Pool Simulation - Impact Calculations for Collisions between Balls

Introduction:

In the pool simulation, the velocities of two balls after impact are calculated using trigonometry and a combination of physics principles. Firstly, the system can be rotated about the center of one of the balls to make the axis of collision horizontal. After this rotation, the vertical component of the velocities becomes tangent to the axis of collision, meaning it will not change after the impact. Conservation of momentum and coefficient of restitution equations can then be applied to calculate the horizontal component of the velocities after impact. Lastly, we convert these velocities back to normal *x* and *y* axes to get the final velocities and angles of the balls.

Initial Velocity and Angle:

The initial *x* and *y* velocities of the balls are used to calculate the initial velocity and initial angle of the balls. The following equations can be used to show this step:

$$\theta_i = \tan^{-1}\left(\frac{v_{iy}}{v_{ix}}\right)^{\circ}$$

$$v_i = \sqrt{{v_{ix}}^2 + {v_{iy}}^2}\ \mathrm{m/s}$$

where v_i is the initial velocity of the ball, v_{ix} is the initial velocity of the ball in the *x* direction, v_{iy} is the initial velocity of the ball in the *y* direction and θ_i is the initial angle from the positive x axis.

Initial Velocities on Rotated Axis of Collision:

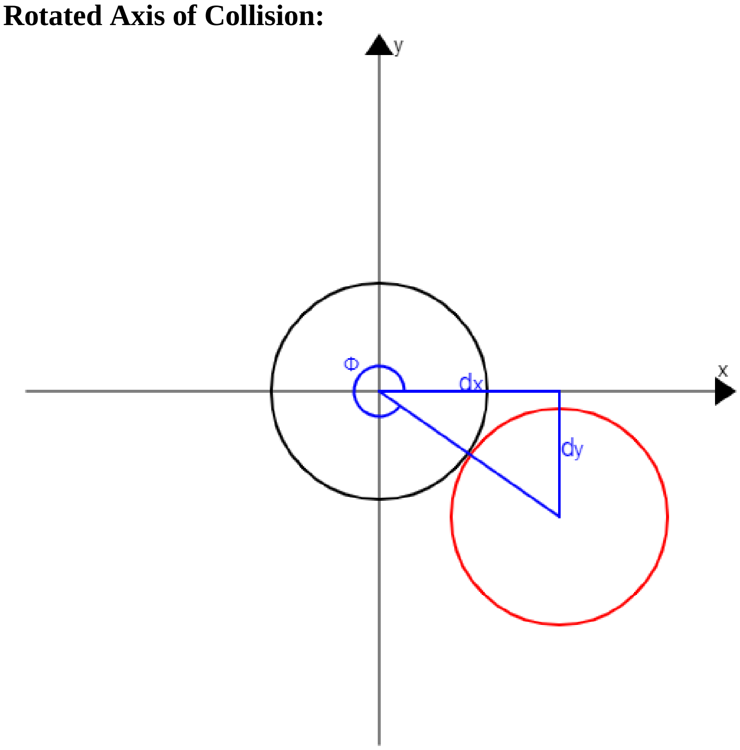

Figure 1: Diagram of the Axis of Collision.

The system is rotated around the center of the first ball to make the axis of collision horizontal. To do this, the angle ϕ is calculated using the following equation:

$$\phi = \tan^{-1}\left(\frac{d_y}{d_x}\right)^{\circ}$$

where d_y is the vertical distance between the balls, d_x is the horizontal distance between the balls and ϕ is the angle of collision from the positive *x* axis.

Now, the following equations are used to find the *x* and *y* velocities on the rotated axis of collision:

$$v_{ix}' = v_i \cos(\theta_i - \phi) \text{ m/s}$$

$$v_{iy}' = v_i \sin(\theta_i - \phi) \text{ m/s}$$

where v_i is the initial velocity of the ball, v_{ix}' is the initial velocity of the ball in the *x* direction after the axis is rotated, v_{iy} is the initial velocity of the ball in the *y* direction after the axis is rotated, θ_i is the initial angle from the positive *x* axis and ϕ is the angle of collision from the positive *x* axis.

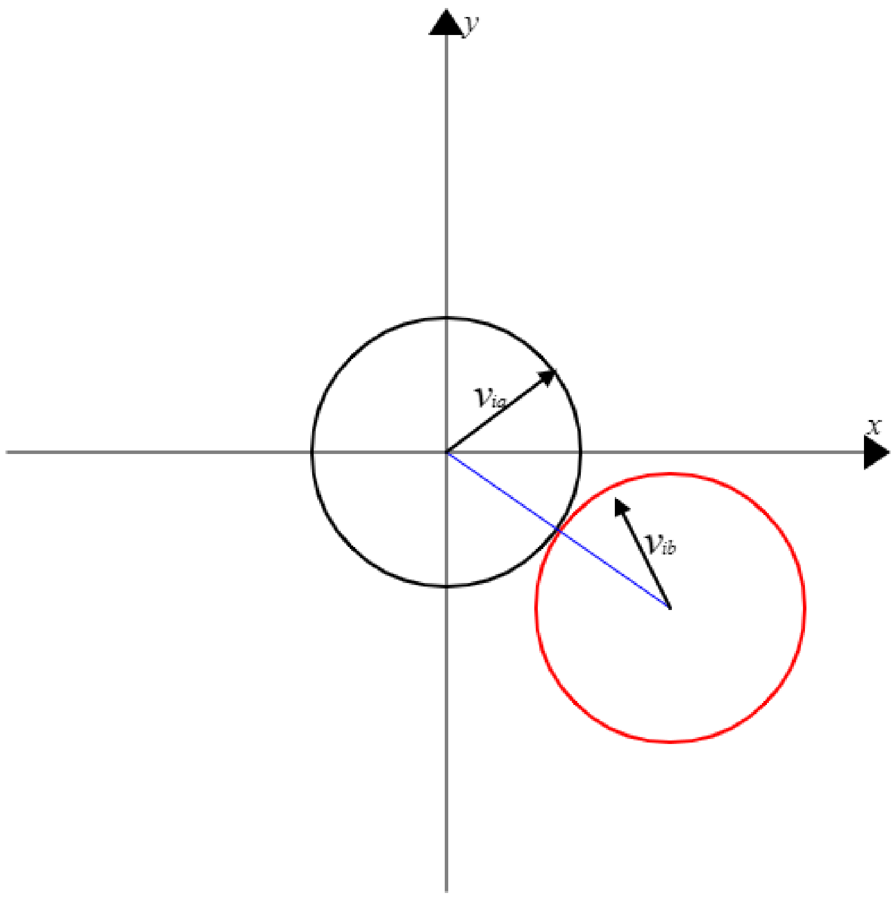

Figure 2a: Diagram showing the velocities before rotation of the Axis of Collision.

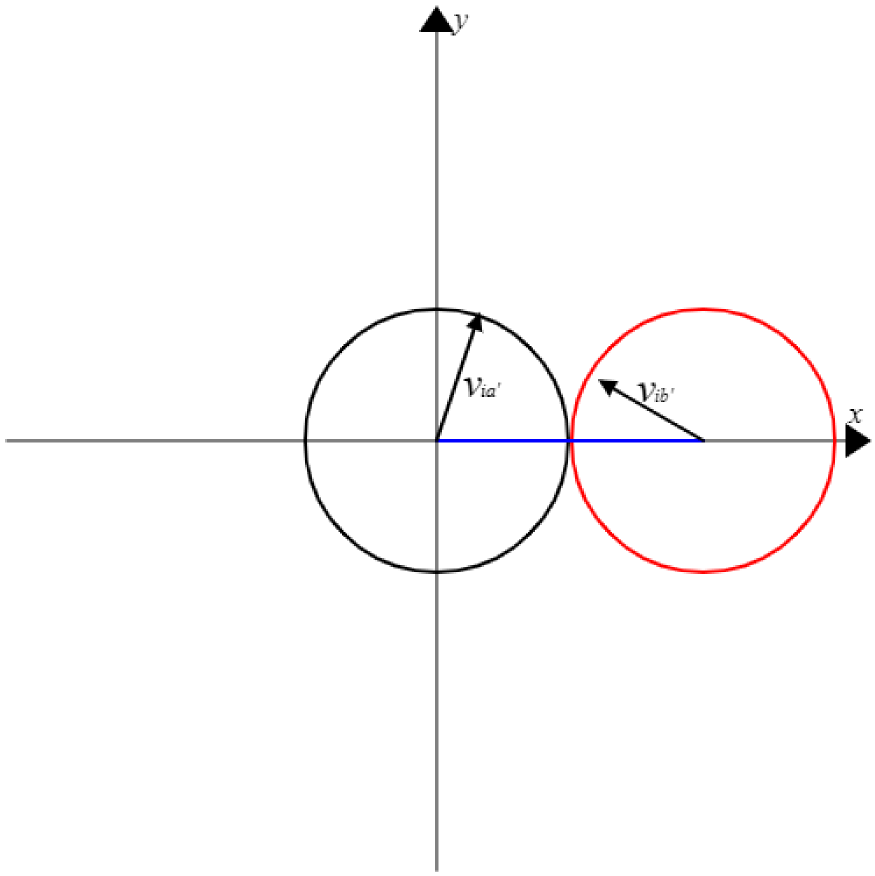

Figure 2b: Diagram showing the velocities after rotation of the Axis of Collision.

Notice that the *y* velocities are now tangent to the axis of collision meaning that they will not change after the collision.

Final Velocities on Rotated Axis of Collision:

The next step is to derive an equation to calculate the *x* velocities after the collision. The following conservation of momentum and coefficient of restitution equations are used:

$$m_a v_{ia} + m_b v_{ib} = m_a v_{fa} + m_b v_{fb}$$

$$e = \frac{|v_{fb} - v_{fa}|}{|v_{ib} - v_{ia}|}$$

where m_a is the mass of the first ball, m_b is the mass of the second ball, v_{ia} is the initial velocity of the first ball, v_{ib} is the initial velocity of the second ball, v_{fa} is the final velocity of the first ball, v_{fb} is the final velocity of the second ball and e is the coefficient of restitution.

After rearranging the equations, we get:

$$v_{fa} = \frac{m_a v_{ia} + m_b v_{ib} - m_b v_{fb}}{m_a} \text{ m/s}$$

$$v_{fb} = e\,(v_{ib} - v_{ia}) + v_{fa} \text{ m/s}$$

Now, the second equation is substituted into the first and the variable v_{fa} is isolated. Since we are only using this equation for the *x* velocities of the balls, we can substitute those into the equation. These steps are shown below:

$$v_{fa} = \frac{m_a v_{ia} + m_b v_{ib} - m_b e\,(v_{ib} - v_{ia})}{m_a + m_b} \text{ m/s}$$

$$v_{fax}' = \frac{m_a v_{iax}' + m_b v_{ibx}' - m_b e\,(v_{ibx}' - v_{iax}')}{m_a + m_b} \text{ m/s}$$

$$v_{fbx}' = \frac{m_a v_{iax}' + m_b v_{ibx}' - m_a e\,(v_{iax}' - v_{ibx}')}{m_a + m_b} \text{ m/s}$$

where v_{fax}' is the final velocity of the first ball in the *x* direction after the axis is rotated, v_{iax}' is the initial velocity of the first ball in the *x* direction before the axis is rotated, v_{fbx}' is the final velocity of the second ball in the *x* direction after the axis is rotated and v_{ibx}' is the initial velocity of the second ball in the *x* direction before the axis is rotated.

Final Velocities on Normal Axis of Collision:

The next step is to rotate the axis of collision back to its original form and get the *x* and *y* velocities in standard cartesian form. This is done using the following equations:

$$v_{fx} = v_{fx}' \cos\phi + v_{fy}' \cos(\phi + 90)$$

$$v_{fy} = v_{fx}' \sin\phi + v_{fy}' \sin(\phi + 90)$$

where v_{fx} is the final velocity of the ball in the x direction and v_{fy} is the final velocity of the ball in the *y* direction.

Final Velocity and Angle:

Lastly, we can use the final *x* and *y* velocities to calculate the final velocity and final angle of the balls. The following equations can be used to show this step:

$$\theta_f = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)^\circ$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} \text{ m/s}$$

where v_f is the final velocity of the ball and θ_f is the final angle from the positive *x* axis.

Conclusion:

Using trigonometry, conservation of momentum and coefficient of restitution equations, the pool simulation can calculate the velocities of two balls after impact.