GATE 2022 BIOMEDICAL ENGINEERING

EE:1205 Signals and systems

Indian Institute of Technology, Hyderabad

Sai Preetam Umesh Sasankota
EE23BTECH11221

I. Question 37

Solution of the differential equation

$$\frac{dy}{dx} - y = cosx \tag{1}$$

is

(A) y = $\frac{sinx-cosx}{2} + ce^{x}$
(B) y = $\frac{sinx+cosx}{2} + ce^{x}$
(C) y = $\frac{sinx-cosx}{2} + ce^{-x}$
(D) y = $\frac{sinx+cosx}{2} + ce^{-x}$

II. Solution

This is a linear differential equation, where every dependent variable and every derivative occurs in the first degree.

The general form of this type of equation is:

$$\frac{dy}{dx} - Py = Q \tag{2}$$

We will be solving this question using Laplace Transforms:

First take Laplace transform at both sides of the equation:

$$\mathcal{L}\left\{\frac{dy}{dx} - y\right\} = \mathcal{L}\{cosx\} \tag{3}$$

$$sY(s) - y(0) - Y(s) = \frac{s}{s^2+1} \tag{4}$$

$$Y(s) = \frac{s}{(s^2+1)(s-1)} + \frac{y(0)}{s-1} \tag{5}$$

Now we compute the inverse Laplace transform at both sides of the equation:

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)(s-1)}\right\} + \mathcal{L}^{-1}\left\{\frac{y(0)}{s-1}\right\} \tag{6}$$

Using results:

$$y(x) = \frac{1}{2}(sinx - cosx + e^{x}) + y(0)\, e^{x} \tag{7}$$

$$y(x) = \frac{sinx - cosx}{2} + e^{x}\left(y(0) + \frac{1}{2}\right) \tag{8}$$

Let

$$\left(y(0) + \frac{1}{2}\right) = c \tag{9}$$

$$\implies y = \frac{sinx - cosx}{2} + ce^{x} \tag{10}$$

Hence the answer is (A)