Example 1

We will explore the issue of cycling of the simplex method using the following example:

$$\begin{aligned} &min && c^T x \\ &\text{St} && Ax = b \\ & && x \geq 0 \end{aligned}$$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{5,6,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (0,0,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 1 and 3 select $k = 1$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = (-0.25, -0.5, 0)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min\left\{\frac{x_i^*}{-d_i}\right\}$ and select r → t $= \min\left(\frac{0}{0.25}, \frac{0}{0.5}\right) = 0$ select $r = 5$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

→ $t = 0$ so no change to BFS

Our new basis B' is made by taking B adding k and taking out r. → $B' = \{1, 6, 7\}$

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Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (-3,0,0)$

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Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = (32, -4, 0)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{ \frac{x_i^*}{-d_i} \right\}$ and select r → t $= \min \left(\frac{0}{4} \right) = 0$ select $r = 6$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

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<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{1,2,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (-1,-1,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 3 select $k = 3$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = \left(-8, -\frac{3}{8}, -1\right)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{\frac{x_i^*}{-d_i}\right\}$ and select r → $t = \min\left(\frac{0}{8}, \frac{0}{3/8}, \frac{1}{1}\right) = 0$ select $r = 1$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

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Our new basis B' is made by taking B adding k and taking out r. → $B' = \{2,3,7\}$

Example 1

We will explore the issue of cycling of the simplex method using the following example:

$$\begin{array}{ll} min & c^T x \\ \text{St} & Ax = b \\ & x \geq 0 \end{array}$$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{2,3,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (2,-3,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 4, 5 select $k = 4$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = \left(-\frac{3}{16}, \frac{21}{2}, -\frac{21}{2}\right)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{\frac{x_i^*}{-d_i}\right\}$ and select r → $t = \min\left(\frac{0}{3/16}, \frac{1}{21/2}\right) = 0$ select $r = 2$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

→ $t = 0$ so no change to BFS

Our new basis B' is made by taking B adding k and taking out r. → $B' = \{3, 4, 7\}$

Example 1

We will explore the issue of cycling of the simplex method using the following example:

$$\begin{array}{ll} min & c^T x \\ \text{St} & Ax = b \\ & x \geq 0 \end{array}$$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{3,4,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (1, -1, 0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 1, 5 select $k = 5$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = \left(-2, -\frac{2}{3}, -2\right)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{\frac{x_i^*}{-d_i}\right\}$ and select r → $t = \min\left(\frac{0}{2}, \frac{0}{2/3}, \frac{1}{2}\right) = 0$ select $r = 3$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

→ $t = 0$ so no change to BFS

Our new basis B' is made by taking B adding k and taking out r. → $B' = \{4, 5, 7\}$

Example 1

We will explore the issue of cycling of the simplex method using the following example:

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$min \qquad c^T x$

St $\qquad Ax = b$

$\qquad x \geq 0$

<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{4,5,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ $\qquad \rightarrow\ y^* = (0,2,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k $\qquad \rightarrow$ Fails 1, 6 $\qquad$ select $k = 6$

Step 3 $\quad$ **a):** Solve: $A_B d_B = -A_k$ $\qquad \rightarrow d_B = \left(-\frac{1}{3}, 3, 0\right)$

b) All d's ≥ 0 means unbounded otherwise. $\qquad \rightarrow$ Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{\frac{x_i^*}{-d_i}\right\}$ and select r $\qquad \rightarrow$ t $= \min\left(\frac{0}{1/3}\right) = 0$ $\qquad$ select $r = 4$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

$\rightarrow t = 0$ so no change to BFS

Our new basis B' is made by taking B adding k and taking out r. $\qquad \rightarrow B' = \{5,6,7\}$ <u>**Note this was our first basis!!!**</u>

Definition: Cycling

We say an algorithm Cycles when it gets into (albeit unintentional) loop.

From our last example, we see that we could end up in an infinite loop if we continued to select k and r in the same way.

Theorem: Bland's Anti-cycling Rule

When running the simplex method, if we ever have a choice of k and/or r, and we always choose k and r to be the smallest index, then we will never cycle.

This is an amazingly simple rule that will always work

Why does this work?

This proof is a bit complicated, and we will actually do the proof (in the proofs section) after seeing the Simplex Tableau so that it will be a bit easier to follow.

For now, we will simply use the rule to avoid cycling.

One thing to note about this rule, even though it prevents cycling, it is not always the most efficient rule to use (and thus can take longer than other rules of switching to different Basic Feasible Solutions). In practice, other "faster pivoting rules" are implemented that might cycle (but rarely do and we can fall on Bland's rule simultaneously where needed).

Strategy: Simplex Method Without Cycling

How To Use It	When To Use It	Why It Works
Start with any BFS x*: (with $x^*_B > 0$) **Step 1:** Find the potential dual solution by solving ${A_B}^T y^* = c_B$ using RREF **Step 2:** See if it is feasible by comparing all constraints not in B in the dual by checking to see if ${A_N}^T y^* \leq c_N$ holds. If it satisfies all of the constraints then x* is optimal, if it doesn't, we will have (at least one) inequality of the form: ${A_k}^T y^* > c_k$. Select **the smallest index** from one of these contradicting inequalities. **Step 3:** Set x'_k = t and solve $x'_B = x^*_B - {A_B}^{-1} A_k t \geq 0$ so that t is as large as possible. To solve this without inverses we should do the following: a) let $d_B = -{A_B}^{-1} A_k$ and solve for d_B by solving: $A_B d_B = -A_k$ b) if $d_B \geq 0$, we can choose t→∞. Stop as the problem is unbounded. c) otherwise, of any $d_B < 0$ choose t = min{x^*_i/-d_i} and **choose the smallest r** that represent the index that produces the minimum t. **Step 4:** Our new BFS is x' where we change $x'_B = x^*_B + td_B$ and change x'_k = t, the rest is unchanged (ie is still 0). Our new basis B' is made by taking B adding k and taking out r. Go to step 1 to see if this new BFS is optimal.	When we want to determine the optimal solution of a linear program or if a linear program is unbounded.	This is simply the simplex method with the new theorem to avoid cycling.

Example 2

Run the simplex method using Bland's Anti-Cycling Rule:

$min \quad c^T x$

St $\quad Ax = b$

$\quad x \geq 0$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{5,6,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ $\quad \rightarrow \; y^* = (0,0,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k $\quad \rightarrow$ Fails 1 and 3 $\quad$ select $k = 1$

Step 3 $\quad$ **a):** Solve: $A_B d_B = -A_k$ $\quad \rightarrow d_B = (-0.25, -0.5, 0)$

b) All d's ≥ 0 means unbounded otherwise. $\quad \rightarrow$ Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{ \frac{x_i^*}{-d_i} \right\}$ and select r $\quad \rightarrow$ t $= \min\left(\frac{0}{0.25}, \frac{0}{0.5}\right) = 0$ $\quad$ select $r = 5$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

$\rightarrow t = 0$ so no change to BFS

Our new basis B' is made by taking B adding k and taking out r. $\quad \rightarrow B' = \{1, 6, 7\}$

Example 2

We will explore the issue of cycling of the simplex method using the following example:

$$\begin{array}{ll} min & c^T x \\ \text{St} & Ax = b \\ & x \geq 0 \end{array}$$

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<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{1,6,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (-3,0,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 2, 3 $\boxed{\text{select } k = 2}$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = (32, -4, 0)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{ \frac{x_i^*}{-d_i} \right\}$ and select r → t $= \min \left(\frac{0}{4} \right) = 0$ $\boxed{\text{select } r = 6}$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

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<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{1, 2, 7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (-1, -1, 0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 3

$\boxed{\text{select } k = 3}$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = \left(-8, -\frac{3}{8}, -1\right)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{ \frac{x_i^*}{-d_i} \right\}$ and select r → $t = \min \left(\frac{0}{8}, \frac{0}{3/8}, \frac{1}{1} \right) = 0$

$\boxed{\text{select } r = 1}$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

→ $t = 0$ so no change to BFS

Our new basis B' is made by taking B adding k and taking out r. → $B' = \{2, 3, 7\}$

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We will explore the issue of cycling of the simplex method using the following example:

$$\begin{array}{ll} min & c^T x \\ \text{St} & Ax = b \\ & x \geq 0 \end{array}$$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{2, 3, 7\}$:

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (2, -3,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 4, 5

select $k = 4$

Step 3

a): Solve: $A_B d_B = -A_k$ → $d_B = \left(-\frac{3}{16}, \frac{21}{2}, -\frac{21}{2}\right)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{\frac{x_i^*}{-d_i}\right\}$ and select r → t $= \min\left(\frac{0}{3/16}, \frac{1}{21/2}\right) = 0$

select $r = 2$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$, the rest is unchanged (ie is still 0).

→ $t = 0$ so no change to BFS

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We will explore the issue of cycling of the simplex method using the following example:

$$\begin{array}{ll} min & c^T x \\ \text{St} & Ax = b \\ & x \geq 0 \end{array}$$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>Use BFS = $x^* = (0,0,0,0,0,0,1)$ with $B = \{3, 4,7\}$:</u>

Step 1: Solve: ${A_B}^T y^* = c_B$ → $y^* = (1, -1,0)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 1, 5 select $k = 1$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = \left(\frac{5}{2}, \frac{1}{4}, -\frac{5}{2}\right)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{\frac{x_i^*}{-d_i}\right\}$ and select r → $\text{t} = \min\left(\frac{1}{5/2}\right) = \frac{2}{5}$ select $r = 7$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$:

$$\rightarrow t = \frac{2}{5} \text{ so BFS becomes: } \left(\frac{2}{5}, 0, 0 + \frac{5}{2}\left(\frac{2}{5}\right), 0 + \frac{1}{4}\left(\frac{2}{5}\right), 0, 0, 1 - \left(\frac{5}{2}\right)\left(\frac{2}{5}\right)\right) = \left(\frac{2}{5}, 0, 1, \frac{1}{10}, 0, 0, 0\right)$$

Our new basis B' is made by taking B adding k and taking out r. → $B' = \{1, 3,4\}$

Example 2

We will explore the issue of cycling of the simplex method using the following example:

$$\begin{array}{ll} min & c^T x \\ \text{St} & Ax = b \\ & x \geq 0 \end{array}$$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Use BFS = $x^* = \left(\frac{2}{5}, 0, 1, \frac{1}{10}, 0, 0, 0\right)$with $B = \{1, 3,4\}$:

Step 1: Solve: ${A_B}^T y^* = c_B$ $\rightarrow$ $y^* = (1.4, -2.2, -0.2)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k → Fails 5 select $k = 5$

Step 3 **a):** Solve: $A_B d_B = -A_k$ → $d_B = \left(\frac{4}{5}, 0, -\frac{2}{15}\right)$

b) All d's ≥ 0 means unbounded otherwise. → Keep going as negatives appear

c) of all d's < 0 then $t = \min \left\{\frac{x_i^*}{-d_i}\right\}$ and select r → $\mathrm{t} = \min\left(\frac{1/10}{2/15}\right) = \frac{3}{4}$ select $r = 4$

Step 4: Our new BFS is $x'_B = x^*_B + td_B$ and change $x'_k = t$:

$$\rightarrow t = \frac{3}{4} \text{ so BFS becomes: } \left(\frac{2}{5} + \frac{4}{5}\left(\frac{3}{4}\right), 0, 1 + 0\left(\frac{3}{4}\right), \frac{1}{10} - \frac{2}{15}\left(\frac{3}{4}\right), \frac{3}{4}, 0, 0\right) = \left(1, 0, 1, 0, \frac{3}{4}, 0, 0\right)$$

Our new basis B' is made by taking B adding k and taking out r. → $B' = \{1, 3,5\}$

Example 2

We will explore the issue of cycling of the simplex method using the following example:

$$\begin{aligned} &min && c^T x \\ &\text{St} && Ax = b \\ & && x \geq 0 \end{aligned}$$

$$A = \begin{bmatrix} 0.25 & -8 & -1 & 9 & 1 & 0 & 0 \\ 0.5 & -12 & -0.5 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.25 & 0.5 & 0 \\ -8 & -12 & 0 \\ -1 & -0.5 & 1 \\ 9 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} -0.75 \\ 20 \\ -0.5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Use BFS = $x^* = \left(1,0,1,0,\frac{3}{4},0,0\right)$with $B = \{1,3,5\}$:

Step 1: Solve: ${A_B}^T y^* = c_B$ $\rightarrow$ $y^* = (0, -1.5, -1.25)$

Step 2: Check : ${A_N}^T y^* \leq c_N$ and select k $\rightarrow$ **none fails, thus optimal solution!**