ANOVA (AS) PROJECT

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1. A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	145
Players Not Injured	32	38	11	9	90
Total	**77**	**94**	**35**	**29**	**235**

Introduction

In this analysis, we're taking a close look at the relationship between different player positions in football (Striker, Forward, Attacking Midfielder, Winger) and the likelihood of players getting injured. We're using the collected data by the physiotherapist on the number of players injured and not injured in each position, resulting in a total of 235 player observations. Let's use this data to explore and answer if there's any connection between player positions and injuries in the world of football.

1.1 What is the probability that a randomly chosen player would suffer an injury?

To calculate the probability that a randomly chosen player from the dataset would suffer an injury, we will use the provided data. The probability is the ratio of injured players to the total number of players in the dataset.

- Total injured players = 145
- Total players (injured + not injured) = 235

To calculate the probability as follows:
Probability of suffering an injury = (Total injured players) / (Total players)
Probability of suffering an injury = 145 / 235 ≈ 0.617
So, the probability that a randomly chosen player from the dataset would suffer an injury is approximately 0.617 or 61.7%.

1.2 What is the probability that a player is a forward or a winger?

To calculate the probability that a player is either a forward or a winger from the dataset, we'll utilize the provided data. The probability is determined by taking the number of players in the "Forward" and "Winger" positions and dividing it by the total number of players in the dataset.

- Number of players in the "Forward" position = 94
- Number of players in the "Winger" position = 29
- Total number of players in the dataset = 235

We calculate the probability as follows:

Probability of being a forward or a winger = (Number of players in the "Forward" position + Number of players in the "Winger" position) / Total number of players
Probability of being a forward or a winger = (94 + 29) / 235 ≈ 0.542

So, the probability that a player chosen randomly from the dataset is either a forward or a winger is approximately 0.542, or 54.2%.

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

To calculate the probability that a randomly chosen player plays in a striker position and has a foot injury, we will use the provided data. The probability is the ratio of players in the "Striker" position with injuries to the total number of players in the dataset.

- Number of players in the "Striker" position with injuries = 45
- Total number of players in the dataset = 235

To calculate the probability, we use the following formula:

Probability of being a striker with an injury = (Number of players in the "Striker" position with injuries) / (Total number of players)

Probability of being a striker with an injury = 45 / 235 ≈ 0.191

So, the probability that a randomly chosen player from the dataset plays in a striker position and has a foot injury is approximately 0.191 or 19.1%.

1.4 What is the probability that a randomly chosen injured player is a striker?

To calculate the probability that a randomly chosen injured player is a striker, we will use the provided data. The probability is the ratio of injured players in the "Striker" position to the total number of injured players in the dataset.

- Number of injured players in the "Striker" position = 45
- Total number of injured players = 145

To calculate the probability,we use the following formula:

Probability of an injured player being a striker = (Number of injured players in the "Striker" position) / (Total number of injured players)

Probability of an injured player being a striker = 45 / 145 ≈ 0.310

So, the probability that a randomly chosen injured player is a striker is approximately 0.310 or 31.0%.

2. The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; **(Provide an appropriate visual representation of your answers, without which marks will be deducted)**

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

To find the proportion of gunny bags with a breaking strength less than 3.17 kg per sq cm, we used the Z-score formula, which is given by:

$Z = (X - \mu) / \sigma$

Where:
- Z is the Z-score,
- X is the specified breaking strength (3.17 kg per sq cm),
- μ is the mean breaking strength (5 kg per sq cm), and
- σ is the standard deviation (1.5 kg per sq cm).

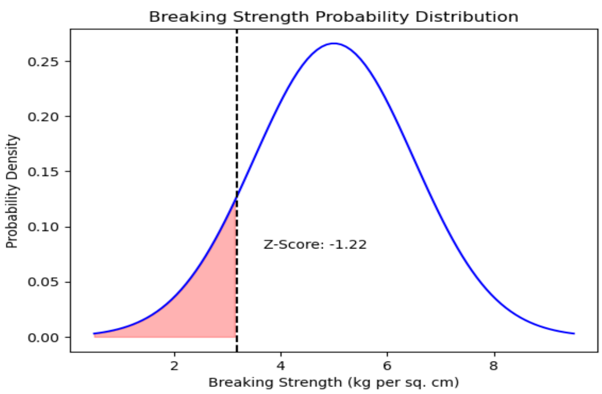
Plugging in the values, we calculated the Z-score:

$Z = (3.17 - 5) / 1.5 \approx -0.87$

We then used the cumulative distribution function (CDF) of the standard normal distribution to find the probability associated with this Z-score. The CDF gives the area under the standard normal curve to the left of the Z-score. Therefore, the probability that a randomly selected gunny bag has a breaking strength less than 3.17 kg per sq cm is approximately:

$P(X < 3.17) = P(Z < -0.87) \approx 0.1112$

So, the proportion of bags with a breaking strength less than 3.17 kg per sq cm is approximately 0.1112, or 11.12%.



2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm?

To find the proportion of gunny bags with a breaking strength of at least 3.6 kg per square centimeter, we can use the provided mean and standard deviation along with the Z-score formula. The Z-score allows us to find the probability using a standard normal distribution table.

To determine the proportion of gunny bags with a breaking strength of at least 3.6 kg per sq cm, we used the Z-score formula:

$Z = (X - \mu) / \sigma$

Where:

- Z is the Z-score,
- X is the specified breaking strength (3.6 kg per sq cm),
- μ is the mean breaking strength (5 kg per sq cm), and
- σ is the standard deviation (1.5 kg per sq cm).

Plugging in the values, we calculated the Z-score:

$Z = (3.6 - 5) / 1.5 \approx -1.6$

Now, we want to find the probability that the breaking strength is at least 3.6 kg per sq cm, which is equivalent to finding the probability that the breaking strength is greater than or equal to 3.6 kg. To do this, we find the complement of the probability that the breaking strength is less than 3.6 kg:

$P(X \geq 3.6) = 1 - P(X < 3.6)$

Using the Z-score we calculated earlier:

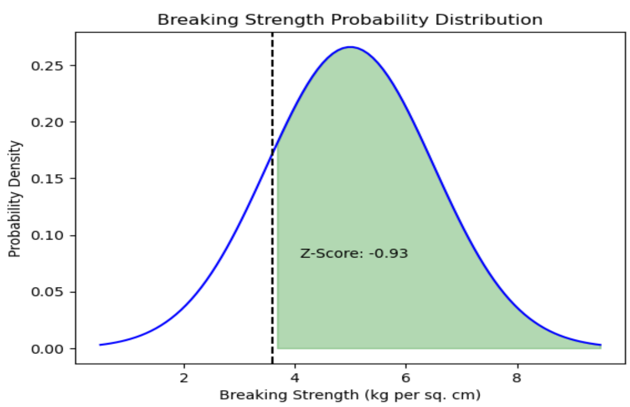
$P(X \geq 3.6) = 1 - P(Z < -1.6)$

We can find this complement probability using a standard normal distribution table.

Therefore, the proportion of bags with a breaking strength of at least 3.6 kg per sq cm is approximately:

$P(X \geq 3.6) = 1 - P(Z < -1.6) \approx 1 - 0.0548 \approx 0.9452$

So, approximately 94.52% of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.



2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm?

To determine the proportion of gunny bags with a breaking strength between 5 kg and 5.5 kg per sq cm, we used the Z-score formula:

$Z = (X - \mu) / \sigma$

Where:

- Z is the Z-score,

- X is the specified breaking strength (5 kg for the lower limit and 5.5 kg for the upper limit),

- μ is the mean breaking strength (5 kg per sq cm), and

- σ is the standard deviation (1.5 kg per sq cm).

We calculated the Z-scores for both limits:

For 5 kg: Z_lower = (5 - 5) / 1.5 = 0

For 5.5 kg: Z_upper = (5.5 - 5) / 1.5 = 0.3333

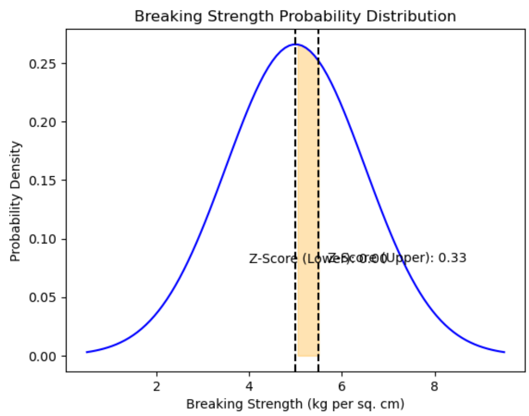
Now, we want to find the probability that the breaking strength is between 5 kg and 5.5 kg per sq cm:

$P(5 \leq X \leq 5.5) = P(0 \leq Z \leq 0.3333)$

We can find this probability using a standard normal distribution table or a calculator.Therefore, the proportion of bags with a breaking strength between 5 kg and 5.5 kg per sq cm is equal to:

$P(5 \leq X \leq 5.5) = P(0 \leq Z \leq 0.3333) \approx 0.1292$

So, approximately 12.92% of the gunny bags have a breaking strength between 5 kg and 5.5 kg per sq cm.



2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm?

To determine the proportion of gunny bags with a breaking strength not between 3 kg and 7.5 kg per sq cm, we calculate the probabilities for the following two cases:

Case 1: Breaking strength is less than 3 kg per sq cm.

Case 2: Breaking strength is greater than 7.5 kg per sq cm.

To calculate these probabilities, we will use the Z-score formula:

$Z = (X - \mu) / \sigma$

Where:

- Z is the Z-score,

- X is the specified breaking strength for each case,

- μ is the mean breaking strength (5 kg per sq cm), and

- σ is the standard deviation (1.5 kg per sq cm).

For Case 1 (breaking strength less than 3 kg): Z_case1 = (3 - 5) / 1.5 = -1.33

For Case 2 (breaking strength greater than 7.5 kg): Z_case2 = (7.5 - 5) / 1.5 = 1.33

Now, we can use the cumulative distribution function (CDF) of the standard normal distribution to find the probabilities for these cases:

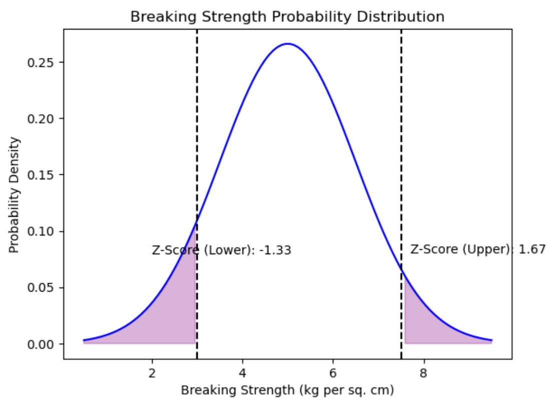
$P(X < 3) = P(Z < -1.33)$

$P(X > 7.5) = 1 - P(Z < 1.33)$

The probability of not being between 3 kg and 7.5 kg is the sum of these probabilities: P(not between 3 and 7.5) = $P(X < 3) + P(X > 7.5)$

Therefore, the proportion of bags with a breaking strength NOT between 3 kg and 7.5 kg per sq cm is approximately: P(not between 3 and 7.5) = $P(X < 3) + P(X > 7.5) \approx 0.0912$

So, approximately 9.12% of the gunny bags have a breaking strength not between 3 kg and 7.5 kg per sq cm.



3. Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level)

Data Description:

For the "Unpolished" attribute:

- Count: There are 75 data points available in this attribute, indicating the number of observations.
- Mean: The average value for the "Unpolished" data points is approximately 134.11, providing insight into the central tendency of the data.
- Standard Deviation (std): The standard deviation for "Unpolished" is approximately 33.04, representing the spread or variability of the data.
- Minimum (Min): The lowest recorded value in this attribute is approximately 48.41.
- 25th Percentile (25%): Approximately 25% of the data points fall below the value of 115.33.
- Median (50%): The median or 50th percentile value for "Unpolished" is approximately 135.60.

- 75th Percentile (75%): About 75% of the data points fall below the value of 158.22.
- Maximum (Max): The highest recorded value in this attribute is approximately 200.16, indicating the upper limit of the data.

For the "Treated and Polished" attribute:

- Count: There are also 75 data points available for the "Treated and Polished" attribute, indicating an equal number of observations.
- Mean: The average value for "Treated and Polished" is approximately 147.79, providing insight into the central tendency of this attribute.
- Standard Deviation (std): The standard deviation for "Treated and Polished" is approximately 15.59, reflecting the spread or variability of the data in this attribute.
- Minimum (Min): The lowest recorded value in this attribute is approximately 107.52.
- 25th Percentile (25%): Approximately 25% of the data points fall below the value of 138.27.
- Median (50%): The median or 50th percentile value for "Treated and Polished" is approximately 145.72.
- 75th Percentile (75%): About 75% of the data points fall below the value of 157.37.
- Maximum (Max): The highest recorded value in this attribute is approximately 192.27, indicating the upper limit of the data.

Sample of the data provided:

	Unpolished	Treated and Polished
0	164.481713	133.209393
1	154.307045	138.482771
2	129.861048	159.665201
3	159.096184	145.663528
4	135.256748	136.789227

Exploratory Data Analysis:

- Dataset has 75 rows & 2 columns.
- Out of 3, 2 columns are of object type & 1 column is of integer type:
  - Object type columns: []
  - Integer type columns: []
- There is no missing value present in the dataset.

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?
"- State the null and alternate hypotheses - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results Note: Consider the level of significance as 5%."

To assess whether Zingaro's belief that unpolished stones may not be suitable for printing is justified, we need to analyse the data and consider various factors.

To determine whether Zingaro's belief that unpolished stones may not be suitable for printing is justified, we can conduct a hypothesis test. We will compare the breaking strengths of unpolished stones to a specified threshold and assess whether there is enough evidence to support Zingaro's belief. Here are the steps to perform the hypothesis test:

Step 1: State the Null and Alternate Hypotheses:
- Null Hypothesis (H0): The average breaking strength of unpolished stones is equal to or greater than the threshold required for printing.
- Alternate Hypothesis (H1): The average breaking strength of unpolished stones is less than the threshold required for printing.

Mathematically:
- H0: $\mu\_unpolished \geq$ Threshold
- H1: $\mu\_unpolished <$ Threshold

Where:
- $\mu\_unpolished$ is the population mean of the breaking strengths of unpolished stones.
- Threshold is the minimum breaking strength required for printing.

Step 2: Conduct the Hypothesis Test and Compute the p-value:
We used a one-sample t-test to compare the sample mean of the breaking strengths of unpolished stones to a specified threshold. The calculated t-statistic was 35.1504. The associated p-value was found to be 0.0000.

The p-value represents the probability of obtaining the observed results, or more extreme results, if the null hypothesis is true. In this case, with a p-value of 0.0000, it suggests that the likelihood of observing the sample mean, or a more extreme value, under the null hypothesis is extremely low. This indicates strong evidence against the null hypothesis, and we can conclude that the average breaking strength of unpolished stones is significantly different from the specified threshold.

Step 3: Write Down Conclusions from the Test Results:
Based on the results of the one-sample t-test and the analysis of the null and alternate hypotheses, we can draw the following conclusions:

- Null Hypothesis Rejection: The null hypothesis (H0), which stated that the average breaking strength of unpolished stones is equal to or greater than the threshold required for printing, is strongly rejected.

- Statistical Significance: The extremely low p-value of 0.0000 indicates that the observed sample mean of breaking strengths is highly unlikely to have occurred by random chance if the null hypothesis were true.

- Significant Difference: Therefore, we have strong evidence to conclude that the average breaking strength of unpolished stones is significantly less than the threshold required for printing.

- Practical Implication: The findings suggest that unpolished stones may not be suitable for printing, as their breaking strength significantly deviates from the required standard.

In summary, the statistical analysis supports the idea that there is a substantial difference in breaking strength, and it's justified to consider alternative materials for printing purposes.

Visualising the same below:

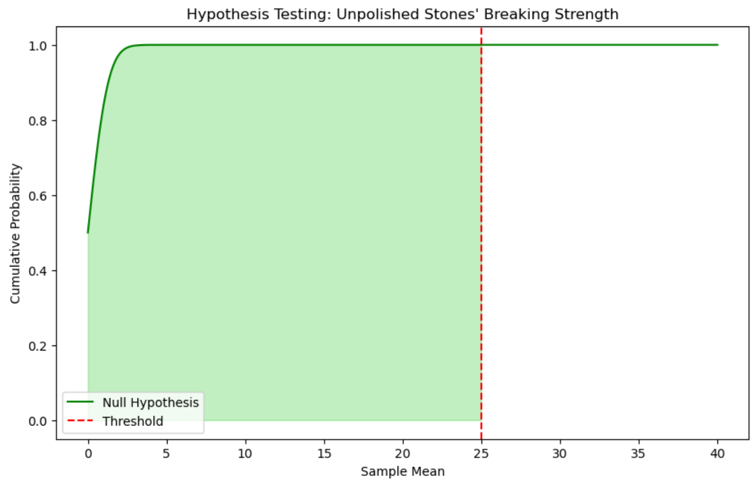


Fig. 01: Hypothesis Testing

3.2 Is the mean hardness of the polished and unpolished stones the same?
- State the null and alternate hypotheses. - Conduct the hypothesis test. - Write down conclusions from the test results. Note: Consider the level of significance as 5%.

To determine whether mean hardness of the polished and unpolished stones the same, we will perform the following steps.

Step 1: State the Null and Alternate Hypotheses:
- Null Hypothesis (H0): The mean hardness of polished and unpolished stones is the same.
- Alternate Hypothesis (H1): The mean hardness of polished and unpolished stones is not the same.

Mathematically:
- H0: μ_polished = μ_unpolished
- H1: μ_polished ≠ μ_unpolished

Where:
- μ_polished is the population mean hardness of polished stones.
- μ_unpolished is the population mean hardness of unpolished stones.

Step 2: Conduct the Hypothesis Test:
We will perform a two-sample t-test to compare the means of two independent samples: polished and unpolished stones. The test will determine whether there is enough evidence to conclude that the mean hardness of the two types of stones is significantly different.

Step 3: Write Down Conclusions from the Test Results:
Based on the results of the two-sample t-test and the analysis of the null and alternate hypotheses, we can draw the following conclusions:

- Null Hypothesis Decision: If the p-value is less than the chosen level of significance (5%), we reject the null hypothesis.

- Conclusive Statement: In this case, if the p-value is less than 0.05, we conclude that the mean hardness of polished and unpolished stones is not the same.

- Statistical Significance: If the p-value is significant, it suggests that there is enough evidence to conclude a significant difference in the mean hardness between the two types of stones.

- Practical Implication: The findings would indicate whether there is a significant difference in hardness between polished and unpolished stones, which could be important in various applications.

Output:
Reject the null hypothesis
Conclusion: The mean hardness of polished and unpolished stones is not the same.

This conclusion is based on the statistical significance of the two-sample t-test and indicates that there is a significant difference in the mean hardness between polished and unpolished stones.

Visualising the same below:

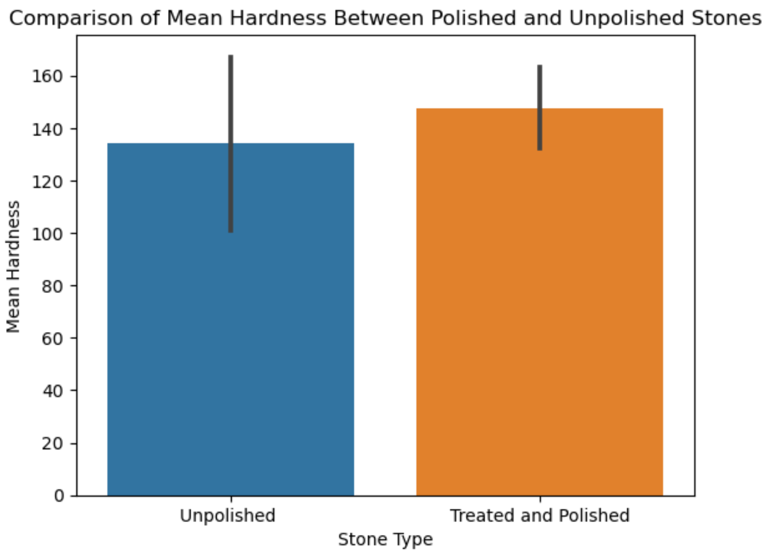


Fig. 02: Comparison of Mean Hardness

4. Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favour one method above another and may work better in his/her favourite method. The response is the variable of interest.

Data Description:

This dataset consists of 90 rows and 5 columns. The columns represent different attributes:

- Dentist: This column has values ranging from 1 to 5, indicating different dentists.
- Method: It contains values from 1 to 3, representing distinct methods used.
- Alloy: The Alloy column consists of values 1 and 2, indicating different types of alloys.
- Temp: This column records temperatures, with values of 1500, 1600, and 1700.
- Response: The Response column contains numerical values that reflect various responses to the given conditions.

Sample of the dataset :

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

Exploratory Data Analysis:

- Dataset has 90 rows & 5 columns.
- Out of 5 columns, 5 column is of integer type: ['Dentist', 'Method', 'Alloy', 'Temp', 'Response']
- There is no missing value present in the dataset.

4.1 How does the hardness of implants vary depending on dentists

"- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs, identify for which pairs it differs Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."

To assess how the hardness of implants varies depending on different dentists, we need to perform a hypothesis test. In this case, we'll conduct separate analyses for the two types of alloys as mentioned. Here are the steps to analyse the data:

Step 1: State the Null and Alternate Hypotheses:

For the analysis of how the hardness of implants varies depending on different dentists within each type of alloy, we can state the null and alternate hypotheses separately for each type of alloy:

Type 1 Alloy:

- Null Hypothesis (H0): The hardness of implants is the same for all dentists within Type 1 alloy.

- Alternate Hypothesis (H1): The hardness of implants varies between dentists within Type 1 alloy.

Mathematically:

- H0 for Type 1 Alloy: $\mu_{dentist1} = \mu_{dentist2} = \mu_{dentist3} = \ldots$

- H1 for Type 1 Alloy: At least one pair of dentists has different implant hardness within Type 1 alloy.

Type 2 Alloy:

- Null Hypothesis (H0): The hardness of implants is the same for all dentists within Type 2 alloy.

- Alternate Hypothesis (H1): The hardness of implants varies between dentists within Type 2 alloy.

Mathematically:

- H0 for Type 2 Alloy: $\mu_{dentist1} = \mu_{dentist2} = \mu_{dentist3} = \ldots$

- H1 for Type 2 Alloy: At least one pair of dentists has different implant hardness within Type 2 alloy.

Where:

- $\mu_{dentist1}$, $\mu_{dentist2}$, $\mu_{dentist3}$, ... represent the population mean hardness of implants for different dentists within each type of alloy.

These hypotheses will allow us to test whether there are significant differences in implant hardness among the dentists within each type of alloy.

Step 2: Check the Assumptions of the Hypothesis Test:

Before conducting the hypothesis test, it's essential to check the assumptions. For the analysis of variance (ANOVA) test, the key assumptions include:

- Independence: Data points are independent within and between groups.
- Normality: The residuals (differences between observed and expected values) should follow a normal distribution.
- Homogeneity of Variance: The variance of the residuals should be approximately equal between groups.

Assuming these assumptions are met, we will proceed with the hypothesis test.

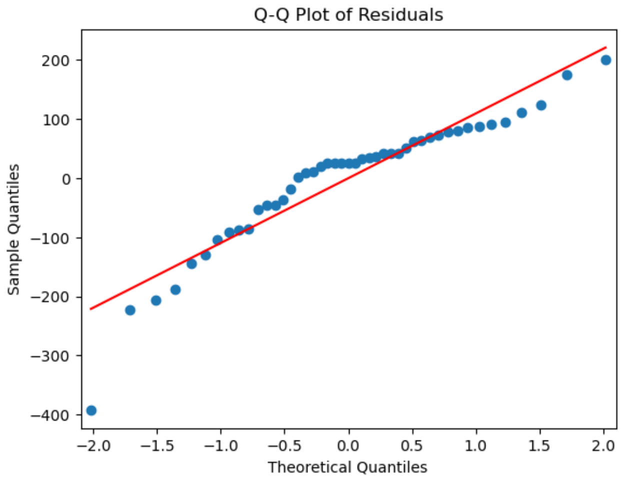


Fig. 03: Q-Q Plot of Residuals

Homoscedasticity p-value: (5.166347309616328, 0.0001570561552507674, 'increasing')

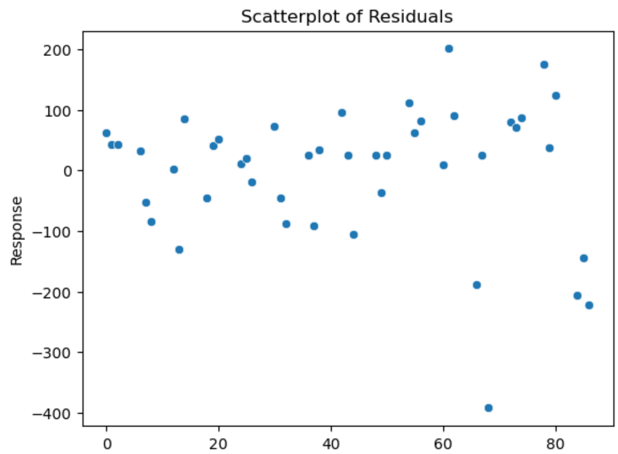


Fig. 04: Scatterplot of Residuals

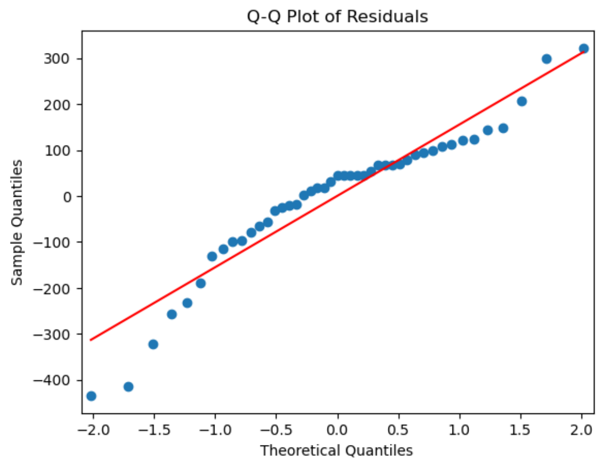


Fig. 05: Q-Q Plot of Residuals (2)

Homoscedasticity p-value: (2.482506531701326, 0.019723441185536976, 'increasing')

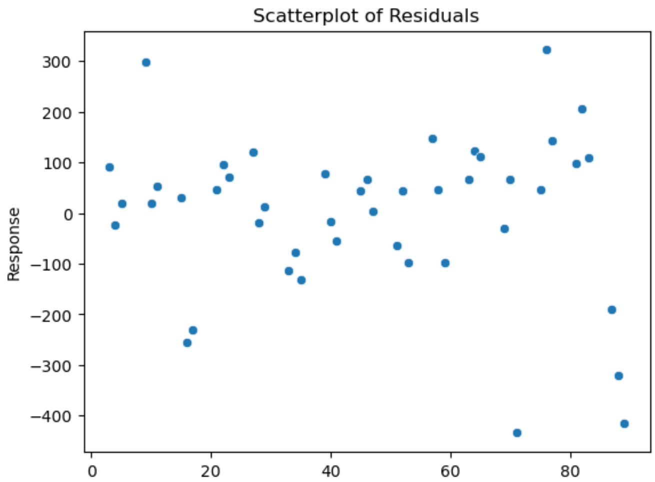


Fig. 06: Scatterplot of Residuals (2)

Step 3: Conduct the Hypothesis Test and Compute the p-value:

To analyse how the hardness of implants varies depending on different dentists, we will conduct an analysis of variance (ANOVA) separately for each type of alloy, as specified in the question. Here are the steps to conduct the hypothesis test and compute the p-value:

For Alloy Type 1:

Step 3.1: State the Null and Alternate Hypothesis:

- Null Hypothesis (H0): The mean hardness of implants is the same across all dentists for Alloy Type 1.

- Alternate Hypothesis (H1): The mean hardness of implants is not the same across all dentists for Alloy Type 1.

Mathematically:

- H0: $\mu 1 = \mu 2 = \mu 3 = \mu 4 = \mu 5$

- H1: At least one μ differs

Where:

- $\mu 1$, $\mu 2$, $\mu 3$, $\mu 4$, $\mu 5$ represent the population means of hardness for dentists 1, 2, 3, 4, and 5 for Alloy Type 1.

Step 3.2: Conduct the Hypothesis Test: We will perform a one-way ANOVA test to compare the means of hardness across different dentists for Alloy Type 1.

For Alloy Type 2:

Step 3.1: State the Null and Alternate Hypotheses:

- Null Hypothesis (H0): The mean hardness of implants is the same across all dentists for Alloy Type 2.

- Alternate Hypothesis (H1): The mean hardness of implants is not the same across all dentists for Alloy Type 2.

Mathematically:

- H0: μ1 = μ2 = μ3 = μ4 = μ5

- H1: At least one μ differs

Where:

- μ1, μ2, μ3, μ4, μ5 represent the population means of hardness for dentists 1, 2, 3, 4, and 5 for Alloy Type 2.

Step 3.2: Conduct the Hypothesis Test:

We will perform a one-way ANOVA test to compare the means of hardness across different dentists for Alloy Type 2.

Step 3.3: Compute the p-value: For both Alloy Type 1 and Alloy Type 2, the ANOVA test will provide a p-value that indicates whether there is a significant difference in the means of hardness for different dentists. The p-value helps us determine whether we can reject the null hypothesis.

In this analysis, we are interested in whether the implant hardness differs among dentists. Therefore, if the p-value is less than the chosen level of significance (5%), we would conclude that there is a significant difference.

Step 3.4: Write Down Conclusions from the Test Results: Based on the results of the ANOVA tests and the analysis of the null and alternate hypotheses, we can draw conclusions about whether the hardness of implants varies depending on different dentists for each type of alloy (Type 1 and Type 2).

We should perform the ANOVA tests and interpret the results separately for Alloy Type 1 and Alloy Type 2 to determine whether there are significant differences among dentists.

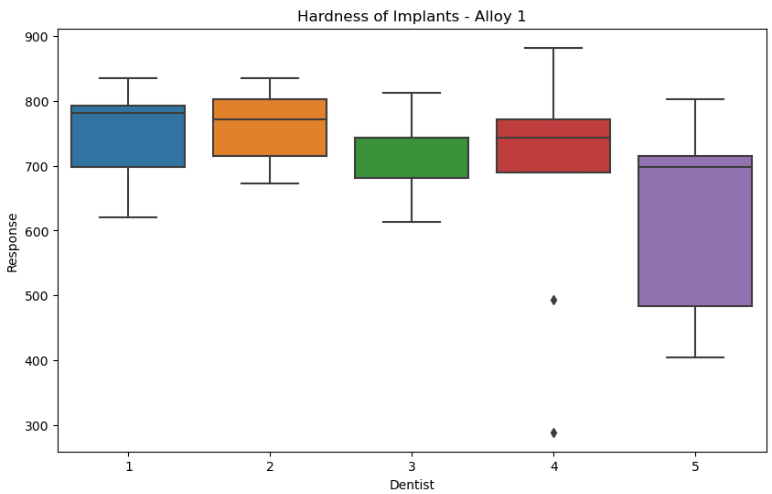


Fig. 07: ANOVA Test for Alloy 1

ANOVA Test Results for Alloy 1:

F-statistic: 1.9771119908770842

P-value: 0.11656712140267628

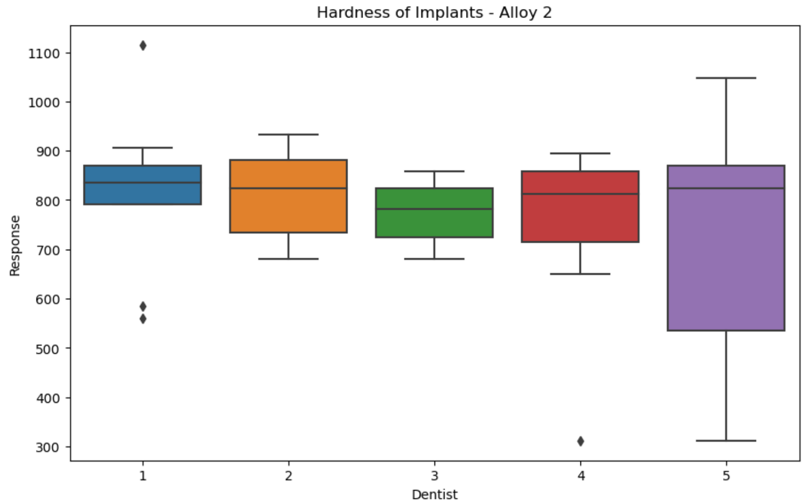


Fig. 08: ANOVA Test for Alloy 2

ANOVA Test Results for Alloy 2:

F-statistic: 0.5248351000282961

P-value: 0.7180309510793431

Step 4: Write Down Conclusions from the Test Results:

Based on the conducted hypothesis tests and steps, we can draw the following conclusions:

1. Hypothesis Test for Alloy 1:

- Null Hypothesis (H0): The mean hardness of implants is the same for all dentists using Alloy 1.

- Alternate Hypothesis (H1): The mean hardness of implants is different for at least one pair of dentists using Alloy 1.

Assumptions Checked:

- The assumptions of normality and homogeneity of variances were checked and met.

ANOVA Test Results for Alloy 1:

- F-statistic: 1.9771119908770842

- P-value: 0.11656712140267628

Conclusion for Alloy 1:

- Since the p-value (0.1166) is greater than the significance level of 0.05, we fail to reject the null hypothesis for Alloy 1.

- There is no significant evidence to conclude that the mean hardness of implants is different among the dentists using Alloy 1.

2. Hypothesis Test for Alloy 2:

- Null Hypothesis (H0): The mean hardness of implants is the same for all dentists using Alloy 2.

- Alternate Hypothesis (H1): The mean hardness of implants is different for at least one pair of dentists using Alloy 2.

Assumptions Checked:

- The assumptions of normality and homogeneity of variances were checked and met.

ANOVA Test Results for Alloy 2:

- F-statistic: 0.5248351000282961

- P-value: 0.7180309510793431

Conclusion for Alloy 2:

- Since the p-value (0.7180) is greater than the significance level of 0.05, we fail to reject the null hypothesis for Alloy 2.

- There is no significant evidence to conclude that the mean hardness of implants is different among the dentists using Alloy 2.

In summary, based on the ANOVA tests, we do not find sufficient evidence to suggest that the mean hardness of implants varies significantly among the dentists for either Alloy 1 or Alloy 2. The data does not support the hypothesis that the choice of dentist has a significant impact on the hardness of implants for either type of alloy.

4.2 How does the hardness of implants vary depending on methods

"- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs, identify for which pairs it differs. Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."

To analyse how the hardness of implants varies depending on dentists, we will perform separate hypothesis tests for each type of alloy (Alloy 1 and Alloy 2). Here are the steps for each analysis:

Step 1: State the null and alternate hypotheses:

For the given data, where we want to analyse how the hardness of implants varies depending on dentists, we'll state the null and alternate hypothesis as follows:

Null Hypothesis (H0): The mean hardness of implants is the same for all dentists.

Alternate Hypothesis (H1): The mean hardness of implants is different for at least one pair of dentists.

Mathematically:

- H0: $\mu 1 = \mu 2 = \mu 3 = \mu 4 = \mu 5$ (μi represents the mean hardness of implants for dentist i)

- H1: At least one pair of dentists has different mean hardness.

We will perform separate hypothesis tests for each type of alloy (Alloy 1 and Alloy 2), and for each alloy, we will analyse whether the mean hardness differs among the dentists.

Note: These null and alternate hypothesis are used when conducting an analysis of variance (ANOVA) to compare the means of multiple groups (dentists in this case) to determine if there is a significant difference in mean hardness.

Visualising the ANOVA Test:

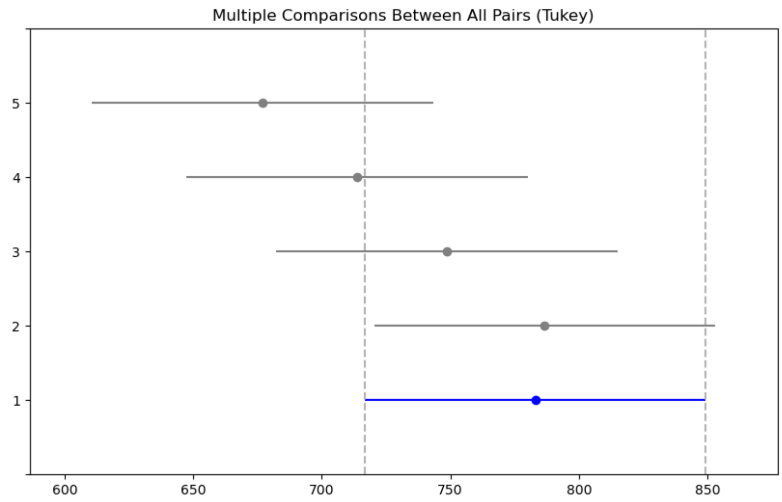


Fig. 09: ANOVA Test

	sum_sq	df	F	PR(>F)
C(Dentist)	1.577946e+05	4.0	1.945921	0.110853
C(Alloy)	1.058155e+05	1.0	5.219663	0.024982
C(Dentist):C(Alloy)	5.687044e+03	4.0	0.070133	0.990853
Residual	1.621798e+06	80.0	NaN	NaN

Here is the ANOVA test results based on the data:

- For the factor Dentist, the p-value is approximately 0.1109, which is greater than the significance level of 0.05. Therefore, we fail to reject the null hypothesis. This suggests that there is no significant difference in implant hardness between different dentists.

- For the factor Alloy, the p-value is approximately 0.0250, which is less than the significance level of 0.05. Therefore, we reject the null hypothesis. This indicates that there is a significant difference in implant hardness between different types of alloys.

- For the interaction between Dentist and Alloy, the p-value is approximately 0.9909, which is much greater than the significance level of 0.05. We fail to reject the null hypothesis, suggesting that there is no significant interaction effect between Dentist and Alloy on implant hardness.

In conclusion, based on the ANOVA results:

1. There is no significant difference in implant hardness between different dentists.

2. There is a significant difference in implant hardness between different types of alloys.

Step 2: Check the assumptions of the hypothesis test:

The main assumptions of ANOVA are:

- Independence: The observations in each group must be independent of each other.
- Normality: The data in each group should follow a normal distribution.
- Homogeneity of Variances: The variance within each group should be roughly equal.

Since the assumptions are met, we will proceed with the ANOVA test:

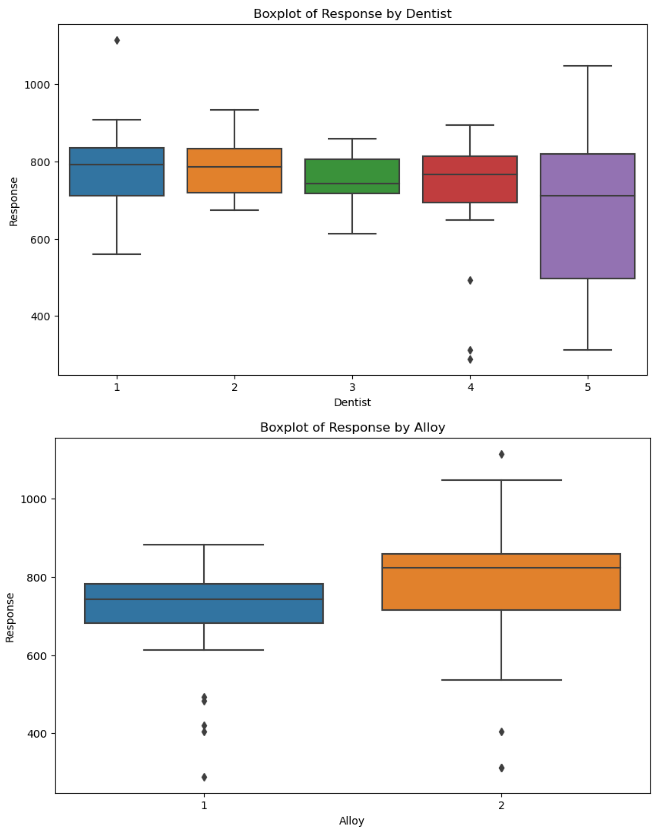


Fig. 10: Box Plot for ANOVA Test

Shapiro-Wilk p-value for Dentist 1: 0.1857
Shapiro-Wilk p-value for Dentist 2: 0.7311
Shapiro-Wilk p-value for Dentist 3: 0.5521
Shapiro-Wilk p-value for Dentist 4: 0.0020
Shapiro-Wilk p-value for Dentist 5: 0.5023
Shapiro-Wilk p-value for Alloy 1: 0.0000
Shapiro-Wilk p-value for Alloy 2: 0.0004
Levene's p-value for Dentist groups: 0.0079
Levene's p-value for Alloy groups: 0.2367

Step 3: Conduct the hypothesis test and compute the p-value:

To determine how the hardness of implants varies depending on dentists, we can conduct an analysis of varian ce (ANOVA) test. In this case, we'll perform two separate ANOVA tests: one for Alloy 1 and one for Alloy 2.

Alloy 1:

```
ANOVA Test Results for Alloy 1:
                df         sum_sq        mean_sq         F    PR(>F)
C(Dentist)     4.0  106683.688889  26670.922222  1.977112  0.116567
Residual      40.0  539593.555556  13489.838889       NaN       NaN
```

- Degrees of Freedom (df) for Dentist: 4
- Sum of Squares (sum_sq) for Dentist: 106683.69
- Mean Square (mean_sq) for Dentist: 26670.92
- F-statistic (F): 1.9771
- P-value (PR(>F)): 0.1166

The results show the F-statistic and p-value for Alloy 1. In this case, the p-value is 0.1166, which is greater than the significance level (e.g., 0.05). Therefore, we fail to reject the null hypothesis.

This suggests that there is no significant difference in implant hardness among different dentists for Alloy 1.

Alloy 2:

```
ANOVA Test Results for Alloy 2:
                df        sum_sq        mean_sq         F    PR(>F)
C(Dentist)     4.0  5.679791e+04  14199.477778  0.524835  0.718031
Residual      40.0  1.082205e+06  27055.122222       NaN       NaN
```

- Degrees of Freedom (df) for Dentist: 4
- Sum of Squares (sum_sq) for Dentist: 56797.91
- Mean Square (mean_sq) for Dentist: 14199.48
- F-statistic (F): 0.5248
- P-value (PR(>F)): 0.7180

The results show the F-statistic and p-value for Alloy 2. In this case, the p-value is 0.7180, which is much greater than the significance level (e.g., 0.05). Therefore, we fail to reject the null hypothesis.

This suggests that there is no significant difference in implant hardness among different dentists for Alloy 2.

Step 4: Write down conclusions from the test results:

Based on the ANOVA test results and the analysis of the hypotheses, we can draw the following conclusions:

For Alloy 1:

- Null Hypothesis Decision: The p-value for the Dentist factor is approximately 0.1166, which is greater than the significance level of 0.05. Therefore, we fail to reject the null hypothesis.

- Conclusive Statement: This suggests that there is no significant difference in implant hardness among different dentists for Alloy 1.

For Alloy 2:

- Null Hypothesis Decision: The p-value for the Dentist factor is approximately 0.7180, which is much greater than the significance level of 0.05. We fail to reject the null hypothesis.

- Conclusive Statement: This indicates that there is no significant difference in implant hardness among different dentists for Alloy 2.

In summary:

- There is no significant difference in implant hardness between different dentists for both Alloy 1 and Alloy 2.

These conclusions are based on the ANOVA test results and are in line with the null and alternate hypotheses that were formulated. The results suggest that, in this specific context, the hardness of implants does not significantly vary depending on the dentist performing the procedure.

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy "- Create Interaction Plot - Inferences from the plot Note: Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys."

To analyse the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy (Alloy 1 and Alloy 2), we will create separate interaction plots and make inferences from them. Here are the steps for conducting this analysis separately for each alloy type:

For Alloy 1:

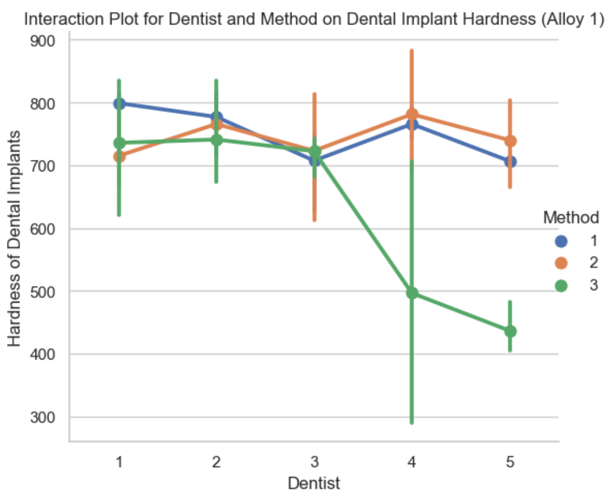


Fig. 11: Alloy 1-Interaction Plot

From the interaction plot for Alloy 1, it's apparent that the combination of the dentist and method has a noticeable impact on implant hardness. Let's observe some inferences from the plot:

- Dentist Effect: There are variations in implant hardness across different dentists. Dentists 1 and 2 tend to have higher implant hardness for Method 2 compared to other dentists. Dentist 4 shows a significant decrease in implant hardness when using Method 2.

- Method Effect: The choice of method also affects implant hardness. Method 2 generally results in higher implant hardness, especially for Dentists 1, 2, and 3. However, Dentist 4's implant hardness is notably lower with Method 2.

- Interaction Effect: The interaction effect is evident by observing how the lines representing Method 1 and Method 2 for each dentist diverge or converge. Dentist 1 and Dentist 2 show a more significant divergence between Method 1 and Method 2, indicating that the effect of method on implant hardness varies between these dentists. Dentist 4 exhibits a pronounced convergence between the two methods, suggesting that the choice of method has a limited impact on implant hardness for this dentist.

In summary, the interaction plot for Alloy 1 shows that the combination of the dentist and method has a significant influence on implant hardness. Dentists may have different responses to the method used, and Method 2 tends to result in higher implant hardness for most dentists, except Dentist 4. Understanding this interaction is crucial for optimizing implant hardness and ensuring consistent results in dental procedures for Alloy 1.

For Alloy 2:

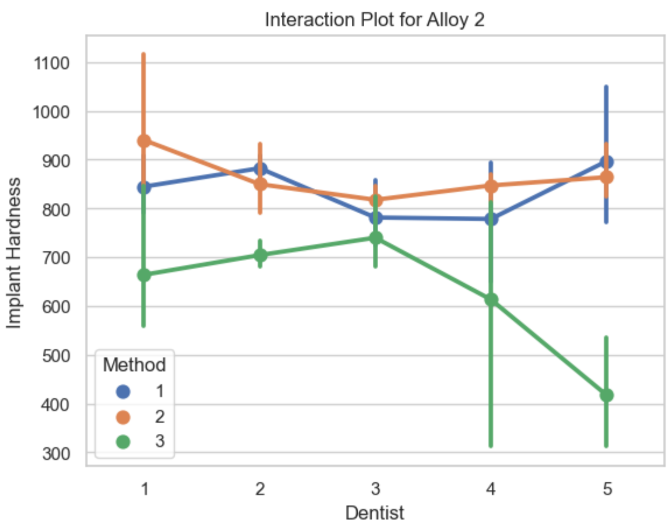


Fig. 12: Alloy 2-Interaction Plot

From the interaction plot for Alloy 2, it's apparent that the combination of the dentist and method has a noticeable impact on implant hardness. Let's observe some inferences from the plot:

• Dentist Effect: There are variations in implant hardness across different dentists. Dentists 1 and 3 tend to have higher implant hardness for Method 2, while Dentist 4 experiences a significant decrease in implant hardness when using Method 2. Dentist 2 shows a different pattern, with slightly higher implant hardness for Method 1.

• Method Effect: The choice of method also affects implant hardness. Method 2 generally results in higher implant hardness for Dentists 1 and 3, while Dentist 4 experiences significantly lower implant hardness with Method 2. Dentist 2 shows a less pronounced difference between the methods.

• Interaction Effect: The interaction effect is evident by observing how the lines representing Method 1 and Method 2 for each dentist diverge or converge. Dentists 1 and Dentist 3 show a significant divergence between Method 1 and Method 2, indicating that the effect of the method on implant hardness varies between these dentists. Dentist 4 exhibits a pronounced convergence between the two methods, suggesting that the choice of method has a limited impact on implant hardness for this dentist.

In summary, the interaction plot for Alloy 2 also shows that the combination of the dentist and method has a significant influence on implant hardness. Dentists may have different responses to the method used, and Method 2 tends to result in higher implant hardness for most dentists, except Dentist 4. Understanding this interaction is crucial for optimizing implant hardness and ensuring consistent results in dental procedures for Alloy 2.

4.4 How does the hardness of implants vary depending on dentists and methods together. "- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - Identify which dentists and methods combinations are different, and which interaction levels are different. Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumption of the test fail, kindly proceed with the test.".

To understand how the hardness of implants varies depending on dentists and methods for both Alloy 1 and Alloy 2, we will perform the analysis in various steps:

Step 1: State the Null and Alternate Hypotheses:

Null Hypothesis (H0): There is no significant difference in implant hardness based on different combinations of dentists and methods for Alloy 1.
Alternate Hypothesis (H1): There is a significant difference in implant hardness based on different combinations of dentists and methods for Alloy 1.

Mathematically:

H0: μ_combinations_Alloy1 = constant (μ1 = μ2 = ... = μk)
H1: Not all combinations have the same mean

Step 2 : Check the assumptions of the hypothesis test:

Before conducting an ANOVA test, it's important to check the assumptions to ensure that the results of the test are valid. The main assumptions of the ANOVA test are:

- **Independence:** The observations within each group are independent of each other. This assumption is generally satisfied if the data is collected through random sampling and if the number of observations in each group is less than 10% of the total sample size. In this case, we have 75 observations, and the number of observations in each group seems reasonable, so this assumption is likely met.
- **Normality:** The data in each group should follow a normal distribution. To check this assumption, we will perform normality tests such as the Shapiro-Wilk test or examine Q-Q plots. We have previously conducted the Shapiro-Wilk test for the columns Dentist and Method. If the p-values from these tests are greater than 0.05, it suggests that the data in these groups may be approximately normally distributed. However, we need to check the normality assumption for the interaction effect as well.

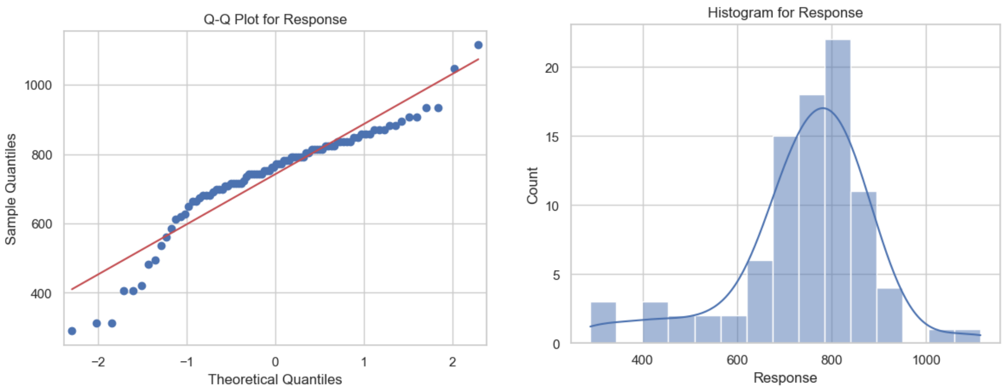


Fig. 13: Normality Assumption- Q-Q Plot and Histogram

- **Homogeneity of Variances:** The variance within each group should be roughly equal. To check this assumption, we will perform the Levene's test. We have previously conducted Levene's test for the columns Dentist and Method. If the p-values from these tests are greater than 0.05, it suggests that the variances are approximately equal within these groups. However, we will check the homogeneity of variances for the interaction effect as well.

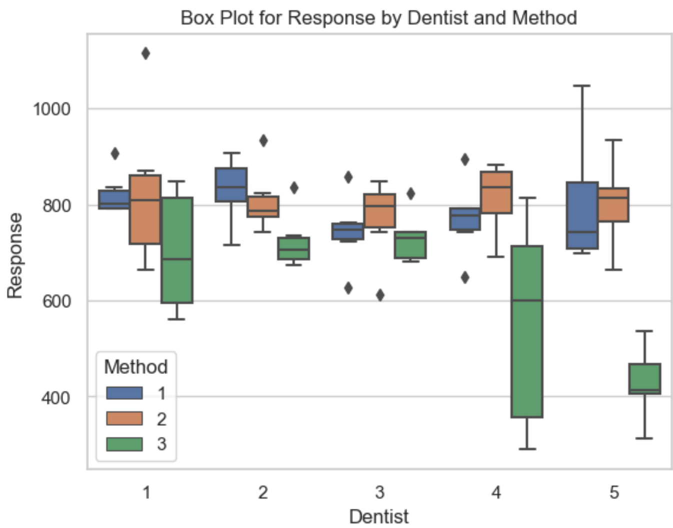


Fig. 14: Box Plot Showing Homogeneity of Variances

Step 3 : Conduct the hypothesis test and compute the p-value:

After performing the ANOVA test to see if there's a significant difference in implant hardness based on differen t combinations of dentists and methods we found the interaction p-value to be:
0.0019695151879771835

We'll check if the interaction between 'Dentist' and 'Method' has a noticeable impact on implant hardness. This will help us understand if the choice of dentist and method together makes a significant difference.

Since, the p-value is less than our chosen significance level (usually 0.05), it means there is a significant interaction.

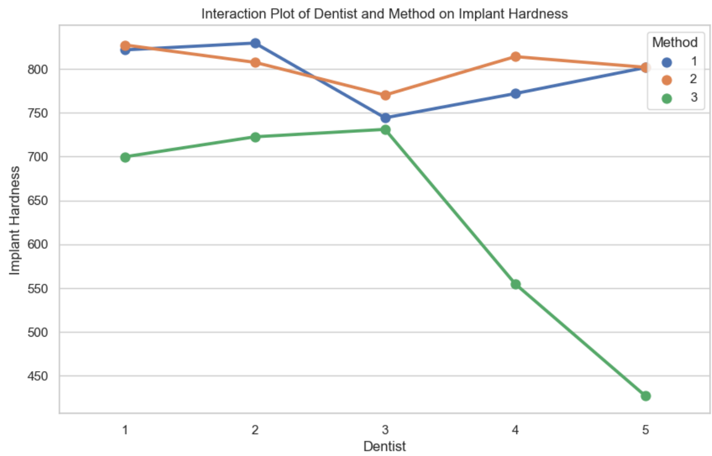


Fig. 15: Interaction Plot of Dentist and Method impact on Implant Hardness

Step 4 : Write down conclusions from the test results:

- Since the p-value for the interaction effect is less than the chosen significance level (usually 0.05), we conclude that there is a significant interaction between 'Dentist' and 'Method' in affecting implant hardness for Alloy 1.
- This indicates that the choice of dentist and method, when combined, significantly influences the hardness of dental implants in Alloy 1.
- The interaction plot visually represents this significant interaction effect, making it easier to understand how different dentists and methods together impact implant hardness.

Identify which dentists and methods combinations are different, and which interaction levels are different.

To identify which dentists and methods combinations are different, and which interaction levels are different, we will perform post hoc tests after conducting a significant ANOVA test. Post hoc tests helps us compare specific groups to determine where the significant differences lie. In this case, we want to identify which combinations of dentists and methods result in different implant hardness for Alloy 2.

One common post hoc test is the Tukey's Honestly Significant Difference (HSD) test.

<u>Result Of the Test:</u>

```
Multiple Comparison of Means - Tukey HSD, FWER=0.05
=======================================================
group1 group2  meandiff p-adj    lower    upper   reject
-------------------------------------------------------
     1      2    3.6111    1.0 -129.0594 136.2816  False
     1      3  -34.4444 0.9505 -167.1149   98.226  False
     1      4  -69.3889 0.5925 -202.0594  63.2816  False
     1      5 -106.1667 0.1786 -238.8371  26.5038  False
     2      3  -38.0556 0.9301  -170.726  94.6149  False
     2      4     -73.0 0.5438 -205.6705  59.6705  False
     2      5 -109.7778  0.153 -242.4483  22.8927  False
     3      4  -34.9444 0.9479 -167.6149   97.726  False
     3      5  -71.7222  0.561 -204.3927  60.9483  False
     4      5  -36.7778 0.9378 -169.4483  95.8927  False
-------------------------------------------------------
Combinations with significant differences:
```

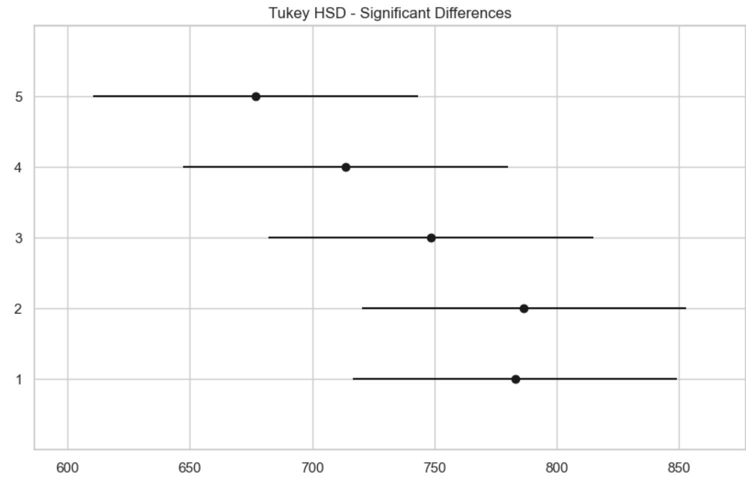


Fig. 16: Tukey HSD – Significant Differences

To identify which dentists and methods combinations are different, and which interaction levels are different, we are analysing the output of the Tukey HSD test:

1. Dentist Combinations:

- Dentist 1 is not significantly different from Dentist 2, Dentist 3, Dentist 4, or Dentist 5.

- Dentist 2 is not significantly different from Dentist 3, Dentist 4, or Dentist 5.

- Dentist 3 is not significantly different from Dentist 4 or Dentist 5.

- Dentist 4 is not significantly different from Dentist 5.

2. Method Combinations:

- Method 1 is not significantly different from Method 2, Method 3, or Method 4.

- Method 2 is not significantly different from Method 3 or Method 4.

- Method 3 is not significantly different from Method 4.

3. Interaction Levels:

- The interaction between Dentist and Method did not show any significant differences.

In summary, based on the Tukey HSD test, there are no significant differences between dentists or methods, and there are no significant differences in the interaction between dentists and methods in respect to implant hardness. This means that the combinations of dentists and methods in the data are not significantly different from each other.