

Assignment-3

EE:1205 Signals and systems
Indian Institute of Technology, Hyderabad

Sai Preetam Umesh Sasankota
EE23BTECH11221

I. QUESTION 11.9.4.2

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \dots$$

II. SOLUTION

Parameter	Description	Value
n	Integer	$\dots -2, -1, 0, 1, 2, \dots$
$x(n)$	General term of sequence	$(n^3 + 3n^2 + 2n) \cdot u(n)$
$y(n)$	Sum of the terms	?
$U(z)$	z-transform of $u(n)$	$\frac{1}{1-z^{-1}}, z > 1$

TABLE 0
VALUES

We know:

$$y(n) = x(n) * u(n) \quad (8)$$

$$Y(z) = X(z) * U(z) \quad (9)$$

$$= \frac{6z^{-1}}{(1-z^{-1})^5}, |z| > |1| \quad (10)$$

Using partial fractions to form z-transform pairs:

$$Y(z) = \frac{6z^{-1}}{(1-z^{-1})} + \frac{24z^{-2}}{(1-z^{-1})^2} + \frac{36z^{-3}}{(1-z^{-1})^3}, \quad (11)$$

$$+ \frac{24z^{-4}}{(1-z^{-1})^4} + \frac{6z^{-5}}{(1-z^{-1})^5}, |z| > |1| \quad (12)$$

Substituting results of inverse z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (1)$$

$$= \sum_{n=-\infty}^{\infty} (n^3 + 3n^2 + 2n) u(n) z^{-n} \quad (2)$$

$$= \sum_{n=-\infty}^{\infty} n^3 u(n) z^{-n} + 3 \sum_{n=-\infty}^{\infty} n^2 u(n) z^{-n} + 2 \sum_{n=-\infty}^{\infty} n u(n) z^{-n} \quad (3)$$

$$y(n) = \frac{n^4 + 6n^3 + 11n^2 + 6n}{4} u(n) \quad (13)$$

$$= \frac{n(n+1)(n+2)(n+3)}{4} u(n) \quad (14)$$

Hence the sum of n terms of the above series is

$$y(n) = \frac{n(n+1)(n+2)(n+3)}{4} \quad (15)$$

Using known results of z-transform:

$$nu(n) \xleftrightarrow{z} \frac{z^{-1}}{(1-z^{-1})^2}, \quad |z| > |1| \quad (4)$$

$$n^2 u(n) \xleftrightarrow{z} \frac{z^{-1}(z^{-1} + 1)}{(1-z^{-1})^3}, \quad |z| > |1| \quad (5)$$

$$n^3 u(n) \xleftrightarrow{z} \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1-z^{-1})^4}, \quad |z| > |1| \quad (6)$$

Hence:

$$X(z) = \frac{6z^{-1}}{(1-z^{-1})^4}, \quad |z| > |1| \quad (7)$$

