Assignment
11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

Question

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.
Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Solution

Symbol	Value	Description
$x(n)$	$(x(0)+nd)u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1^{st} term of the A.P
d	d	Common difference
$y(n)$	$x(n)*u(n)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of first p terms of the AP
b	$y(q-1)$	Sum of first q terms of the AP
c	$y(r-1)$	Sum of first r terms of the AP

TABLE 0
Variable description

$$y(n)=\frac{n+1}{2}(2x(0)+nd)u(n) \tag{1}$$

Using y(n),

$$a=\frac{p}{2}(2x(0)+(p-1)d) \tag{2}$$
$$b=\frac{q}{2}(2x(0)+(q-1)d) \tag{3}$$
$$c=\frac{r}{2}(2x(0)+(r-1)d) \tag{4}$$

The equations (2),(3) and (4) can be represented as,

$$\begin{bmatrix}x(0)\\d\end{bmatrix}=\left[\begin{array}{cc|c}1 & \frac{p-1}{2} & \frac{a}{p}\\1 & \frac{q-1}{2} & \frac{b}{q}\\1 & \frac{r-1}{2} & \frac{c}{r}\end{array}\right] \tag{5}$$

Performing row reduction,

$$\xrightarrow[R_2=R_2-R_1]{R_3=R_3-R_1}\begin{bmatrix}x(0)\\d\end{bmatrix}=\left[\begin{array}{cc|c}1 & \frac{p-1}{2} & \frac{a}{p}\\0 & \frac{q-p}{2} & \frac{b}{q}-\frac{a}{p}\\0 & \frac{r-p}{2} & \frac{c}{r}-\frac{a}{p}\end{array}\right] \tag{6}$$

$$\xrightarrow{R_2=\frac{R_2}{\frac{q-p}{2}}}\begin{bmatrix}x(0)\\d\end{bmatrix}=\left[\begin{array}{cc|c}1 & \frac{p-1}{2} & \frac{a}{p}\\0 & 1 & \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p}\\0 & \frac{r-p}{2} & \frac{c}{r}-\frac{a}{p}\end{array}\right] \tag{7}$$

$$\xrightarrow[R_1=R_1-\frac{p-1}{2}R_2]{R_3=R_3-\frac{r-p}{2}R_2}\left[\begin{array}{cc|c}1 & 0 & \frac{a}{p}-\frac{\left(\frac{b}{q}-\frac{a}{p}\right)(p-1)}{q-p}\\0 & 1 & \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p}\\0 & 0 & \left(\frac{c}{r}-\frac{a}{p}\right)-\frac{\left(\frac{b}{q}-\frac{a}{p}\right)(r-p)}{q-p}\end{array}\right] \tag{8}$$

$$\implies\left[\begin{array}{cc|c}1 & 0 & \frac{aq(q-1)-bp(p-1)}{pq(q-p)}\\0 & 1 & \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p}\\0 & 0 & \frac{\frac{a}{p}(r-q)+\frac{b}{q}(p-r)+\frac{c}{r}(q-p)}{q-p}\end{array}\right] \tag{9}$$

Effectively the third row of the matrix can be written as,

$$0+0=\frac{\frac{a}{p}(r-q)+\frac{b}{q}(p-r)+\frac{c}{r}(q-p)}{q-p} \tag{10}$$

For the equations to be consistent,

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0 \tag{11}$$

Effectively the second row of the matrix can be written as,

$$d = \left(\frac{b}{q} - \frac{a}{p}\right)\frac{2}{q-p} \tag{12}$$

Effectively the first row of the matrix can be written as,

$$x(0) = \frac{aq(q-1) - bp(p-1)}{pq(q-p)} \tag{13}$$

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{14}$$

$$X(z) = \frac{aq(q-1) - bp(p-1)}{pq(q-p)(1-z^{-1})} x(0) + \frac{2\left(\frac{b}{q} - \frac{a}{p}\right)z^{-1}}{(q-p)(1-z^{-1})^2} d \tag{15}$$

$$R.O.C\,(|z| > 1) \tag{16}$$

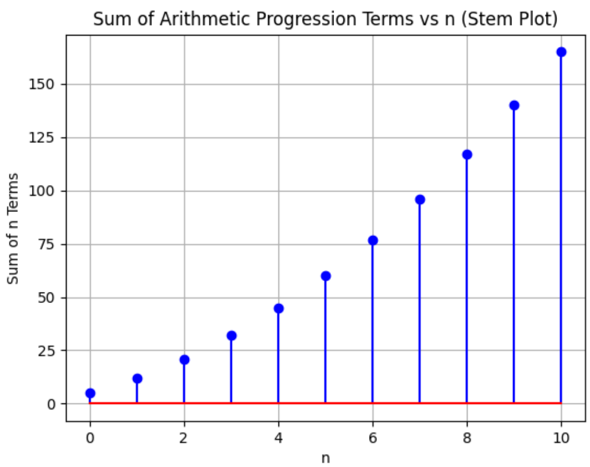

Fig. 0. Plot of x(n) *vs* n

$x(0)$	5
d	2
p	8
q	10
r	4
a	96
b	140
c	32

TABLE 0
VERIFIED VALUES