Assignment-2

EE:1205 Signals and systems
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I. Question 1.2.4

How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

II. Solution

Parameter	Description	Value
x(0)	First Term	9
d	Common Difference	8

TABLE 0
Parameter Table

$$x(n) = (9 + 8n)(u(n)) \tag{1}$$

$$X(z) = \frac{9}{1 - z^{-1}} + \frac{8z^{-1}}{(1 - z^{-1})^2} \quad \textbf{(??)} \tag{2}$$

$$X(z) = \frac{6}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$Y(z) = X(z)\, U(z) \tag{5}$$

$$= \frac{9}{(1 - z^{-1})^2} + \frac{8}{(1 - z^{-1})^3}, \quad |z| > 1 \tag{6}$$

Using contour integration to find the inverse Z-transform:

$$\implies y(n) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{n-1}\, dz \tag{7}$$

$$= \frac{1}{2\pi j} \oint_C \frac{9z^{n+1}}{(z-1)^2} + \frac{8z^n}{(z-1)^3}\, dz \tag{8}$$

Let us take $y(n) = y_1(n) + y_2(n)$
We know,

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{9}$$

For $y_1(n)$ we have a two time repeating pole at z = 1

$$= \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{9z^{n+1}}{(z-1)^2}\right) \tag{10}$$

$$= \lim_{z \to 1} \frac{d}{dz} \left(9z^{n+1}\right) \tag{11}$$

$$= 9(n+1) \tag{12}$$

and for $y_2(n)$ we have a three time repeating pole at z = 1

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{8z^{n+1}}{(z-1)^3}\right) \tag{13}$$

$$= \frac{1}{2} \lim_{z \to 1} \frac{d^2}{dz^2} \left(8z^{n+1}\right) \tag{14}$$

$$= 4n(n+1) \tag{15}$$

Adding both of the above equations,

$$y(n) = 9(n+1) + 4n(n+1) \tag{16}$$

$$y(n) = 4n^2 + 13n + 9 \tag{17}$$

We know $y(n) = 636$
$\therefore$

$$636 = 4n(n+1) + 9(n+1) \tag{18}$$

$$4n^2 + 13n - 627 = 0 \tag{19}$$

On solving the quadratic equation, we get the positive value of n as 11
$\therefore$ there are 12 terms in the AP

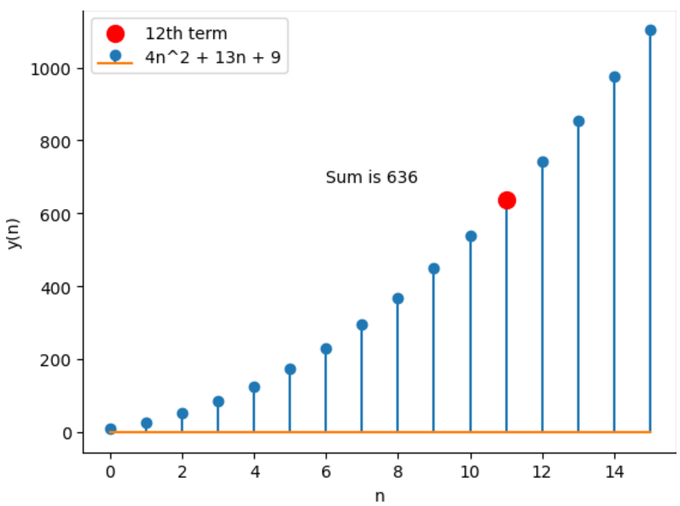

Fig. 0. Plot of y(n) vs n