Sub. Code: 216'D'

HSEB - GRADE XII
2070 (2013)
Mathematics
(New Course)

It is for those student whose first two digit of registration number starts from 68 or greater than.

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Time - 3 hrs.

Full Marks:- 100
Pass Marks:- 35

Note : Group A is compulsory and select another one group either B or C.

Group 'A'

1. a) In how many ways can the letters of the word "ELEMENT" be arranged ?   2

   b) Show that $\log_e 2 = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \cdots$ 2

   c) Show that the multiplication is a binary operation on the set $S = \{-1, 0, 1\}$ 2

2. a) Find the eccentricity and the foci of the ellipse $3x^2 + 4y^2 = 36$.   2

   b) Find the equation of the plane which makes equal intercepts on the axes and passes through the point $(2, 3, 4)$.   2

   c) The vertices A, B, C of a triangle are $(2, -1, -3)$, $(4, 2, 3)$ and $(6, 3, 4)$ respectively. Show that $\overrightarrow{AB} = (2, 3, 6)$ and $AC = 9$.   2

3. a) Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 8$ at $(2, 4)$.   2

   b) Evaluate : $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$, $(\beta > \alpha)$.   2

   c) Find the sine of the angle between the two vectors $2\vec{i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$.   2

4. a) Solve : $x^2 dy - y^2 dx = 0$.   2

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b) If $n = 10$, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. 2

c) A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is a King or a Diamond? 2

5. a) From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include atleast two gentlemen? 4

b) Show that the set of integers Z forms a group under the operation of addition. 4

Or

If a and b are the elements of a group (G, O) prove that the equation $a \circ x = b$ has a unique solution in (G, O).

6. a) Prove that the line $3x + 4y + 6 = 0$ is tangent to the parabola $2y^2 = 9x$. Find its point of contact. 4

Or

Deduce the equation of a hyperbola with a focus at (6, 0) and a vertex at (4, 0).

b) Find the angle between two straight lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2. 4

7. a) Evaluate: $\int \dfrac{dx}{1 + 2\sin x}$ 4

b) Solve: $(1 + x^2)\dfrac{dy}{dx} + 2xy = 4x^2$. 4

Or

Solve: $(x^2 + y^2)dy = xy\,dx$

8. a) Consider the following distribution.

	Distribution A	Distribution B
Arithmetic mean:	100	90
Median:	90	80
Standard deviation:	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skewness? 4

b) A coin is tossed 5 times. Find the probability of getting
   i) two heads   ii) atleast two heads.   4

9. If $(1+x)^n = C_o + C_1x + C_2x^2 + ... + c_nx^n$, prove that :

$$C_o C_n + C_1 C_{n-1} + ... + C_n C_o = \frac{2n!}{(n!)^2} .$$ 6

10. Define scalar product of two vectors.   6
    Prove by vector method that :

    $\cos(A-B) = \cos A \cos B + \sin A \sin B.$

11. State mean value theorem. Interpret it geometrically. Verify mean value
    theorem for the function $f(x) = x(x-1)^2$ in $[0, 2]$.   6
                              Or
    Find from first principles, the derivative of x^x.

Group 'B'

12. a) Two forces acting at an angle of $45°$ have a resultant equal to

       $\sqrt{10}$ N; if one of the forces be $\sqrt{2}$ N, find the other force.   2

    b) Find two like parallel forces acting at a distance of 2.5m apart, which
       are equivalent to a given force of 30N. The line of action of one being
       at a distance of 50cm from the given force.   2

    c) A ball is projected vertically upwards with a velocity of 40m/s. Find
       its velocity and position at the end of 3s. $(g = 10m/s^2)$ 2

13. a) A body of weight 65N is suspended by two strings of lengths 5 and
       12m attached to two points in the same horizontal line whose distance
       apart is 13m; find the tensions of the string.   4
                              Or
       State and prove Lami's theorem.

    b) If a,b,c be the spaces described by a particle during the pth, q^{th}, r^{th}
       seconds of its motion respectively, prove that:

       $a(q-r) + b(r-p) + c(p-q) = 0.$ 4

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14. A projectile thrown from a point in a horizontal plane comes back to the plane in 4 sec. at a distance of 60m in front of the point of projection. Find the velocity of projection. $(g=10m/s^2)$ 6

Or

A bullet of mass 20g is fired horizontally into a suspended stationary wooden block of mass 380g with a velocity of 200m/s. What is the common velocity of the bullet and the block if the bullet is embedded into the block ? Find the loss of K.E. by the impact. $(g=10m/s^2)$

15. Define moment of a force about a point. Prove that the algebraic sum of the moments of the moment of two like parallel forces about any point in their plane is equal to the moment of their resultant about the same point. 6

Group 'C'

16. a) Draw the graph of the following inequalities
    $$x + y \le 6, \qquad 2x + y \ge 8, \qquad y \ge 0,$$
    Shade the feasible region. 2

    b) Convert the decimal number 3058 to hexadecimal form. 2

    c) Using trapezoidal rule, evaluate $\int_0^3 (3x^2 - 4x)dx$, $n=3$. 2

17. a) Solve the following system of equations by Gaussian elimination method.
    $$x + 3y - 2z = 5, \quad 3x + 5y + 6z = 7, \quad 2x + 4y + 3z = 8.$$ 4

Or

Solve the following system of equations by Gauss Seidel method
    $$3x + y - z = 2, \quad 2x - 5y + z = 20, \quad x - 3y - 8z = 3.$$

    b) Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^4 \frac{dx}{1+x}$, $n=4$. 4

18. Using Simplex method,
    $$\text{Max. } P = 50x_1 + 80x_2$$
    subject to $\quad x_1 + 2x_2 \le 32$
    $$3x_1 + 4x_2 \le 84$$
    $$x_1, x_2 \ge 0.$$ 6

19. Applying the method of successive bisection, find the root of the equation $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to 2 places of decimals. 6

Or

Using Newton-Raphson method, find the positive root of $x^3 - 18 = 0$ in (2, 3).

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