Assignment-3
Class: 12
Subject: Maths

Answer all the questions.

1. a. How many 5 different digit numbers can be formed with the digits 0, 1, 2, 3, 4? How many of them start with 0?

   b. Prove that: $\frac{1}{2} - \frac{1}{2.2^2} + \frac{1}{3.2^3} - \frac{1}{4.2^4} \ldots = log_e\left({}^3\!/_2\right)$

   c. Show that multiplication is binary operation on the set $s = \{-1, 0, 1\}$.

2. a. Find the equation of parabola with focus at (-1, 2) and directrix $x = -5$.

   b. Find the ratio in which the yz-plane divides the line joining (4, 6, 7) and (-1, 2, 5). Also, find the coordinates of the point in yz-plane.

   c. If $\vec{a} = (1, 2, 3)$ and $\vec{b} = (-1, 2, 1)$, find the projection of $\vec{a}$ and $\vec{b}$.

3. a. Find the sine of the angle between the two vectors $2\vec{\imath} - \vec{\jmath} + \vec{K}$ and $3\vec{\imath} + 4\vec{\jmath} - \vec{K}$.

   b. Find the regression coefficient of x and y for the data $\sum x = 25$, $\sum y = 32$, $\sum xy = 104$, $\sum x^2 = 75$, $\sum y^2 = 230, n = 10$.

4. An examination paper consists of 12 questions divided into two parts A and B. Part A contains 7 questions and part B contains 5 questions. A candidate required to attempt 8 questions selecting at least 3 from each part. In how many ways he can select the questions.

5. Show that the set {1, -1, i, -i}, where i is imaginary unit forms a group under complex number multiplication.

   OR

   Prove a group (G, o) is a abelian if and only if $(aob)^{-1} = a^{-1}ob^{-1}$.

6. Find the equation of tangent at the point (x_1, y_1) to the parabola y^2 = 4ax. Also deduce its equation in m-form.

   OR

   Find the eccentricity, vertices, foci, length of latus rectum of the ellipse $25x^2+4y^2=100$.

7. Find the equation of plane passing through the points (1, 1, 0), (-2, 2, -1) and (1, 2, 1).

8. Calculate the correlation coefficient for the age of husband and age of wife.

X	32	35	36	38	34	33	37
Y	27	28	29	31	26	28	27

9. If $(1+x)^n = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$, prove that $c_0c_n + c_1c_{n-1} + \ldots + c_nc_0 = \frac{2n!}{n!n!}$. [6]

   OR

   Sum to infinity $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \cdots$.

10. Define dot product of two vectors with example. Prove $cos(A + B) = cosAcosB - sinAsinB$ using vector method. [6]

11. a. Using L Hospital's rule, evaluate $\lim_{x \to 0} \frac{x - sinxcosx}{x^3}$.

    b. Evaluate: $\int \frac{dx}{\sqrt{2ax+x^2}}$

12. a. Solve: $(x^2 - 1)\frac{dy}{dx} = xy$

    b. Two dice are rolled once, what is the probability of getting a total of 9 or 6?

13. a. Evaluate: $\int \frac{dx}{(x-1)^2(x-2)^3}$

    b. Reduce the equation $(1 + x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$ in linear form and solve it.

    OR

    Solve: $\frac{dy}{dx} = \frac{y+1}{x+y+1}$

c. A sample of 100 fuses is known to have an average 5 defective fuses. Three fuses of sample are tested. What is the probability that (i) none of them is defective (ii) exactly one of them is defective?

14. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x)=x(x-3)^2$ for $x \in [0, 3]$.

OR

Find from first principles, the derivative of $log(tanx)$.

15. a. Draw the graph of the following inequalities and shade the region.

$x + y \leq 6,\ x - y \geq -2,\ x \geq 0, y \geq 0$

b. Examine whether the following system of equations are ill-conditioned or well-conditioned.

$2x + y = 25$

$2.001x + y = 25.01$

c. Evaluate $\int_1^2 \frac{1}{x^2} dx, n = 4$ using trapezoidal rule.

16. a. Solve the following system of equations by using Gauss-Seidal method.

$2x_2 + 3x_3 = 7, \quad 3x_1 - 2x_2 + 2x_3 = 1, \quad 2x_1 + 3x_2 - 3x_3 = 5$

OR

Solve the following system of equations by using Gauss-Elimination method with partial pivoting.

$x - 2y + 3z = 2$

$2x - 3y + z = 1$

$3x - y + 2z = 9$

b. Evaluate the approximate value of $\int_0^\pi sinx dx$ with n=6 using Simpson's $^1/_3$ rule.

17. Using Simplex method,

Maximize $z = 5x + 3y$

Subject to constraints

$2x + y \leq 40$

$x + 2y \leq 50$

and $x, y \geq 0$

18. Find the root of the equation $x^3 - 18 = 0$ in (2, 3) up to three places of decimal by using Newton-Raphson method.

OR

Using the method of successive bisection, find the square root of 3 within 2 places of decimal in (1, 2).

The End