GATE 2022 BIOMEDICAL ENGINEERING

EE:1205 Signals and systems
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I. Question 37

Solution of the differential equation

$$\frac{dy}{dx} - y = cosx \tag{1}$$

is

(A) y = $\frac{sinx-cosx}{2} + ce^{x}$
(B) y = $\frac{sinx+cosx}{2} + ce^{x}$
(C) y = $\frac{sinx-cosx}{2} + ce^{-x}$
(D) y = $\frac{sinx+cosx}{2} + ce^{-x}$

II. Solution

This is a linear differential equation, where every dependent variable and every derivative occurs in the first degree.

The general form of this type of equation is:

$$\frac{dy}{dx} - Py = Q \tag{2}$$

And the general solution of this equation is given by:

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c \tag{3}$$

Comparing both the equations,

$$P = -1 \tag{4}$$
$$Q = cosx \tag{5}$$

Solving:

$$ye^{\int -dx} = \int cosx\, e^{\int -dx}dx + c \tag{6}$$
$$y\, e^{-x} = \int cosx\, e^{-x}dx + c \tag{7}$$
$$y\, e^{-x} = \frac{e^{-x}}{2}(sinx - cosx) + c \tag{8}$$
$$\tag{9}$$
$$\implies y = \frac{sinx + cosx}{2} + ce^{x} \tag{10}$$

Hence the solution of this question is (B)