Assignment
GATE-EE-50

EE23BTECH11034 - Prabhat Kukunuri

I. Question

The Fourier transform X($j\omega$) of the signal $x(t) = \frac{t}{(1+t^2)^2}$ is ________.

(A) $\frac{\pi}{2j}\omega e^{-|\omega|}$
(B) $\frac{\pi}{2}\omega e^{-|\omega|}$
(C) $\frac{\pi}{2j}e^{-|\omega|}$
(D) $\frac{\pi}{2}e^{-|\omega|}$

Solution: Let x(t) be a signal such that,

$$x(t) \stackrel{\text{F.T}}{\longleftrightarrow} X(\omega) \tag{1}$$

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{2}$$

$$\frac{d}{d\omega}X(\omega) = \int_{t=-\infty}^{\infty} x(t)(-jt) e^{-j\omega t} dt \tag{3}$$

$$j\frac{d}{d\omega}X(\omega) = \int_{t=-\infty}^{\infty} tx(t) e^{-j\omega t} dt \tag{4}$$

$$tx(t) \stackrel{\text{F.T}}{\longleftrightarrow} j\frac{d}{d\omega}X(\omega) \tag{5}$$

This is known as the "Differentiation in frequency domain property".
From inverse Fourier transform we get,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \tag{6}$$

Replacing t by -t and multiplying 2π on both sides we get,

$$2\pi x(-t) = \int_{-\infty}^{\infty} X(\omega) e^{-j\omega t} d\omega \tag{7}$$

$$X(t) \stackrel{\text{F.T}}{\longleftrightarrow} 2\pi x(-\omega) \tag{8}$$

This is called the "Duality property of Fourier transform".
The Fourier transform of the form x(t)=$e^{-a|t|}$ is

$$x(t) \stackrel{\text{F.T}}{\longleftrightarrow} X(\omega) \tag{9}$$

$$X(\omega) = \frac{2a}{a^2+\omega^2} \tag{10}$$

Consider,

$$x(t) = e^{-|t|} \tag{11}$$

$$X(\omega) = \frac{2}{1+\omega^2} \tag{12}$$

By using differentiation property in frequency domain,

$$tx(t) \stackrel{\text{F.T}}{\longleftrightarrow} j\frac{d}{d\omega}X(\omega) \tag{13}$$

$$tx(t) \stackrel{\text{F.T}}{\longleftrightarrow} j\left[\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right)\right] \tag{14}$$

$$te^{-|t|} \stackrel{\text{F.T}}{\longleftrightarrow} \frac{-4j\omega}{(1+\omega^2)^2} \tag{15}$$

Applying duality property,

$$\frac{-4jt}{(1+t^2)^2} \stackrel{\text{F.T}}{\longleftrightarrow} 2\pi(-\omega) e^{-|-\omega|} \tag{16}$$

$$\frac{t}{(1+t^2)^2} \stackrel{\text{F.T}}{\longleftrightarrow} \frac{-2\pi\omega e^{-|\omega|}}{-4j} \tag{17}$$

$$\frac{t}{(1+t^2)^2} \stackrel{\text{F.T}}{\longleftrightarrow} \frac{\pi}{2j}\omega e^{-|\omega|} \tag{18}$$