Negative viscosity and stochastic parameterizations of Kinetic Energy Backscatter (KEB) for NEMO 3.6. Implementation details.

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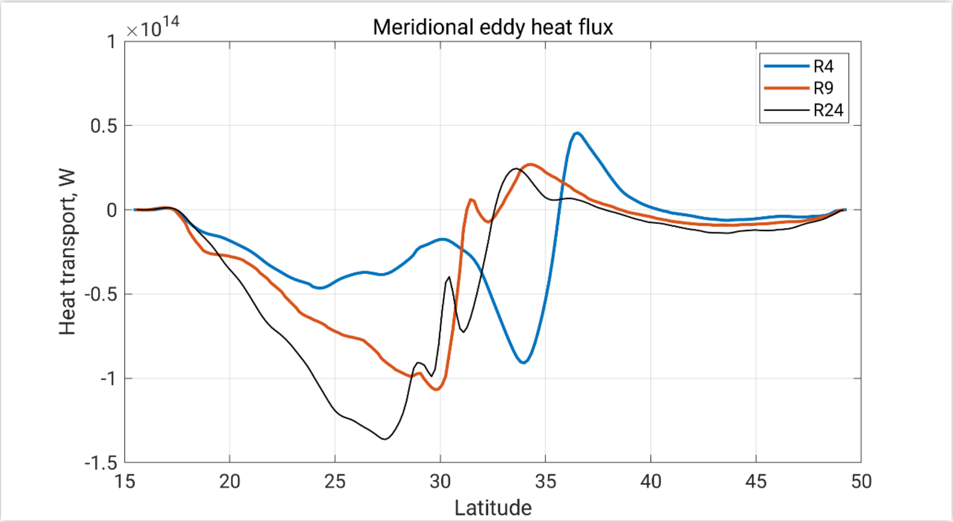
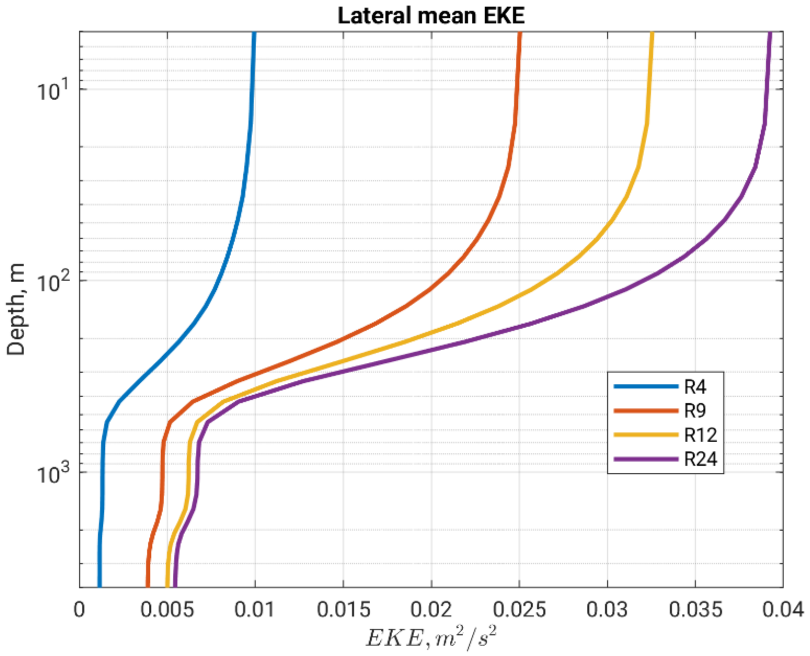
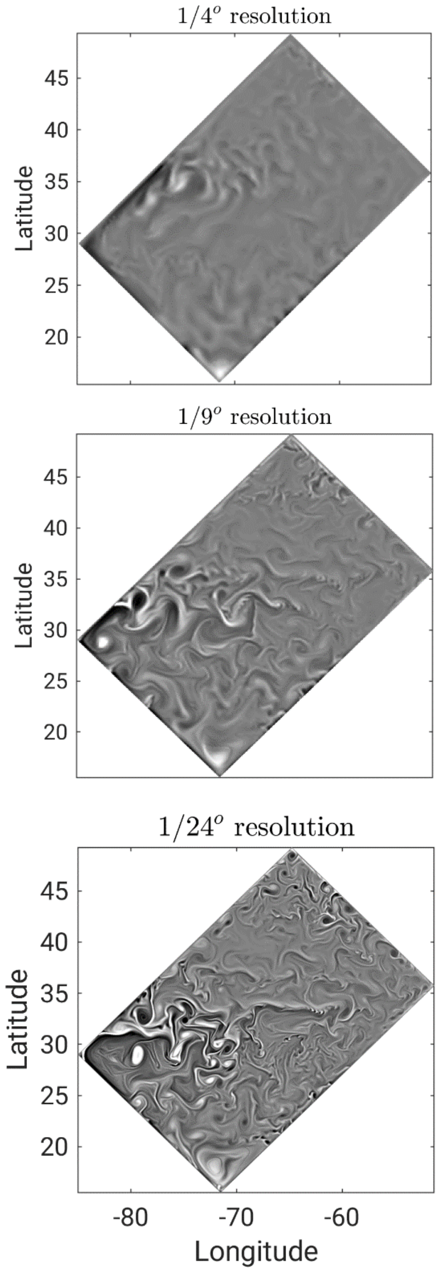
Introduction

- Mesoscale eddies form the most of the kinetic energy in ocean
- Mesoscale eddies only weakly resolved in eddy-permitting simulations
- Eddy kinetic energy and eddy heat fluxes strongly depend on the grid resolution







Eddy-permitting models weakly resolve energy cycle

- The energy of mesoscale eddies is given by two energy pathways:
  - APE->KE conversion
  - Inverse energy cascade
- Both pathways are weakly resolved in eddy-permitting simulations

Fox-Kemper, B., & Marsland, S. (Eds.). (2020). Sources and sinks of ocean mesoscale eddy energy.

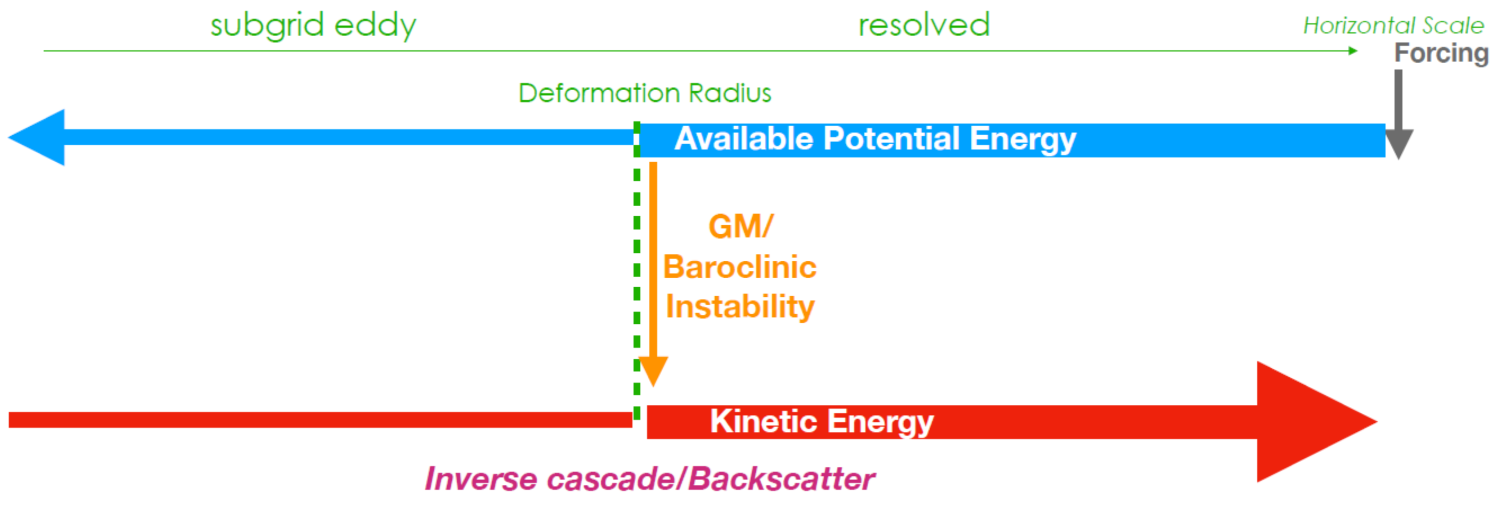


Figure 1. Schematic of energy transfer between reservoirs — potential (blue) and kinetic (red) energy — and horizontal scales (green).

Subgrid stress and backscatter

- The role of subgrid eddies in resolved dynamics is guided by the subgrid stress tensor:

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$$

$\overline{(\cdot)}$ – spatial filter

- Pointwise energy exchange with subgrid eddies can be of either sign:

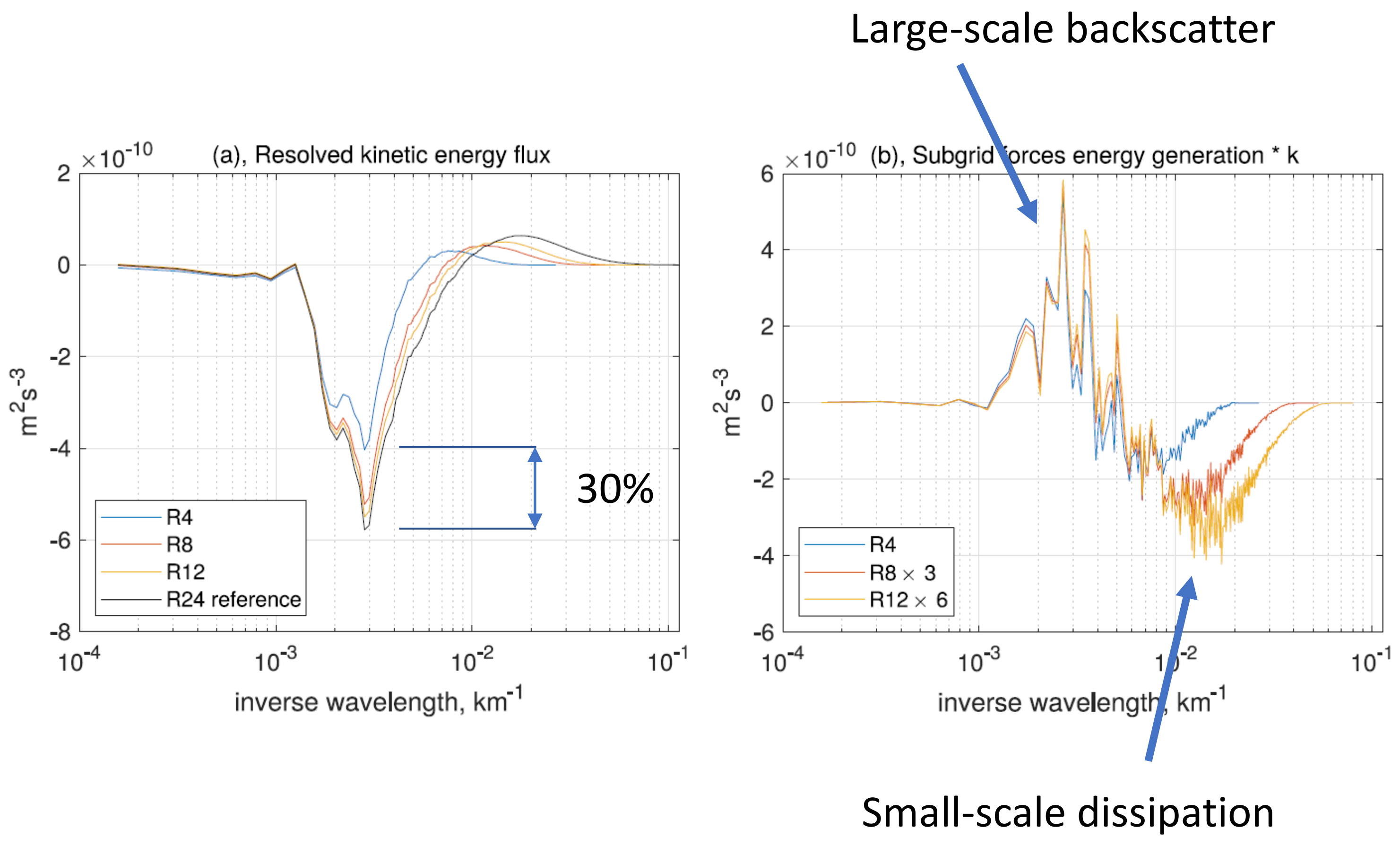
$-\tau_{ij} S_{ij} > 0$ – dissipation

$-\tau_{ij} S_{ij} < 0$ – **backscatter**

Definition for 3D turbulence. Meneveau 2000

Backscatter enhances inverse energy cascade

- *Hi-res* simulations show:
    - Resolved part of energy redistribution toward large scales (*due to resolved advection*) decreases with coarsening the resolution
    - Subgrid eddies compensate for unresolved inverse cascade
    - **Backscatter:** Subgrid eddies inject kinetic energy into large scales



Backscatter is more important in baroclinic fluids

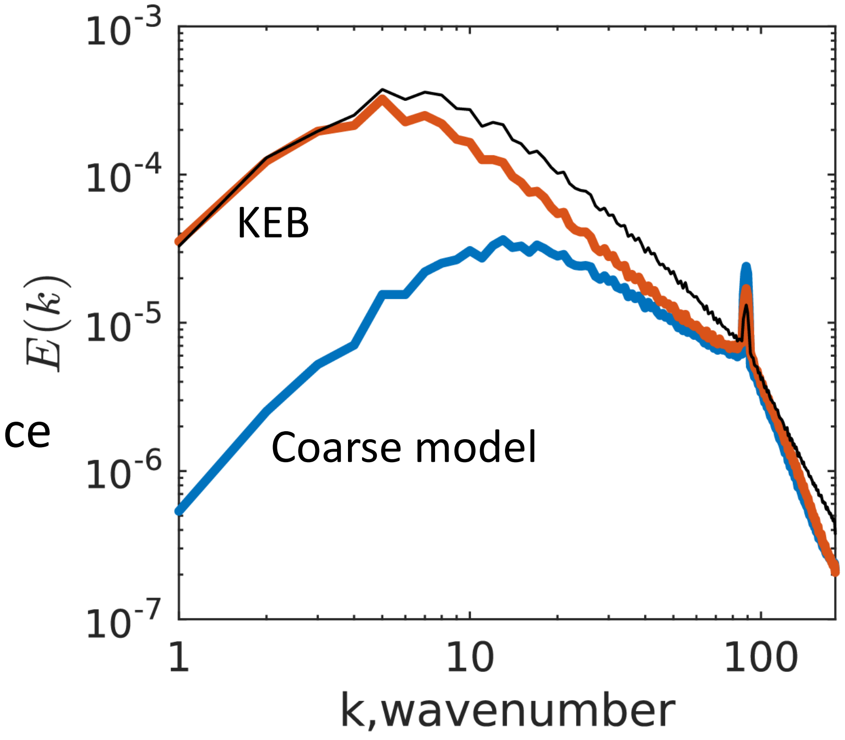
Barotropic fluid:

Improved kinetic energy in large scales

Baroclinic fluid:

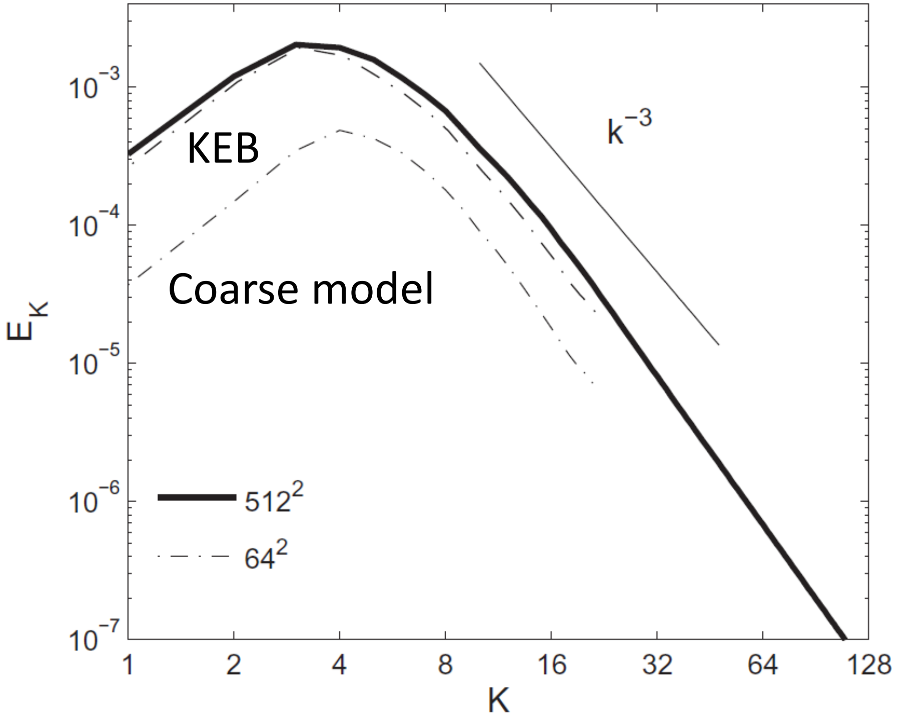
Energizing eddies, we enhance energy extraction from the mean flow

Barotropic forced turbulence



Perezhogin et. al. 2019

Baroclinic fluid



Jansen Held 2014

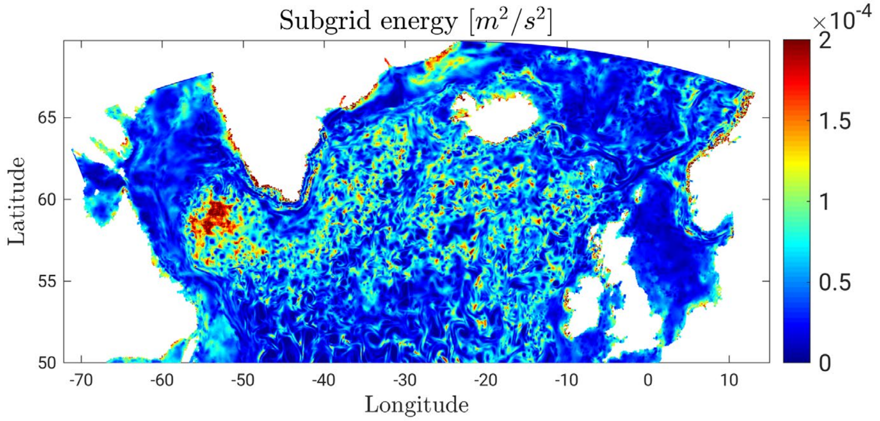
To conclude the introduction

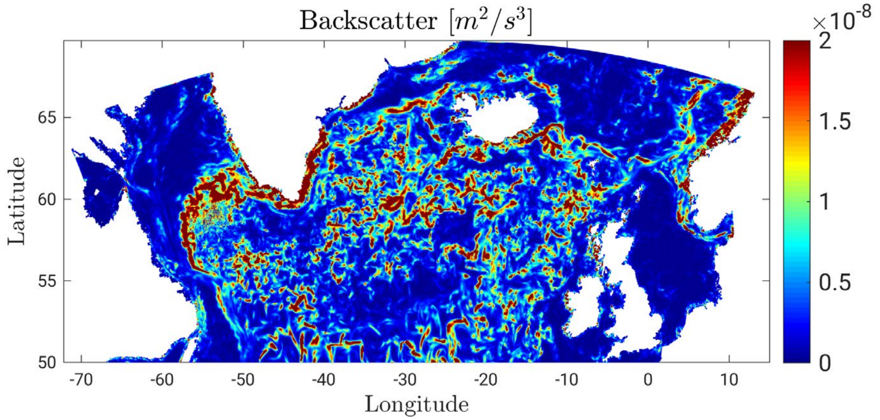
- There are two popular approaches to simulate KEB:
  - Stochastic: Berner 2009, Jansen & Held 2014, Grooms & Majda 2015
  - Negative viscosity: Jansen & Held 2014, Juricke et.al. 2019, Bachman 2019
- Some elements of these approaches are implemented in NEMO 3.6

History of development

- Early 2019
  - KEB in cartesian coordinates **in NEMO 3.6**
  - Perezhogin, P. "Deterministic and stochastic parameterizations of kinetic energy backscatter in the NEMO ocean model in Double-Gyre configuration." IOP Conference Series. 2019.
  - Perezhogin, Pavel A. "Testing of kinetic energy backscatter parameterizations in the NEMO ocean model." RJNAMM. 2020.
- Late 2019
  - Curvilinear coordinates
  - Cooperation with *Polina Verezemskaya* from Shirshov Institute of Oceanology, Moscow
  - 1 model run in North Atlantic **NNATL12 in NEMO 4**
- August 2021
  - Cooperation with *Colin Jones* from Metoffice
  - Large update of the code for **NEMO 3.6**
  - **https://github.com/Pperezhogin/Kinetic-energy-backscatter-for-NEMO**

Two years old picture





Code overview

$\dot{E}_{diss} = \nu_4(\Delta \boldsymbol{u})^2$

dynldf_bilap.F90

$\dot{E}_{source} = F(c_{diss}\dot{E}_{diss})$

KEB_apply( )

$\frac{de}{dt} = \dot{E}_{source} - \dot{E}_{back}$

KEB_negvisc_tendency( )

$\psi(x,y,z) \sim \sqrt{\dot{E}_{source}}\phi(x,y)$

KEB_AR1_tendency( )

Statistics is collected

KEB_statistics( )

stp	99.5%	98.156s
tra_adv	19.2%	18.899s
zdf_tke	15.1%	14.882s
keb_apply	12.7%	12.544s
keb_ar1_tendency	10.2%	10.047s
filter_laplace_t3d_ntimes	2.1%	2.061s
[loop in keb_apply at KEB_module.f90:2	0.3%	0.296s
[loop in keb_apply at KEB_module.f90:2	0.1%	0.128s
mpp_lnk_3d	0.0%	0.012s
dyn_zdf	7.9%	7.830s
dyn_ldf	7.3%	7.184s

(b) EXP_AR1

stp	99.5%	101.104s
tra_adv	18.5%	18.809s
keb_apply	15.5%	15.714s
keb_negvisc_tendency	10.2%	10.354s
klower_cdiss	2.8%	2.870s
filter_laplace_t3d_ntimes	2.1%	2.143s
[loop in keb_apply at KEB_module.f90:2	0.3%	0.327s
keb_statistics	0.0%	0.020s
zdf_tke	14.8%	15.005s
dyn_ldf	6.9%	6.990s
dyn_zdf	6.6%	6.742s

(c) EXP_negvisc

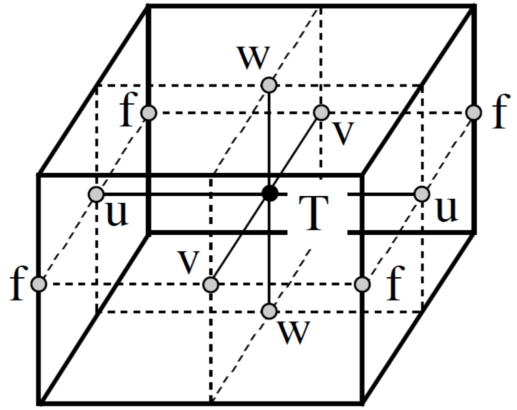
Design of the filters

- Filter is based on the diffusion operator:

$$F(\psi(x,y,\sigma)) = 1 + \nabla_\sigma \left(\begin{bmatrix} \frac{dx^2}{8} & 0 \\ 0 & \frac{dy^2}{8} \end{bmatrix} \nabla_\sigma \psi \right)$$

∇_σ – Nabla operator along vertical coordinate isosurface $\sigma = const$

- In Cartesian coordinates:
  - $F(\psi_{i,j}) = \frac{1}{8}\left(4\psi_{i,j} + \psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1}\right)$
  - Removes check-board noise
- Neumann or Dirichlet B.C. are possible
- Conservative



Source of subgrid energy ($\dot{E}_{source}$)

- Dissipation by biharmonic eddy viscosity has sign-definite form:

$$\dot{E}_{diss}(x, y, \sigma) = |\nu_4|(\Delta_\sigma \boldsymbol{u})^2$$

- Source of subgrid energy:

$$\dot{E}_{source}(x, y, \sigma) = F^n(c_{diss}\dot{E}_{diss})$$

- Parameter $c_{diss} \leq 1$ may be constant or given by Klöwer et.al. 2018:

$$c_{diss}(x, y, \sigma) = \left(1 + \frac{R_o}{R_{diss}}\right)^{-1}, R_o = \frac{|D|}{f}$$

Only biharmonic viscosity is provided in the code. Easily extended to the Smagorinsky biharmonic viscosity

```
ndiss = 2
dirichlet_filter = .true.
```

```
constant_cdiss = .false.
R_diss=0.5
```

Negative viscosity KEB. Operator for returning energy

- Vector-invariant form of Laplacian operator:

$$\frac{\partial \boldsymbol{u}_h}{\partial t} = -\nabla_\sigma \times (\nu_2 F^n(\omega)\boldsymbol{k})$$

- B.C.: zero vorticity ω on the boundary
- Divergence-free
- Sign-definite formula for returning energy:

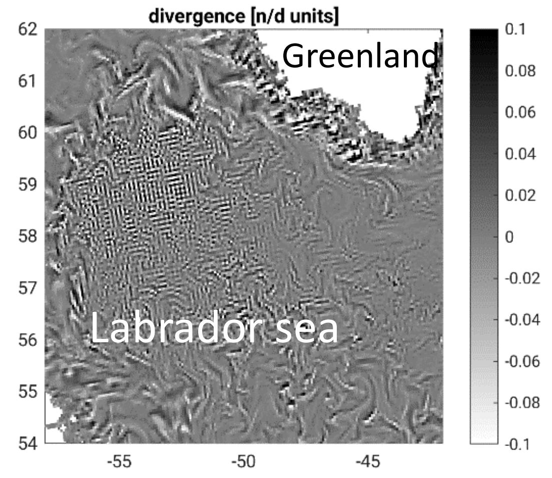
$$\dot{E}_{back}(x, y, \sigma, t) = |\nu_2| \left(F^{\frac{n}{2}}(\omega)\right)^2$$

note $n/2$ as filter is self-adjoint

- Formula for negative viscosity:

$$\nu_2(x, y, \sigma, t) = c_{back}\sqrt{dxdy}\sqrt{\max(2\mathrm{e}, 0)}$$

nback = 4



c_back = 0.1

Negative viscosity KEB. Subgrid energy equation

- Time evolution for 3D subgrid energy:

$$\frac{\partial e(x, y, \sigma, t)}{\partial t} + \nabla(\boldsymbol{u}e) = \dot{E}_{source} - \dot{E}_{back} + \nu_e \Delta_\sigma e$$

- Due to sign-definite conversion terms $\dot{E}_{source}$ and $\dot{E}_{back}$, energy is positive:

$$e \geq 0$$

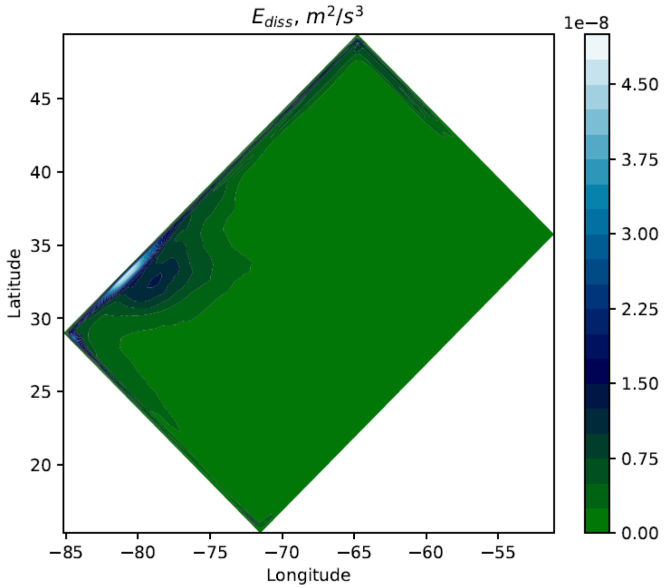
- Advantages of positive e:
  - Fast check for bugs in new configurations
  - Negative energy can accumulate along boundaries

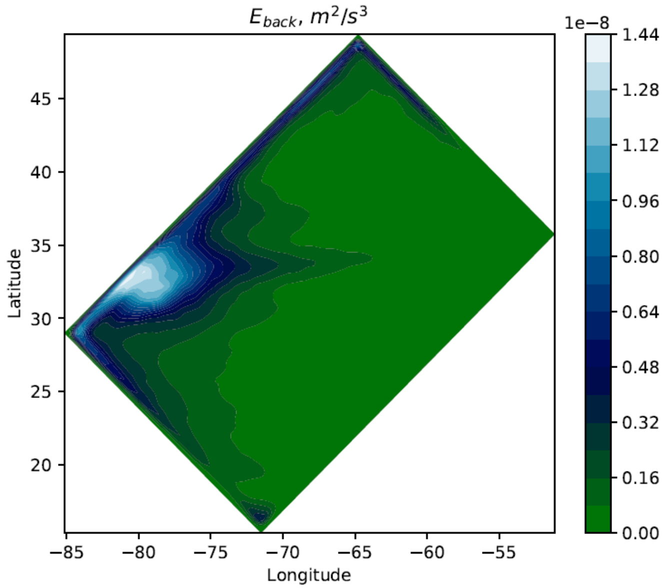
```
tke_adv = .true.
dirichlet_TKE = .true.
nu_TKE = 300.
```

Is advection of subgrid energy important?

$$\frac{\partial e}{\partial t} + \nabla(\boldsymbol{u}e) = \dot{E}_{source} - \dot{E}_{back} + \nu_e \Delta_\sigma e$$

- There are evidences that subgrid energy can be transported on large distances [Grooms 2013]
- Advection of subgrid energy makes KEB to be Galilean invariant:
  - Backscattered energy is injected downstream the mean flow





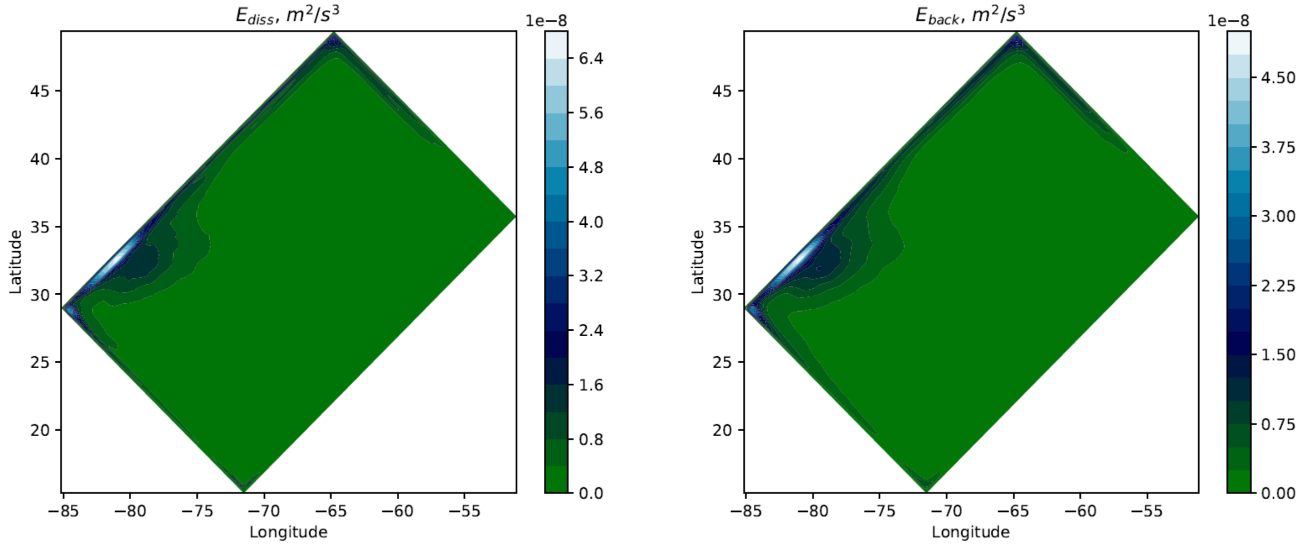
How to choose c_{back}?

$$\nu_2(x, y, \sigma, t) = c_{back}\sqrt{dxdy}\sqrt{\max(2e, 0)}$$

- Parameter c_{back} controls amount of subgrid energy [Jansen 2015]
- Also we may want to optimize subgrid energy lifetime:

$$T = \frac{\langle e \rangle}{\langle \dot{E}_{back} \rangle} \sim \langle e \rangle$$

- For lifetime of interest ($T \approx 30days$), when advection on large distances is possible, the level of subgrid energy is too large
- For realistic level of subgrid energy lifetime is only $T \approx 2days$



The current model does not allow to simulate both long subgrid energy propagation and realistic subgrid energy level

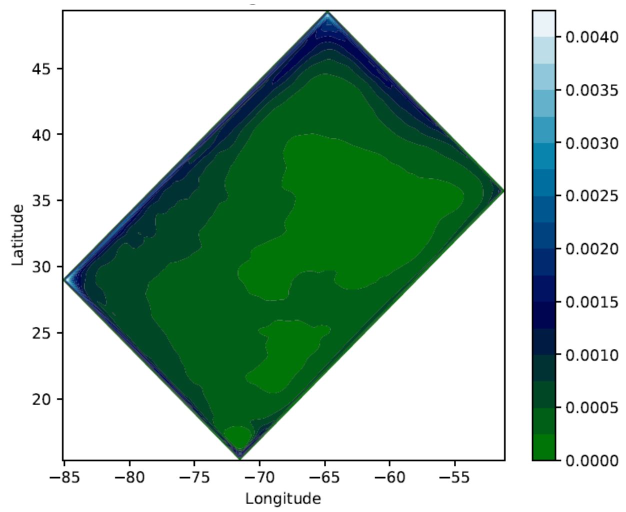
How to calibrate parameters of subgrid energy equation?

- Compare subgrid energy with estimation [Pomraning&Rutland2002]

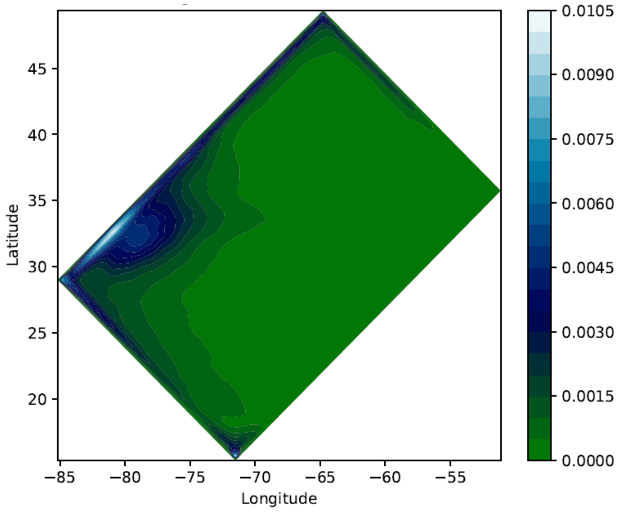
$$e \equiv \frac{1}{2} tr\left(\tau_{ij}\right) \approx \frac{1}{2} tr\left(\tau_{ij}^{VGM}\right) = \frac{\Delta_f^2}{48}\left(|S|^2 + \omega^2\right)$$

- Too much subgrid energy near boundaries?
  - Apply Dirichlet B.C. for subgrid energy and filters, increase R_{diss} in Klöwer formula
- Too much subgrid energy?
  - Increase c_{back}

Simulated e



Estimated e



Stochastic KEB

- Return energy as follows:

$$\frac{\partial \boldsymbol{u}_h}{\partial t} = \begin{bmatrix} f_x(x, y, \sigma, t) \\ f_y(x, y, \sigma, t) \end{bmatrix}$$

f_x, f_y – stochastic fields

- Amount of returning energy:

$$\tau\left(\langle f_x^2 \rangle + \langle f_y^2 \rangle\right)$$

T_decorr=86400.

where τ – correlation time

- Desired properties:
  - Divergence-free
  - Correlated in space
  - Prescribed energy input: $\dot{E}_{source}$

Stochastic KEB. Generation of noise with desired properties

- The proposed form for stochastic field:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \nabla_\sigma \times (\psi \boldsymbol{k})$$

- Where streamfunction is given by:

$$\psi(x, y, \sigma) = F^n\big(A(x, y, \sigma)\phi(x, y)\big)$$

nstoch=6

$\phi(x, y)$ – grid white noise $N(0,1)$

- According to O'Neil "A new stochastic backscatter model for large-eddy simulation of neutral atmospheric flows" 2016:

$$\langle f_x^2 \rangle + \langle f_y^2 \rangle \approx A^2 \cdot 2\rho_0(1 - \rho_1)\left(\frac{1}{dx^2} + \frac{1}{dy^2}\right)$$

- Final formula for A:

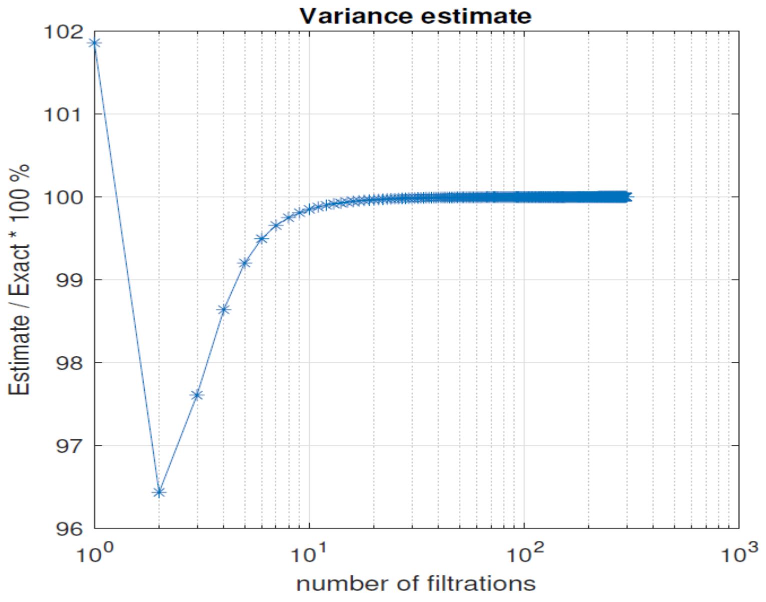
$$A(x, y, \sigma) = \sqrt{\dot{E}_{source}}\left(2\tau\rho_0(1 - \rho_1)\left(\frac{1}{dx^2} + \frac{1}{dy^2}\right)\right)^{-1/2}$$

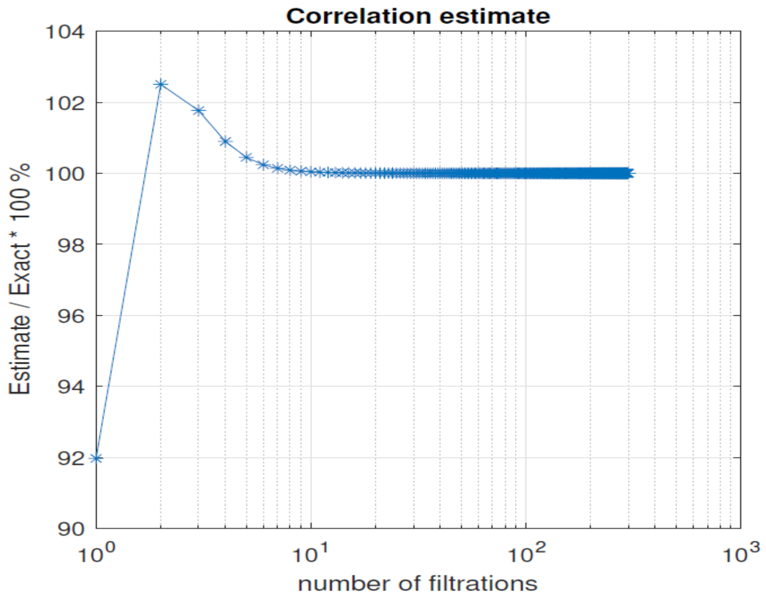
Moments of the filter

- For filter F^n define the moments:

$$\rho_0 = \sum_{i,j} w_{i,j}^2 \to \frac{1}{n\pi}$$

$$\rho_1 = \frac{\sum_{i,j} w_{i,j} w_{i+1,j}}{\rho_0} \to \exp\left(-\frac{1}{n}\right)$$





Correlation in time

- Numerical integration with modified Leap-Frog scheme
- Introduce time correlation without changing variance [Schumann1995]:

$$\hat{\psi}^{n+1} = \left(1 - \frac{dt}{\tau}\right) \hat{\psi}^{n} + \left(\frac{dt}{\tau}\left(2 - \frac{dt}{\tau}\right)\right)^{1/2} \psi$$

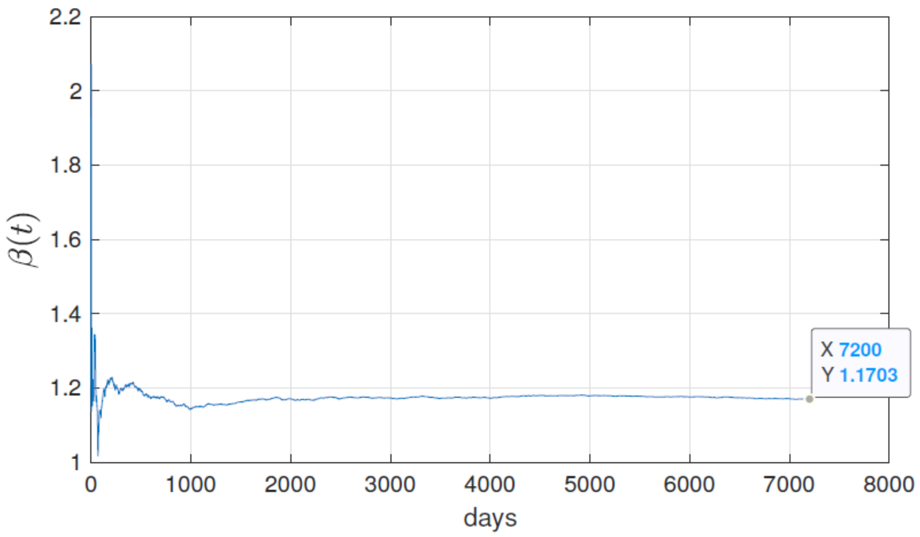
- A posteriori correction:

$$\beta(t) = \frac{\langle \dot{E}_{source} \rangle_{space \times time}}{\langle \hat{\boldsymbol{f}} \cdot \boldsymbol{u}_h \rangle_{space \times time}}$$

- Final tendency:

$$\frac{\partial \boldsymbol{u}_h}{\partial t} = \beta(t) \begin{bmatrix} \hat{f}_x \\ \hat{f}_y \end{bmatrix}$$

Correction increases input +37%



$$\frac{dE}{dt} \overset{!}{\approx} \tau\left(\langle f_x^2 \rangle + \langle f_y^2 \rangle\right)$$

How to be sure that in complex configuration the code works as expected? KEB_test

- Reference values for some integrals provided

```
! ------------------------- diffusion ------------------------ !
conservation:
no-flux  (1e-20 ok): -1.064180091878790E-021
Dirichlet (1e-4 ok): -1.252268632356181E-004

positiveness:
min ok:   1.850456085375350E-007  1.856420384649364E-007
max ok:   8.217884550423646E-002  8.206124773202066E-002

! -------------------------- z-filter ------------------------ !
positiveness:
min ok:   1.850456085375350E-007  1.850865339105703E-007
max ok:   8.217884550423646E-002  8.066954583786658E-002
conservation  (1e-20 ok): -7.289300561377718E-020


! --------------------- local c_diss -------------------------- !
min value for c_diss (0 ok):  0.220658658338855
max value for c_diss (1 ok):  0.999355240226306

! ------------------- laplace filter ------------------------- !
filter_laplace_T3D_ntimes, no-flux:
conservation  (1e-20 ok): -9.063963607859050E-022
min ok:   9.955149248888630E-013  8.190910945193417E-012
max ok:   1.660738531266926E-006  9.642326610140910E-007
```

Results.

$$\frac{\partial T}{\partial t} + \boldsymbol{U} \cdot \nabla T = F_T, \frac{\partial S}{\partial t} + \boldsymbol{U} \cdot \nabla S = F_S$$

$$\rho = \rho_0(1 - a(T - T_0) + b(S - S_0))$$

$$\frac{\partial p}{\partial z} = -\rho g$$

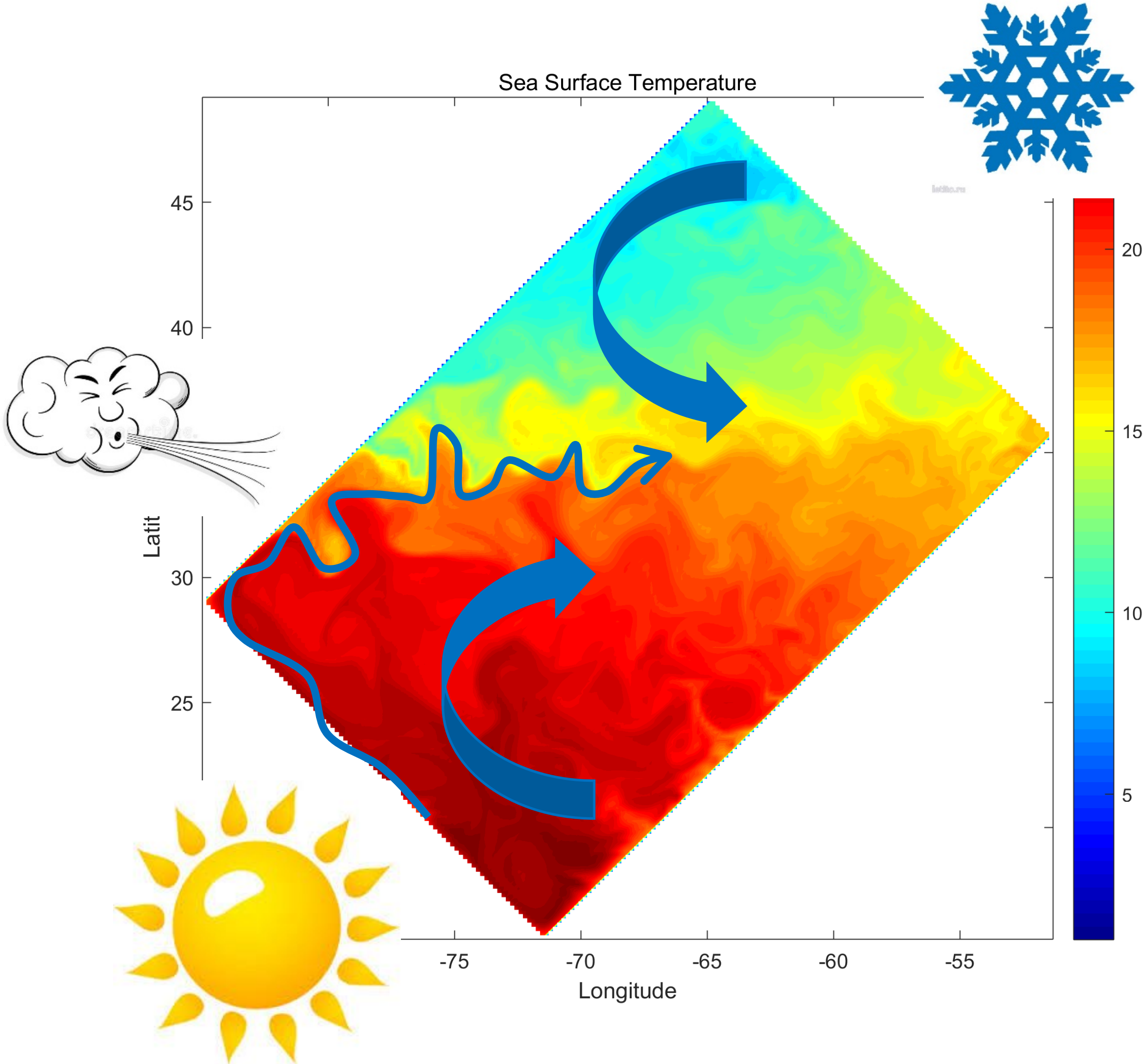
$$\frac{\partial \boldsymbol{U}_h}{\partial t} + \boldsymbol{U} \cdot \nabla \boldsymbol{U}_h + \mathbf{cor}_h = -\frac{1}{\rho_0} \nabla_h p + F_U$$

$$\nabla \cdot \boldsymbol{U} = 0$$

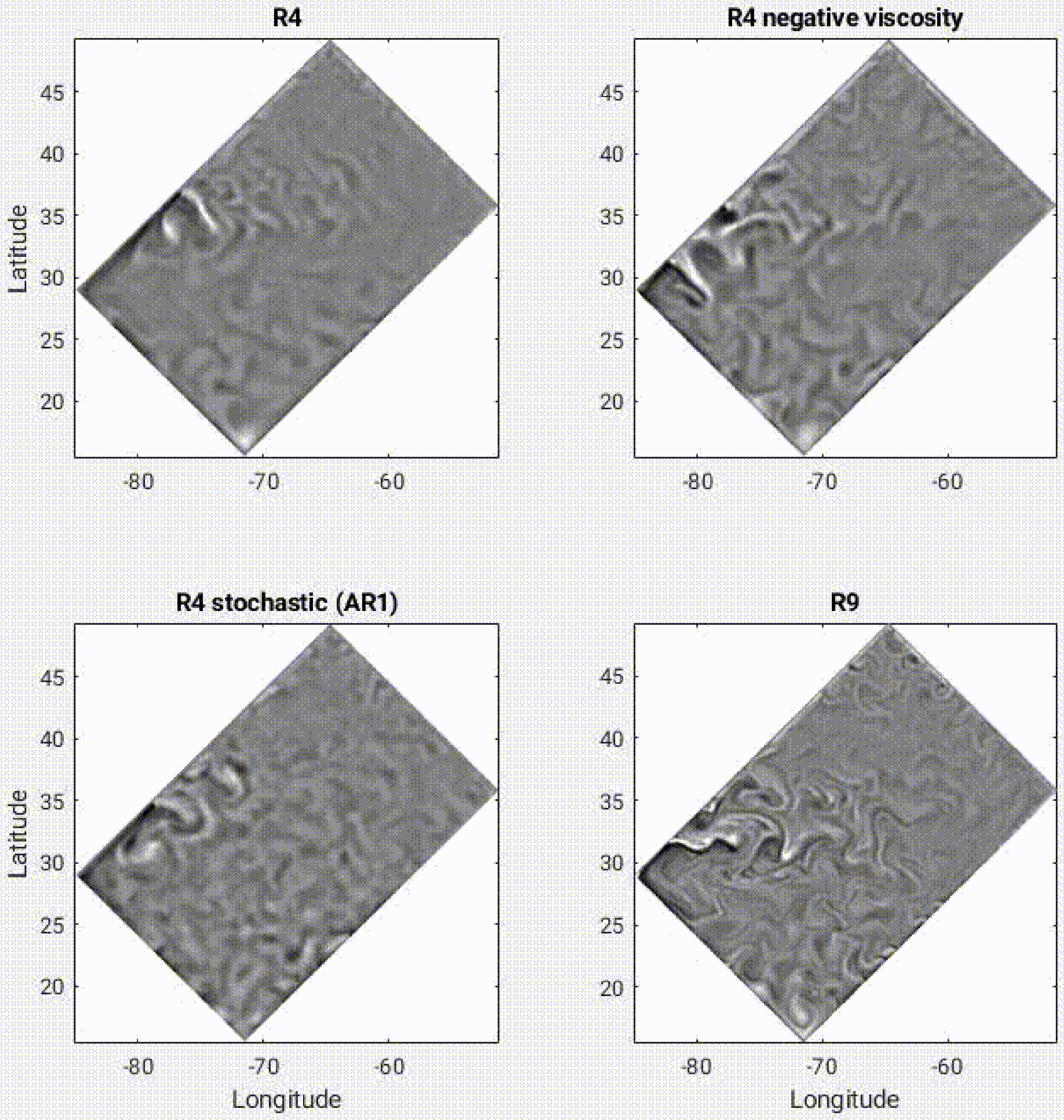
$$\frac{\partial \eta}{\partial t} = -H \nabla_h \cdot (\overline{\boldsymbol{U}_h})$$

$T, S, \boldsymbol{U}, \boldsymbol{U_h}, \overline{\boldsymbol{U_h}}, p, \eta, \rho, H$ – temperature; salinity; full, horizontal and vertically-averaged velocity; pressure; sea surface height; density; depth.

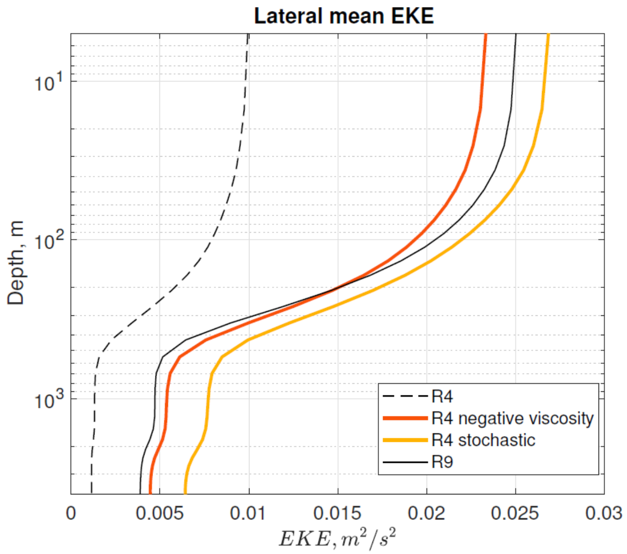
Lateral B.C.: zero heat and salinity flux, free-slip

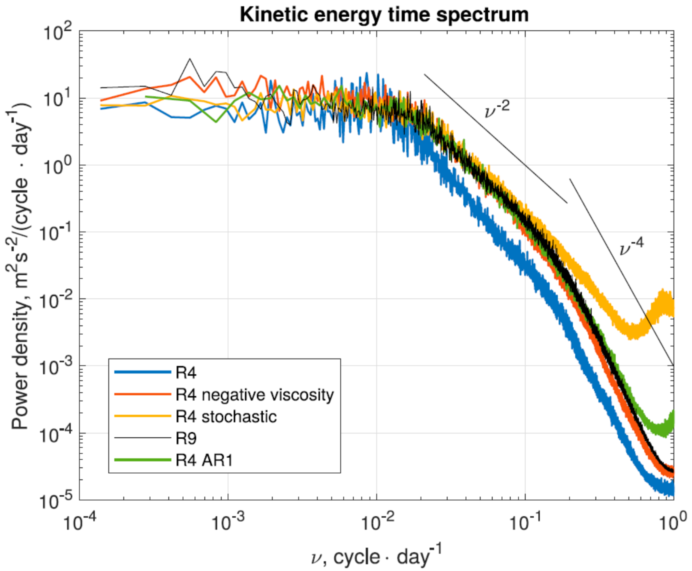


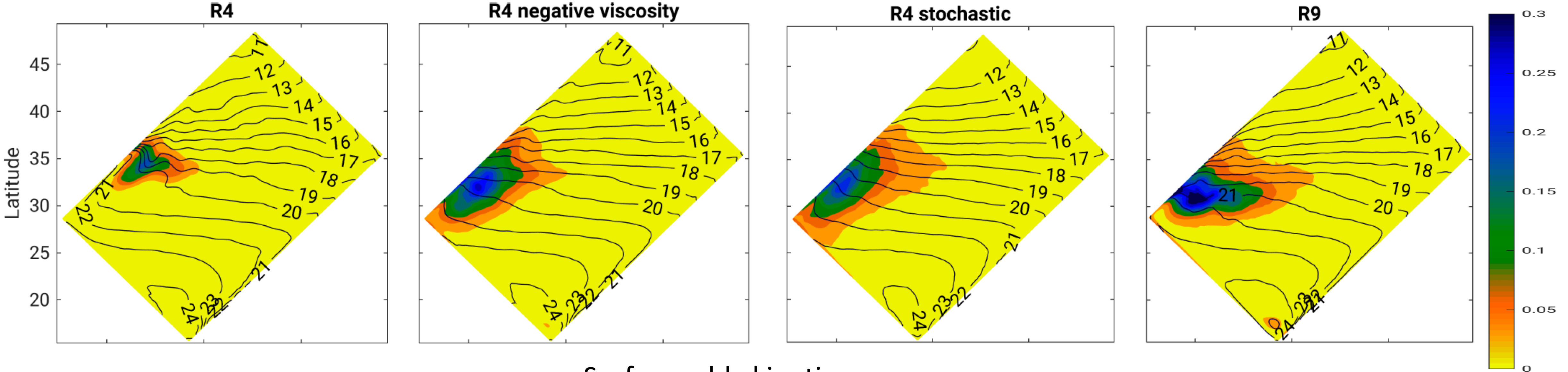
Surface relative vorticity in f (Coriolis) units



Eddy kinetic energy



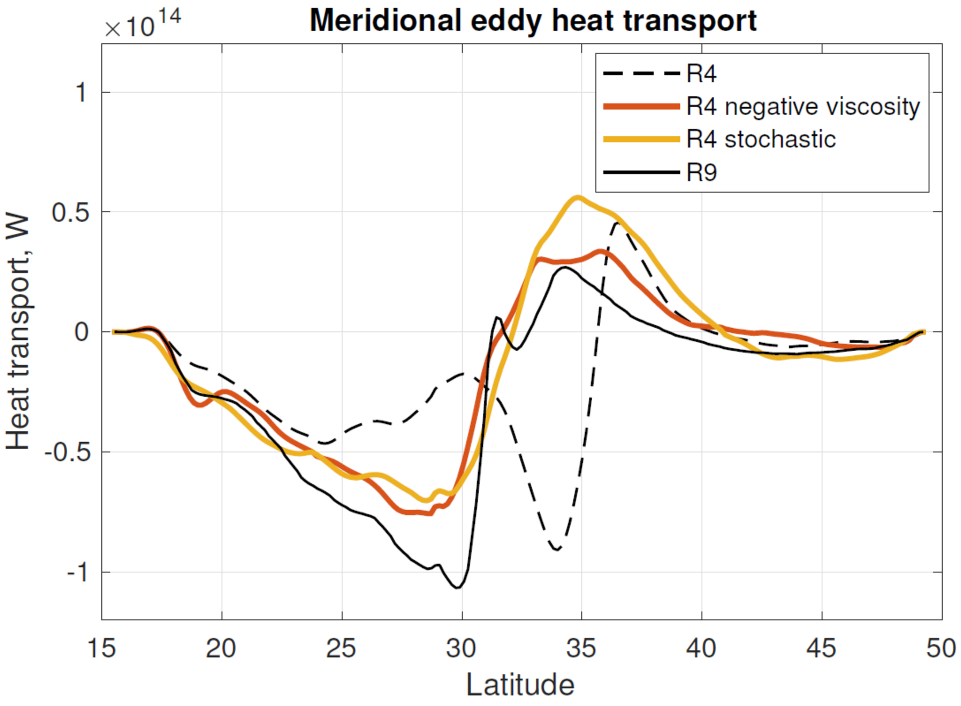


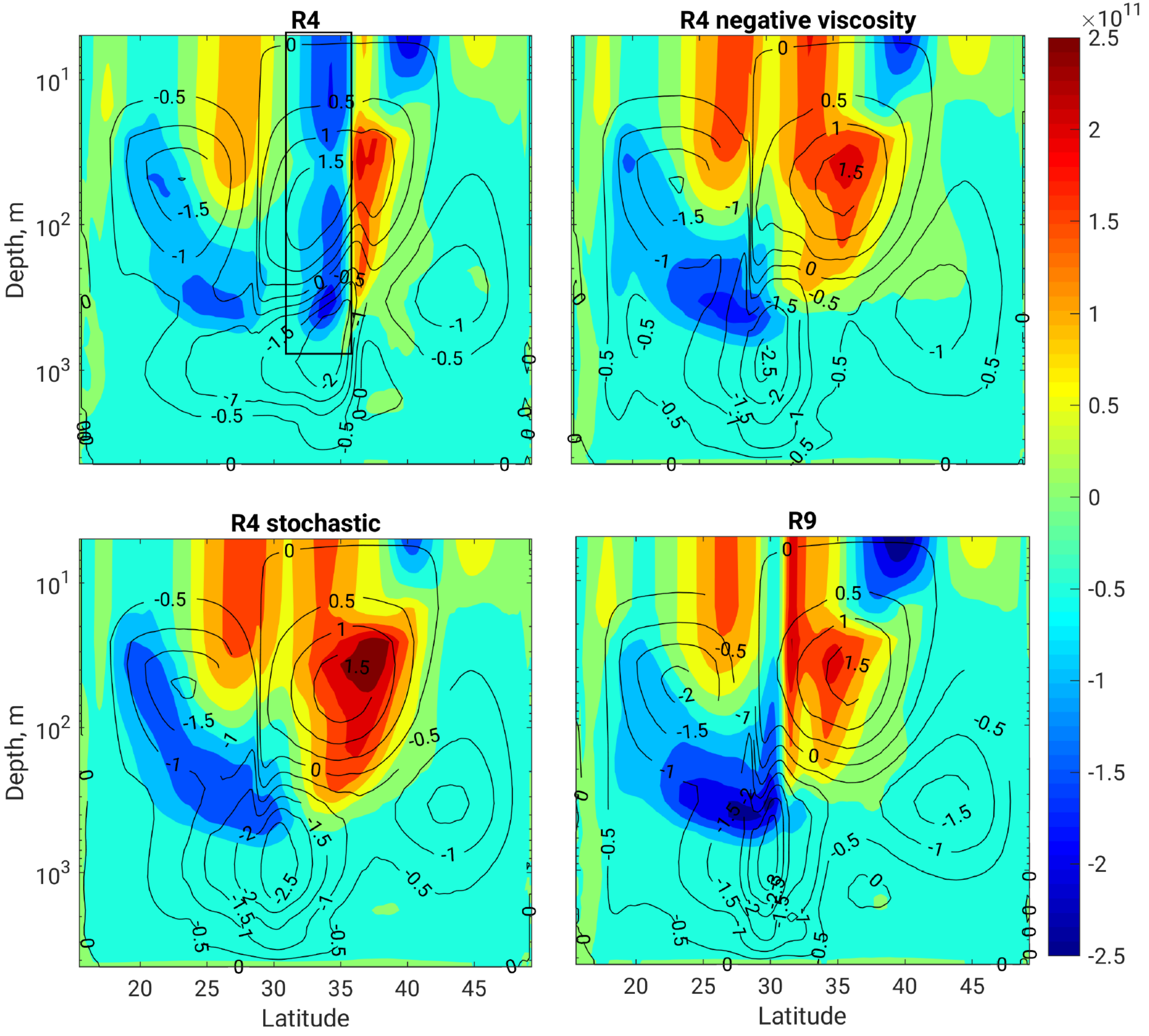


Surface eddy kinetic energy

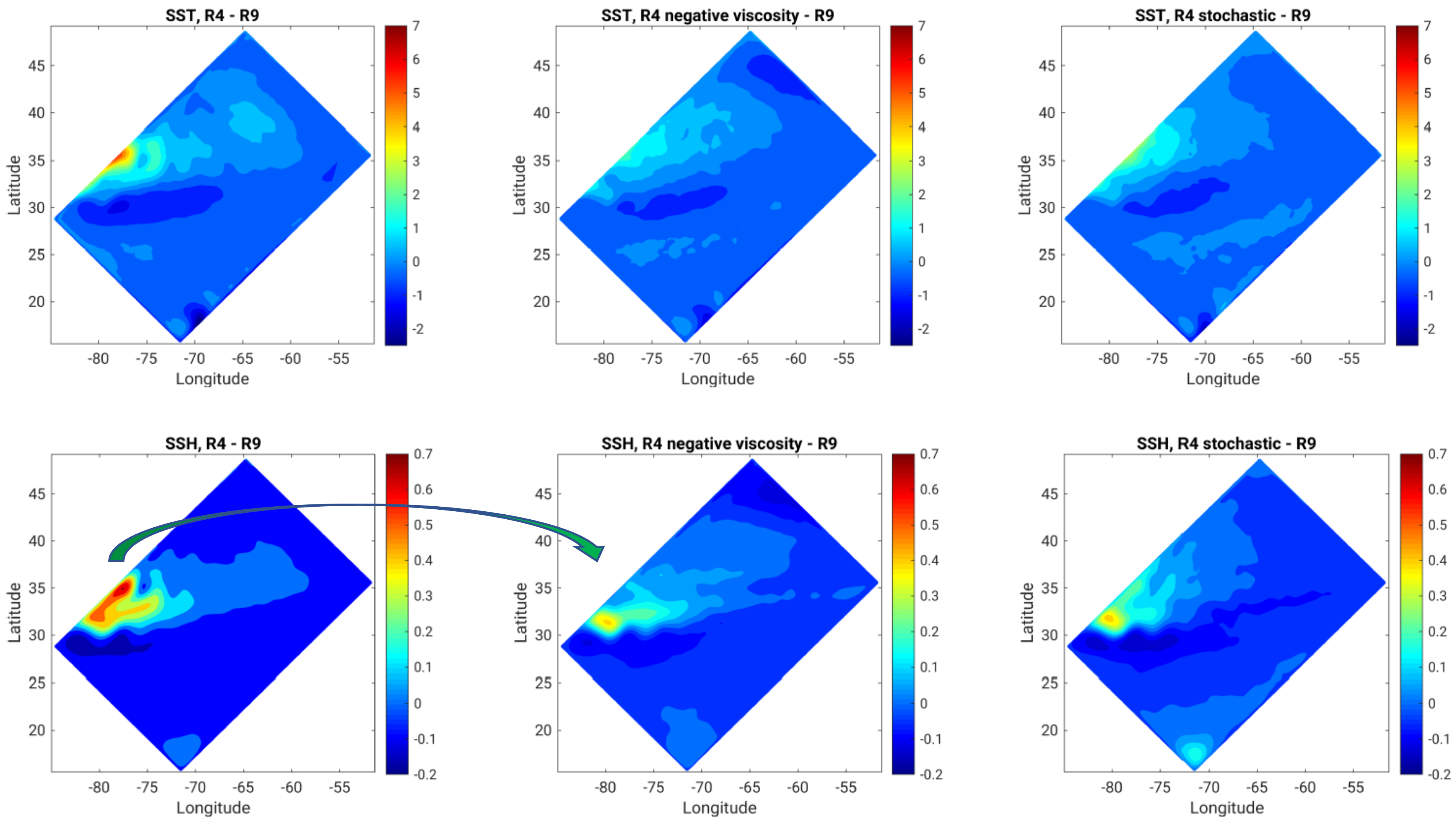
Meridional eddy heat flux and MOC

- The latitude of extreme value shifts southward
- Eddy heat flux at the surface is closer to reference
- Bottom MOC cell shifts southwards





An error in averaged fields is reduced in WBC separation region

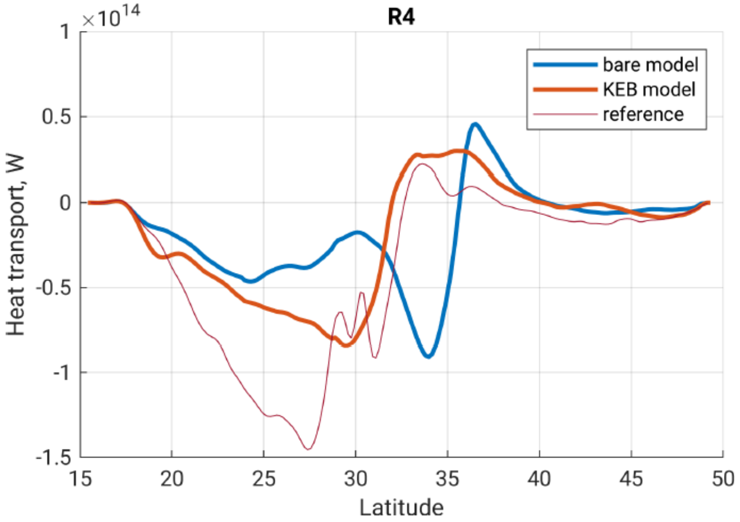


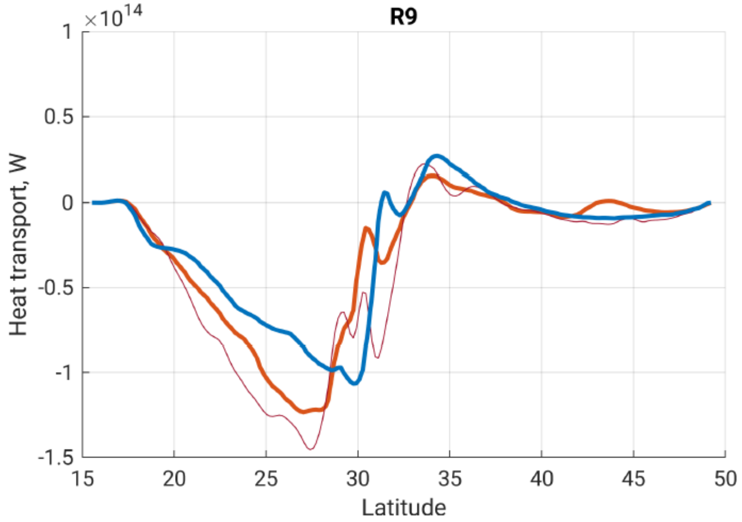
Negative viscosity at higher resolutions

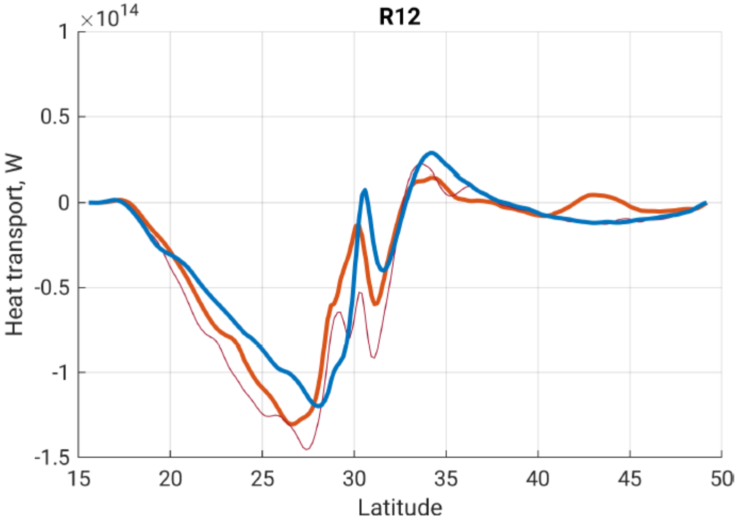
- Integral eddy heat flux is better reproduced

max norm | L2 norm

	SST, C^o	SSH, м	SSS, psu
R4	8.8; 0.54	0.75; 0.1	0.65; 0.16
R4 negvisc	5.3; 0.45	0.63; 0.08	0.45; 0.15
R9	3.8; 0.22	0.6; 0.04	0.37; 0.06
R9 negvisc	2.15; 0.2	0.18; 0.03	0.26; 0.06
R12	2.0; 0.13	0.21; 0.024	0.22; 0.04
R12 negvisc	1.1; 0.17	0.24; 0.03	0.21; 0.06







Conclusions

- Two types of kinetic energy backscatter parameterizations are implemented in NEMO 3.6 model
- They work in curvilinear coordinates and generalized vertical coordinate
- They can be used in complex configurations at eddy-permitting resolution
- The implementation is the most close to Juricke2019 and Berner2009, and a little less close to Jansen2019 and Bachman2019