MG207

Managerial Economics

Michaelmas Term

Lecture 4

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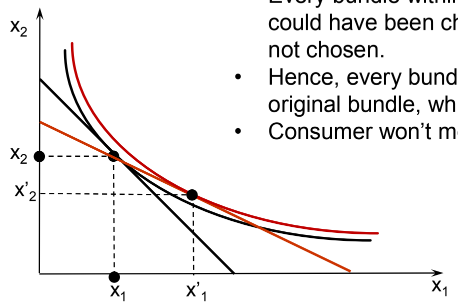
Today

- Applying the Slutsky Equation.
- Intertemporal Choice
  - The intertemporal budget constraint.
  - Borrowers and lenders.
- Assets and Asset Markets

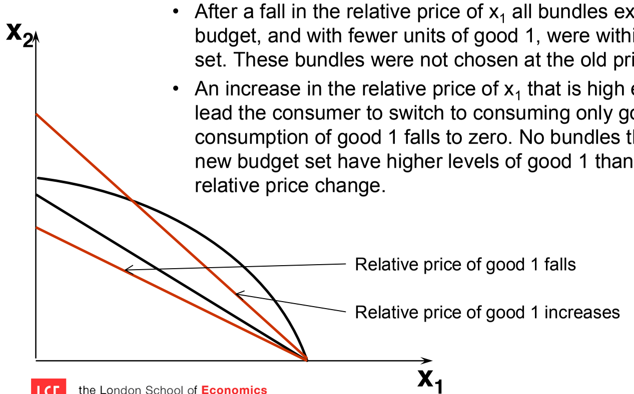
Substitution effects: Recap

- The Slutsky substitution effect is negative when the indifference curve is convex. That is, if p_1 falls, x_1 increases; p_1 rises, x_1 decreases.
- How do we know?
  - Here, p_1 falls. The red budget constraint reflects only the fall in relative prices.
  - Every bundle within the new set that contains less x_1 could have been chosen under the old prices, but was not chosen.
  - Hence, every bundle with less x_1 is dominated by original bundle, which is still in the feasible set.
  - Consumer won't move to any bundle with less x_1.



Substitution effects: Recap

- Suppose preferences give concave indifference curves.
- Is the substitution effects still negative? Yes.
- Say consumer was at a corner solution, consuming only x_1, so consumption was $(m/p_1,0)$.
  - After a fall in the relative price of x_1 all bundles exhausting the new budget, and with fewer units of good 1, were within the old budget set. These bundles were not chosen at the old prices.
  - An increase in the relative price of x_1 that is high enough might lead the consumer to switch to consuming only good 2, so his consumption of good 1 falls to zero. No bundles that exhaust the new budget set have higher levels of good 1 than before the relative price change.

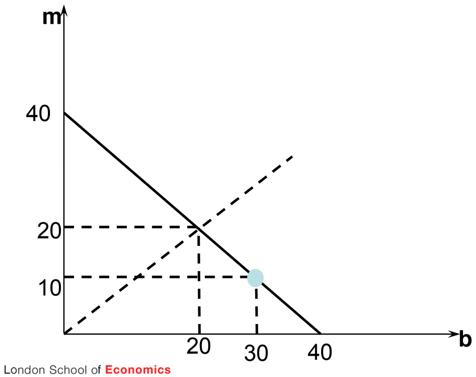


Application: Trading

- A small country produces oil and machines. Both are needed to provide power for the country.
- These two commodities are perfect complements in this economy because 1 million barrels of oil are required for each machine.
- In a certain year, the country produces 30 million barrels of oil and 10 machines. At this time, the world price for a million barrels of oil is \$5m, and the price of each machine is also \$5m.
- First, find the value of the endowment and the net demands for oil and machines from this country in the world market.

Application: Trading

- Endowment value: $p_o\omega_o+p_m\omega_m = 5 \times 30 + 5 \times 10 = 200$
- Budget constraint: slope is $-p_o/p_m = -5/5 = -1$
- We know (30,10) lies on the budget constraint.
- On the constraint, $p_ob+p_mm = 200$. $m=200/p_m-(p_o/p_m)b = 40 - b$.



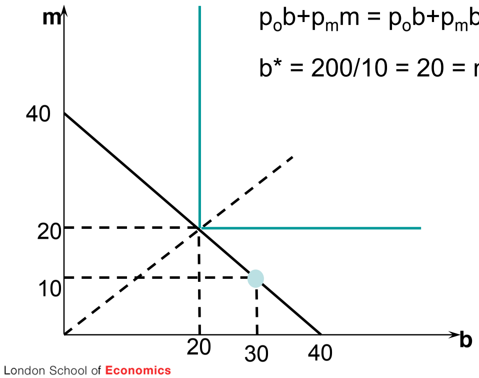
Application: Trading

- On the constraint, $p_ob+p_mm = 200$. $m=200/p_m-(p_o/p_m)b = 40 - b$.
- Perfect complements means optimal bundle is when country has a million barrels of oil per machine.

This means $b^*=m^*$, so the constraint becomes:

$p_ob+p_mm = p_ob+p_mb = b(p_m+p_o) = 200$

$b^* = 200/10 = 20 = m^*$



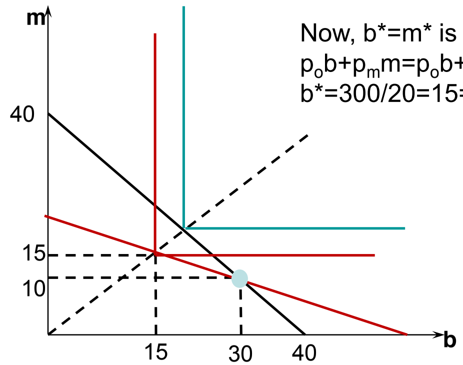
Application: Trading

- Suppose that before the country trades, the price of machines rises to \$15m while the price of oil remains at \$5m.
- What is the value of the endowment now and what will the consumption bundle be?
- Endowment value: $p_o\omega_o+p_m\omega_m = 5 \times 30 + 15 \times 10 = 300$.

Now, $b^*=m^*$ is such that

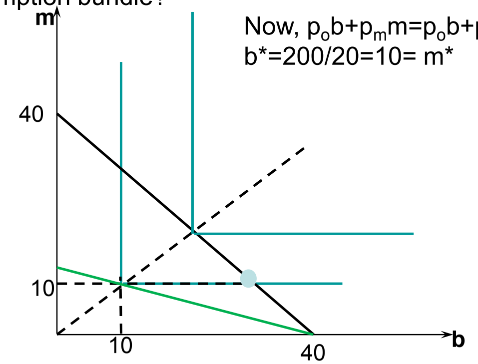
$p_ob+p_mm=p_ob+p_mo=b(p_m+p_o)=300$

$b^*=300/20=15= m^*$



Application: Trading

- Suppose the country had sold its entire endowment at the original prices so had $200m, intending to buy back oil and machines.
- Before it had a chance to buy back any commodities, the price of machines rose to $15m while the price of oil stayed at $5m.
- What is the budget constraint in this scenario? What is the consumption bundle?



Now, $p_o b+p_m m=p_o b+p_m b=b(p_m+p_o)=200$
$b^*=200/20=10= m^*$

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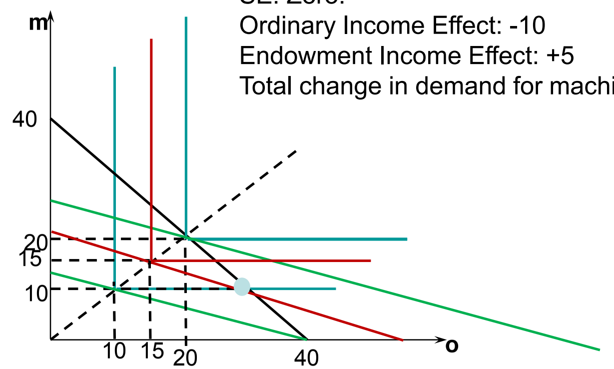
Application: Trading

- Assuming the price of machines roles before this country made any transactions.
- The change in demand for machines is the substitution effect, the ordinary income effect and the endowment income effect. Here:

SE: Zero!
Ordinary Income Effect: -10
Endowment Income Effect: +5
Total change in demand for machines: -5



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Application: Labor Supply Decision

- A worker is endowed with \$0 of nonlabor income and $\overline{R}$ hours of time which can be used for labor or leisure (relaxation), so $\boldsymbol{\omega} = (\overline{R},0)$.
- If the consumer supplies labor, she earns an hourly wage w.
- She can spend her wage income on consumption goods, priced at p_c per unit.
- This means she trades off consumption and leisure. These activities give her utility.
- The worker's budget constraint is

$$p_c C = w(\overline{R} - R)$$

where C, R denote gross demands for the consumption good and for leisure.

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Application: Labor Supply Decision

- Note, rearranging the budget constraint gives:

$$p_c C + wR = w\overline{R}$$

- Where the left hand side is the value of expenditure and the right hand side is the value of the endowment.
- Rearranging again to get C as a function of other variables gives us the worker's budget constraint in a form that we can plot on axes of C against R.

$$C = \frac{w\overline{R}}{p_c} - \frac{w}{p_c} R$$

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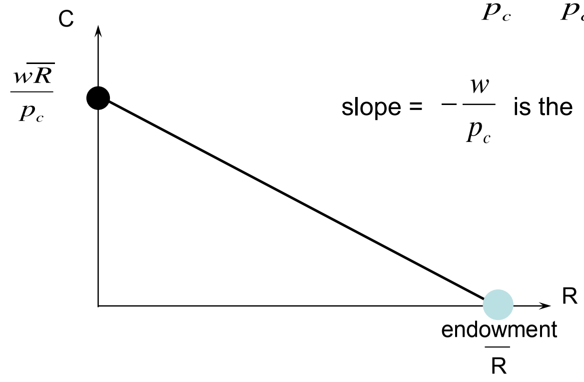
Application: Labor Supply Decision

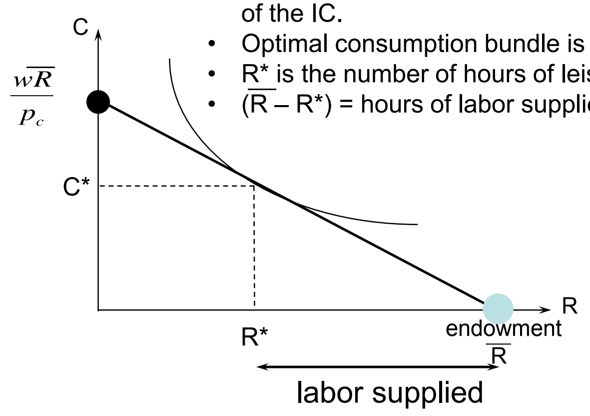
$$C = \frac{w\overline{R}}{p_c} - \frac{w}{p_c}R$$

slope = $-\dfrac{w}{p_c}$ is the ‘real wage rate’



Application: Labor Supply Decision

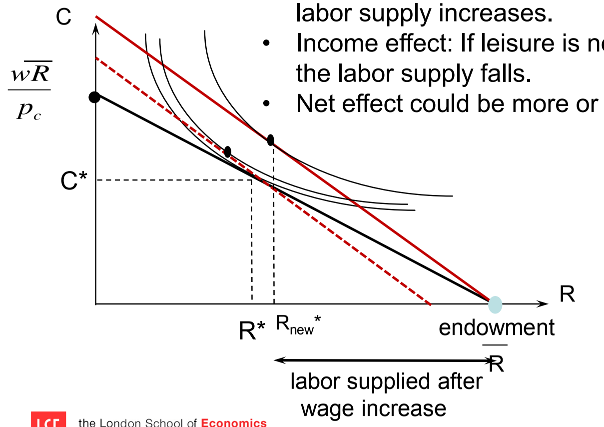
- In these axes, we can add her preferences, summarized in an indifference curve.
- The MRS between C and R is given by the slope of the IC.
- Optimal consumption bundle is where MRS=-w/p_c.
- R* is the number of hours of leisure.
- ($\overline{R}$ – R*) = hours of labor supplied.



Labor Supply: Wage Change

$$C = \frac{w\overline{R}}{p_c} - \frac{w}{p_c}R$$

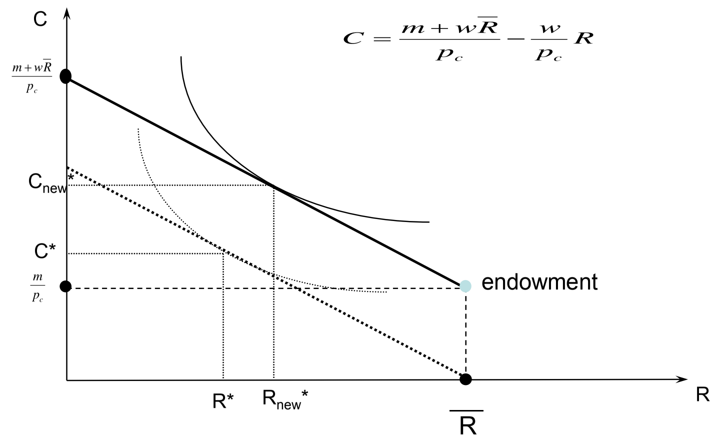
- Let us say that hourly wages rise.
- Substitution effect: C rises and R falls, so the labor supply increases.
- Income effect: If leisure is normal, R rises and the labor supply falls.
- Net effect could be more or less labor supply.



Labor Supply Decision

- Let’s introduce a universal basic income.
- New budget constraint is a parallel vertical shift, by the amount (m/p_c):

$$C = \frac{m + w\overline{R}}{p_c} - \frac{w}{p_c}R$$

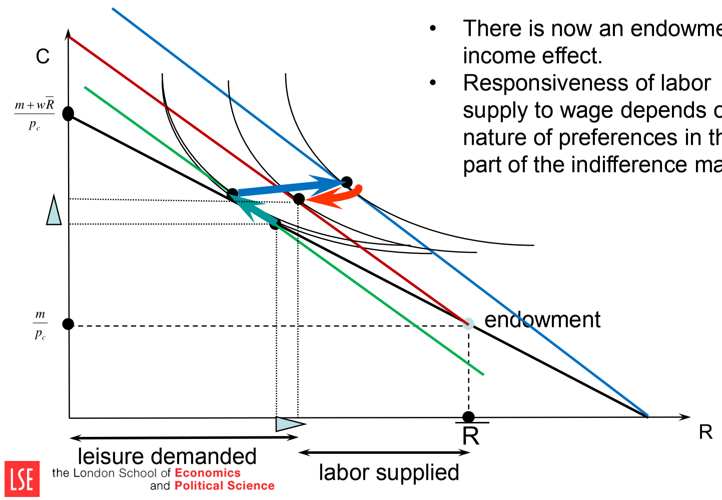


Intertemporal Choice

- People often receive income in “lump sums”; *e.g.* monthly salary.
- How is a lump of income spread over the following month, saving now for consumption later?
- Or, how is consumption financed by borrowing now against income to be received at the end of the month?
- Choices of consumption over time are known as intertemporal choices.
- These choices are constrained by an intertemporal budget line.
- Endowments, “prices”, and preferences determine consumption outcomes.

Labor Supply Decision

- How does the introduction of a universal basic income affect the elasticity of the labor supply with respect to wages?



- There is now an endowment income effect.
- Responsiveness of labor supply to wage depends on nature of preferences in this part of the indifference map.

Present and Future Values

- Begin with some simple financial arithmetic:
- Take just two periods: 1 and 2.
- Let *r* denote the interest rate per period.
- Then, for example, if r = 0.1 then \$100 saved at the start of period 1 becomes \$110 at the start of period 2.
- The value next period of \$1 saved now is the future value of that dollar.
- Given an interest rate *r*:
  - The future value one period from now of \$1 is FV = (1 + *r*).
  - The future value one period from now of \$m is FV= m(1 + *r*).
  - The present value of \$1 available at the start of the next period is PV= 1 / (1 + *r*).
  - And the present value of \$m available at the start of the next period is PV= m / (1 + *r*).

Intertemporal Choice

- Given an expected income during my working life, how much should I consume while working and how much should I save for retirement?
- If the nominal interest rate or expected inflation rate were to change, how would that impact my decision?
- What kinds of financial securities should I invest in to facilitate my future consumption?
- How are security prices determined?

The Intertemporal Choice Problem

- Let m_1 and m_2 be income received in periods 1 and 2.
- Let c_1 and c_2 be consumption in periods 1 and 2.
- Let p_1 and p_2 be the prices of consumption in periods 1 and 2.
- The intertemporal choice problem is:

  Given incomes m_1 and m_2, and given consumption prices p_1 and p_2, what is the most preferred intertemporal consumption bundle (c_1, c_2)?
- For an answer we need to know:
  - the intertemporal budget constraint.
  - intertemporal consumption preferences.

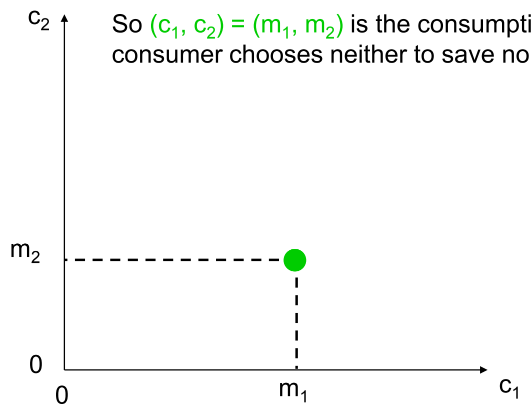
The Intertemporal Budget Constraint

- To start, let's ignore price effects by supposing that

  $$p_1 = p_2 = \$1.$$
- Suppose that the consumer chooses not to save or to borrow.
- Q: What will be consumed in period 1?
- A: $c_1 = m_1$.
- Q: What will be consumed in period 2?
- A: $c_2 = m_2$.

The Intertemporal Budget Constraint

So $(c_1, c_2) = (m_1, m_2)$ is the consumption bundle if the consumer chooses neither to save nor to borrow.

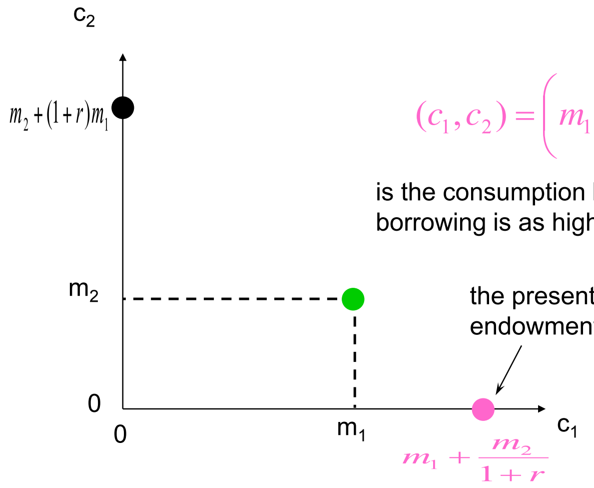


The Intertemporal Budget Constraint

- Now suppose that the consumer spends nothing on consumption in period 1; that is, $c_1 = 0$ and the consumer saves $s_1 = m_1$.
- The interest rate is r.
- What will be period 2's consumption level now?
- Period 2 income is m_2.
- Savings plus interest from period 1 sum to $(1 + r)m_1$.
- So total income available in period 2 is $m_2 + (1 + r)m_1$.
- So period 2 consumption expenditure is:

$$(p_2c_2) = c_2 = m_2 + (1 + r)m_1$$

The Intertemporal Budget Constraint

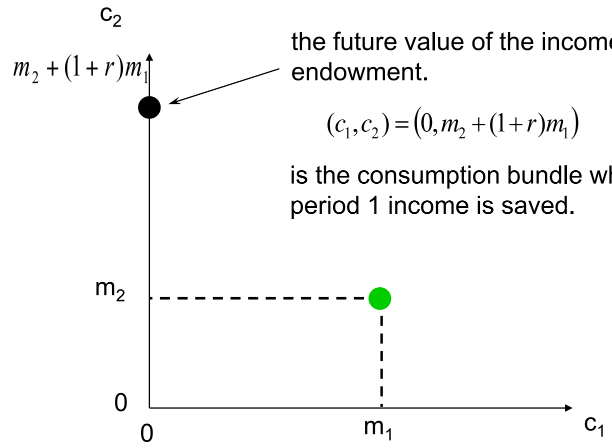


$$(c_1, c_2) = \left(m_1 + \frac{m_2}{1+r}, 0\right)$$

is the consumption bundle when period 1 borrowing is as high as possible.

the present value of the income endowment

The Intertemporal Budget Constraint



the future value of the income endowment.

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when all period 1 income is saved.

The Intertemporal Budget Constraint

- Suppose that c_1 units are consumed in period 1. This costs \$$c_1$ and leaves (m_1- c_1) saved. Period 2 consumption will then be:

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

- which is

$$c_2 = \underbrace{m_2 + (1+r)m_1}_{\text{intercept}} - \underbrace{(1+r)}_{\text{slope}}c_1.$$

The Intertemporal Budget Constraint

- Now suppose that the consumer spends everything possible on consumption in period 1, so c_2 = 0.
- What is the most that the consumer can borrow in period 1 against her period 2 income of \$$m_2$?
- Let b_1 denote the amount borrowed in period 1.
- Only \$$m_2$ will be available in period 2 to pay back \$$b_1$ borrowed in period 1.
- So $b_1(1 + r) = m_2$.
- That is, $b_1 = m_2 / (1 + r)$.
- So the largest possible period 1 consumption level is

$$c_1 = m_1 + m_2 / (1 + r)$$

The Intertemporal Budget Constraint

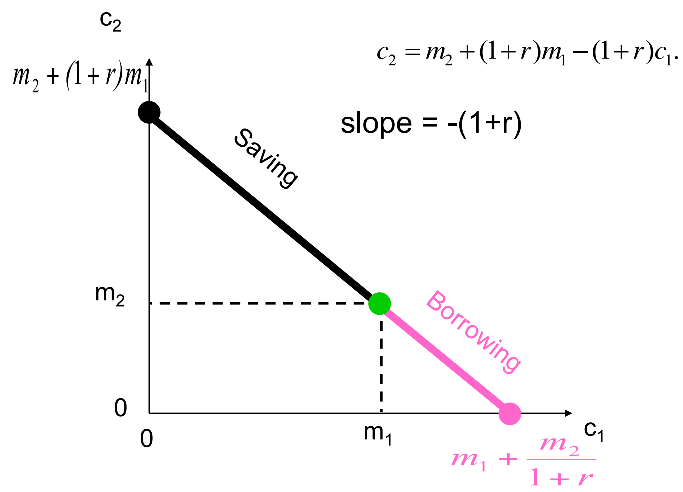
- Now let's add prices p_1 and p_2 for consumption in periods 1 and 2.
- How does this affect the budget constraint?
- Given her endowment (m_1,m_2) and prices p_1, p_2 what intertemporal consumption bundle (c_1^*,c_2^*) will be chosen by the consumer?
- Maximum possible expenditure in period 2 is

$$m_2 + (1+r)m_1$$

- so maximum possible consumption in period 2 is

$$c_2 = \frac{m_2 + (1+r)m_1}{p_2}.$$

The Intertemporal Budget Constraint



Intertemporal Choice

- Similarly, maximum possible expenditure in period 1 is

$$m_1 + \frac{m_2}{1+r}$$

so maximum possible consumption in period 1 is

$$c_1 = \frac{m_1 + m_2/(1+r)}{p_1}.$$

- Finally, if c_1 units are consumed in period 1 then the consumer spends p_1c_1 in period 1, leaving $(m_1 - p_1c_1)$ saved for period 1. Available income in period 2 will then be

$$m_2 + (1+r)(m_1 - p_1c_1)$$

so

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1).$$

The Intertemporal Budget Constraint

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

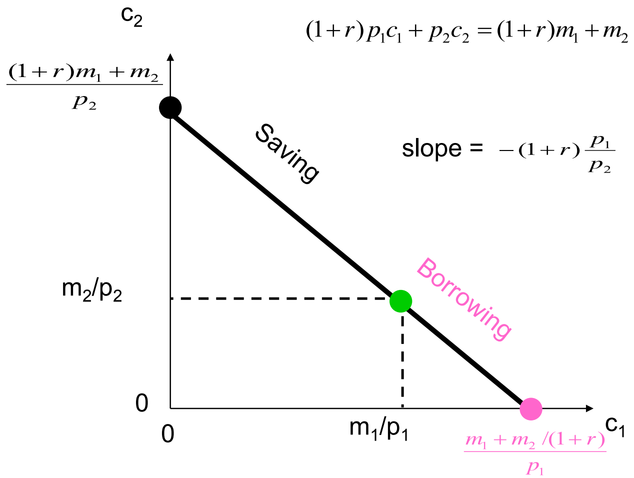
is the "future-valued" form of the budget constraint since all terms are in period 2 values.

This is equivalent to:

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

which is the "present-valued" form of the constraint since all terms are in period 1 values.

The Intertemporal Budget Constraint



Intertemporal Choice

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1)$$

rearranged is

$$(1+r)p_1c_1 + p_2c_2 = (1+r)m_1 + m_2.$$

This is the “future-valued” form of the budget constraint since all terms are expressed in period 2 values. Equivalent to it is the “present-valued” form

$$p_1c_1 + \frac{p_2}{1+r}c_2 = m_1 + \frac{m_2}{1+r}$$

where all terms are expressed in period 1 values.

Price Inflation

- Define the inflation rate by π where

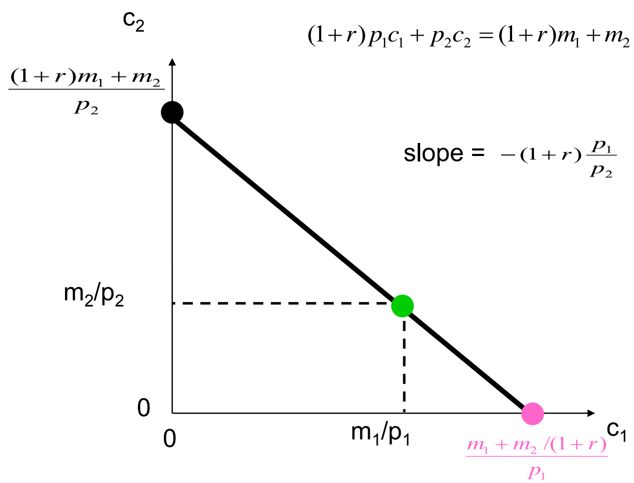
$$p_1(1+\pi) = p_2.$$

- For example,

π = 0.2 means 20% inflation, and

π = 1.0 means 100% inflation.

The Intertemporal Budget Constraint



Real Interest Rate

r	0.30	0.30	0.30	0.30	0.30
π	0.0	0.05	0.10	0.20	1.00
$r - \pi$	0.30	0.25	0.20	0.10	-0.70
ρ	0.30	0.24	0.18	0.08	-0.35

Price Inflation

- We lose nothing by setting $p_1=1$, so that $p_2 = 1+\pi$.
- Then we can rewrite the budget constraint:

$$p_1 c_1 + \frac{p_2}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

as

$$c_1 + \frac{1+\pi}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

- And this can be rearranged to give:

$$c_2 = \frac{1}{1+\pi}((1+r)m_1 + m_2) - \frac{1+r}{1+\pi} c_1$$

- So the slope of the intertemporal budget constraint is:

$$-\frac{1+r}{1+\pi}.$$

Comparative Statics

- The slope of the intertemporal budget constraint is

$$-(1+\rho) = -\frac{1+r}{1+\pi}.$$

- The constraint becomes flatter if the interest rate r falls or the inflation rate π rises (both decrease the real rate of interest).
- See Assignment 3, in Classes for Week 5.

Price Inflation and the Real Interest Rate

- When there was no price inflation ($p_1=p_2=1$) the slope of the budget constraint was $-(1+r)$.
- Now, with price inflation, the slope of the budget constraint is $-(1+r)/(1+\pi)$. This can be written as

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

ρ is known as the real interest rate.

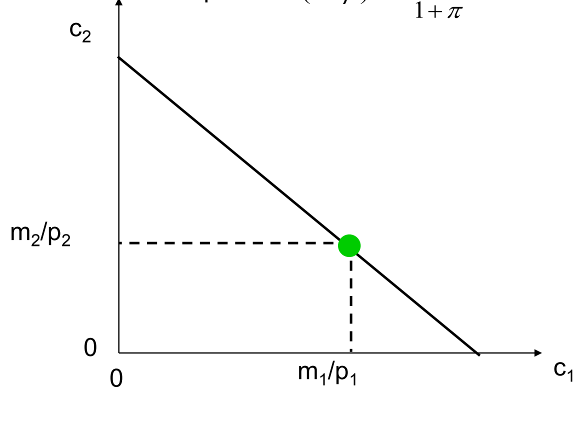
- And:

$$\rho = \frac{r-\pi}{1+\pi}.$$

- For low inflation rates ($\pi \approx 0$), $\rho \approx r - \pi$. For higher inflation rates this approximation becomes poor.

Comparative Statics

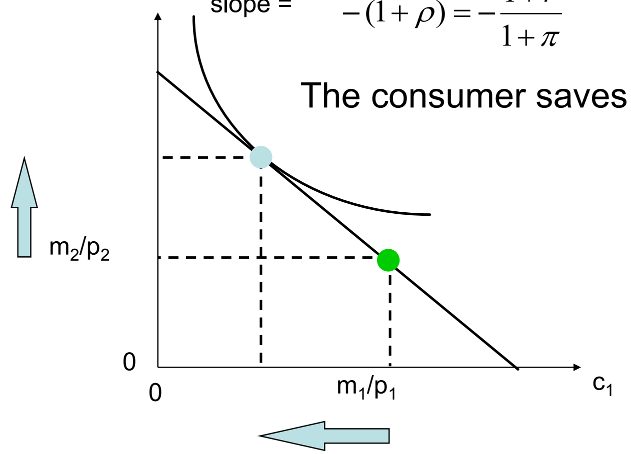
slope = $-(1+\rho) = -\dfrac{1+r}{1+\pi}$



Comparative Statics

slope = $-(1+\rho) = -\dfrac{1+r}{1+\pi}$

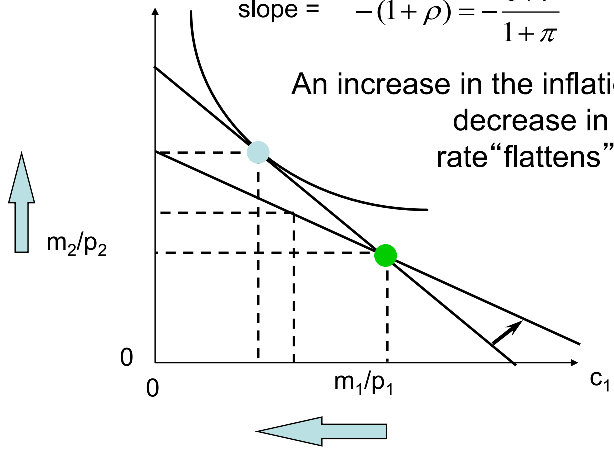
The consumer saves.



Comparative Statics

slope = $-(1+\rho) = -\dfrac{1+r}{1+\pi}$

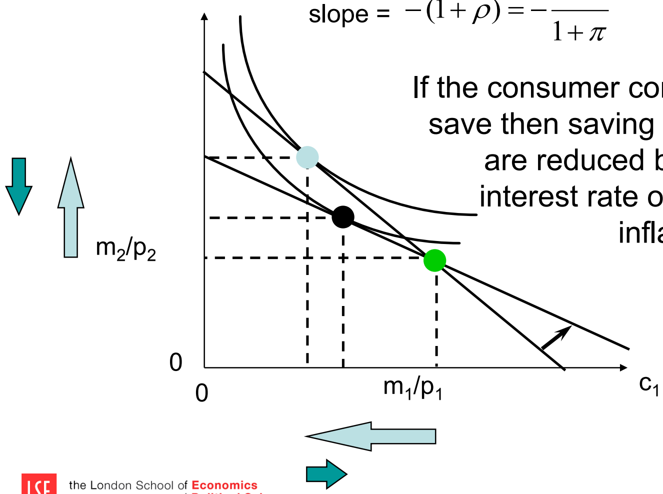
An increase in the inflation rate or a decrease in the interest rate "flattens" the budget constraint.



Comparative Statics

slope = $-(1+\rho) = -\dfrac{1+r}{1+\pi}$

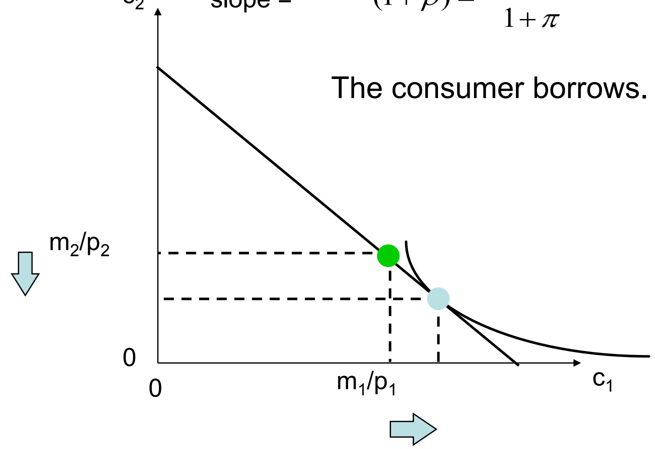
If the consumer continues to save then saving and utility are reduced by a lower interest rate or a higher inflation rate.



Comparative Statics

slope = $-(1+\rho) = -\frac{1+r}{1+\pi}$

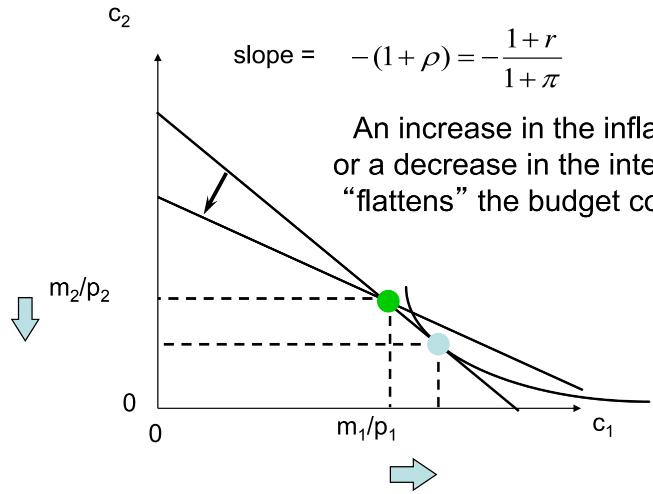
The consumer borrows.



Comparative Statics

slope = $-(1+\rho) = -\frac{1+r}{1+\pi}$

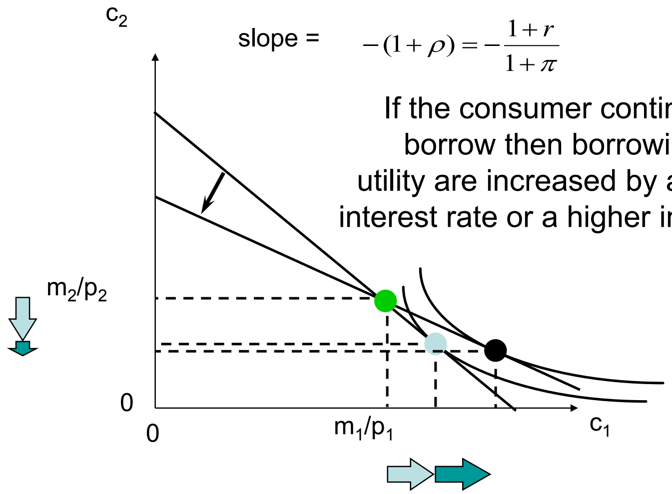
An increase in the inflation rate or a decrease in the interest rate “flattens” the budget constraint.



Comparative Statics

slope = $-(1+\rho) = -\frac{1+r}{1+\pi}$

If the consumer continues to borrow then borrowing and utility are increased by a lower interest rate or a higher inflation rate.



Valuing Securities

- A financial security is a financial instrument that promises to deliver an income stream.
- E.g.; a security that pays
  - \$$m_1$ at the end of year 1,
  - \$$m_2$ at the end of year 2, and
  - \$$m_3$ at the end of year 3.
- What is the most that should be paid now for this security?
- The PV of \$$m_1$ paid 1 year from now is $\mathbf{m_1 / (1+r)}$
- The PV of \$$m_2$ paid 2 years from now is $\mathbf{m_2 / (1+r)^2}$
- The PV of \$$m_3$ paid 3 years from now is $\mathbf{m_3 / (1+r)^3}$
- The PV of the security is therefore

$$\mathbf{m_1 / (1+r) + m_2 / (1+r)^2 + m_3 / (1+r)^3.}$$

Arbitrage

- Arbitrage is trading commodities that are not used for consumption for profit.
- E.g. buying and selling stocks, or bonds.
- If there is no uncertainty, all profit opportunities will be found. What does this imply for prices over time?
- The price today of an asset is p_0. Its price tomorrow will be p_1. Should it be sold now?
- The rate-of-return from holding the asset is

$$R = \frac{p_1 - p_0}{p_0}$$

or,

$$(1+R)p_0 = p_1.$$

Valuing Bonds and Consols

- A bond is a special type of security that pays a fixed amount \$x for T years (its maturity date) and then pays its face value \$F.
- What is the most that should now be paid for such a bond?

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots + \frac{x}{(1+r)^{T-1}} + \frac{F}{(1+r)^T}.$$

- A consol is a bond which never terminates, paying \$x per period forever.
- What is a consol's present-value?

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \ldots$$
$$= \frac{1}{1+r}\left[x + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots\right] = \frac{1}{1+r}[x + PV]$$

- Solving for PV gives PV=x/r

Arbitrage

- Sell the asset now for \$$p_0$, put the money in the bank to earn interest at rate r and tomorrow you have

$$(1+r)p_0.$$

- When is not selling best? When

$$(1+R)p_0 > (1+r)p_0.$$

- i.e. if the rate-or-return to holding the asset $R > r$ the interest rate, then keep the asset.
- And if $R < r$ then

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

so sell now for \$$p_0$.

Assets

- An asset is a commodity that provides a flow of services over time.
- E.g. a house, or a computer.
- A financial asset provides a flow of money over time, that is, a security.
- Typically asset values are uncertain. Incorporating uncertainty is difficult at this stage so we will instead study assets assuming that we can see the future with perfect certainty.
- Q: When should an asset be sold?
- It is typically not when the value of the asset is at a maximum.
- For example, for an asset where returns are falling over time, the asset should be sold when the rate of return on the asset falls below the interest rate.

Recap of Today

- Understanding the endowment income effect in the Slutsky decomposition: Whenever a consumer is choosing a bundle that is not equal to his endowment bundle, and then there is price change, the endowment income effect becomes relevant.
- Intertemporal Choice: The intertemporal budget constraint with interest rate and inflation.
- Borrowing and saving, comparative statics.
- Valuing securities: Bonds and consuls.
- “No arbitrage” conditions in asset markets.

Arbitrage

- If all asset markets are in equilibrium then $R = r$ for every asset.
- Hence, for every asset, today’s price p_0 and tomorrow’s price p_1 satisfy

$$p_1 = (1+r)\, p_0.$$

- i.e. tomorrow’s price is the future-value of today’s price. Equivalently,

$$p_0 = \frac{p_1}{1+r}.$$

- i.e. today’s price is the present-value of tomorrow’s price.

Financial Intermediaries

- Banks, brokerages etc.
  - facilitate trades between people with different levels of impatience.
  - patient people (savers) lend funds to impatient people (borrowers) in exchange for a rate-of-return on the loaned funds.
  - both groups are better off.
- What happens when the buyers and sellers of financial assets aren’t evenly matched? E.g. if more people want to sell consumption tomorrow than to buy it? Just as in any market, the price of consumption tomorrow will fall. Since the price of consumption tomorrow is (1/1+r), the interest rate will rise, people will save more and consume less today, tending to equate demand and supply.