

Sub.Code : 216'D'

**HSEB - GRADE XII**

**2071(2014)**

**Mathematics**

**(New Course)**

It is for those students whose first two digit of registration number starts from 68 or greater than.

*Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.*

Time - 3 hrs.

Full Marks:- 100

Pass Marks:- 35

**Note: Group A is compulsory and select another one group either B or C.**

**Group 'A'**

Attempt all questions.

1. a. A man has 5 friends. In how many ways can he invite one or more of them to a dinner ? 2

✓ b. Find the coefficient of  $x$  in the expansion of  $\left(x^2 + \frac{a^2}{x}\right)^5$ . 2

- c. Show that the multiplication is a binary operation on the set  $S = \{-1, 0, 1\}$ . 2

2. a. Find the eccentricity and the foci of the hyperbola  $\frac{x^2}{9} - \frac{y^2}{16} = 1$ . 2

- ✓ b. Find the direction cosines of a line passing through the points  $M(-2, 4, 3)$  and  $N(-1, 2, 5)$ . 2

- c. Show that the three points whose position vectors are  $7\vec{j} + 10\vec{k}$ ,  $-\vec{i} + 6\vec{j} + 6\vec{k}$  and  $-4\vec{i} + 9\vec{j} + 6\vec{k}$  form an isosceles triangle. 2

- 3 a. Using L Hospital's rule, evaluate :  $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$ . 2

✗ b. Evaluate :  $\int \frac{dx}{\sqrt{2ax + x^2}}$ . 2

Contd...



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- c. Find a unit vector perpendicular to each of the vectors  
 $3\vec{i} + \vec{j} + 2\vec{k}$  and  $2\vec{i} - 2\vec{j} + 4\vec{k}$ . 2

4. a. Solve :  $\frac{dy}{dx} + 4x = 2e^{2x}$ . 2

- b. If  $n=10$ ,  $\sum x=120$ ,  $\sum x^2=1530$ , find the standard deviation and the coefficient of variation. 2

- c. Two coins are tossed simultaneously. Find the sample space. Find the probability that both are heads. 2

5. a. In how many ways can the letter of the word "COMPUTER" be arranged so that i) all vowels are always together? ii) the relative positions of the vowels and consonants are not changed? 4

- b. A binary operation  $*$  defined on the set  $S=\{a, b, c\}$  is presented in the following Cayley's table

$*$	$a$	$b$	$c$
$a$	$a$	$b$	$c$
$b$	$b$	$c$	$a$
$c$	$c$	$a$	$b$

Show that :  $(S, *)$  forms a group. 4

Or

Let  $a, b, c$  are the elements of a group  $(G, *)$ .

i) if  $a * b = b$ , prove that :  $a = e$

ii) if  $a * b = e$ , prove that :  $b = a^{-1}$ .

6. a. Find the equation of the parabola in the standard form  $y^2 = 4ax$ . 4

Or

Find the equation of the ellipse whose distance between two foci is 8 and the semi-latus rectum is 6.

- b. Find the equation of the plane through the points  $(-1, 1, 1)$  and  $(1, -1, 1)$  and perpendicular to the plane  $x + 2y + 2z = 5$ . 4

7. a. Evaluate :  $\int \frac{dx}{1-3 \sin x}$  4

- b. Solve :  $xy \frac{dy}{dx} = x^2 + y^2$  4

Or



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Solve :  $\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{1}{(1+x^2)^2}$

8. a. Calculate Karl Pearson's correlation coefficient between the two variables height (in cms) and weight (in kg) from the data given below:

Height	160	162	165	161	163
Weight	63	62	64	60	61

4

- b. Suppose that in a certain city 60% of all recorded births are males, suppose we select 5 birth records from the population. What is the probability that : i) three of them are males ii) more than 4 are males.

4

9. Show that:  $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \dots = \frac{3e}{2}$

6

10. Define scalar product of two vectors. Prove by vector method that:  $\cos(A+B) = \cos A \cos B - \sin A \sin B$ .

6

11. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function  $f(x) = x(x-3)^2$  for  $x \in [0, 3]$ .

6

Or

Find from first principle, the derivative of :  $\ln\left(\sin \frac{x}{a}\right)$ .

Group 'B'

12. a. Forces equal to 7p, 5p and 8p acting on a particle are in equilibrium. Find the angle between latter pair of forces.

2

- b. Two unlike parallel forces, the greater of which is 75N, have a resultant 25N. Find the ratio of the distances of the resultant from the component forces.

2

- c. A ball is thrown vertically upwards with a velocity of 30m/s. Find the time taken by the ball to reach the ground again. ( $g=10\text{m/s}^2$ )

2

13. a. The resultant of two forces P and Q is equal to  $\sqrt{3}Q$  and makes an angle of  $30^\circ$  with the direction of P; Show that P is either equal to Q or is double of Q.

4

Or

State and prove Lami's theorem.



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- b. A gun of mass 400kg fires a shot of mass 3kg, with a velocity of 200m/s, find the constant force which acting on the gun would stop it after a recoil of 2.5 meters. 4
14. Define potential energy and kinetic energy of a body. Prove that the sum of the K.E. and P.E. of a freely falling body at any instant is constant. 6
- Or
- A cannon ball has the same range R on a horizontal plane for two different angles of projection. If H and H<sup>1</sup> are the greatest heights in two paths for which this is possible, prove that :  $R^2 = 16HH^1$ .
15. Define moment of a force about a point. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. 6

**Group 'C'**

16. a. Determine graphically the feasible region determined by the following inequalities:  $3x + 4y \leq 24$ ,  $x \geq 2$ ,  $y \geq 1$ . 2
- b. Convert the hexadecimal number AB5<sub>16</sub> to the decimal number. 2
- c. Using the trapezoidal rule, evaluate :  $\int_0^2 (2x^2 - 1) dx$ ,  $n = 4$ . 2
17. a. Using Gauss -elimination method, solve the following system of equation:  $x - 2y + 3z = 2$ ,  $2x - 3y + z = 1$ ,  $3x - y + 2z = 9$ . 4
- Or
- Solve the following system of equations using inverse matrix method:
- $$x_1 - 2x_2 - x_3 = 1, \quad x_1 - x_2 + 2x_3 = 9, \quad 2x_1 - 3x_2 - x_3 = 4.$$
- b. Estimate the following integral using Simpson's 1/3 rule,
- $$\int_0^\pi \sin x dx, \quad n = 6. \quad 4$$

18. Using simplex method,
- maximize  $U = 25x + 45y$  subject to  $x + 3y \leq 21$   
 $2x + 3y \leq 24$   
 $x, y \geq 0$  6
19. Using the bisection method, find a root of the equation:  
 $f(x) = 2x^3 - 5x + 2 = 0$ , between 1 and 2 with error less than  $10^{-2}$ . 6
- Or
- Derive the formula for Newton-Raphson method. Using Newton Raphson method, find a positive root of  $x^3 + 3x - 5 = 0$  lying between 1 and 2 correct to three places of decimals.

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