GATE 2022 BIOMEDICAL ENGINEERING

EE:1205 Signals and systems
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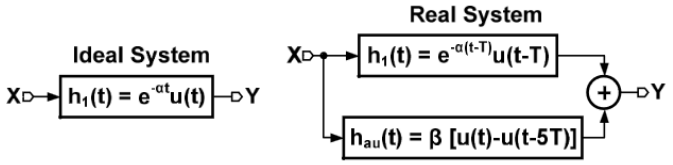
I. Question 40

The block diagrams of an ideal system and a real system with their impulse responses are shown below. An auxiliary path is added to the delayed impulse response in the real system.

For a unit impulse input ($x(t) = \delta(t)$) to both systems, gain β is chosen such that $y(4T)$ is same for both systems. The value of β is:



(A) $e^{-3\alpha T}\left(1 - e^{-2\alpha T}\right)$

(B) $-e^{-\alpha T}\left(1 - e^{-3\alpha T}\right)$

(C) $e^{-3\alpha T}\left(1 - e^{-\alpha T}\right)$

(D) $e^{-2\alpha T}\left(1 - e^{-2\alpha T}\right)$

II. Solution

Let the output of the ideal system be y_I and output of the real system be y_R

$$y_I = e^{-\alpha t}u(t) \tag{1}$$

$$y_R = \left[\beta(u(t) - u(t-5T)) + e^{-\alpha(t-T)}u(t-T)\right] \tag{2}$$

At time t = 4T, both the signals are equal (for a unit impulse)

If $t > 0$:

$$u(t) = 1 \tag{3}$$

Else:

$$u(t) = 0 \tag{4}$$

For both signals to be equal:

$$e^{-\alpha t}u(t) = \left[\beta(u(t) - u(t-5T)) + e^{-\alpha(t-T)}u(t-T)\right] \tag{5}$$

Putting t = 4T:

$$e^{-\alpha 4T} = \beta + e^{-\alpha 3T} \tag{6}$$

$$\implies \beta = e^{-3\alpha T}\left(1 - e^{-\alpha T}\right) \tag{7}$$

Hence the answer is (C)