ASSIGNMENT 2

Class: 12

Subject: Mathematics

1. a. How many numbers of at most 3 different digits can be formed from the integers 1, 2, 3, 4, 5 and 6? [2]

   b. Find the middle terms in the expansion of $\left(2x + \frac{1}{3x^2}\right)^9$. [2]

   c. Define group. Give an example of it. [2]

2. a. Find the vertices and foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. [2]

   b. Find the point where the line through the points (1, 2, 3) and (4, -4, 9) meets the zx-plane. [2]

   c. Determine the unit vector of $2\vec{a} - 3\vec{b}$ where $\vec{a} = 4\vec{\imath} + 3\vec{\jmath}$ and $\vec{b} = 2\vec{\imath} + 3\vec{\jmath}$. [2]

3. a. Find the personian coefficient of skewness when, Σx=735, Σx^2=28750, Mode=35.25, n=20. [2]

   b. Given P(A)=0.4, P(AUB)=0.56, P(B)=0.3. Are A and B independent? [2]

   c. Find the area of the parallelogram formed by the vectors $\vec{\imath} + 2\vec{\jmath} + 3\vec{k}$ and $-3\vec{\imath} - 2\vec{\jmath} + \vec{k}$. [2]

4. a. In how many ways can the letters of the word 'Monday' be arranged? How many arrangements do not begin with M? How many of these arrangements begin with M and do not end with N? [4]

   b. If C_0, C_1, C_2, …., C_n are binomial coefficients in the expansion of $(1+x)^n$, then prove that $C_0C_n + C_1C_{n-1} + \ldots + C_nC_0 = \frac{2n!}{n!n!}$. [6]

   OR

   Prove that: $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \cdots to\ \infty = \frac{3e}{2}$

5. a. Show that following algebraic structures Z_3={0, 1, 2} under addition modulo 3($+_3$) forms abelian group. [4]

   b. Show that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$. [4]

   OR

   Find the equation of plane passing through the points (2, 2, 1) and (9, 3, 6) and is normal to the plane $2x + 6y + 6z = 9$.

6. a. Prove $cos(A + B) = cosAcosB - sinAsinB$ using vector method. Also define dot product of two vectors and interpret it. [6]

   b. Find the correlation coefficient between the variables x and y.

X	6	2	10	4	8
Y	9	11	?	8	7

   Arithmetic means of X and Y series are 6 and 8 respectively. [4]

7. a. State and prove theorem of compound probability. [4]

   OR

   If three dice are thrown, what is the probability of getting (i) exactly 2 sixes (ii) exactly 3 sixes.

   b. Find the condition under which the line $lx + my + n = 0$ is tangent to the parabola $y^2 = 4ax$. For what value of 'a' will the straight line $y = 2x + 3$ touch the parabola $y^2 = 4ax$. [4]

8. a. Find the derivative of $2tanh^{-1}\left(tan\frac{x}{2}\right)$. [2]

   b. Evaluate: $\int \frac{dx}{4x^2 - 4x + 3}$ [2]

   c. Solve the differential equation $x^2dy - y^2dx = 0$. [2]

9. a. Evaluate: $\int \frac{dx}{a + bcosx} \quad (when\ a > b)$ [4]

   b. Solve the differential equation $(x^2 + y^2)dy = xydx$. [4]

   OR

   Solve the differential equation $tanx\frac{dy}{dx} + y = secx$.

10. Find the first principle the derivative of $\sqrt{tanx}$. [6]

    OR

State mean value theorem. Interpret it geometrically. Use Lagrange's mean value theorem to find he point on the curve $f(x) = x^2 - 2x$ where the tangent is parallel to the chord joining the points (1, -1) and (4, 8).

11. a. Draw the graph of following inequalities and shade the region. [2]
$x + 4 \leq 6,\ 2x + y \geq 8,\ y \geq 0$

b. Examine whether the following system of equations are ill-conditioned or well-conditioned.
$2x + y = 25$
$2.001x + y = 25.01$ [2]

c. Evaluate $\int_1^3 (2x - 1)dx, n = 4$ using trapezoidal rule. [2]

12. a. Solve the following system of equations by using Gauss-Seidel method. [4]
$5x + 2y + z = 12$
$x + 4y + 2z = 15$
$x + 2y + 5z = 20$

OR

Solve the following system of equations by using Gauss-Elimination method with partial pivoting.
$x - 2y + 3z = 2$
$2x - 3y + z = 1$
$3x - y + 2z = 9$

b. Evaluate the approximate value of $\int_0^1 \frac{dx}{1+x^2}$ with $n = 4$ using Simpson's $\frac{1}{3}$ rule. [4]

13. Using Simplex method,
Maximize: $z = 2x + 3y$
Subject to constraints
$2x + y \leq 14$
$x + y \leq 10$
where $x, y \geq 0$. [6]

14. Find the root of the equation $x^3 - x - 4 = 0$ between 1 and 2 correct upto three places of decimal by using Newton-Raphson method. [6]

OR

Using the method of successive bisection method, find the square root of 3 within 2 places of decimal in (1, 2).

The End