Assignment-3

EE:1205 Signals and systems
Indian Institute of Technology, Hyderabad

Sai Preetam Umesh Sasankota
EE23BTECH11221

I. Question 1.2.4

$1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + ...$

II. Solution

Parameter	Description	Value
n	Integer	$... -2, -1, 0, 1, 2...$
$x(n)$	General term of sequence	$\left(n^3 + 3n^2 + 2n\right).u(n)$
$y(n)$	Sum of the terms	?
$U(z)$	z-transform of $u(n)$	$\frac{1}{1-z^{-1}}, \lvert z\rvert > 1$

TABLE 0
Values

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{1}$$

$$= \sum_{n=-\infty}^{\infty} \left(n^3 + 3n^2 + 2n\right) u(n) z^{-n} \tag{2}$$

$$= \sum_{n=-\infty}^{\infty} n^3 u(n) z^{-n} + 3 \sum_{n=-\infty}^{\infty} n^2 u(n) z^{-n} + 2 \sum_{n=-\infty}^{\infty} nu(n) z^{-n} \tag{3}$$

Using known results of z-transform:

$$nu(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}}{\left(1-z^{-1}\right)^2}, \quad |z| > |1| \tag{4}$$

$$n^2 u(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^3}, \quad |z| > |1| \tag{5}$$

$$n^3 u(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^2}, \quad |z| > |1| \tag{6}$$

Hence:

$$X(z) = \frac{6z^-1}{\left(1-z^{-1}\right)^4}, \quad |z| > |1| \tag{7}$$

We know:

$$y(n) = x_1(n) * u(n) \tag{8}$$

$$Y(z) = X_1(z) U(z) \tag{9}$$

$$= \frac{6z^{-1}}{\left(1-z^{-1}\right)^5}, |z| > 1 \tag{10}$$

Using partial fractions to form z-transform pairs:

$$Y(z) = \frac{6z^{-1}}{\left(1-z^{-1}\right)} + \frac{24z^{-2}}{\left(1-z^{-1}\right)^2} + \frac{36z^{-3}}{\left(1-z^{-3}\right)^3}, \tag{11}$$

$$+ \frac{24z^{-4}}{\left(1-z^{-1}\right)^4} + \frac{6z^{-5}}{\left(1-z^{-1}\right)^5}, |z| > |1| \tag{12}$$

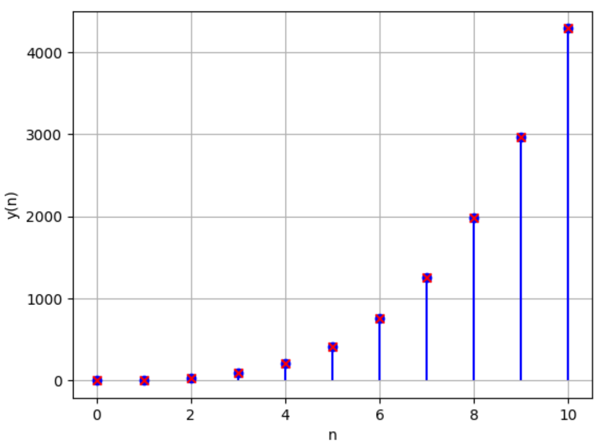
Substituting results of inverse z-transform:

$$y(n) = \frac{n^4 + 6n^3 + 11n^2 + 6n}{4} u(n) \tag{13}$$

$$= \frac{n(n+1)(n+2)(n+3)}{4} u(n) \tag{14}$$

Hence the sum of n terms of the above series is

$$y(n) = \frac{n(n+1)(n+2)(n+3)}{4} \tag{15}$$


4000
3000
2000
1000
0
y(n)
0
2
4
6
8
10
n