

Real Numbers

★ Fundamental Theorem of Arithmetic (Prime Factorisation)

Every composite number can be expressed as product of primes and this factorisation is unique, apart from order in which prime factors occur.

- Prime Numbers - have 2 factors
 - these are '1' & no. itself
 - ex. 2, 3, 5, 7, ...

- Composite Numbers - have more than 2 factors
 - ex. 4, 9, 16, 20, ...

EX-1.1

Q1) Express each no. as product of its prime factors

Sol i) $140 = 2 \times 2 \times 5 \times 7$

$$= 2^2 \times 5 \times 7$$

ii) $156 = 2 \times 2 \times 3 \times 13$

$$= 2^2 \times 3 \times 13$$

iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17$

$$= 3^2 \times 5^2 \times 17$$

iv) $5005 = 5 \times 7 \times 11 \times 13$

v) $7429 = 17 \times 19 \times 23$

(Q2) Find the LCM & HCF of the following pairs of integers and verify $LCM \times HCF = \text{product of 2 nos.}$

Sol

i) 96 and 91

$$96 = 2 \times 13$$

$$91 = 7 \times 13$$

$$HCF = 13$$

$$\begin{aligned} LCM &= 13 \times 2 \times 7 \\ &= 182 \end{aligned}$$

Verification

$$\begin{array}{c|c} LCM \times HCF = a \times b & \\ \hline LHS & RHS \\ LCM \times HCF & a \times b \\ 13 \times 182 & 96 \times 91 \\ 2366 & 2366 \\ \hline LHS = RHS & \end{array}$$

Hence verified

ii) 510 & 92

$$92 = 2 \times 2 \times 23$$

$$510 = 2 \times 3 \times 5 \times 17$$

$$HCF = 2$$

$$\begin{aligned} LCM &= 2 \times 3 \times 5 \times 17 \times 23 \\ &= 13770 \end{aligned}$$

Verification

$$\begin{array}{c|c} LHS & RMS \\ \hline LCM \times HCF & a \times b \\ 13770 \times 2 & 510 \times 92 \\ 27540 & 27540 \\ \hline LHS = RHS & \end{array}$$

Hence verified

iii) 336 & 54

$$336 = 2^4 \times 3 \times 7$$

$$54 = 2^1 \times 3^3$$

$$\text{HCF} = 2 \times 3 = 6$$

$$\begin{aligned}\text{LCM} &= 2^4 \times 3^3 \times 7 \\ &= 3024\end{aligned}$$

Verification

$$\text{LCM} \times \text{HCF} = a \times b$$

LHS	RHS
$\text{LCM} \times \text{HCF}$	$a \times b$
3024×6	336×54
18144	18144

$$\text{LHS} = \text{RHS}$$

Mence verified

Q3) Find LCM & HCF of following by prime factorisation method.

Sol i) 12, 15 & 21

$$12 = 2 \times 2 \times 3$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\text{HCF} = 3$$

$$\begin{aligned}\text{LCM} &= 3 \times 2 \times 5 \times 7 \times 3 \\ &= 420\end{aligned}$$

ii) 17, 23, 29

$$17 = 1 \times 17$$

$$23 = 1 \times 23$$

$$29 = 1 \times 29$$

$$\text{HCF} = 1$$

$$\begin{aligned}\text{LCM} &= 17 \times 23 \times 29 \\ &\approx 11339\end{aligned}$$

iii) 8, 9, 25

$$8 = 2 \times 2 \times 2$$

$$9 = 3 \times 3$$

$$25 = 5 \times 5$$

$$\text{HCF} = 1$$

$$\begin{aligned}\text{LCM} &= 8 \times 9 \times 25 \\ &= 1800\end{aligned}$$

Q4.) Given that $\text{HCF}(306, 657) = 9$, Find LCM.

Sol. We know that by Fundamental theorem of arithmetic

$$\text{LCM} \times \text{HCF} = a \times b$$

$$\text{LCM} \times 9 = 306 \times 657$$

$$\text{LCM} = \frac{306 \times 657}{9}$$

$$\begin{aligned}\text{LCM} &= 34 \times 657 \\ &= 22338\end{aligned}$$

Q5.) Check whether 6^n can end with digit '0'.

Sol. If 6^n ends with digit zero it must contain 2 and 5 as prime factors.

$$\text{Now, P.F of } 6^n = (2 \times 3)^n = 2^n \times 3^n$$

It contains 2's & 3's only.

\therefore According to fundamental theorem of arithmetic it never ends with digit zero.

Q6.) Explain why following nos. are composite.

Sol. i) $7 \times 11 \times 13 + 13$

$$= 7 \times 11 \times 13 + 13$$

$$= 13 [7 \times 11 + 1]$$

$$= 13 \times [77 + 1]$$

$$= 13 \times 78$$

Since 13 is a prime factor of a given no.
 \therefore It is composite.

$$\begin{aligned} \text{ii) } & 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 \\ & = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 \\ & = 5[7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1] \\ & = 5[1008 + 1] \\ & = 5 \times 1009 \end{aligned}$$

Since 5 is the prime factor of given no.
 \therefore It is composite.

Q7) There is a circular path around a sports field. Sonia takes ----- at the starting point?

Sol Time taken by Sonia = 18 minutes

" " " Ravi = 12 minutes

Minimum time taken by both -

to meet at starting point = LCM(18, 12)

$$= 6 \times 6$$

$$\begin{array}{r} 2 | 18-12 \\ 3 | 9-6 \\ \hline 3-2 \end{array}$$

$$= 36 \text{ min}$$

Ex 3 Find HCF(96, 404) and then LCM.

Sol $96 = 2^5 \times 3$

$$404 = 2^2 \times 101$$

$$HCF = 4$$

$$LCM = \frac{96 \times 404}{4} = 9696 \quad \left\{ \because LCM \times HCF = a \times b \right\}$$

Ex - 1.2

Q1) Prove that $\sqrt{5}$ is irrational.

Sol. Proof:- let us suppose that $\sqrt{5}$ is rational.

$$\therefore \frac{\sqrt{5}}{1} = \frac{p}{q} \text{ where } p \text{ & } q \text{ are co-prime.}$$

$$q \times \sqrt{5} = p$$

Squaring both sides

$$(q \times \sqrt{5})^2 = p^2$$

$$5q^2 = p^2 \quad \text{--- (1)}$$

$$q^2 = \frac{p^2}{5}$$

5 divides p^2

5 divides p also

$$\therefore p = 5x \quad \text{--- (2)}$$

Putting (2) in (1)

$$5q^2 = (5x)^2$$

$$5q^2 = 25x^2$$

$$q^2 = 5x^2$$

$$\frac{q^2}{5} = x^2$$

5 divides by q^2

5 divides by q also.

$$\therefore q = 5y \quad \text{--- (3)}$$

From (2) & (3)

HCF of P and q is 5

∴ Our supposition is wrong
Hence $\sqrt{5}$ is irrational.

Clearly $\frac{1-R^2}{-2R}$ is rational but $\sqrt{2}$ is irrational.

$\therefore \sqrt{2} + \sqrt{5}$ is irrational.

Q2) Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

Let us suppose that $\sqrt{3} + \sqrt{5}$ is rational.

$\therefore \sqrt{3} + \sqrt{5} = R$ [where R is rational no.]

$$\sqrt{3} = R - \sqrt{5}$$

Squaring Both Sides

$$(\sqrt{3})^2 = (R - \sqrt{5})^2$$

$$3 = R^2 + 5 - 2\sqrt{5}R$$

$$3 - R^2 - 5 = -2\sqrt{5}R$$

$$-2R^2 = -2\sqrt{5}R$$

$$\frac{-2R^2}{2R} = \sqrt{5}$$

$$-R = \sqrt{5}$$

Clearly $\frac{-R^2}{-2R}$ is rational no. but $\sqrt{5}$ is irrational.

$\therefore \sqrt{3} + \sqrt{5}$ is irrational.

Q3) Find the largest no. which divides 70 & 195, leaving remainder 5 & 8 in each case.

Let

$$70 - 5 = 65, 195 - 8 = 117$$

Largest Number = HCF (65, 117)

$$65 = 13 \times 5$$

$$117 = 3 \times 3 \times 13$$

$$\text{HCF} = 13$$

ii) $7\sqrt{5}$

let us suppose $7\sqrt{5}$ as rational.

$\therefore 7\sqrt{5} = R$ [where R is a rational no.]

$$\sqrt{5} = \frac{R}{7}$$

Clearly $\frac{R}{7}$ is rational and $\sqrt{5}$ is irrational.

$\therefore 7\sqrt{5}$ is irrational.

iii) $6 + \sqrt{2}$

let us suppose $6 + \sqrt{2}$ as rational.

$\therefore 6 + \sqrt{2} = R$ [where R is a rational no.]

$$\sqrt{2} = R - 6$$

Clearly $R - 6$ is rational but $\sqrt{2}$ is irrational.

$\therefore 6 + \sqrt{2}$ is irrational

EXTRA QUESTIONS

Q1) Prove that $\sqrt{2} + \sqrt{3}$ is irrational

Sol Let us suppose that $\sqrt{2} + \sqrt{3}$ is rational

$\therefore \sqrt{2} + \sqrt{3} = R$ [where R is rational no.]

$$\sqrt{3} = R - \sqrt{2}$$

Squaring both sides

$$(\sqrt{3})^2 = (R - \sqrt{2})^2$$

$$3 = R^2 + (\sqrt{2})^2 - 2 \times \sqrt{2} \times R$$

$$3 = R^2 + 2 - 2\sqrt{2}R$$

$$3 - R^2 - 2 = -2\sqrt{2}R$$

$$1 - R^2 = -2\sqrt{2}R$$

$$\frac{1 - R^2}{-2R} = \sqrt{2}$$

Clearly $\frac{1-R^2}{-2R}$ is rational but $\sqrt{2}$ is irrational.

$\therefore \sqrt{2} + \sqrt{3}$ is irrational.

Q2) Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

Sol let us suppose that $\sqrt{3} + \sqrt{5}$ is rational.

$\therefore \sqrt{3} + \sqrt{5} = R$ [where R is rational no.]

$$\sqrt{3} = R - \sqrt{5}$$

Squaring Both sides

$$(\sqrt{3})^2 = (R - \sqrt{5})^2$$

$$3 = R^2 + 5 - 2\sqrt{5}R$$

$$3 - R^2 - 5 = -2\sqrt{5}R$$

$$-2R^2 = -2\sqrt{5}R$$

$$\frac{-2R^2}{-2R} = \sqrt{5}$$

$$\frac{R}{2}$$

Clearly $\frac{-2R^2}{-2R}$ is rational no. but $\sqrt{5}$ is irrational.

$\therefore \sqrt{3} + \sqrt{5}$ is irrational.

Q3) Find the largest no. which divides 70 & 125, leaving remainder 5 & 8 in each case.

Sol

$$70 - 5 = 65 \quad , \quad 125 - 8 = 117$$

Largest Number = HCF (65, 117)

$$65 = 13 \times 5$$

$$117 = 3 \times 3 \times 13$$

$$\text{HCF} = 13$$

Q 4.) Find the least no. divisible by all natural no. from 1 - 10.

Sol.

Required no. is $\text{LCM}(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$

$$\begin{aligned}\text{LCM} &= 2 \times 3 \times 3 \times 5 \times 7 \times 2 \\ &= 2520\end{aligned}$$

Q 5.) If $\text{LCM}(72, 120) = 24$, Find LCM.

Sol.

$$\text{HCF} \times \text{LCM} = a \times b$$

$$24 \times \text{LCM} = 72 \times 120$$

$$\text{LCM} = \frac{72 \times 120}{24}$$

$$\text{LCM} = 360$$

Q 6.) If $\text{LCM}(9, 96) = 182$, Find HCF.

Sol.

$$\text{HCF} \times \text{LCM} = a \times b$$

$$\text{HCF} = \frac{9 \times 96}{182}$$

$$\text{HCF} = 13$$

Q 7.) If the HCF & LCM of two natural nos. are 12 and 180 and one of the no. is 36. Find other.

Sol.

$$\text{HCF} = 12$$

$$\text{LCM} = 180$$

$$a = 36$$

$$b = ?$$

$$\text{HCF} \times \text{LCM} = a \times b$$

$$\frac{12 \times 180}{36} = b$$

$$b = 60$$

(Q8) If $HCF(a, 8) = 7$ & $LCM(a, 8) = 24$. Find a .

Sol $HCF \times LCM = a \times b$

$$4 \times 24 = a \times 8$$

$$\frac{4 \times 24}{8} = 9$$

$$a = 12.$$

(Q9) Given that $HCF(2520, 6600) = 120 \cdot LCM = 2520k$. Find k .

Sol $HCF \times LCM = a \times b$

$$120 \times 2520k = 2520 \times 6600$$

$$k = \frac{2520 \times 6600}{2520 \times 120}$$

$$k = 550.$$

(Q10) If $HCF(65, 117)$ is expressible in form of $65m - 117$. Find m .

Sol $HCF = 13$

$$65 = 13 \times 5$$

$$117 = 3 \times 3 \times 13$$

$$HCF = 13$$

ATQ

$$65m - 117 = 13$$

$$65m = 13 + 117$$

$$m = \frac{130}{65}$$

$$m = 2.$$

(Q11.) 3 bells toll at intervals of 9, 12, 15 minutes. If they start tolling together, after what time will they toll together again?

Sol Time taken by bells to toll together again = LCM(9, 12, 15)

$$\text{LCM}(9, 12, 15) = 3 \times 3 \times 4 \times 5$$

$$= 180 \text{ mins}$$

$$= \frac{180}{60} = 3 \text{ hrs.}$$

(Q12.) 3 traffic lights at 3 road crossings change after every 72s - 96s - 120s. If they changed together at 9:00 AM, after what time they will change together again?

Sol Next instance of together change = LCM(72, 96, 120).

$$\text{LCM} = 8 \times 9 \times 20$$

$$= \frac{1440}{60} = 24 \text{ minutes}$$

Time required = 9:24 AM.

(Q13.) If HCF(6591) The length, breadth and height of a room are 8m 50cm, 6m 25cm and 4m 75cm. Find length of longest rod that can measure dimensions of room exactly.

Sol length of room = 8m 50cm = 850cm
 Breadth " " = 6m 25cm = 625cm
 Height " " = 4m 75cm = 475cm
 length of longest rod = HCF(850, 625, 475)

$$= 25\text{cm}$$

(Q14.) A circular track around a sports ground has circumference 1080 m. 2 cyclists Sonia & Parvi starts together and cycled at constant speed of 6m/s & 9m/s around the track. After how many minutes will they meet again at the starting point.

STATISTICS

(Central Tendencies of Statistics :-

- i) Mean
 - ii) Median
 - iii) Mode
-] Empirical relation :- $3 \text{ Medians} = \text{Mode} + 2 \text{ Mean}$

Class size = upper limit - lower limit

Class marks = ~~Upper limit + lower limit~~

Mean 3 methods :-

$$\text{i) Direct method} \rightarrow \bar{x} = \frac{\sum f x}{\sum f}$$

$$\text{ii) Assumed Mean method} \rightarrow \bar{x} = A + \frac{\sum f d}{\sum f} \quad \{ d = x - A \}$$

$$\text{iii) Step deviation} \rightarrow \bar{x} = A + \frac{\sum f U}{\sum f} \quad \left\{ \begin{array}{l} U = \frac{x-A}{h} \\ h = \text{class size} \end{array} \right.$$

EX-13.1

Q1.) A survey

C/I	X	$U = \frac{x-A}{h}$	f	$\sum f U$
0-2	1	-3	1	-3
2-4	3	-2	9	-4
4-6	5	-1	1	-1
6-8	7	0	5	0
8-10	9	1	6	6
10-12	11	2	8	4
12-14	13	3	3	9
$n = 30$			$\sum f = 30$	$\sum f U = 11$

$$\text{No. of boys} = \frac{375}{15} = 31$$

$$\text{" " girls} = \frac{495}{15} = 31$$

(17) Milkman has 2 vessels containing 720 ml and 405 ml of milk. Milk from these containers is poured into glasses of equal capacity to their full brim. Find minimum no. of glasses that can be filled.

Sol

$$\begin{aligned}\text{Minimum no. of glasses that can be filled} &= \text{HCF}(720, 405) \\ &= 3 \times 3 \times 5 \\ &= 9 \times 5 = 45\end{aligned}$$

\therefore These 2 vessels can fill minimum of 45 glasses.

$$\text{HCF of } (720, 405) = 45 = 3 \times 5$$

$$\text{No. of glasses of Type I} = \frac{720}{45} = 16$$

$$\text{" " " " " II} = \frac{405}{45} = 9$$

$$\text{Total glasses required} = 16 + 9 = 25$$

~~11/04~~

Statistics

Central Tendencies of Statistics :-

- i) Mean
 - ii) Median
 - iii) Mode
-] Empirical relation :- $3 \text{Median} = \text{Mode} + 2 \text{Mean}$

Class size = upper limit - lower limit

Class marks = Upper limit + lower limit
 $\frac{x}{2}$

Mean 3 methods :-

$$\text{i) Direct method} \rightarrow \bar{x} = \frac{\sum f x}{\sum f}$$

$$\text{ii) Assumed Mean method} \rightarrow \bar{x} = A + \frac{\sum f v}{\sum f} x_h \quad \left\{ d = x - A \right\}$$

$$\text{iii) Step deviation} \rightarrow \bar{x} = A + \frac{\sum f d}{\sum f} \quad \left\{ U = \frac{x-A}{h} \right\} \left\{ h = \text{class size} \right\}$$

EX-13.1

Q1.) A survey

Sol	C/I	X	$U = \frac{x-A}{h}$	f	FU
0-2	1	-3	1	-3	
2-4	3	-2	9	-4	
4-6	5	-1	1	-1	
6-8	7	0	5	0	
8-10	9	1	6	6	
10-12	11	2	8	4	
12-14	13	3	3	9	
			$\sum f = 70$	$\sum fU = 11$	

$$\bar{x} = A + \frac{\sum f_U x_h}{\sum f} = 7 + \frac{11}{10} \times 2 = \frac{70 + 11}{10} = \frac{81}{10} = 8.1$$

We used step deviation method : it is easy and convenient.

Q2)

Consider

Sol.

C/I	X	$U = \frac{x-A}{h}$	f	f_U	
500-520	510	-3	17	-24	-38
520-540	530	-1	14	-14	
540-560	550 ^A	0	8	0	
560-580	570	1	6	6	26
580-600	590	2	10	20	
$h = 20$			$\sum f = 50$	$\sum f_U = -12$	

$$\begin{aligned}
 \bar{x} &= A + \frac{\sum f_U x_h}{\sum f} \\
 &= 550 + \frac{(-12)}{50} \times 20 \\
 &= 550 - \frac{240}{50} \\
 &= 550 - 4.8 \\
 &= 545.2
 \end{aligned}$$

Q4.)

Thirty

Sol.

C/I	X	$U = \frac{x-A}{h}$	f	f_U	
65-68	66.5	-3	2	-6	
68-71	69.5	-2	4	-8	-17
71-74	72.5	-1	3	-3	
74-77	75.5 ^A	0	8	0	
77-80	78.5	1	7	7	
80-83	81.5	2	4	8	21
83-86	84.5	3	3	6	
$h = 3$			$\sum f = 30$	$\sum f_U = 4$	

$$\bar{x} = A + \frac{\sum fU}{\sum f} x_h$$

$$= 75.5 + \frac{4}{26} \times 8$$

$$= 75.5 + \frac{9}{5}$$

$$= 75.5 + 0.4$$
~~$$= 75.9$$~~

Q5.) In a - - - - -

Class Interval	X	$U = \frac{x-A}{h}$	f	fU	
50-52	51	-3	15.	-30	-140
53-55	54	-1	10.	-10	
56-58	57 ¹	0	135	0	
59-61	60	1	115.	115	165
62-64	63	2	25.	50	
			$\sum f = 900$	$\sum fU = 25$	

$h = 2$

$$\bar{x} = A + \frac{\sum fU}{\sum f} x_h$$

$$= 57 + \frac{25}{900} \times 8$$
~~$$= 57 + \frac{200}{900}$$~~

$$= 57 + \frac{1}{8}$$

$$= 57 + 0.125$$

$$= 57.125$$

Q6)

<u>Sol</u>	<u>C/I</u>	<u>X</u>	<u>$U = \frac{x-A}{h}$</u>	<u>f</u>	<u>fU</u>	
	100-150	125	-2	4	-8	
	150-200	175	-1	5	-5	
	200-250	225	0	12	0	
	250-300	275	1	3	2	
	300-350	325	2	2	4	
				$\sum f = 25$	$\sum fU = -7$	
		$h = 50$				

$$\bar{x} = A + \frac{\sum fU}{\sum f} \times h$$

$$= 225 + \frac{(-7)}{25} \times 50$$

$$= 225 - 14 \\ = 211$$

Q7)

To find

<u>Sol</u>	<u>C/I</u>	<u>X</u>	<u>$U = \frac{x-A}{h}$</u>	<u>f</u>	<u>fU</u>	
	0.00-0.04	0.02	-2	4	-8	
	0.04-0.08	0.06	-1	9	-9	
	0.08-0.12	0.10	0	9	0	
	0.12-0.16	0.14	1	2	2	
	0.16-0.20	0.18	2	4	8	
	0.20-0.24	0.22	3	2	6	
		$h = 0.04$		$\sum f = 30$	$\sum fU = -1$	

$$\bar{x} = A + \frac{\sum fU}{\sum f} \times h = 0.10 + \frac{-1}{30} \times 0.04 = 0.10 + \frac{4}{3000} = \frac{3004}{3000}$$

$$= \frac{0.10 + 0.13}{750} = 0.10 + 0.13 = 0.93$$

$$0.10 - \frac{4}{3000}$$

$$= \frac{10}{100} - \frac{4}{3000}$$

$$= \frac{300 - 4}{3000} = \frac{296}{3000}$$

$$\approx 0.296 \\ \overline{3} \\ \approx 0.98666$$

$$= 6.99$$

Q9) The following

Age	C/I	X	$U = \frac{x-4}{h}$	f	fU	
45-55	50		-2	3	-6	-16
55-65	60		-1	10	-10	
65-75	70	A	0	11	0	
75-85	80		1	8	8	14
85-95	90		2	3	6	
				$\sum f = 35$	$\sum fU = -2$	
		$h = 10$				

$$\bar{x} = A + \frac{\sum fU}{\sum f} \times h = 70 + \frac{(-2)}{35} \times 10 = 70 - \frac{20}{35} = \frac{490 - 20}{7} = \frac{470}{7} = 69.42$$

Q8) A class

Sol

C/I	X	f	$\sum fX$
0-6	3	11	33
6-10	8	10	80
10-14	12	7	84
14-20	17	4	68
20-28	24	4	96
28-38	33	3	99
38-40	39	1	39

$$\sum f = 40 \quad \sum fX = 499$$

$$\bar{x} = \frac{\sum fX}{\sum f} = \frac{499}{40} = 12.475$$

Q3) The

Sol

C/I	X	$U = \frac{x-A}{h}$	f	$\sum fU$
11-13	12	-3	7	-21
13-15	14	-2	6	-12
15-17	16	-1	9	-9
17-19	18	0	13	0
19-21	20	1	8	8
21-23	22	2	5	10
23-25	24	3	4	12

$$h = 2$$

$$\sum f = 44 + f \quad \sum fU = -20 + f$$

$$\text{Mean} = 18$$

$$\bar{x} = A + \frac{\sum fU}{\sum f} \times h$$

$$18 = 18 + \frac{(-20 + f)}{44 + f} \times 2$$

$$18 - 18 = \frac{-40 + f}{44 + f} \times 2$$

$$0 = \frac{-40 + 2f}{44 + f}$$

$$0 \times (44 + f) = -40 + 2f$$

$$0 = -40 + 2f$$

$$2f = 40$$

$$f = \frac{40}{2}$$

$$f = 20$$

Ex - 13.2

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

L = lower limit of modal class

f_0 = frequency of class preceding the modal class.

f_1 = " " " "

f_2 = " " " " succeeding "

h = class size.

(Q1) The -----

Sol	C/I	frequency
	5-15	6
	15-25	11
	25-35	9
	35-45	23
	45-55	14
	55-65	5

$$h = 10$$

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= L + \frac{23 - 9}{2 \times 23 - 9 - 14} \times h$$

$$= L + \frac{14}{11} \times h$$

$$= 35 + \frac{14}{11} \times 10$$

$$= 35 + \frac{140}{11} = 35 + 12.727 = 36.81$$

(Q2) The following - - - - -

Sol	C/I	frequency
	0 - 20	10
	20 - 40	35
	40 - 60	52
	60 - 80	61
	80 - 100	38
	100 - 120	29
	$h = 20$	

$$\begin{aligned}
 \text{Mode} &= L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \\
 &= 60 + \frac{61 - 52}{9 \times 61 - 52 - 38} \times 20 \\
 &= 60 + \frac{9}{122 - 90} \times 20 \\
 &= 60 + \frac{9}{32} \times 20^5 \\
 &= 60 + \frac{45}{8} \\
 &= 60 + 5.625 \\
 &= 65.625
 \end{aligned}$$

Q3)
Dol

The following -
C/I frequency

Class Interval	Frequency	Modality
1000 - 1500	24	f_1
1500 - 2000	40	f_0
2000 - 2500	33	f_1
2500 - 3000	28	f_2
3000 - 3500	30	
3500 - 4000	22	
4000 - 4500	16	
4500 - 5000	7	
$h = 500$		

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$

$$= 1500 + \frac{40 - 24}{2 \times 40 - 24 - 33} \times 500$$

$$= 1500 + \frac{16}{80 - 57} \times 500$$

$$= 1500 + \frac{16}{23} \times 500$$

$$= 1500 + \frac{8000}{23}$$

$$= 1500 + 347.82$$

$$= 1847.82$$

Q4) The following

C/I	frequency	Model Class
15-20	3	f_0
20-25	8	f_1
25-30	9	f_2
30-35	10	
35-40	3	
40-45	0	
45-50	0	
50-55	2	

$h = 5$

$$\begin{aligned}
 \text{Mode} &= L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\
 &= 30 + \frac{10 - 9}{2 \times 10 - 9 - 3} \times 5 \\
 &= 30 + \frac{1}{18} \times 5 \\
 &= 30 + \frac{5}{18} \\
 &= 30 + 0.625 \\
 &= 30.625
 \end{aligned}$$

Q5)
Sol

The given

C/I	frequency	f_0	f_1	f_2
3000 - 4000	4			
4000 - 5000	18	Model Class		
5000 - 6000	9			
6000 - 7000	7			
7000 - 8000	6			
8000 - 9000	3			
9000 - 10000	1			
10000 - 11000	1			

$$h = 1000$$

$$\begin{aligned}
 \text{Mode} &= L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \\
 &= 4000 + \frac{18 - 4}{2 \times 18 - 4 - 9} \times 1000 \\
 &= 4000 + \frac{14}{23} \times 1000 \\
 &= 4000 + \frac{14000}{23} \\
 &= 4000 + \cancel{6666}^{608.7} \\
 &= \cancel{46666}^{4608.7}
 \end{aligned}$$

A student -	frequency
0 - 10	7
10 - 20	14
20 - 30	13
30 - 40	12
40 - 50	20
50 - 60	11
60 - 70	15
70 - 80	8

$$h = 10$$

$$\begin{aligned}
 \text{Mode} &= L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \\
 &= 40 + \frac{20 - 13}{2 \times 20 - 13 - 11} \times 10 \\
 &= 40 + \frac{8}{40 - 23} \times 10 \\
 &= 40 + \frac{8}{17} \times 10 \\
 &= 40 + \frac{80}{17} \\
 &= 40 + 4.705 \\
 &= 44.705
 \end{aligned}$$

Ex - 13.3

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.F.}{f} \right] \times h \quad (\sum f = N)$$

L = lower limit of median class.

N = $\sum f$ → sum of frequencies

C.F. = Cumulative frequency of class preceding median class

h = class size

f = class size

Q1) The -----

Sl.	C/I	f	C.f
	65-85	4	4
	85-105	5	9
	105-125	13	22
	125-145	20	42
	145-165	14	56
	165-185	8	64
	185-205	4	68
	$h = 20$	$N = 68$	

$$\frac{N}{2} = \frac{68}{2} = 34$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.F.}{f} \right] \times h$$

$$= 125 + \frac{34 - 22}{20} \times 20$$

$$= 125 + 12$$

$$= 137$$

$$\text{Mode} = 135.7$$

Median = 137

$$3\text{Mean} = 3\text{Median} - \text{Mode} \quad \{ \text{By relation} \}$$

$$= 3 \times 137 - 135.7$$

$$= 411 - 135.7 = \underline{\underline{275.3}} = 137.6$$

(Q6) 100 surnames -

Sol	C/I	f	C.F
1-4		6	6
4-7		30	36
7-10		40	76
10-13		16	92
13-16		4	96
16-19		4	100
<u>h = 3</u>	<u>N = 100</u>		

$$\frac{N}{2} = \frac{100}{2} = 50$$

$$\begin{aligned}
 \text{Median} &= L + \left[\frac{\frac{N}{2} - C.F}{f} \right] \times h \\
 &= 7 + \frac{50 - 36}{40} \times 3 \\
 &= 7 + \frac{14}{40} \times 3 \\
 &= 7 + \frac{31}{20} \\
 &= 7 + 1.05 \\
 &= 8.05
 \end{aligned}$$

Q 2) The distribution -

Sol

C/I	f	C.F
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	3	30
$\lambda = 5$	$N = 30$	

$$\frac{N}{2} = \frac{30}{2} = 15$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.F}{f} \right] \times h$$

$$= 55 + \frac{15 - 13}{6} \times 5$$

$$= 55 + \frac{2}{6} \times 5$$

$$= 55 + \frac{5}{3}$$

$$= 55 + 1.66\ldots$$

$$= 56.66\ldots$$

$$= 56.67$$

(Q5) The following

<u>C/I</u>	<u>f</u>	<u>C.f</u>	<u>C.f</u>
1500 - 2000	14	14	
2000 - 2500	56	70	
2500 - 3000	60	130	
3000 - 3500	86	216	Median Class
3500 - 4000	79	290	
4000 - 4500	62	252	
4500 - 5000	48	200	
$h = 500$	$N = 300$		

$$\frac{N}{2} = \frac{300}{2} = 150 \quad \frac{N}{2} = 200$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.f}{f} \right] \times h$$

$$= 3000 + \frac{150 - 130}{86} \times 500$$

$$= 3000 + \frac{20}{86} \times 500$$

$$= 3000 + \frac{1000}{43}$$

$$= 3000 + 116.2$$

$$= 3116.2$$

(Q4)

The lengths

of

117.5 - 126.5	3	3
126.5 - 135.5	5	8
135.5 - 144.5	9	17
144.5 - 153.5	12	29
153.5 - 162.5	5	34
162.5 - 171.5	4	38
171.5 - 180.5	2	40
$h = 9$	$N=40$	

$$\frac{N}{2} = \frac{40}{2} = 20$$

$$\begin{aligned}
 \text{Median} &= L + \left[\frac{\frac{N}{2} - C.F.}{f} \right] \times h \\
 &= 144.5 + \frac{20 - 17}{12} \times 9 \\
 &= 144.5 + \frac{3}{12} \times 9 \\
 &= 144.5 + \frac{9}{4} \\
 &= 144.5 + 2.25 \\
 &= 146.75
 \end{aligned}$$

(Q3) Sol The life -

C.I	C.I	f	C.f
below 20	15 - 20	3	3
11 25	20 - 25	6 - 3 = 4	6
11 30	25 - 30	24 - 6 = 18	24
11 35	30 - 35	45 - 24 = 21	45
11 40	35 - 40	78 - 45 = 33	78
11 45	40 - 45	11	89
11 50	45 - 50	3	92
11 55	50 - 55	6	98
11 60	55 - 60	9	100.
$M = 5$		$N = 100$	

C.f Median Class f

$$\frac{N}{2} = \frac{100}{2} = 50$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.F}{f} \right] \times h$$

$$= 35 + \frac{50 - 45}{33} \times 5$$

$$= 35 + \frac{5}{33} \times 5$$

$$= 35 + \frac{25}{33}$$

$$= 35 + 0.75$$

$$= 35.75$$

Q2.)
Sol

<u>C.I</u>	<u>f</u>	<u>C.F</u>
0-10	5	5
10-20	x	$5+x$
20-30	20	$25+x$
30-40	15	$40+x$
40-50	y	$40+x+y$
50-60	5	$45+x+y$
$\sum f = 60$		

$$\text{Median} = 28.5 \quad , \frac{N}{2} = \frac{60}{2} = 30$$

$$5+x+20+15+4+5 = 60$$

$$45+x+y = 60$$

$$x+y = 60-45 = 15 \quad \text{--- (1)}$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.F}{f} \right] \times h$$

$$28.5 = 30 + \frac{30 - (5+x)}{2} \times 10$$

$$28.5 - 30 = \frac{30 - 5 - x}{2}$$

$$8.5 = \frac{25 - x}{2}$$

$$8.5 \times 2 = 25 - x$$

$$17 = 25 - x$$

$$x = 25 - 17$$

$$x = 8$$

Putting $x = 8$ in ①

$$x+y = 15$$

$$8+y = 15$$

$$y = 15 - 8$$

$$y = 7$$

EXTRA QUESTIONS

Q1) The median of data is 14.4, find x & y if total frequency is 20.

C/I	f	C.f
0-6	4	4
6-12	x	$4+x$
12-18	5	$9+x$
18-24	y	$9+x+y$
24-30	1	$10+x+y$
$h = 6$	$N = 20$	

] Median class f

$$\text{Median} = 14.4$$

$$\frac{N}{2} = \frac{20}{2} = 10$$

$$4+x+5+y+1 = 20$$

$$10+x+y = 20$$

$$x+y = 20-10$$

$$x+y = 10 \quad \text{--- ①}$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.f}{f} \right] \times h$$

$$14.4 = 12 + \left(\frac{10 - (4+x)}{5} \right) \times 6$$

$$14.4 - 12 = \frac{10 - 4 - x}{5} \times 6$$

$$2.4 = \frac{6 - x}{5} \times 6$$

$$2.4 = \frac{36 - 6x}{5}$$

$$2.4 \times 5 = 36 - 6x$$

$$6x = 36 - 12$$

$$x = \frac{24}{6}$$

$$x = 4 \quad \text{--- (2)}$$

Putting (2) in (1)

$$x + y = 10$$

$$4 + y = 10$$

$$y = 10 - 4$$

$$y = 6$$

$$\therefore x = 4, y = 6$$

(Q2) Find value of x & y if median of data is 31 & sum of frequencies is 40.

C/I	f	C.f	
0-10	5	5	
10-20	x	5+x	
20-30	6	11+x	
L (30)-40	y	11+x+y	Median Class
40-50	6	17+x+y	f
50-60	5	22+x+y	
$h = 10$	$N = 40$		

$$\text{Median} = 31, \quad \frac{N}{2} = \frac{40}{2} = 20$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C_f}{f} \right] \times h$$

$$31 = 30 + \frac{20 - 11 - x}{y} \times 10$$

$$31 - 30 = \frac{9 - x}{y} \times 10$$

$$y = 90 - 10x$$

$$10x + y = 90 \quad \text{--- (2)}$$

$$5 + x + 6 + y + 6 + 5 = 40$$

$$33 + x + y = 40$$

$$x + y = 40 - 33$$

$$x + y = 18 \quad \text{--- (1)}$$

$$x = 18 - y \quad \text{--- (3)}$$

Putting (3) in (2)

$$10(18 - y) + y = 90$$

$$180 - 10y + y = 90$$

$$-9y = 90 - 180$$

$$y = \frac{180}{9}$$

$$y = 10 \quad \text{--- (4)}$$

Putting (4) in (1)

$$x + y = 18$$

$$x + 10 = 18$$

$$x = 18 - 10$$

$$x = 8$$

$$\therefore x = 8 \quad y = 10$$

(Q3) Median of the data is 50. Find p & q if $N=90$.

Sol

C.I	f	C.F	C.Y
20-30	1	1	
30-40	15	15+p	
40-50	25	40+p	
L (50)-60	20	60+p	Median Class
60-70	9	60+p+q	
70-80	8	68+p+q	
80-90	10	78+p+q	
$h = 10$	$N = 90$		

$$\text{Median} = 50, \frac{N}{2} = \frac{90}{2} = 45$$

$$\text{Median} = L + \left[\frac{\frac{N}{2} - C.F}{f} \right] \times h$$

$$50 = 50 + \frac{45 - 40 - p}{10}$$

$$50 - 50 = 5 - p$$

$$p = 5 \quad -\textcircled{2}$$

$$p + 15 + 25 + 20 + q + 8 + 10 = 90$$

$$78 + p + q = 90$$

$$p + q = 90 - 78$$

$$p + q = 12 \quad -\textcircled{1}$$

Letting $\textcircled{2}$ in $\textcircled{1}$

$$5 + q = 12$$

$$q = 12 - 5 = 7$$

$$\therefore p = 5, q = 7$$

(Q4) Find the value of x & y if mode of data is 55 & sum of frequencies is 51.

<u>Class Interval</u>	<u>f</u>
0-15	6
15-30	7
30-45	x
45-60	15
60-75	10
75-90	y
$h=15$	51

$$6 + 7 + x + 15 + 10 + y = 51$$

$$38 + x + y = 51$$

$$x + y = 51 - 38$$

$$x + y = 13 \quad \text{--- (1)}$$

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$55 = 45 + \frac{15 - x}{30 - x - 10} \times 15$$

$$55 - 45 = \frac{15 - x}{30 - x} \times 15$$

$$10 = \frac{225 - 15x}{30 - x}$$

$$300 - 10x = 225 - 15x$$

$$300 = 225 - 15x + 10x$$

$$300 - 225 = -5x$$

$$x = \frac{-25}{-5}$$

$$x = 5$$

Putting ② in ①

$$5+y = 13$$

$$y = 13 - 5$$

$$y = 8$$

$$\therefore x = 5, y = 8$$

(Q5) The mode of the data is 65. Find x & y if sum of frequencies is 70.

Sol

C/I	f	
0 - 30	8	
30 - 40	11	
40 - 60	x	
L (60) - 80	12	Model Class
80 - 100	y	
100 - 120	9	
120 - 140	9	
140 - 160	5	
h = 20	70	

$$\text{Mode} = 65$$

$$8 + 11 + x + 12 + y + 9 + 9 + 5 = 70$$

$$54 + x + y = 70$$

$$x + y = 70 - 54$$

$$x + y = 16 \quad \text{--- (1)}$$

$$x = 16 - y \quad \text{--- (2)}$$

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$65 = 60 + \frac{12-x}{24-x-y} \times 20$$

$$65 - 60 = \frac{240 - 20x}{24-x-y}$$

$$5(24-x-y) = 240 - 20x$$

$$120 - 5x - 5y = 240 - 20x$$

$$-5y = 240 - 120 - 20x + 5x$$

$$-5y = 120 - 15x$$

$$15x - 5y = 120 \quad \text{--- (3)}$$

Putting (2) in (3)

$$15(16-y) - 5y = 120$$

$$240 - 15y - 5y = 120$$

$$-20y = 120 - 240$$

$$y = \frac{-120}{-20}$$

$$y = 6 \quad \text{--- (4)}$$

Putting (4) in (1)

$$x+y = 16$$

$$x+6 = 16$$

$$x = 16 - 6$$

$$x = 10$$

$$\therefore x = 10, y = 6$$

(Q6)

The ~~mode~~^{mean} of the data is 69.8 and the sum of frequencies is 50. Find f_1 and f_2 .

Sol

C/I	X	$U = \frac{x-A}{h}$	f	f_U
0-20	10	-2	5	-10
20-40	30	-1	f_1	$-f_1$
40-60	50 ^A	0	10	0
60-80	70	1	f_2	f_2
80-100	90	2	7	14
100-120	110	3	8	24
$h = 20$			$\sum f = 30 + f_1 + f_2$	$\sum f_U = 28 - f_1 + f_2$

$$5 + f_1 + 10 + f_2 + 7 + 8 = 50$$

$$30 + f_1 + f_2 = 50$$

$$f_1 + f_2 = 50 - 30$$

$$f_1 + f_2 = 20 \quad \text{---(1)}$$

$$f_1 = 20 - f_2 \quad \text{---(2)}$$

$$\bar{x} = A + \frac{\sum f_U}{\sum f} \times h$$

$$62.8 = 50 + \frac{28 - f_1 + f_2}{30 + f_1 + f_2} \times 20$$

$$62.8 - 50 = \frac{560 - 20f_1 + 20f_2}{30 + f_1 + f_2}$$

$$12.8 = \frac{560 - 20f_1 + 20f_2}{30 + f_1 + f_2}$$

$$12.8(30 + f_1 + f_2) = 560 - 20f_1 + 20f_2$$

$$38.4 + 12.8f_1 + 12.8f_2 = 560 - 20f_1 + 20f_2$$

$$12.8f_1 + 12.8f_2 = 560 - 38.4 - 20f_1 + 20f_2$$

$$12.8f_1 + 20f_1 + 12.8f_2 - 20f_2 = 521.6$$

$$32.8f_1 - 7.2f_2 = 521.6 \quad \text{---(3)}$$

Putting (2) in (3)

$$32.8(20 - f_2) - 7.2f_2 = 521.6$$

$$656.0 - 32.8f_2 - 7.2f_2 = 521.6$$

$$-40f_2 = 521.6 - 656$$

$$f_2 = \frac{-134.4}{-40} \quad \text{---(4)}$$

$$f_2 = 12 \quad \text{---(4)}$$

Putting (4) in (1)

$$f_1 + f_2 = 20$$

$$f_1 + 12 = 20$$

$$f_1 = 20 - 12$$

$$f_1 = 8$$

$$\therefore f_1 = 8 \quad \text{and} \quad f_2 = 12$$

Q 7.)

The daily expenditure of 100 families are given below.
Calculate f_1 & f_2 if mean daily expenditure is 188 Rs.

Sol

C/I	x	$V = \frac{x-1}{n}$	f	fV	
140-160	150	-3	5	-10	-35
160-180	170	-1	25	-25	
180-200	(190) ^A	0	f_1	0	$10 + f_2$
200-220	210	1	f_2	f_2	
220-240	230	3	5	10	
$\bar{h} = 20$			35 $f_1 + f_2$	$\sum fV = -25 + f_2$	

$$5 + 25 + f_1 + f_2 + 5 = 100$$

$$35 + f_1 + f_2 = 100$$

$$f_1 + f_2 = 100 - 35$$

$$f_1 + f_2 = 65 \quad \text{--- (1)}$$

$$f_1 = 65 - f_2 \quad \text{--- (2)}$$

$$\bar{x} = A + \frac{\sum fV}{\sum f} \times h$$

$$188 = 190 + \frac{(-25 + f_2)}{35 + f_1 + f_2} \times 20$$

$$188 - 190 = \frac{-25 + f_2}{35 + f_1 + f_2} \times 20$$

$$-2(35 + f_1 + f_2) = -500 + 20f_2$$

$$-70 - 2f_1 - 2f_2 = -500 + 20f_2$$

$$-2f_1 - 2f_2 - 20f_2 = -500 + 70$$

$$-2f_1 - 22f_2 = 430$$

$$-2[f_1 + 11f_2] = 430$$

$$f_1 + 11f_2 = \frac{430}{-2} \quad ?15$$

$$f_1 + 11f_2 = ?15 \quad \text{--- (3)}$$

Putting ② in ③

$$65 - f_2 + 11f_2 = 215$$

$$10f_2 = 215 - 65$$

$$f_2 = \frac{150}{10}$$

$$f_2 = 15 - ④$$

Putting ④ in ①

$$f_1 + f_2 = 65$$

$$f_1 + 15 = 65$$

$$f_1 = 65 - 15$$

$$f_1 = 50$$

$$\therefore f_1 = 50, f_2 = 15$$

Ans 1804

Probability

$$P(E) = \frac{\text{Favorable outcome}}{\text{Total outcomes}} = \frac{a}{b} = 0 - 1$$

$$P(E) + P(\text{not } E) = 1$$

Ex - 14.1

Q1.) Complete the following:-

- i) Probability of an event E + Probability of an event 'not E' = 1
- ii) The probability of an event that cannot happen is 0. Such an event is called Impossible event.
- iii) The probability of an event that is certain to happen is 1. Such an event is called Sure event.
- iv) The sum of the probabilities of all the elementary events of an experiment is 1.
- v) The probability of an event is greater than or equal to 0 and less than or equal to 1.

- Q2.) Which of the following experiments have equally likely outcomes?
- i) A driver attempts to start a car. The car starts or doesn't start. False
 - ii) A player attempts to shoot a basketball. He shoots or misses the shot. False
 - iii) Trial is made to answer a True/False question. The answer is right or wrong. True.
 - iv) A child is born a boy or girl. True.

Q3) Why is tossing a coin considered to be a fair way of deciding?

Ans : Tossing a coin is an equally likely outcome.

Q4) Which of following can't be probability of an event?

- Sol a) $\frac{2}{3}$ b) -1.5 c) 15% d) 0.7

~~c) 15%~~

Q5) If $P(E) = 0.05$, what is probability of not E?

Sol $P(E) = 0.05$

$P(\text{not } E) = ?$

$$P(E) + P(\text{not } E) = 1$$

$$0.05 + P(\text{not } E) = 1$$

$$P(\text{not } E) = 1 - 0.05$$

$$= 0.95$$

Q6) A bag -----

Sol i) 0

ii) 1

Q7) It is -----

Sol $P(\text{not } E) = 0.992$

$$P(E) = ?$$

$$P(E) + P(\text{not } E) = 1$$

$$P(E) = 1 - 0.992$$

$$= 0.008$$

Q8) A bag - - - - -

Ques. No. of Red balls = 3

" " Black " = 5

Total " = 8

i) $P(\text{Red})$

Favourable Cases = 3

$$P(E) = \frac{\text{Favourable Cases}}{\text{Total Cases}} = \frac{3}{8}$$

$$\text{ii) } P(\text{not Red}) = 1 - \frac{3}{8} = \frac{8-3}{8} = \frac{5}{8}$$

Q9) A box - - - - -

Ques. T. C = 17 = 5 + 8 + 4

$$P(\text{Red}) = \frac{5}{17}$$

$$P(\text{White}) = \frac{8}{17}$$

$$P(\text{not green}) = \frac{5+8}{17} = \frac{13}{17}$$

Q10) A piggy - - - - -

Ques. 50p coins = 100

1 Re " = 50

2 Re " = 20

5 Re " = 10

Total " = 180

$$\text{i) } P(50p) = \frac{5}{180} = \frac{1}{36}$$

$$\text{ii) } P(\text{not a } 5\text{ Re coin}) = \frac{170}{180} = \frac{17}{18}$$

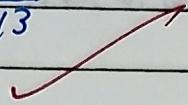
Q(1) Gopi -

Sol Male fishes = 5

Female " = 8

Total " = 13

$$P(\text{Male}) = \frac{5}{13}$$



Q(2)

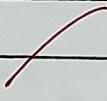
A game - - - - -

Sol Total cases = 8

i) $P(8) = \frac{1}{8}$

ii) Odd cases = 4

$$P(\text{odd}) = \frac{4}{8} = \frac{1}{2}$$



iii) No. greater than 2 = 6

$$P(\text{no. greater than } 2) = \frac{6}{8} = \frac{3}{4}$$

iv) No. less than 9 = 8

$$P(\text{no. less than } 9) = \frac{8}{8} = 1$$



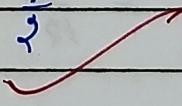
Q(3)

A die - - - - -

Sol T.C = 6

i) $P(\text{prime}) = \frac{3}{6} = \frac{1}{2}$

ii) $P(\text{lts } 2 \& 6) = \frac{2}{6} = \frac{1}{3}$



iii) $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$



Q14) One card -----

Sol Composition of cards :-

$$\text{Total} = 52$$

Black

Red

Types = 4 { Spade ♠, Club ♦, Heart ♥, Diamond ♦ }

All 4 cards have - Ace, 2 to 10 nos and 3 face cards { J, Q, K }

$$i) P(\text{Red King}) = \frac{1}{52}$$

$$ii) P(\text{Face Cards}) = \frac{12}{52} = \frac{3}{13}$$

$$iii) P(\text{Red Face Card}) = \frac{6}{52} = \frac{3}{26}$$

$$iv) P(\text{Jack of hearts}) = \frac{1}{52}$$

$$v) P(\text{Spade}) = \frac{13}{52} = \frac{1}{4}$$

$$vi) P(\text{Queen of diamonds}) = \frac{1}{52}$$

Q15.) Five cards -----

Sol i) $P(\text{green}) = \frac{1}{5}$

ii) a) $P(\text{an ace}) = \frac{1}{4}$, b) $P(\text{queen}) = 0$

Q16.) 12 defective -----

Sol T.C = 144

$$P(\text{Good fm}) = \frac{132}{144} = \frac{11}{12}$$

Q17) i) A lot - - - - - - - - -

$$\text{Sol} \quad i) \quad T.C = 20 \\ F.C = 4$$

$$P(\text{defective}) = \frac{4}{20} = \frac{1}{5}$$

$$ii) T.C = 19$$

$$\underline{F.C = 15}$$

$$P(\text{good one}) = \frac{15}{19}$$

Q18.) A box - - -

Set T.C = 90

$$i) F.C = 81$$

$$P(\text{2 digit no.}) = \frac{81}{90} = \frac{9}{10}$$

$$ii) F.C = 89$$

$$P(\text{perfect square no.}) = \frac{1}{40} = \frac{1}{\cancel{4}\cancel{0}} = \frac{1}{10}$$

$$iii) F.C = 18$$

$$P(\text{Mo. divisible by 5}) = \frac{18}{90} = \frac{1}{5}$$

A 19.) A child

~~Sol~~ T.C = 5

$$i) F.C = 2$$

$$P(A) = \frac{2}{5} = \underline{\underline{\frac{2}{5}}}$$

$$\text{ii) } F.C = 1$$

$$P(D) = \frac{1}{5}$$

(Q20) Suppose - - -

Sol

$$\text{Area of rectangular sheet} = l \times b$$

$$= 3 \times 2 = 6 \text{ m}^2$$

$$\text{Diameter of circle} = 1 \text{ m}$$

$$\text{Radius } \therefore r = \frac{1}{2}$$

$$\text{Area of circle} = \pi r^2 = \pi \times \frac{1}{2} \times \frac{1}{2} = \frac{\pi}{4}$$

$$\text{T.C} = 6 \text{ m}^2$$

$$\text{F.C} = \frac{\pi}{4} \text{ m}^2$$

$$P(C \text{ disc will land in circle}) = \frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$$

(Q21) A lot - - - - -

Sol

$$\text{Total Pens} = 144$$

$$\text{defective } \therefore = 20$$

$$\text{Good } \therefore = 144 - 20 = 124$$

$$i) \text{ T.C} = 144$$

$$\text{F.C} = 124$$

$$P(\text{she will buy}) = \frac{124}{144} = \frac{31}{36}$$

$$ii) \text{ T.C} = 144$$

$$\text{F.C} = 20$$

$$P(\text{she won't buy}) = \frac{20}{144} = \frac{5}{36}$$

(Q2) Refer - - - - -
Sol $T.C = 6^n = 36$

	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7						
2	3	4	5	6	7	8						
3	4	5	6	7	8	9						
4	5	6	7	8	9	10						
5	6	7	8	9	10	11						
6	7	8	9	10	11	12						

i) Sum 9

$$F.C = 1$$

$$P(\text{Sum } 9) = \frac{1}{36}$$

ii) Sum 3

$$F.C = 2$$

$$P(\text{Sum } 3) = \frac{2}{36} = \frac{1}{18}$$

iii) Sum 4

$$F.C = 3$$

$$P(\text{Sum } 4) = \frac{3}{36} = \frac{1}{12}$$

iv) Sum 5

$$F.C = 4$$

$$P(\text{Sum } 5) = \frac{4}{36} = \frac{1}{9}$$

v) Column 6

$$F.C = 5$$

$$P(\text{Column 6}) = \frac{5}{36}$$

vi) Column 7

$$F.C = 6$$

$$P(\text{Column 7}) = \frac{6}{36} = \frac{1}{6}$$

vii) Column 8

$$F.C = 5$$

$$P(\text{Column 8}) = \frac{5}{36}$$

viii) Column 9

$$F.C = 4$$

$$P(\text{Column 9}) = \frac{4}{36} = \frac{1}{9}$$

ix) Column 10

$$F.C = 3$$

$$P(\text{Column 10}) = \frac{3}{36} = \frac{1}{12}$$

x) Column 11

$$F.C = 2$$

$$P(\text{Column 11}) = \frac{2}{36} = \frac{1}{18}$$

x) When 12

$$F.C = 1$$

$$P(\text{When } 12) = \frac{1}{36}$$

Q ii) No, ∵ they are not no. but sum of 2 no.

Q3) A game - -----

Sol Note - Tossing coin general formula = 2^n {n is no. of times/coins}

$$T.C = 2^3 = 8 \{ \text{HHH, HHT, HTT, TTH, } \}$$

$$F.C = 6 \quad \{ \text{TTT, THT, HTT, TTH} \}$$

$$P(C \text{ Manif will lose}) = \frac{6}{8} = \frac{3}{4} \quad \{ 0, 1, 1, 1, 2, 2, 2, 3 \}$$

Q4) A die - -----

Sol i) $T.C = 36$

$$F.C = 36 - 11 = 25$$

$$P(C \text{ 5 will come}) = \frac{25}{36}$$

ii) $T.C = 36$

$$F.C = 11$$

$$P(C \text{ 5 will come}) = \frac{11}{36}$$

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Q5) Which - -----

Sol i) False ∵ it has 4 possible outcomes.

ii) True ∵ it is an equally likely outcome.

Extra Questions

(i) 3 coins are tossed simultaneously. Find probability of getting -

- i) 1 Tail
- ii) 2 heads
- iii) 2 Tails
- iv) No head
- v) Exactly 3 tails

Soln

$$T.C = 2^3 = 8$$

$$\{0, 1, 1, 1, 2, 2, 2, 2\}$$

$$j) F.C = 3$$

$$P(1 \text{ tail}) = \frac{3}{8}$$

$$ii) F.C = 3$$

$$P(2 \text{ heads}) = \frac{3}{8}$$

$$iii) F.C = 3$$

$$P(2 \text{ tails}) = \frac{3}{8}$$

$$iv) F.C = 1$$

$$P(3 \text{ no head}) = \frac{1}{8}$$

$$v) F.C = 1$$

$$P(3 \text{ tails}) = \frac{1}{8}$$

$$vi) F.C = 7$$

$$P(\text{At least 1 tail}) = \frac{7}{8}$$

$$vii) F.C = 4$$

$$P(\text{At least 2 heads}) = \frac{4}{8} = \frac{1}{2}$$

- vi) At least 1 tail
- vii) At least 2 heads
- viii) At most 2 tails
- ix) At most 1 tail

vii) $F.C = 7$

$$P(C \text{ At most 2 tails}) = \frac{7}{8}$$

ix) $F.C = 4$

~~$$P(C \text{ At most 1 tail}) = \frac{4}{8} = \frac{1}{2}$$~~

Q2) 2 dice are thrown simultaneously. Find the probability of getting doublet.

Ans

$$T.C = 36$$

$$F.C = 6$$

$$P(C \text{ Doublet}) = \frac{6}{36} = \frac{1}{6}$$

Q3) 3 dice are thrown simultaneously. Find the probability of getting a triplet.

Ans

$$T.C = 6^n = 6^3 = 216$$

$$F.C = 6$$

$$P(C \text{ getting triplet}) = \frac{6}{216} = \frac{1}{36}$$

Q4) What is probability of getting 53 sundays in a non leap year?

Ans

$$\text{Total days in a year} = 365$$

$$\text{No. of weeks} = \frac{365}{7} = 52 \text{ weeks} + 1 \text{ day}$$

This 1 day can be arranged as

{S, M, Tu, W, Th, Fr, Sat}

$$T.C = 7$$

$$F.C = 1$$

$$P(C \text{ 53 Sundays}) = \frac{1}{7}$$

(Q5) What is the probability of getting 53 Sundays in leap year?

Sol

Total Days in year = 366

$$\text{1 week} \Rightarrow \frac{366}{7} = 52 \text{ weeks} + 2 \text{ days}$$

These 2 days can be arranged as:-

(Sun, Mon) (Mon, Tue) (Tue, Wed) (Wed, Thu) (Thu, Fri) (Fri, Sat)
(Sat, Sun)

$$T.C = 7$$

$$F.C = 2$$

$$P(C \text{ 53 Sundays}) = \frac{2}{7}$$

(Q6) Cards numbered from 1 to 30 are put in a bag. A card is drawn at random. What is probability of getting -

- i) a prime no. bigger than 7
- ii) not a perfect square
- iii) Not divisible by 3.

Sol

$$T.C = 30$$

$$i) F.C = 6$$

$$P(C \text{ 7+ prime}) = \frac{6}{30} = \frac{1}{5}$$

$$ii) F.C = 30 - 5 = 25$$

$$P(C \text{ not perfect square}) = \frac{25}{30} = \frac{5}{6}$$

$$iii) F.C = 30 - 10 = 20$$

$$P(C \text{ not divisible by 3}) = \frac{20}{30} = \frac{2}{3}$$

Q7) A letter of English alphabet is chosen at random. Determine the probability that the letter is -

- i) consonant ii) vowel

Sol T.C = 26

i) F.C = 26 - 5 = 21

$$P(\text{consonant}) = \frac{21}{26} = \frac{21}{26}$$

ii) F.C = 5

$$P(\text{vowel}) = \frac{5}{26}$$

Q8) 2 dice are thrown simultaneously. Find the probability of no. appearing on each sum is -

- i) 9 or 11 iv) At least 10
 ii) 6 or 7 v) no. less than or equal to 10
 iii) a multiple of 3 is the sum vi) even

Sol T.C = 36

i) F.C = 6

$$P(9 \text{ or } 11) = \frac{6}{36} = \frac{1}{6}$$

vi) F.C = 18

$$P(\text{even}) = \frac{18}{36} = \frac{1}{2}$$

ii) F.C = 11

$$P(6 \text{ or } 7) = \frac{11}{36}$$

iii) F.C = 12

$$P(\text{multiple of 3}) = \frac{12}{36} = \frac{1}{3}$$

iv) F.C = 6

$$P(\text{At least } 10) = \frac{6}{36} = \frac{1}{6}$$

v) F.C = 33

$$P(\text{no. } \leq 10) = \frac{33}{36} = \frac{11}{12}$$

- Q9) 2 different dice tossed together. Find probability that product of 2 no. at step are -
- i) 6
 - ii) perfect square
 - iii) even
 - iv) less than 16

Sol $T.C = 36$

i) $F.C = 4$

$$P(6) = \frac{4}{36} = \frac{1}{9}$$

ii) $F.C = 8$

$$P(\text{perfect square}) = \frac{8}{36} = \frac{2}{9}$$

iii) $F.C = 27$

$$P(\text{even}) = \frac{27}{36} = \frac{3}{4}$$

iv) $F.C = 26$

$$P(< 16) = \frac{26}{36} = \frac{13}{18}$$

- Q10) A die is numbered in such a way that its faces are 1, 2, 2, 3, 3, 6. It is thrown twice and total score in 2 throws is noted. Find the probability that total score is -

i) Even

ii) six

iii) At least 6

Sol $T.C = 36$

i) $F.C = 18$

$$P(\text{Even}) = \frac{18}{36} = \frac{1}{2}$$

i) $F.C = 4$

$$P(\text{six}) = \frac{4}{36} = \frac{1}{9}$$

ii) $F.C = 15$

$$P(\text{At least } 6) = \frac{15}{36} = \frac{5}{12}$$

All) 3 coins are tossed simultaneously find probability of getting -

i) 3 heads

ii) 2 heads

iii) 1 head

iv) No head

v) At least 1 head

Sol

$$T.C = 8$$

i) $F.C = 1$

$$P(3 \text{ heads}) = \frac{1}{8}$$

ii) $F.C = 3$

$$P(2 \text{ heads}) = \frac{3}{8}$$

iii) $F.C = 3$

$$P(1 \text{ head}) = \frac{3}{8}$$

iv) $F.C = 1$

$$P(\text{no head}) = \frac{1}{8}$$

v) $F.C = 7$

$$P(\text{At least 1 head}) = \frac{7}{8}$$

vi) At least 2 heads

vii) At most 2 heads

viii) At most 1 tail

vi) F.C = 4

$$P(C \text{ At least } 2 \text{ heads}) = \frac{7}{8} = \frac{1}{2}$$

vii) F.C = 7

$$P(C \text{ At most } 2 \text{ heads}) = \frac{7}{8}$$

viii) F.C = 7

$$P(C \text{ At most } 1 \text{ tail}) = \frac{7}{8} = \frac{1}{2}$$

Q.R) A card is drawn from a well shuffled deck of 52 cards. Find probability that -

- i) Neither King nor ace
- ii) Either King or ace
- iii) Spade or King
- iv) Spade and King
- v) Neither Red nor green
- vi) 10 of hearts
- vii) King or green
- viii) King and green

Sol

$$T.C = 52$$

i) F.C = 44

$$P(C \text{ neither king nor ace}) = \frac{44}{52} = \frac{11}{13}$$

ii) F.C = 8

$$P(C \text{ either King or ace}) = \frac{8}{52} = \frac{2}{13}$$

iii) F.C = 16

$$P(C \text{ spade or King}) = \frac{16}{52} = \frac{4}{13}$$

v) $F.C = 1$

$$P(C \text{ shade & King}) = \frac{1}{52}$$

v) $F.C = 24$

$$P(\text{neither red nor green}) = \cancel{\frac{24}{52}} = \frac{6}{13}$$

vi) $F.C = 1$

$$P(10 \text{ of hearts}) = \frac{1}{52}$$

vii) $F.C = 8$

$$P(\text{King or queen}) = \frac{8}{52} = \frac{2}{13}$$

viii) $F.C = 0$

$$P(\text{King and queen}) = \frac{0}{52} = 0$$

~~Simple~~ 24/02

Polynomials

Zero - A value of variable that makes polynomial 0.

Degree = highest power of variable
no. of zeroes = highest degree

Ex-2.1

- Q1.) The _____
- Sol i) 0
ii) 1
iii) 3
iv) 2
v) 4
vi) 3

General form of quadratic polynomial - $ax^2 + bx + c$

Relation b/w 2 zeroes and coeff.

If α and β are zeroes of polynomial

i) sum of 2 zeroes $\Rightarrow \alpha + \beta = -\frac{b}{a}$

ii) Product of 2 zeroes $\Rightarrow \alpha \cdot \beta = \frac{c}{a}$

iii) If α & β are the zeroes of the polynomial then

$$P(x) = x^2 - (\alpha + \beta)x + \alpha \cdot \beta$$

$\alpha + \beta$ = sum of zeroes, $\alpha \cdot \beta$ = Product of zeroes

Q1) Find -----

Sol i) $x^2 - 2x - 8$

$= x^2 - 4x + 3x - 8$

$= x(x-4) + 3(x-4)$

$= (x+3)(x-4)$

$x-4 = 0$

$x = 4$

$\alpha = 4$

$x+2 = 0$

$x = -2$

$\beta = -2$

Verification

$\alpha + \beta = \frac{-b}{a}$

$\alpha + \beta = 4 + (-2)$
 $= 4 - 2$
 $= 2$

$\frac{-b}{a} = \frac{-(-2)}{1}$

$= 2$

$\alpha \beta = \frac{c}{a}$

$\alpha \beta = 4 \times -2 = -8$

$\frac{c}{a} = \frac{-8}{1} = -8$

$\therefore \alpha \beta = \frac{c}{a}$

$\therefore \alpha + \beta = \frac{-b}{a}$

Mence verified

$$ii) 4s^2 - 4s + 1$$

$$\begin{aligned} &= 4s^2 - 2s - 2s + 1 \\ &= 2s(2s-1) - 1(2s-1) \\ &= (2s-1)(2s-1) \end{aligned}$$

$$\left. \begin{array}{l} 2s-1=0 \\ 2s=1 \\ s=\frac{1}{2} \\ \alpha=\frac{1}{2} \end{array} \right| \quad \left. \begin{array}{l} 2s-1=0 \\ 2s=1 \\ s=\frac{1}{2} \\ \beta=\frac{1}{2} \end{array} \right|$$

Verification

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = \frac{1}{2} + \frac{1}{2}$$

$$= \frac{2}{2} = 1$$

$$-\frac{b}{a} = -\frac{-4}{4}$$

$$= \frac{4}{4} = 1$$

$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\alpha \beta = \frac{c}{a}$$

$$\alpha \beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{c}{a} = \frac{1}{4}$$

$$\therefore \alpha \beta = \frac{c}{a}$$

Hence verified

$$\begin{aligned}
 \text{iii)} \quad & 6x^2 - 3 - 7x \\
 &= 6x^2 - 7x - 3 \\
 &= 6x^2 - 9x + 2x - 3 \\
 &= 6x^2 + 2x - 9x - 3 \\
 &= 2x(3x+1) - 3(3x+1) \\
 &= (2x-3)(3x+1)
 \end{aligned}$$

$2x-3 = 0$	$3x+1 = 0$
$2x = 3$	$3x = -1$
$x = \frac{3}{2}$	$x = -\frac{1}{3}$
$\alpha = \frac{3}{2}$	$\beta = -\frac{1}{3}$

Verification

$\alpha + \beta = -\frac{b}{a}$	$\alpha\beta = \frac{c}{a}$
$ \begin{aligned} \alpha + \beta &= \frac{3}{2} + (-\frac{1}{3}) \\ &= \frac{9-2}{6} \\ &= \frac{7}{6} \\ -\frac{b}{a} &= -\left(-\frac{7}{6}\right) \\ &= \frac{7}{6} \end{aligned} $	$ \begin{aligned} \alpha\beta &= \frac{3}{2} \times -\frac{1}{3} = -\frac{1}{2} \\ \frac{c}{a} &= \frac{-1}{2} = -\frac{1}{2} \\ \therefore \alpha\beta &= \frac{c}{a} \end{aligned} $
$\therefore \alpha + \beta = -\frac{b}{a}$	Mence verified.

$$\text{iv}) = 4v^2 + 8v \\ = 4v(v+2) \\ = (4v)(v+2)$$

$$\left. \begin{array}{l} 4v=0 \\ v=0 \\ \alpha=0 \end{array} \right| \quad \left. \begin{array}{l} v+2=0 \\ v=-2 \\ \beta=-2 \end{array} \right.$$

Verification

$$\left. \begin{array}{l} \alpha+\beta = -\frac{b}{a} \\ \alpha+\beta = 0+(-2) \\ = -2 \end{array} \right| \quad \left. \begin{array}{l} \alpha\beta = -\frac{c}{a} \\ \alpha\beta = 0 \times -2 \\ = 0 \end{array} \right.$$

$$\left. \begin{array}{l} -\frac{b}{a} = -\frac{2}{1} = -2 \\ \therefore \alpha+\beta = -\frac{b}{a} \end{array} \right| \quad \left. \begin{array}{l} -\frac{c}{a} = -\frac{0}{1} = 0 \\ \therefore \alpha\beta = -\frac{c}{a} \end{array} \right.$$

Hence verified.

$$\text{v) } t^2 - 15$$

$$\bullet t^2 - 15 = 0$$

$$t^2 = 15$$

$$t = \pm\sqrt{15}$$

$$\alpha = \sqrt{15}, \beta = -\sqrt{15}$$

Verification

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha + \beta = \sqrt{15} - \sqrt{15} \\ = 0$$

$$\frac{-b}{a} = \frac{0}{1} = 0$$

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\alpha \beta = \frac{-c}{a}$$

$$\alpha \beta = \sqrt{15} \times (-\sqrt{15}) = -15$$

$$\frac{c}{a} = \frac{-15}{1} = -15$$

$$\therefore \alpha \beta = \frac{c}{a}$$

Hence verified

$$\text{vi) } 3x^2 - x - 4$$

$$= 3x^2 - 4x + 3x - 4$$

$$= 3x^2 + 3x - 4x - 4$$

$$= 3x(x+1) - 4(x+1)$$

$$= (3x-4)(x+1)$$

$$3x-4=0$$

$$x = \frac{4}{3}$$

$$\alpha = \frac{4}{3}$$

$$x+1=0$$

$$x = -1$$

$$\beta = -1$$

Verification

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = \frac{4}{3} - 1 = \frac{1}{3}$$

$$\frac{-b}{a} = \frac{-(-1)}{3} = \frac{1}{3}$$

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\alpha \beta = \frac{c}{a}$$

$$\alpha \beta = \frac{4}{3} \times 1 = \frac{4}{3}$$

$$\frac{c}{a} = \frac{-4}{3}$$

$$\therefore \alpha \beta = \frac{c}{a}$$

Mence verified.

Q2) Find - - - - -

Sol i) $\frac{1}{4}, -1$

$$S = \frac{1}{4}, P = -1$$

$$\begin{aligned} P(x) &= x^2 - Sx + P \\ &= x^2 - \frac{1}{4}x + (-1) \\ &= x^2 - \frac{1}{4}x - 1 \\ &= \frac{1}{4} [4x^2 - 4x - 4] \\ &= \frac{1}{4} [4x^2 - x - 4] \end{aligned}$$

$$P(x) = 4x^2 - x - 4$$

ii) $\sqrt{2}, \frac{1}{3}$

$$\alpha = \sqrt{2}, \beta = \frac{1}{3}$$

$$p(x) = x^2 - \alpha x + \beta$$

$$= x^2 - \sqrt{2}x + \frac{1}{3}$$

$$= \frac{1}{3} \left[3x^2 - 3\sqrt{2}x + 3 \cdot \frac{1}{3} \right]$$

$$= \frac{1}{3} [3x^2 - 3\sqrt{2}x + 1]$$

$$p(x) = 3x^2 - 3\sqrt{2}x + 1$$

iii) 0, $\sqrt{5}$

$$p(x) = x^2 - 0x + \sqrt{5}$$

$$= x^2 + \sqrt{5}$$

iv) 1, 1

$$p(x) = x^2 - 2x + \beta$$

$$= x^2 - x + 1$$

v) $-\frac{1}{4}, \frac{1}{4}$

$$p(x) = x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$$

$$= x^2 + \frac{1}{4}x + \frac{1}{4}$$

$$= \frac{1}{4} \left[4x^2 + 4x + 4 \right]$$

$$p(x) = 4x^2 + x + 1$$

v) 4, 1

$$p(x) = x^2 + -2x + p$$

$$= x^2 - 4x + 1$$

EXTRA QUESTIONS

Q1.) The quadratic polynomial whose zeroes are $(2, \frac{-3}{2})$

Sol. $P = 2x - 3 = -3$

$$S = 2 + \frac{-3}{2} = \frac{4-3}{2} = \frac{1}{2}$$

$$p(x) = x^2 + -2x + P$$

$$= x^2 - \frac{1}{2}x - 3$$

$$= \frac{1}{2} [2x^2 - x - 6]$$

$$p(x) = 2x^2 - x - 6$$

Q2.) The quadratic polynomial whose zeroes are (-3, 4).

Sol. $P = -3 \times 4 = -12$

$$S = -3 + 4 = 1$$

$$p(x) = x^2 - 1x + 12$$

$$= x^2 - x - 12$$

$$= x^2 - x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - \frac{12}{2}$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

Q3) Find the zeroes of polynomial and verify the relation between zeroes & coefficient.

$$i) 2x^2 + \frac{7}{2}x + \frac{3}{4}$$

$$ii) 4x^2 + 5\sqrt{2}x - 3$$

$$iii) y^2 + \frac{3}{8}\sqrt{5}y - 5$$

$$iv) 4x^2 + 9x - 3$$

$$v) x^2 + \frac{1}{6}x - 2$$

$$vi) v^2 + 4\sqrt{3}v - 15$$

Sol i) $2x^2 + \frac{7}{2}x + \frac{3}{4}$

$$= \frac{1}{4} [8x^2 + 14x + 3]$$

$$= 8x^2 + 14x + 3$$

$$= 8x^2 + 12x + 2x + 3$$

$$= 8x^2 + 2x + 12x + 3$$

$$= 2x(4x+1) + 3(4x+1)$$

$$= (2x+3)(4x+1)$$

$$2x+3 = 0$$

$$x = -\frac{3}{2}$$

$$\alpha = -\frac{3}{2}$$

$$4x+1 = 0$$

$$x = -\frac{1}{4}$$

$$\beta = -\frac{1}{4}$$

Verification

$$\alpha + \beta = -\frac{3}{2} - \frac{1}{4} = -\frac{6-1}{4}$$

$$= -\frac{7}{4}$$

$$-\frac{b}{a} = -\frac{7}{4} = -\frac{7}{4}$$

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\alpha \beta = \frac{13}{2} \times \frac{1}{4} = \frac{3}{8}$$

$$\frac{\alpha \beta}{a} = \frac{3}{8}$$

$$\therefore \alpha \beta = \frac{c}{a}$$

Please verified

$$\text{ii) } 4x^2 + 5\sqrt{2}x - 3 \\ = 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3$$

~~$\cancel{6\sqrt{2}x}$~~

$$= \cancel{4x^2 + 6\sqrt{2}x} - 3 \\ = \cancel{2\sqrt{2}x(\sqrt{2}x + 3)} + 6$$

$$= 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3)$$

$$= (2\sqrt{2}x - 1)(\sqrt{2}x + 3)$$

$$2\sqrt{2}x - 1 = 0$$

$$2\sqrt{2}x = 1$$

$$x = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$\alpha = \frac{\sqrt{2}}{4}$$

$$\sqrt{2}x + 3 = 0$$

$$\sqrt{2}x = -3$$

$$x = \frac{-3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-3\sqrt{2}}{2}$$

$$\beta = -\frac{3\sqrt{2}}{2}$$

Verification :-

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha + \beta = \frac{\sqrt{2}}{4} + \left(-\frac{3\sqrt{2}}{2}\right)$$

$$\alpha\beta = \frac{\sqrt{2}}{4} \times -\frac{3\sqrt{2}}{2}$$

$$= \frac{\sqrt{2} - 6\sqrt{2}}{4}$$

$$= -\frac{3\sqrt{2}}{2}$$

$$= -\frac{5\sqrt{2}}{4}$$

$$= -\frac{3}{4}$$

$$-\frac{b}{a} = -\frac{5\sqrt{2}}{4}$$

$$\frac{c}{a} = -\frac{3}{4}$$

$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\therefore \alpha\beta = \frac{c}{a}$$

Mence verified

$$\text{iii) } y^2 + \frac{3}{2}\sqrt{5}y - 5$$

$$= \left[2y^2 + 3\sqrt{5}y - 10 \right]$$

$$= 2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10$$

$$= 2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5})$$

$$= (y + 2\sqrt{5})(2y - \sqrt{5})$$

$$y + 2\sqrt{5} = 0$$

$$y = -2\sqrt{5}$$

$$\alpha = -2\sqrt{5}$$

$$2y - \sqrt{5} = 0$$

$$2y = \sqrt{5}$$

$$y = \frac{\sqrt{5}}{2}$$

$$\beta = \frac{\sqrt{5}}{2}$$

Verification :-

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = -\frac{3\sqrt{5}}{2} + \frac{\sqrt{5}}{2}$$

$$= \frac{-4\sqrt{5} + \sqrt{5}}{2}$$

$$= -\frac{3\sqrt{5}}{2}$$

$$-\frac{b}{a} = -\frac{3\sqrt{5}}{2 \times 1}$$

$$= -\frac{3\sqrt{5}}{2}$$

$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = -\frac{3\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}$$

$$= -\frac{5}{1}$$

$$\frac{c}{a} = -\frac{5}{1}$$

$$\therefore \alpha\beta = \frac{c}{a}$$

Mence Verified

$$\begin{aligned}
 \text{iv) } 4x^2 + 4x - 3 &= 4x^2 - 9x + 6x - 3 \\
 &= 2x(2x-1) + 3(2x-1) \\
 &= (2x+3)(2x-1)
 \end{aligned}$$

$$2x+3=0$$

$$2x = -3$$

$$x = \frac{-3}{2}$$

$$\alpha = \frac{-3}{2}$$

$$2x-1=0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$\beta = \frac{1}{2}$$

Verification :-

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha + \beta = \frac{-3}{2} + \frac{1}{2}$$

$$\alpha\beta = \frac{-3}{2} \times \frac{1}{2}$$

$$= \frac{-3+1}{2}$$

$$= \frac{-3}{4}$$

$$= \frac{-2}{2} = -1$$

$$\frac{c}{a} = \frac{-3}{4}$$

$$\frac{-b}{a} = \frac{-4}{4} = -1$$

$$\therefore \alpha\beta = \frac{c}{a}$$

$$\therefore \alpha + \beta = -\frac{b}{a}$$

Hence verified

$$v) x^2 + \frac{1}{6}x - 2$$

$$= \frac{1}{6} [6x^2 + x - 12]$$

$$= 6x^2 + x - 12$$

$$= 6x^2 + 9x - 8x - 12$$

$$= 3x(2x - 3) - 4(2x - 3)$$

$$= (3x - 4)(2x - 3)$$

$$3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

$$\alpha = \frac{4}{3}$$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$\beta = \frac{3}{2}$$

Verification :-

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha \beta = \frac{c}{a}$$

$$\alpha + \beta = \frac{4}{3} + \frac{3}{2}$$

$$\alpha \beta = \frac{4^2}{3} \times \frac{3}{2} = 2$$

$$= \frac{8 + 9}{6}$$

$$\frac{c}{a} = \frac{12}{6} = 2$$

$$= -\frac{1}{6}$$

$$\therefore \alpha \beta = \frac{c}{a}$$

$$-\frac{b}{a} = -\frac{1}{6}$$

$$\therefore \alpha + \beta = -\frac{b}{a}$$

Hence verified.



Page No. 80
Date: 26/4/2018

$$\text{vi) } v^2 + 4\sqrt{3}v + 15 \\ = v^2 + 5\sqrt{3}v - \sqrt{3}v - 15 \\ = v(v+5\sqrt{3}) - \sqrt{3}(v+5\sqrt{3}) \\ = (v-\sqrt{3})(v+5\sqrt{3})$$

$$v - \sqrt{3} = 0$$

$$v = \sqrt{3}$$

$$\alpha = \sqrt{3}$$

$$v + 5\sqrt{3} = 0$$

$$v = -5\sqrt{3}$$

$$\beta = -5\sqrt{3}$$

Verification :-

$$\alpha + \beta = -\frac{b}{a}$$

$$\begin{aligned}\alpha + \beta &= \sqrt{3} + (-5\sqrt{3}) \\ &= \sqrt{3} - 5\sqrt{3} \\ &= -4\sqrt{3}\end{aligned}$$

$$\frac{-b}{a} = -\frac{4\sqrt{3}}{1}$$

$$= -4\sqrt{3}$$
$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\alpha \beta = \frac{c}{a}$$

$$\begin{aligned}\alpha \beta &= \sqrt{3} \times (-5\sqrt{3}) \\ &= -5 \times 3 \\ &= -15\end{aligned}$$

$$\frac{c}{a} = \frac{-15}{1} = -15$$

$$\therefore \alpha \beta = \frac{c}{a}$$

Hence verified

- Q4) If α and $\frac{1}{\alpha}$ are the zeroes of the polynomial $6x^2 + 11x - (k-2)$
then find the value of k .
Sol. $p(x) = 6x^2 + 11x - (k-2)$
 $a = 6, b = 11, c = -(k-2)$

Since zeroes are reciprocal of each other

$$\therefore \alpha \beta = \frac{c}{a}$$

$$\frac{\alpha \times \frac{1}{\alpha}}{\alpha} = \frac{-(k-2)}{6}$$

$$1 = \frac{-k+2}{6}$$

$$6 = -k + 2$$

$$\begin{aligned} k &= 2 - 6 \\ &= -4 \end{aligned}$$

- Q5) If one zero of $p(x) = (a^2 + 9)x^2 + 13x + 6a$ is reciprocal of other find 'a'

Sol. $p(x) = (a^2 + 9)x^2 + 13x + 6a$
 $a = a^2 + 9, b = 13, c = 6a$

Since zeroes are reciprocal of each other

$$\therefore \alpha \beta = \frac{c}{a}$$

$$\frac{\alpha \times \frac{1}{\alpha}}{\alpha} = \frac{6a}{a^2 + 9}$$

$$a^2 + 9 = 6a$$

$$a^2 - 6a + 9 = 0$$

$$a^2 - 3a - 3a + 9$$

$$a(a-3) - 3(a-3) = 0$$

$$(a-3)(a-3) = 0$$

$$a - 3 = 0$$

$$a = 3$$

Q6) If the zeroes of $p(x) = x^2 - 5x + k$ are reciprocal of each other find k .

Sol

$$p(x) = x^2 - 5x + k$$

$$a = 1, b = -5, c = k$$

Since zeroes are reciprocal of each other,

$$\therefore \alpha \beta = \frac{c}{a}$$

$$\cancel{\alpha} \cancel{x} = \frac{k}{1}$$

$$k = 1$$

Q7) If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, then find k .

Sol

$$a = 3, b = -8, c = 2k + 1$$

$$\text{let } \alpha = x, \beta = 7x$$

$$\alpha + \beta = -\frac{b}{a}$$

$$x + 7x = -\frac{-8}{3}$$

$$8x = \frac{8}{3}$$

$$x = \frac{8x}{3}$$

$$x = \frac{1}{3}$$

$$\alpha \beta = \frac{c}{a}$$

$$x \cdot 7x = \frac{2k+1}{3}$$

$$7x^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$\frac{7}{9} = \frac{2k+1}{3}$$

$$91 = 18k + 9$$

$$18k = 21 - 9$$

$$18k = 12$$

$$k = \frac{12}{18}$$

$$k = \frac{2}{3}$$

Q8) If sum of zeroes of $p(x) = 3x^2 - kx + 6$ is 3 find k .
Sol. $a = 3, b = -k, c = 6$

$$\alpha + \beta = 3$$

$$\frac{-b}{a} = 3$$

$$\frac{-(-k)}{3} = 3$$

$$k = 3 \times 3$$

$$k = 9$$

Q9) -3 is the zero of $p(x) = (k-1)x^2 + kx - 3$. Find k .

Given $x = -3$

$$0 = (k-1)(-3)^2 + k(-3) - 3$$

$$0 = 9k - 9 - 3k - 3$$

$$0 = 6k - 12$$

$$6k = 12$$

$$k = \frac{12}{6}$$

$$k = 2$$

Identities :- i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

ii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

iii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

iv) $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

- Q.b) If α & β are zeroes of $p(x) = 3x^2 + 3x - 6$. Find value of i) $\alpha^2 + \beta^2$ ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ iv) $\alpha^2 + \beta^2 - \alpha\beta$
 v) $\alpha^2\beta + \beta^2\alpha$ vi) $\alpha^3 + \beta^3$ vii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ viii) $\alpha - \beta$

Sol $a = 3, b = 3, c = -6$

$$\alpha + \beta = -\frac{b}{a} = -\frac{3}{3} = -1, \quad \alpha\beta = \frac{-6}{3} = -2$$

$$\text{i) } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \\ = \left(-\frac{3}{3}\right)^2 - 2(-3) \\ = \frac{9}{4} + 6 = \frac{33}{4}$$

$$\text{ii) } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{3}{3}}{-2} = \frac{-\frac{3}{3}}{-2} = \frac{1}{2}$$

$$\text{iii) } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{\frac{33}{4}}{-3} = \frac{33}{4} \times -\frac{1}{3} = -\frac{11}{4}$$

$$\text{iv) } \alpha^2 + \beta^2 - \alpha\beta = \frac{33}{4} - (-3) = \frac{33}{4} + 3 = \frac{33+12}{4} = \frac{45}{4}$$

$$\text{v) } \alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$$

$$= -3 \left(\frac{-3}{2} \right)$$

$$= \frac{-9}{2}$$

~~$$\text{vi) } \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= \left(\frac{3}{4} \right)^3 - 3 \times -3 \left(\frac{3}{4} \right)$$

$$= \frac{35937}{64} + \frac{9 \times 3}{4}$$

$$= \frac{35937}{64} + \frac{27}{4}$$

$$= \frac{35937 + 432}{64} = \frac{40689}{64} =$$~~

$$\text{vii) } \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= \left(\frac{-3}{2} \right)^3 - 3(-3) \left(\frac{-3}{4} \right)$$

$$= \frac{-27}{8} - \frac{27}{4}$$

$$= \frac{-27 - 108}{8} = \frac{-135}{8}$$

$$\text{viii) } \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{-135}{8} = \frac{125}{8} \times \frac{1}{-5} = \frac{25}{8}$$

$$\text{ix) } \alpha - \beta = \left(\frac{-3}{2} \right)^2 - 4 \times -3$$

$$= \frac{9}{4} + 12 = \frac{9 + 48}{4} = \sqrt{57} = \frac{\sqrt{57}}{4}$$

- (Q1) If α & β are the zeroes of $p(x) = x^2 - x - 6$. Find
- $\alpha^2 + \beta^2$
 - $\frac{1}{\alpha} + \frac{1}{\beta}$
 - $\frac{\alpha + \beta}{\beta \alpha}$
 - $\alpha^2 + \beta^2 - \alpha\beta$
 - $\alpha^2\beta + \beta^2\alpha$
 - $\alpha^3 + \beta^3$
 - $\frac{\alpha^2 + \beta^2}{\beta}$
 - $\alpha - \beta$

Sol

$$p(x) = x^2 - x - 6$$

$$a = 1, b = -1, c = -6$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{1} = \frac{1}{1} = 1$$

$$\alpha\beta = \frac{c}{a} = \frac{-6}{1} = -6$$

$$\begin{aligned} i) \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 1^2 - 2 \times -6 \\ &= 1 + 12 = 13 \end{aligned}$$

$$ii) \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{1}{-6} = -\frac{1}{6}$$

$$iii) \frac{\alpha + \beta}{\beta \alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{13}{-6} = -\frac{13}{6}$$

$$iv) \alpha^2 + \beta^2 - \alpha\beta = 13 - (-6) = 13 + 6 = 19$$

$$v) \alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta) = -6(1) = -6$$

$$\begin{aligned} vi) \alpha^3 + \beta^3 &= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \\ &= 1^3 - 3 \times -6 \times 1 \\ &= 1 + 18 \\ &= 19 \end{aligned}$$

$$\text{vii) } \frac{\alpha^2 + \beta^2}{\alpha \beta} = \frac{\alpha^3 + \beta^3}{\alpha \beta} = \frac{-19}{6} .$$

$$\begin{aligned}\text{viii) } \alpha - \beta &= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \\ &= \sqrt{1^2 - 4x - 6} \\ &= \sqrt{1 + 24} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

(Q) If α & β are the zeroes of the $p(x) = 3x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, find value of k .
Sol

$$\alpha + \beta = -\frac{b}{a} = -\frac{5}{3}, \quad \alpha\beta = \frac{c}{a} = \frac{k}{3}$$

$$\alpha^2 + \beta^2 + \alpha\beta = ?$$

$$(\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta = ?$$

$$(\alpha + \beta)^2 - \alpha\beta = ?$$

$$\left(-\frac{5}{3}\right)^2 - \frac{k}{3} = \frac{21}{4}$$

$$\frac{25}{9} - \frac{k}{3} = \frac{21}{4}$$

$$\frac{25}{9} - \frac{21}{4} = \frac{k}{3}$$

$$\frac{25}{9} - \frac{21}{4} = k$$

$$\frac{1}{2}k = b$$

$$k = 2$$

Q13) If α & β are zeroes of $p(x) = kx^2 + 4x + 4$

Page No. 89
Date: 29/4/2025

satisfying the relation $\alpha^2 + \beta^2 = 24$. Find k .

Sol

$$a = k, b = 4, c = 4$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{4}{k} \quad \Rightarrow \quad \alpha \beta = \frac{c}{a} = \frac{4}{k}$$

$$\alpha^2 + \beta^2 = 24$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 24$$

$$\left(\frac{-4}{k}\right)^2 - 2 \times \frac{4}{k} = 24$$

$$\frac{16}{k^2} - \frac{8}{k} = 24$$

$$\frac{16 - 8k}{k^2} = 24$$

$$16 - 8k = 24k^2$$

$$24k^2 + 8k - 16 = 0$$

$$3k^2 + k - 2 = 0$$

$$3k^2 + 3k - 2k - 2 = 0$$

$$3k(k+1) - 2(k+1) = 0$$

$$(3k-2)(k+1) = 0$$

$$3k - 2 = 0$$

$$3k = 2$$

$$k = \frac{2}{3}$$

$$k+1=0$$

$$k=-1$$

(Q4) Find the quadratic $p(x)$ whose sum & product of zeroes is $-2\sqrt{3}$ and -9 . Also find zeroes of $p(x)$

Sol

$$\begin{aligned} p(x) &= x^2 - Sx + P \\ &= x^2 - (-2\sqrt{3})x + -9 \\ &= x^2 + 2\sqrt{3}x - 9 \end{aligned}$$

$$\begin{aligned} &= x^2 + 2\sqrt{3}x - 9 \\ &= x^2 + 3\sqrt{3}x - \sqrt{3}x - 9 \\ &= x^2 - \sqrt{3}x + 3\sqrt{3}x - 9 \\ &\cancel{=} x(x - \sqrt{3}) + 3(\cancel{x} + \cancel{\sqrt{3}}x - 3) \\ &= x^2 + 3\sqrt{3}x - \sqrt{3}x - 9 \\ &= x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) \\ &= (x - \sqrt{3})(x + 3\sqrt{3}) \end{aligned}$$

$$\left. \begin{array}{l} x - \sqrt{3} = 0 \\ x = \sqrt{3} \\ \alpha = \sqrt{3} \end{array} \right| \quad \left. \begin{array}{l} x + 3\sqrt{3} = 0 \\ x = -3\sqrt{3} \\ \beta = -3\sqrt{3} \end{array} \right|$$

(Q5) Find the quadratic $p(x)$ whose zeroes are $\frac{3-\sqrt{5}}{5}$ & $\frac{3+\sqrt{5}}{5}$.

$$\alpha = \frac{3-\sqrt{5}}{5}, \beta = \frac{3+\sqrt{5}}{5}$$

$$\alpha + \beta = \frac{3-\sqrt{5}}{5} + \frac{3+\sqrt{5}}{5} = \frac{3+3-\sqrt{5}+\sqrt{5}}{5} = \frac{6}{5}$$

$$\alpha \beta = \frac{3-\sqrt{5}}{5} \times \frac{3+\sqrt{5}}{5} = \frac{3^2 - \sqrt{5}^2}{25} = \frac{9-5}{25} = \frac{4}{25}$$

$$p(x) = x^2 - \frac{6}{5}x + \frac{4}{25}$$

$$= 25x^2 - 30x + 4$$

CH-4

Quadratic Equations

Nature of Roots :-

Discriminant :- (D)

$$D = b^2 - 4ac$$

$$D > 0 \quad D=0 \quad D < 0$$

$D > 0$
Real &
Distinct

$D=0$
Real &
Equal

$D < 0$
Imaginary roots /
no real roots

To find roots if $D > 0, D=0$

$$x = \frac{-b + \sqrt{D}}{2a}$$

[EX-4.]

Q1) Check _____

Ans i) $(x+1)^2 = 2(x-3)$

$$x^2 + 1^2 + 2x \times x = 2x - 6$$

$$x^2 + 1 + 2x = 2x - 6$$

$$x^2 + 1 + 6 = 0$$

$$x^2 + 7 = 0$$

∴ Yes

$$i) x^2 - 3x = -2(3-x)$$

$$x^2 - 3x = -6 + 2x$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x^2 - 4x + 6 = 0$$

\therefore Yes

$$ii) (x-2)(x+1) = (x-1)(x+3)$$

$$x^2 + x - 3x - 2 = x^2 + 3x - x - 3$$

$$x^2 - x^2 + x + x - 3x + 3x - 2 + 3 = 0$$

$$3x + 2 = 0$$

\therefore No

$$iv) (x-3)(3x+1) = x(x+5)$$

$$3x^2 + x - 6x - 3 = x^2 + 5x$$

$$3x^2 - x^2 + x - 5x - 6x - 3 = 0$$

$$x^2 - 10x - 3 = 0$$

\therefore Yes

$$v) (3x-1)(x-3) = 6x(x-1)$$

$$9x^2 - 6x - x + 3 = 6x^2 - 6x + 5x - 5$$

$$9x^2 - x^2 - 6x - 5x - x + 3 + 5 = 0$$

$$x^2 - 13x + 8 = 0$$

\therefore Yes

$$vi) x^2 + 3x + 1 = (x+2)^2$$

$$x^2 + 3x + 1 = x^2 + 4 - 4x$$

$$x^2 - x^2 + 3x - 4x + 1 - 4 = 0$$

$$-x - 3 = 0$$

\therefore No

$$vii) (x+3)^3 = 2x(x^2 - 1)$$

$$x^3 + 8 + 3 \cdot 6x(x+2) = 2x^3 - 2x$$

$$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$$

$$x^3 - 2x^3 + 8 + 6x^2 + 12x + 2x = 0$$

$$-x^3 + 8 + 6x^2 + 14x = 0$$

\therefore No

$$viii) x^3 - 4x^2 - x + 1 = (x-3)^3$$

$$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x(x-2)$$

$$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$$

$$x^3 - x^3 - 4x^2 + 6x^2 - x - 12x + 1 + 8 = 0$$

$$9x^2 - 13x + 9 = 0$$

\therefore Yes

Q2) Represent

Sol i) Let $B = x_m$

$$L = (1+3x)_m$$

$$A = 528_m^2$$

$$L \times B = A$$

$$(1+3x) \times x = 528$$

$$3x^2 + x = 528$$

$$2x^2 + x - 528 = 0$$

$$a=2, b=1, c=-528$$

$$\begin{aligned}D &= b^2 - 4ac \\&= 1^2 - 4 \times 2 \times -528 \\&= 1 + 4224 \\&= 4225\end{aligned}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-1 \pm \sqrt{4225}}{2 \times 2}$$

$$x = \frac{-1 + 65}{4}$$

$$= \frac{64}{4}$$

$$x = 16$$

$$x = \frac{-1 - 65}{4}$$

$$= \frac{-66}{4}$$

Not possible

$$B = 16 \text{ km}$$

$$L = 1 + 2x$$

$$= 1 + 32$$

$$= 33 \text{ km}$$

i) Let first no. = x
 Second no. = $x+1$
 Product = 306

$$x(x+1) = 306$$

$$x^2 + x - 306 = 0$$

$$a = 1, b = 1, c = -306$$

$$\begin{aligned} D &= \cancel{b^2} - 4ac \\ &= 1^2 - 4 \times 1 \times -306 \\ &= 1 + 1224 \\ &= 1225 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{-1 \pm \sqrt{1225}}{2 \times 1} \\ &= \frac{-1 \pm 35}{2} \end{aligned}$$

∴

$$\begin{aligned} x &= \frac{-1 + 35}{2} \\ &= \frac{34}{2} = 17 \end{aligned}$$

$$x = \frac{-1 - 35}{2}$$

$$= \frac{-36}{2} = -18$$

Not possible

$$x = 17$$

$$\text{First no.} = 17$$

$$\begin{aligned} \text{Second no.} &= 17 + 1 \\ &= 18 \end{aligned}$$

(iv) Let present age of Rohan = x
 " " " mother = $x + 26$

3 years later age of Rohan = $x + 3$

" " " mother = $x + 26 + 3 = x + 29$
 Product = 360

$$(x+3)(x+29) = 360$$

$$x^2 + 29x + 3x + 87 = 360$$

$$x^2 + 32x + 87 = 0$$

$$x^2 + 32x - 273 = 0$$

~~$$x^2 + 39x - 7x - 273 = 0$$~~

~~$$D = b^2 - 4ac$$~~

$$= 32^2 - 4 \times 1 \times -273$$

$$= 1024 + 1092$$

$$= 2116$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-32 \pm \sqrt{46}}{2 \times 1}$$

$$x = \frac{-32 + 46}{2}$$

$$= \frac{14}{2} = 7$$

$$x = 7$$

$$x = \frac{-32 - 46}{2}$$

$$= \frac{-78}{2} = -39$$

not feasible

Rohan Age = 7

Mother age = $7 + 26 = 33$

iv) Let speed = x km/h

Distance = 480 km

$$T_1 = \frac{480}{x} - \textcircled{1}$$

New Speed = $(x-8)$ km/h

$$T_2 = \frac{480}{x-8} - \textcircled{2}$$

Difference = 3 hrs

$$T_2 - T_1 = 3$$

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$\frac{480x - 480(x-8)}{x(x-8)} = 3$$

$$x(x-8)$$

$$\frac{480x - 480x + 3840}{x^2 - 8x} = 3$$

$$\frac{3840}{x^2 - 8x} = 3$$

$$(x^2 - 8x)3 = 3840$$

$$3x^2 - 24x - 3840 = 0$$

$$x^2 - 8x - 1280 = 0$$

$$D = b^2 - 4ac$$

$$= (-8)^2 - 4 \times 1 \times -1280$$

$$= 64 + 5120$$

$$= 5184$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-8) \pm \sqrt{5184}}{2 \times 1} = \frac{8 \pm 72}{2}$$

$$x = \frac{8+72}{2}$$

$$= \frac{80}{2} = 40$$

$$x = \frac{8-72}{2}$$

$$= \frac{-64}{2} = -32$$

Not possible

∴ Speed of train = 40 km/h

(ii) A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken one hr less for same journey. Find speed of train.

Let speed = x km/h

Distance = 360 km

$$T_1 = \frac{360}{x} \quad \text{(1)}$$

New speed = $(x+5)$ km/h

$$T_2 = \frac{360}{x+5} \quad \text{(2)}$$

Difference = 1 hr

$$T_1 - T_2 = 1$$

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$$\frac{360x + 1800 - 360x}{x(x+5)} = 1$$

$$\frac{1800}{x^2 + 5x} = 1$$

$$1800 = x^2 + 5x$$

$$x^2 + 5x - 1800 = 0$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 25 - 4 \times 1 \times -1800 \\ &= 25 + 7200 \\ &= 7225 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{-5 \pm \sqrt{7225}}{2 \times 1} \\ &= \frac{-5 \pm 85}{2} \end{aligned}$$

$$\begin{aligned} x &= \frac{-5 + 85}{2} \\ &= \frac{80}{2} = 40 \end{aligned}$$

$$\begin{aligned} x &= \frac{-5 - 85}{2} \\ &= \frac{-90}{2} = -45 \end{aligned}$$

Not possible

\therefore Speed of Train = 40 km/h

(Q2) An express train takes 1 hr less than a passenger train to travel 132 km b/w Mysore and Bangalore. If average speed of express is 11 km/h more than passenger train. Find avg. speed of both trains.

Sol Let speed of express train = x km/h

$$\text{Distance} = 132 \text{ Km}$$

$$T_1 = \frac{132}{x} - ①$$

$$\text{Speed of Passenger train} = \frac{x}{x-11} (x-11) \text{ km/h}$$

$$T_2 = \frac{132}{x-11} - ②$$

Difference = 1 hr.

$$T_2 - T_1 = 1$$

$$\frac{132}{x-11} - \frac{132}{x} = 1$$

$$\frac{132x - 132}{x(x-11)} + 1452 = 1$$

$$1452 = x^2 - 11x$$

$$x^2 - 11x - 1452 = 0$$

$$D = b^2 - 4ac$$

$$= 11^2 - 4 \times 1 \times -1452$$

$$= 121 + 5808$$

$$= 5929$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-11) \pm \sqrt{5929}}{2} = \frac{11 \pm 77}{2}$$

$$x = \frac{11 + 77}{2}$$

$$= \frac{88}{2} = 44$$

$$x = \frac{11 - 77}{2}$$

$$= \frac{-66}{2} = -33$$

Not possible

Speed of express train = 44 km/h
 " " passenger " = 33 km/h