Assignment
11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

Question

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.
Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Solution

Symbol	Value	Description
$x(n)$	$(x(0)+nd)u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1^{st} term of the A.P
d	d	Common difference
$u(n)$	unit step function	$u(n) = 0 \ (n<0)$ $u(n)=1 \ (n \geq 0)$
$y(n)$	$x(n)*u(n)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of first p terms of the AP
b	$y(q-1)$	Sum of first q terms of the AP
c	$y(r-1)$	Sum of first r terms of the AP

TABLE 0
VARIABLE DESCRIPTION

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{1}$$
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{2}$$
$$X(z) = \sum_{n=-\infty}^{\infty} (x(0)+nd)u(n)z^{-n} \tag{3}$$
$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} U(z) = \frac{1}{1-z^{-1}}, |z|>1 \tag{4}$$
$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{5}$$
$$y(n) \overset{\mathcal{Z}}{\longleftrightarrow} Y(z) \tag{6}$$
$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)z^{-n} \tag{7}$$
$$y(n) = x(n) * u(n) \tag{8}$$
$$Y(z) = X(z)\,U(z) \tag{9}$$
$$Y(z) = \left(\frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right), |z|>1 \tag{10}$$

By performing Z transform on Y(z) using contour integration we get,

$$y(n) = x(0)(n+1)u(n) + d\left(\frac{n(n+1)}{2}\right)u(n) \tag{11}$$
$$y(n) = \frac{n+1}{2}(2x(0)+nd)u(n) \tag{12}$$
$$a = \frac{p}{2}(2x(0)+(p-1)d) \tag{13}$$
$$b = \frac{q}{2}(2x(0)+(q-1)d) \tag{14}$$
$$c = \frac{r}{2}(2x(0)+(r-1)d) \tag{15}$$

Back substituting values into the term $\frac{a}{p}(q-r)$ it can be rewritten as $\left(\frac{p}{2}\right)\left(\frac{1}{p}(q-r)(2x(0)+(p-1)d)\right)$

On further simplification it can be rewritten as

$$\frac{(q-r)}{2}(2x(0)-d+pd) \tag{16}$$

Assuming $2x(0)-d$ as a constant k

$$\frac{a}{p}(q-r)=\frac{(q-r)}{2}(k+pd) \tag{17}$$

$$\frac{(q-r)}{2}(k+pd)=\frac{kq+pqd-kr-prd}{2} \tag{18}$$

$$\frac{(r-p)}{2}(k+qd)=\frac{kr+qrd-kp-pqd}{2} \tag{19}$$

$$\frac{(p-q)}{2}(k+rd)=\frac{kp+prd-kq-qrd}{2} \tag{20}$$

Upon on addition of (18), (19) and (20) the total sum adds up to 0.

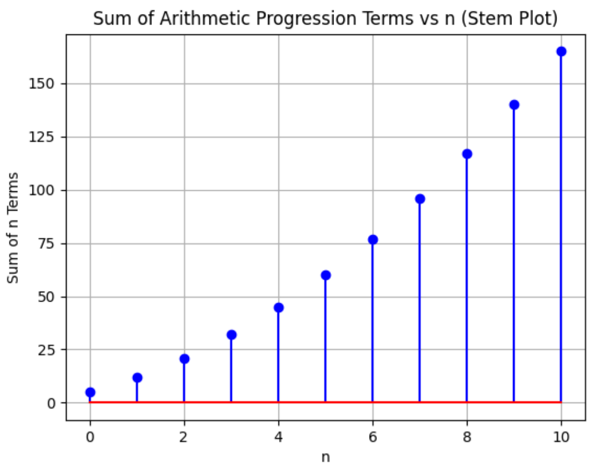

Fig. 0. Plot of x(n) *vs* n

$x(0)$	5
d	2
p	8
q	10
r	4
a	96
b	140
c	32

TABLE 0
VERIFIED VALUES