NEB-GRADE XII
2076 (2019)
Mathematics

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Time: 3 hrs.

Full Marks: 100

Pass Marks: 35 (Only for partial student)

Note: Group A is compulsory and select another one Group either B or C.

Group 'A'

Attempt all the questions.

1. a) In how many ways the letters of the word ALGEBRA can be arranged so that repeated letter are never together ? (2)

   b) Show that $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \ldots = 1$. (2)

   c) Prepare a Cayley's table for $S = \{0,2,3\}$, under addition modulo 4. (2)

2. a) Find the eccentricity and foci of the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$. (2)

   b) Find the ratio in which the line joining the points $P\ (-2,4,7)$ and $Q\ (3,-5,-1)$ is divided by the ZX- plane. (2)

   c) If $\vec{a} = (3,-1,-4), \vec{b} = (-2, 4,-3)$ and $\vec{c} = (-5, 7,-1)$, find $\left| 2\vec{a} + \vec{b} - \vec{c} \right|$. (2)

3. a) If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, find the projection of $\vec{b}$ on $\vec{a}$. (2)

   b) Evaluate, using L' Hopital rule: $\displaystyle \lim_{x \to 0} \dfrac{x - \sin x . \cos x}{x^3}$. (2)

   c) Evaluate: $\displaystyle \int \dfrac{x}{(x-a)\,(x-b)} \, dx$. (2)

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4. a) Solve : $\dfrac{dy}{dx} + \dfrac{1+\cos 2y}{1-\cos 2x} = 0$. (2)

b) Calculate the mean deviation from mean of the data: (2)
3, 5, 9, 11, 7, 6.

c) The chance that A can solve the problem is $\dfrac{1}{3}$ and the chance that B can solve is $\dfrac{2}{3}$. Find the probability that the problem is solved by A and B. (2)

5. a) A committee of five persons is to be formed from 5 men and 3 women. In how many ways can this be done so that at least two women are included ? (4)

Or

Show that the number of combinations of 'n' different objects taken 'r' at a time is given by $C(n, r) = \dfrac{n!}{(n-r)!\, r!}$. Also show that $C(n, r) + C(n, r-1) = C(n+1, r)$

b) Define abelian group. If $(G, *)$ is an abelian group, prove that $(a * b)^{-1} = a^{-1} * b^{-1}$ $\forall\, a, b \in G$. (4)

6. a) Find the condition that a line $ax + by + c = 0$ may be normal to the parabola $y^2 = 4mx$. (4)

Or

Find the vertices and foci of the ellipse $\dfrac{(x+2)^2}{16} + \dfrac{(y-5)^2}{9} = 1$.

b) Show that the plane $2x + 3y - 4z = 3$ is parallel to the plane $10x + 15y - 20z = 12$ and is perpendicular to $3x + 2y + 3z = 5$. (4)

7. a) Evaluate: $\displaystyle\int \dfrac{dx}{1+\sin x + \cos x}$. (4)

b) Solve: $\cos^2 x \dfrac{dy}{dx} + y = 1$. (4)

Or

Solve: $(1+x)\dfrac{dy}{dx} - xy = 1-x$.

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8. a) Find correlation coefficient of the following two sets of data
    A and B :
    (4)

A	56	72	48	64	81	69
B	63	74	45	82	66	57

    b) Find the probability of getting three heads in six tosses of a coin. (4)

9. Show that : $1-\dfrac{1}{4}+\dfrac{1.3}{4.8}-\dfrac{1.3.5}{4.8.12}+...to \infty =\sqrt{\dfrac{2}{3}}$.   (6)

10. Define cross product of two vectors and interpret the product geometrically. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors, prove that:

    $$\vec{a}\times (\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$$
    (6)

11. From definition, find the derivative of $e^{tan.x}$.   (6)

Or

State Mean value theorem. Verify it for the function $f(x) = 2x^2 - 10x +29$ in $[2, 7]$.

Group 'B'

12. a) If the resultant of two equal forces is equal to the given force, find angle between the forces.   (2)

    b) A ball is thrown vertically upwards at a rate of $40ms^{-1}$. Find the time taken to attain the maximum height. ($g = 10ms^{-2}$)   (2)

    c) Find the mass of an object which on earth weighs 98N. ($g=9.8ms^{-2}$)   (2)

13. a) Two men carry a weight 50N between two strings fixed to the weight, one string is inclined at 30^0 to the vertical and the other at 60^0, find the tension of each string.   (4)

    b) A body slides down from rest from the top of a smooth plane of height 44.1 m and inclination 30^0 with the horizon. Divide the plane into three parts so that the body at the top of the plane may describe each part in equal interval of time. ($g=9.8ms^{-2}$)   (4)

Or

A stone is dropped into a well and the sound of its striking the water is heard in $4\dfrac{2}{9}$ seconds. If the velocity of the sound is $352.8ms^{-1}$, find the depth of the well. ($g=9.8ms^{-2}$)

14. Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is *250m* off and *125 m* high.$(g = 9.8ms^{-2})$

(6)

15. Deduce the resultant of two parallel forces.

(6)

Or

Define Moment geometrically. Also state and prove the Varignon's theorem for two intersecting forces.

Group 'C'

16. a) In graph shade the feasible region under the constraints.

$2x + y \leq 40$, $x + 2y \leq 50$, $x \geq 0$, $y \geq 0$.

(2)

b) Convert the decimal number 31923 into hexadecimal number.

(2)

c) Examine whether the system of equations $3x + 12y - z = 28$, $x + 4y + 7z = 2$ and $10x + 4y - 2z = 20$ is diagonally dominant.

(2)

17. a) Solve by **Gauss elimination** or **Gauss seidel** method:

(4)

$2x + 2y + z = 6$, $x - y + z = 0$ and $4x + 2y + 3z = 4$.

b) Use the **Bisection** method to find solutions accurate to within 10^{-2} for

$x^3 - 7x^2 + 14x - 6 = 0$ in $(0, 1)$.

(4)

18. By **Simplex** method maximize

$F = 15x_1 + 10x_2$ subject to $2x_1 + x_2 \leq 10$, $x_1 + 3x_2 \leq 10$; $x_1, x_2 \geq 0$. (6)

19. Compute $\int_{0}^{1} (1 + x^2)^{-1} \, dx$, by **Simpson's** $\dfrac{1}{3}$ rule. Also compare it with true solution.

(6)

Or

Approximate the value for $\int_{-1}^{1} e^x dx$ using **Trapezoidal rule** with $n = 2$.

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