Linear Regression for Boston House Price Prediction

In [1]:
```
# Importing necessary libraries
import pandas as pd
import numpy as np
from sklearn.datasets import load_boston
from sklearn.model_selection import train_test_split
import matplotlib.pyplot as plt
boston = load_boston()
```

In [2]:
```
print(boston.data.shape)
```

```
(506, 13)
```

In [3]:
```
print(boston.feature_names)
```

```
['CRIM' 'ZN' 'INDUS' 'CHAS' 'NOX' 'RM' 'AGE' 'DIS' 'RAD' 'TAX' 'PTRATIO'
 'B' 'LSTAT']
```

In [4]:
```
print(boston.target)
```

```
[24.  21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 15.  18.9 21.7 20.4
 18.2 19.9 23.1 17.5 20.2 18.2 13.6 19.6 15.2 14.5 15.6 13.9 16.6 14.8
 18.4 21.  12.7 14.5 13.2 13.1 13.5 18.9 20.  21.  24.7 30.8 34.9 26.6
 25.3 24.7 21.2 19.3 20.  16.6 14.4 19.4 19.7 20.5 25.  23.4 18.9 35.4
 24.7 31.6 23.3 19.6 18.7 16.  22.2 25.  33.  23.5 19.4 22.  17.4 20.9
 24.2 21.7 22.8 23.4 24.1 21.4 20.  20.8 21.2 20.3 28.  23.9 24.8 22.9
 23.9 26.6 22.5 22.2 23.6 28.7 22.6 22.  22.9 25.  20.6 28.4 21.4 38.7
 43.8 33.2 27.5 26.5 18.6 19.3 20.1 19.5 19.5 20.4 19.8 19.4 21.7 22.8
 18.8 18.7 18.5 18.3 21.2 19.2 20.4 19.3 22.  20.3 20.5 17.3 18.8 21.4
 15.7 16.2 18.  14.3 19.2 19.6 23.  18.4 15.6 18.1 17.4 17.1 13.3 17.8
 14.  14.4 13.4 15.6 11.8 13.8 15.6 14.6 17.8 15.4 21.5 19.6 15.3 19.4
 17.  15.6 13.1 41.3 24.3 23.3 27.  50.  50.  50.  22.7 25.  50.  23.8
 23.8 22.3 17.4 19.1 23.1 23.6 22.6 29.4 23.2 24.6 29.9 37.2 39.8 36.2
 37.9 32.5 26.4 29.6 50.  32.  29.8 34.9 37.  30.5 36.4 31.1 29.1 50.
 33.3 30.3 34.6 34.9 32.9 24.1 42.3 48.5 50.  22.6 24.4 22.5 24.4 20.
 21.7 19.3 22.4 28.1 23.7 25.  23.3 28.7 21.5 23.  26.7 21.7 27.5 30.1
 44.8 50.  37.6 31.6 46.7 31.5 24.3 31.7 41.7 48.3 29.  24.  25.1 31.5
 23.7 23.3 22.  20.1 22.2 23.7 17.6 18.5 24.3 20.5 24.5 26.2 24.4 24.8
 29.6 42.8 21.9 20.9 44.  50.  36.  30.1 33.8 43.1 48.8 31.  36.5 22.8
 30.7 50.  43.5 20.7 21.1 25.2 24.4 35.2 32.4 32.  33.2 33.1 29.1 35.1
 45.4 35.4 46.  50.  32.2 22.  20.1 23.2 22.3 24.8 28.5 37.3 27.9 23.9
 21.7 28.6 27.1 20.3 22.5 29.  24.8 22.  26.4 33.1 36.1 28.4 33.4 28.2
 22.8 20.3 16.1 22.1 19.4 21.6 23.8 16.2 17.8 19.8 23.1 21.  23.8 23.1
 20.4 18.5 25.  24.6 23.  22.2 19.3 22.6 19.8 17.1 19.4 22.2 20.7 21.1
 19.5 18.5 20.6 19.  18.7 32.7 16.5 23.9 31.2 17.5 17.2 23.1 24.5 26.6
 22.9 24.1 18.6 30.1 18.2 20.6 17.8 21.7 22.7 22.6 25.  19.9 20.8 16.8
 21.9 27.5 21.9 23.1 50.  50.  50.  50.  50.  13.8 13.8 15.  13.9 13.3
 13.1 10.2 10.4 10.9 11.3 12.3  8.8  7.2 10.5  7.4 10.2 11.5 15.1 23.2
  9.7 13.8 12.7 13.1 12.5  8.5  5.   6.3  5.6  7.2 12.1  8.3  8.5  5.
 11.9 27.9 17.2 27.5 15.  17.2 17.9 16.3  7.   7.2  7.5 10.4  8.8  8.4
 16.7 14.2 20.8 13.4 11.7  8.3 10.2 10.9 11.   9.5 14.5 14.1 16.1 14.3
 11.7 13.4  9.6  8.7  8.4 12.8 10.5 17.1 18.4 15.4 10.8 11.8 14.9 12.6
 14.1 13.  13.4 15.2 16.1 17.8 14.9 14.1 12.7 13.5 14.9 20.  16.4 17.7
 19.5 20.2 21.4 19.9 19.  19.1 19.1 20.1 19.9 19.6 23.2 29.8 13.8 13.3
 16.7 12.  14.6 21.4 23.  23.7 25.  21.8 20.6 21.2 19.1 20.6 15.2  7.
  8.1 13.6 20.1 21.8 24.5 23.1 19.7 18.3 21.2 17.5 16.8 22.4 20.6 23.9
 22.  11.9]
```

In [5]:
```
print(boston.DESCR)
```

```
Boston House Prices dataset
===========================

Notes
------
Data Set Characteristics:

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive

    :Median Value (attribute 14) is usually the target

    :Attribute Information (in order):
        - CRIM     per capita crime rate by town
        - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
        - INDUS    proportion of non-retail business acres per town
        - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
        - NOX      nitric oxides concentration (parts per 10 million)
        - RM       average number of rooms per dwelling
        - AGE      proportion of owner-occupied units built prior to 1940
        - DIS      weighted distances to five Boston employment centres
        - RAD      index of accessibility to radial highways
        - TAX      full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
        - LSTAT    % lower status of the population
        - MEDV     Median value of owner-occupied homes in $1000's

    :Missing Attribute Values: None

    :Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.
http://archive.ics.uci.edu/ml/datasets/Housing


This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic
prices and the demand for clean air', J. Environ. Economics & Management,
vol.5, 81-102, 1978.   Used in Belsley, Kuh & Welsch, 'Regression diagnostics
...', Wiley, 1980.   N.B. Various transformations are used in the table on
pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address regression
problems.

**References**

   - Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of Collinearity', Wile
y, 1980. 244-261.
   - Quinlan,R. (1993). Combining Instance-Based and Model-Based Learning. In Proceedings on the Tenth International
Conference of Machine Learning, 236-243, University of Massachusetts, Amherst. Morgan Kaufmann.
   - many more! (see http://archive.ics.uci.edu/ml/datasets/Housing)
```

In [6]:
```
# Loading data into pandas dataframe
bos = pd.DataFrame(boston.data)
print(bos.head())
```

```
         0     1     2    3      4      5     6       7    8      9    10  \
0  0.00632  18.0  2.31  0.0  0.538  6.575  65.2  4.0900  1.0  296.0  15.3
1  0.02731   0.0  7.07  0.0  0.469  6.421  78.9  4.9671  2.0  242.0  17.8
2  0.02729   0.0  7.07  0.0  0.469  7.185  61.1  4.9671  2.0  242.0  17.8
3  0.03237   0.0  2.18  0.0  0.458  6.998  45.8  6.0622  3.0  222.0  18.7
4  0.06905   0.0  2.18  0.0  0.458  7.147  54.2  6.0622  3.0  222.0  18.7

       11    12
0  396.90  4.98
1  396.90  9.14
2  392.83  4.03
3  394.63  2.94
4  396.90  5.33
```

In [7]:
```
bos['PRICE'] = boston.target
X = bos.drop('PRICE', axis = 1)
Y = bos['PRICE']
```

In [8]:
```
# Split data into train and test
X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size = 0.2, random_state = 5)
print(X_train.shape)
print(X_test.shape)
print(Y_train.shape)
print(Y_test.shape)
```

```
(404, 13)
(102, 13)
(404,)
(102,)
```

In [10]:
```
# Standardization
from sklearn.preprocessing import StandardScaler
std = StandardScaler()
X_train = std.fit_transform(X_train)
X_test = std.transform(X_test)
```

In [11]:
```
import warnings
warnings.filterwarnings('ignore')
from sklearn.linear_model import SGDRegressor
from sklearn.metrics import mean_squared_error, r2_score
clf = SGDRegressor()
clf.fit(X_train, Y_train)
Y_pred = clf.predict(X_test)

print("Coefficients: \n", clf.coef_)
print("Y_intercept", clf.intercept_)
```

```
Coefficients: 
 [-0.87124896  0.65471297 -0.44191942  0.88490447 -0.63460155  2.98328602
 -0.34713416 -2.15542918  0.85903213 -0.3325135  -1.75839589  1.06975581
 -3.63978981]
Y_intercept [22.26755422]
```

Stochastic Gradient Decent(SGD) for Linear Regression

In [12]:
```
# Imported necessary libraries
from sklearn.datasets import load_boston
from sklearn.model_selection import train_test_split
import pandas as pd
import numpy as np
```

In [13]:
```
# Data loaded
bostan = load_boston()
# Convert it into pandas dataframe
data = pd.DataFrame(bostan.data, columns = bostan.feature_names)
data.head()
```

Out[13]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [14]:
```
# Summary
data.describe()
```

Out[14]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PT
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.
mean	3.593761	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.549407	408.237154	18.4
std	8.596783	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.707259	168.537116	2.16
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.000000	187.000000	12.6
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.000000	279.000000	17.4
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.000000	330.000000	19.0
75%	3.647423	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.000000	666.000000	20.2
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.000000	711.000000	22.0

In [15]:
```
#standardize for fast convergence to minima
data = (data - data.mean())/data.std()
data.head()
```

Out[15]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	L
0	-0.417300	0.284548	-1.286636	-0.272329	-0.144075	0.413263	-0.119895	0.140075	-0.981871	-0.665949	-1.457558	0.440616	-1.07
1	-0.414859	-0.487240	-0.592794	-0.272329	-0.739530	0.194082	0.366803	0.556609	-0.867024	-0.986353	-0.302794	0.440616	-0.49
2	-0.414861	-0.487240	-0.592794	-0.272329	-0.739530	1.281446	-0.265549	0.556609	-0.867024	-0.986353	-0.302794	0.396035	-1.20
3	-0.414270	-0.487240	-1.305586	-0.272329	-0.834458	1.015298	-0.809088	1.076671	-0.752178	-1.105022	0.112920	0.415751	-1.36
4	-0.410003	-0.487240	-1.305586	-0.272329	-0.834458	1.227362	-0.510674	1.076671	-0.752178	-1.105022	0.112920	0.440616	-1.02

In [16]:
```
# MEDV(median value is usually target), change it to price
data["PRICE"] = bostan.target
data.head()
```

Out[16]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	L
0	-0.417300	0.284548	-1.286636	-0.272329	-0.144075	0.413263	-0.119895	0.140075	-0.981871	-0.665949	-1.457558	0.440616	-1.07
1	-0.414859	-0.487240	-0.592794	-0.272329	-0.739530	0.194082	0.366803	0.556609	-0.867024	-0.986353	-0.302794	0.440616	-0.49
2	-0.414861	-0.487240	-0.592794	-0.272329	-0.739530	1.281446	-0.265549	0.556609	-0.867024	-0.986353	-0.302794	0.396035	-1.20
3	-0.414270	-0.487240	-1.305586	-0.272329	-0.834458	1.015298	-0.809088	1.076671	-0.752178	-1.105022	0.112920	0.415751	-1.36
4	-0.410003	-0.487240	-1.305586	-0.272329	-0.834458	1.227362	-0.510674	1.076671	-0.752178	-1.105022	0.112920	0.440616	-1.02

In [17]:
```
# Target and features
Y = data["PRICE"]
X = data.drop("PRICE", axis = 1)
```

In [18]:
```
from sklearn.model_selection import train_test_split
x_train, x_test, y_train, y_test = train_test_split(X, Y, test_size = 0.2)
print(x_train.shape, x_test.shape, y_train.shape, y_test.shape)
```

```
(404, 13) (102, 13) (404,) (102,)
```

In [19]:
```
x_train["PRICE"] = y_train
#x_test["PRICE"] = y_test
```

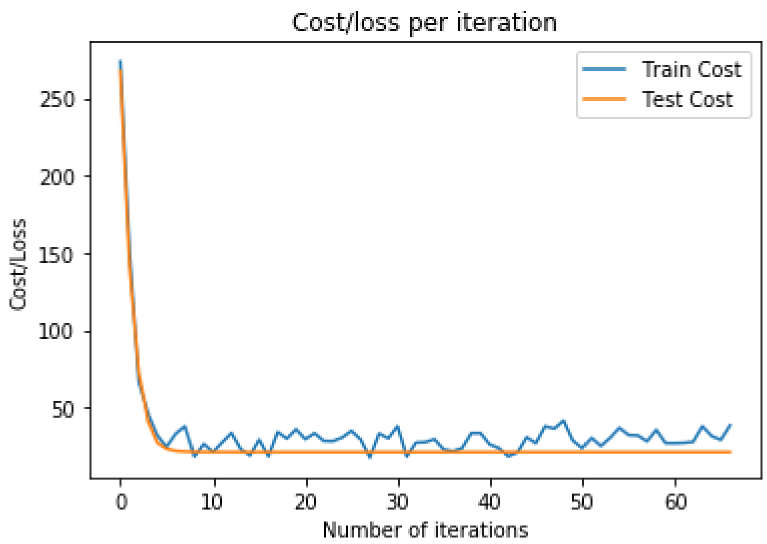
In [20]:
```
def cost_function(b, m, features, target):
    totalError = 0
    for i in range(0, len(features)):
        x = features
        y = target
        totalError += (y[:,i] - (np.dot(x[i] , m) + b)) ** 2
    return totalError / len(x)
```

In [21]:
```
# The total sum of squares (proportional to the variance of the data)i.e. ss_tot
# The sum of squares of residuals, also called the residual sum of squares i.e. ss_res
# the coefficient of determination i.e. r^2(r squared)
def r_sq_score(b, m, features, target):
    for i in range(0, len(features)):
        x = features
        y = target
        mean_y = np.mean(y)
        ss_tot = sum((y[:,i] - mean_y) ** 2)
        ss_res = sum(((y[:,i]) - (np.dot(x[i], m) + b)) ** 2)
        r2 = 1 - (ss_res / ss_tot)
    return r2
```

In [22]:
```
def gradient_decent(w0, b0, train_data, x_test, y_test, learning_rate):
    n_iter = 1000
    partial_deriv_m = 0
    partial_deriv_b = 0
    cost_train = []
    cost_test = []
    for j in range(1, n_iter):

        # Train sample
        train_sample = train_data.sample(160)
        y = np.asmatrix(train_sample["PRICE"])
        x = np.asmatrix(train_sample.drop("PRICE", axis = 1))
        for i in range(len(x)):
            partial_deriv_m += np.dot(-2*x[i].T , (y[:,i] - np.dot(x[i] , w0) + b0))
            partial_deriv_b += -2*(y[:,i] - (np.dot(x[i] , w0) + b0))

        w1 = w0 - learning_rate * partial_deriv_m
        b1 = b0 - learning_rate * partial_deriv_b

        if (w0==w1).all():
            #print("W0 are\n", w0)
            #print("\nW1 are\n", w1)
            #print("\n X are\n", x)
            #print("\n y are\n", y)
            break
        else:
            w0 = w1
            b0 = b1
            learning_rate = learning_rate/2


        error_train = cost_function(b0, w0, x, y)
        cost_train.append(error_train)
        error_test = cost_function(b0, w0, np.asmatrix(x_test), np.asmatrix(y_test))
        cost_test.append(error_test)

    #print("After {0} iteration error = {1}".format(j, error_train))
        #print("After {0} iteration error = {1}".format(j, error_test))

    return w0, b0, cost_train, cost_test
```

In [23]:
```
# Run our model
learning_rate = 0.001
w0_random = np.random.rand(13)
w0 = np.asmatrix(w0_random).T
b0 = np.random.rand()

optimal_w, optimal_b, cost_train, cost_test = gradient_decent(w0, b0, x_train, x_test, y_test, learning_rate)
print("Coefficient: {} \n y_intercept: {}".format(optimal_w, optimal_b))

# Plot train and test error in each iteration
plt.figure()
plt.plot(range(len(cost_train)), np.reshape(cost_train,[len(cost_train), 1]), label = "Train Cost")
plt.plot(range(len(cost_test)), np.reshape(cost_test, [len(cost_test), 1]), label = "Test Cost")
plt.title("Cost/loss per iteration")
plt.xlabel("Number of iterations")
plt.ylabel("Cost/Loss")
plt.legend()
plt.show()
```

```
Coefficient: [[ 3.93816973e-01]
 [-1.97545126e-03]
 [-6.49377910e-01]
 [ 4.31315251e-01]
 [-1.51670156e+00]
 [ 3.91016564e+00]
 [ 3.47536552e-01]
 [-1.03333789e+00]
 [ 4.13280615e-01]
 [ 1.68738667e-01]
 [-2.54908057e+00]
 [ 3.92544621e-01]
 [-2.32710738e+00]]
 y_intercept: [[21.28452626]]
```



Comparison :- sklearn SGD and SGD implementation in python

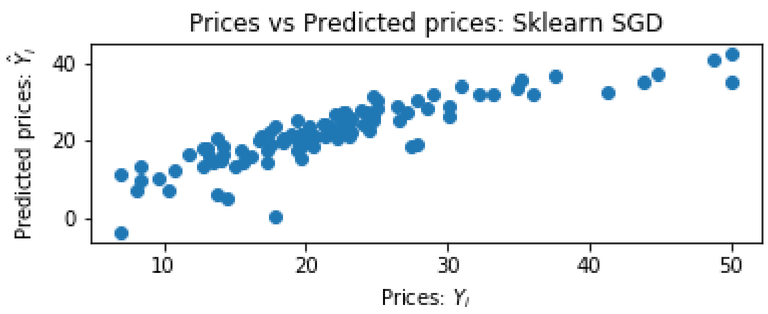
In [24]:
```
# The mean squared error
print("Mean squared error: %.2f" % mean_squared_error(Y_test, Y_pred))
# Explained variance score: 1 is perfect prediction
print("Variance score: %.2f" % r2_score(Y_test, Y_pred))
```

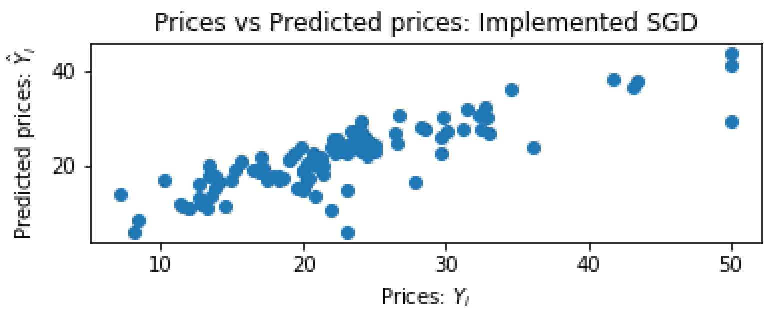
```
Mean squared error: 19.70
Variance score: 0.75
```

In [25]:
```
# Implemented SGD
# The mean squared error
error = cost_function(optimal_b, optimal_w, np.asmatrix(x_test), np.asmatrix(y_test))
print("Mean squared error: %.2f" % (error))
# Explained variance score : 1 is perfect prediction
r_squared = r_sq_score(optimal_b, optimal_w, np.asmatrix(x_test), np.asmatrix(y_test))
print("Variance score: %.2f" % r_squared)
```

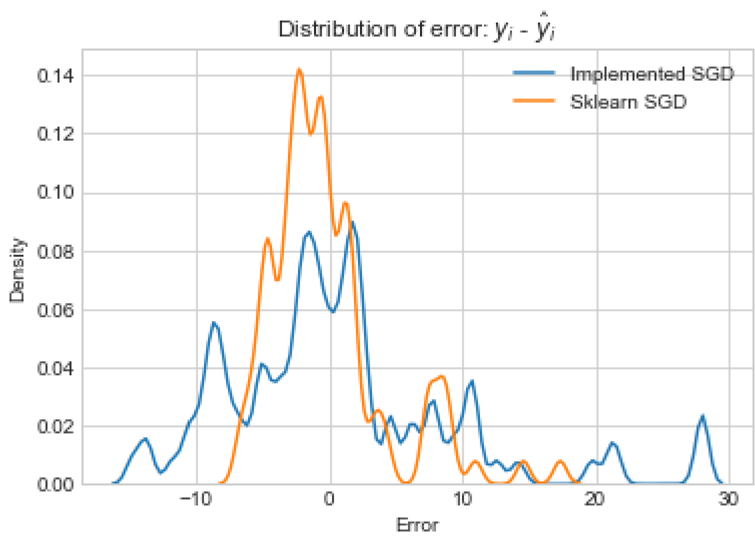
```
Mean squared error: 21.69
Variance score: -0.84
```

In [26]:
```
# Scatter plot of test vs predicted
# sklearn SGD
plt.figure(1)
plt.subplot(211)
plt.scatter(Y_test, Y_pred)
plt.xlabel("Prices: $Y_i$")
plt.ylabel("Predicted prices: $\hat{Y}_i$")
plt.title("Prices vs Predicted prices: Sklearn SGD")
plt.show()

# Implemented SGD
plt.subplot(212)
plt.scatter([y_test], [(np.dot(np.asmatrix(x_test), optimal_w) + optimal_b)])
plt.xlabel("Prices: $Y_i$")
plt.ylabel("Predicted prices: $\hat{Y}_i$")
plt.title("Prices vs Predicted prices: Implemented SGD")
plt.show()
```

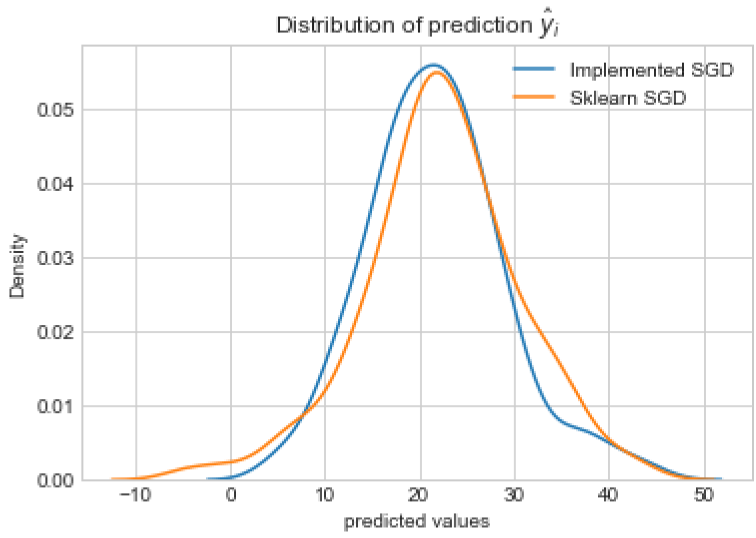




In [27]:
```
# Distribution of error
delta_y_im = np.asmatrix(y_test) - (np.dot(np.asmatrix(x_test), optimal_w) + optimal_b)
delta_y_sk = Y_test - Y_pred
import seaborn as sns;
import numpy as np;
sns.set_style('whitegrid')
sns.kdeplot(np.asarray(delta_y_im)[0], label = "Implemented SGD", bw = 0.5)
sns.kdeplot(np.array(delta_y_sk), label = "Sklearn SGD", bw = 0.5)
plt.title("Distribution of error: $y_i$ - $\hat{y}_i$")
plt.xlabel("Error")
plt.ylabel("Density")
plt.legend()
plt.show()
```



In [28]:
```
# Distribution of predicted value
sns.set_style('whitegrid')
sns.kdeplot(np.array(np.dot(np.asmatrix(x_test), optimal_w) + optimal_b).T[0], label = "Implemented SGD")
sns.kdeplot(Y_pred, label = "Sklearn SGD")
plt.title("Distribution of prediction $\hat{y}_i$")
plt.xlabel("predicted values")
plt.ylabel("Density")
plt.show()
```



Conclusions

1. MSE should be zero as this is our goal , but as observed the Regression output lines are not fitting the data up to that extent. It is just fine.

2. As Comparison between SKlearn SGD Linear Regression and SGD implemented in python are almost similar, but SKlearn SGD Linear Regression performed slightly better.

3. Both the Models performed fine but they are not the best models.