

LAB 4

Linear Regression

- It computes the linear relationship between the dependent variable and one or more independent features by fitting a linear equation with observed data.
- It predicts the continuous output variable based on the independent input variable.
- To find $y = mx + c$ minimize the error b/w predicted values \hat{y} and actual values y .

Algorithm for Linear Regression

- Given a dataset with n samples, each having an input x and output y .
- $y = \beta_0 + \beta_1 x + \epsilon$ that minimizes the error ϵ .
- Initialize Parameters:
Start with random values for β_0 and β_1 (aka)

Hypothesis Function:

Single feature: $\hat{y}_i = \beta_0 + \beta_1 x_i$

Multiple feature: $\hat{y} = \beta_0 + X\beta$

Calculate mean squared error

$$MSE = \frac{1}{N} \sum_{i=1}^N (\hat{y}_i - y_i)^2$$

Make Prediction

Use trained model - $y = \beta_0 + \beta_1 x$ to predict output value for new inputs

$N \rightarrow$ observation

$\hat{y}_i \rightarrow$ predicted values

$y_i \rightarrow$ actual values

Applications

- Marketing and Advertising
- Business and Economics
- Environmental Science
- Sports Analytics

Pseudocode

Function LinearRegression(x, y):

Step 1: Add column of ones to x for the intercept term

$x = \text{Add Column of Ones}(x)$

Step 2: Compute the coefficients using OLS formula

$$\text{beta} = (X^T X)^{-1} X^T y$$

$X_{\text{transpose}} = \text{transpose}(x)$

$X^T X = \text{Multiply}(X_{\text{transpose}}, x)$

$X^T X_{\text{inverse}} = \text{inverse}(X^T X)$

$X^T y = \text{Multiply}(X_{\text{transpose}}, y)$

$\text{beta} = \text{Multiply}(X^T X_{\text{inverse}}, X^T y)$

Step 3: Return coefficients

Return beta

Multi-linear Regression

involves more than one independent variable and one dependent variable.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

y is dependent variable.

x_1, x_2, \dots, x_n are independent variables

β_0 is intercept

$\beta_1, \beta_2, \dots, \beta_n$ are the slopes

Let data points be $(x_1, x_2, \dots, x_n, y_i)$

$\forall i \in \{0, \dots, m\}$ where $x_i \forall i \in \{0, \dots, m\}$

represents independent variables as x values
are dependent variables

In the form of matrix

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{bmatrix} 1 + x_{11} + x_{12} + \dots + x_{1n} \\ 1 + x_{21} + \dots + x_{2n} \\ \vdots \\ 1 + x_{n1} + \dots + x_{nn} \end{bmatrix}$$

where $\beta = ((x^T x)^{-1} x^T) y$

The above values can be used to plot the
best fit line and can be used to
predict future values.

Logistic Regression

- Logistic regression approach operates on
Sigmoid curve rather than best fit line.
we get a value $\in [0, 1]$ (Binary
classification) and then classify into +ve
or -ve by comparing with median

let data points be $(x_i, y_i) \forall i \in \{0, n\}$
Finding best fit line through previously
mentioned methods

$$v = \frac{1}{1 + e^{-(mx+c)}} = \frac{1}{1 + e^{-(b_0 + b_1 x)}}$$

Classification will be based on the
obtained value v

- If $v < 0.5 \rightarrow$ then "no"
- If $v > 0.5 \rightarrow$ then "Yes"