Nevertheless, the transmissions in both cases are similar. **(b)** Measurements in the non-linear regime. Similar measurements to (a) but the same applied *source voltage* range is kept in both cases (the noiseless current is some ten times larger for the same voltage). The transmission is found to be strongly current dependent when the impinging current is noiseless.



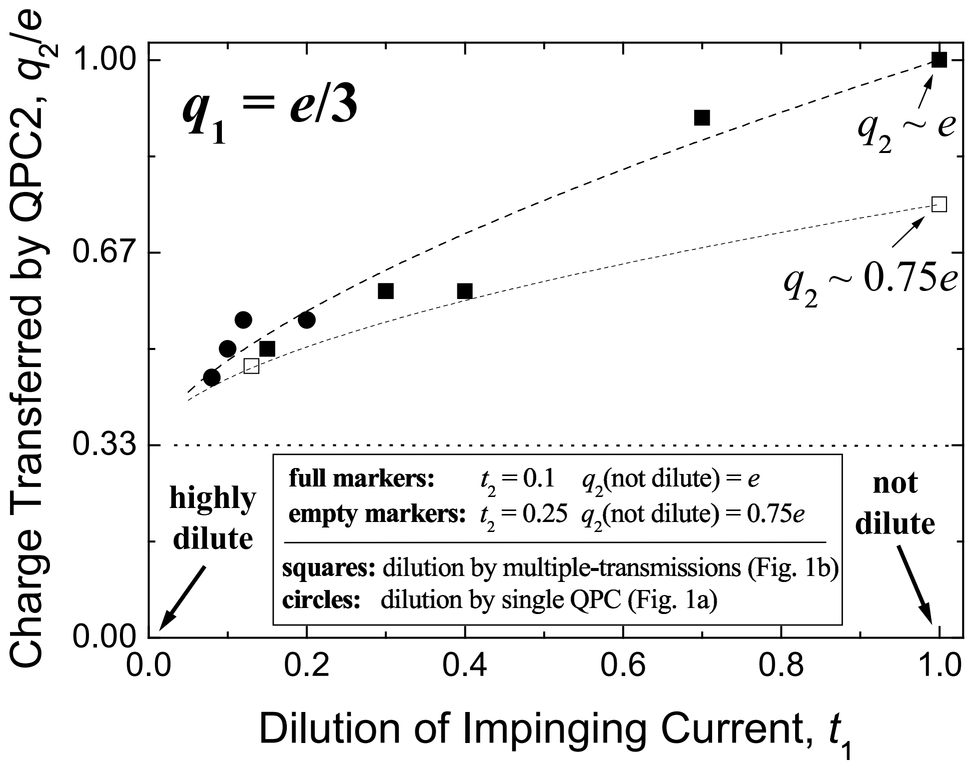
Comforti *et al.* - Figure 1



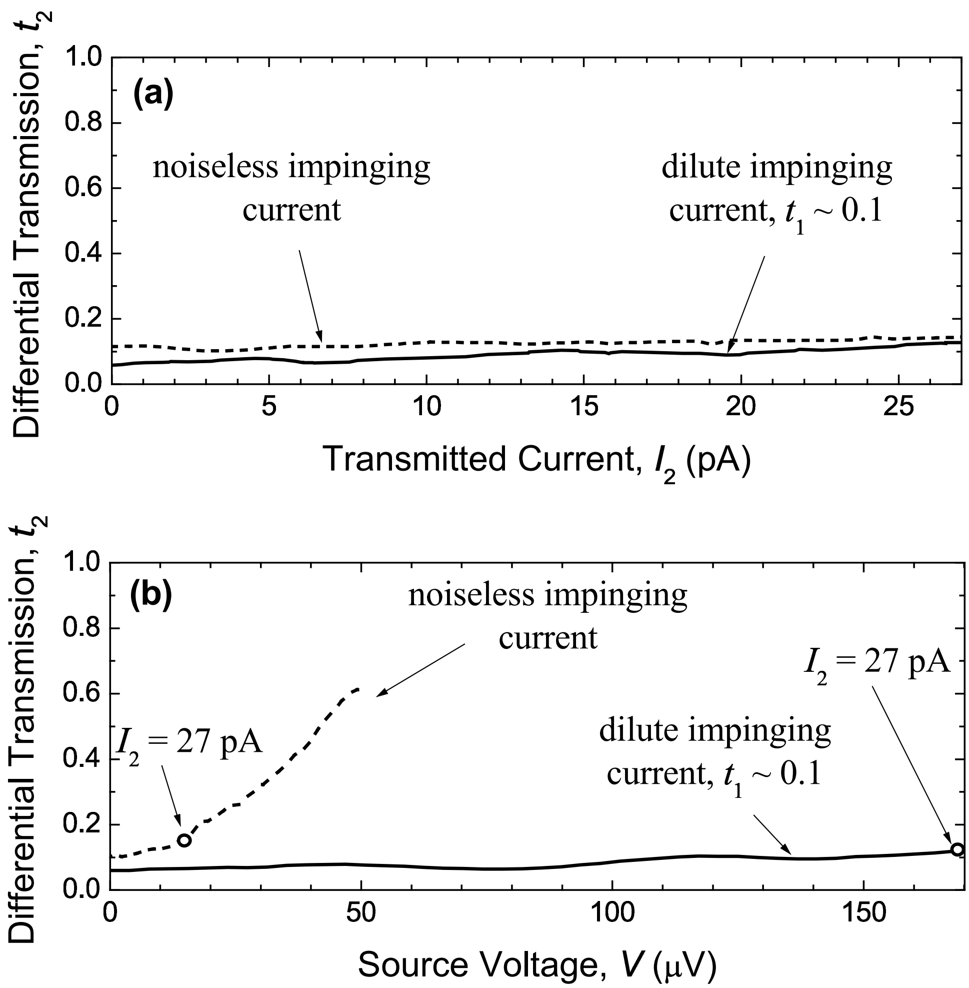
Comforti *et al.* - Figure 2



Comforti *et al.* - Figure 3



Comforti *et al.* - Figure 4



Comforti *et al.* - Figure 5