Sub.Code : 216'D'

HSEB-GRADE XII
2072 (2015)
Mathematics
(New Course)

It is for those students whose first two digit of registration number starts from 68 or greater than.

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Time : 3 hrs.

Full Marks:- 100
Pass Marks:- 35

Note: Group A is compulsory and select another one group either B or C.

Group 'A'

Attempt **all** the questions.

1. (a) In how many ways the letters of the word ELEMENT can be arranged so that vowels are always together? 2

   (b) Prove that $\frac{2}{1!}+\frac{4}{3!}+\frac{6}{5!}+....=e$. 2

   (c) In a Caley's table for a finite group, why does each element occur exactly once in each row and exactly once in each column? 2

2. (a) Find the equation of the hyperbola with vertex (8, 0) and passing through the point $(8\sqrt{2}, 4)$. 2

   (b) If P and Q denote the coordinates (2, 6, 2) and (4, 5, 0) respectively, find the direction cosines of the line PQ. 2

   (c) If $\vec{a}=(3,-1,-4)$, $\vec{b}=(-2,4,-3)$ find unit vector along $\vec{a}-2\vec{b}$. 2

3. (a) Compute the integral $\int\frac{\coth x\ dx}{\sinh x-9\ \text{cosech}\, x}$ 2

   (b) Using L' Hospital's rule, evaluate: $\lim_{x\to 0}\frac{e^x-x-1}{x^2}$ 2

   (c) If $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=0$, prove that $|\vec{a}|=|\vec{b}|$ 2

4. (a) Solve : $\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2y}=0$. 2

Contd...

(b) In the distribution of data 20, 25, 30, 36, 32, 43; find standard deviation. 2

(c) In a draw of a card from well shuffled deck of 52 cards what is the probability that it is a king or a queen? 2

5. (a) A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives? 4

(b) Let $(G, *)$ be a group. If $a, b \in G$, then prove that
(i) $(a * b)^{-1} = b^{-1} * a^{-1}$ and (ii) $(a^{-1})^{-1} = a$ 4

Or

Define a group. Let a, b, c and x be elements of a group G. Solve the following for x:
$x^2 = a^2$ and $x^5 = e$

6. (a) If the tangent to the parabola $y^2 = 12x$ makes an angle 45° with the straight line $x - 2y + 3 = 0$, find its equation and the point of contact. 4

Or

Find the eccentricity and coordinates of the foci of the curve $\frac{(x+6)^2}{4} + \frac{y^2}{36} = 1$.

(b) Show that the plane $2x + 3y - 4z = 3$ is parallel to the plane $10x + 15y - 20z = 12$ and is perpendicular to the plane $3x + 2y + 3z = 5$. 4

7. (a) Evaluate: $\int \frac{dx}{(x-1)^2(x-2)^3}$ 4

(b) Reduce the equation $\frac{dy}{dx} + \frac{y}{x} = y^2$ in linear form hence solve it. 4

Or

Solve: $\frac{dy}{dx} = \frac{y+1}{x+y+1}$

8. (a) Define correlation. Find Karl Pearson's coefficient of correlation of the marks of the following distribution. 4

X	20	30	40	50	60
Y	50	46	30	24	8

Contd...

(b) The probability of hitting a target is found to be 0.25. If eight hits are made, find the probability that (i) none will hit the target (ii) exactly two will hit the target. 4

9. State Binomial theorem. In the expansion of $(1 + x)^n$ prove that the sum of the coefficients of the odd terms is equal to the sum of coefficients of the even terms and each equals to $2^n - 1$. 6

10. Define Vector product of two Vectors. Prove by Vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 6

11. State Mean Value theorem. Verify the mean value theorem for the function $f(x) = \sqrt{x^2 - 4}, x \in [2, 4]$. 6

Or

Find from first principles the derivative of $\ln \cos^{-1} x$.

Group 'B'

12 (a) Show that the resultant of two equal forces bisects the angle between them. 2

(b) A uniform beam AB is 16 m long and weighs 50 kg weights of 20 kg and 50 kg are suspended from A and B respectively. At what point must the beam be supported so that it may rest horizontally? 2

(c) A particle slides down a smooth inclined plane 10 m long and acquires a velocity $10\sqrt{2}$ ms^{-1}. Find the inclination of the plane. ($g = 10\ ms^{-2}$) 2

13. (a) State and prove Lami's theorem. 4

Or

A body of weight 68N is suspended by two strings of length 8 m and 15 m respectively, and the other ends of the strings are attached to two fixed points in a horizontal line 17 m apart, find the tensions of the strings.

(b) State laws of motion. Use Newton's Law to define an absolute unit of force. 4

14. Define coplanar forces. Forces equal to P, 2P, 3P and 4P act along the sides of a square ABCD taken in order; find the magnitude, direction and the line of action of the resultant. 6

Contd...

15. Define energy. State principle of conservation of energy. Also prove that the sum of the kinetic and potential energy of a moving body remains constant throughout the motion. 6

Or,

Describe motion of a projectile. A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. Find where it will strike the level ground through the foot of the tower and also find the striking velocity.

Group 'C'

16 (a) Draw the graph of the inequality: $3x - 3 \leq 5x - y$. 2

(b) Convert hexadecimal number $70A_{16}$ into binary form. 2

(c) Test whether the system of equations
$12x + 3y - 5z = 1$, $x + 5y + 3z = 28$ and $3x + 7y + 13z = 1$
is diagonally consistent? 2

17. (a) Using Gauss Seidel method, solve:
$3x + 4y + 8z = 7$, $x + 20y + z = -18$, $25x + y - 5z = 19$. 4

Or

Use Gauss elimination method to solve:
$4x - y + z = 8$, $2x + 5y + 2z = 3$, $x + 2y + 4z = 11$.

(b) Using the bisection method find the root of the equation $x^2 + x - 4 = 0$ in (1, 2) correct to two places of decimals. 4

18. Using Simplex method, find the optimal solution of $z = 7x_1 + 5x_2$ subject to $x_1 + 2x_2 \leq 6$, $4x_1 + 3x_2 \leq 12$, $x_1, x_2 \geq 0$. 6

19. Approximate the value using trapezoidal rule for $\int_{-1}^{1} e^x dx$, $n = 2$. 6

Or

Evaluate $\int_0^1 \sqrt{1+x^3}\, dx$ using Simpson's $\frac{1}{3}$ rule with $n = 4$.

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