

Assignment

11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

QUESTION

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

SOLUTION

Symbol	Value	Description
$x(n)$	$(x(0) + nd) \times u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1 st term of the A.P
d	d	Common difference
$u(n)$	unit step function	$u(n) = 0 \ (n < 0)$ $u(n) = 1 \ (n \geq 0)$

TABLE 0
 n^{th} TERM OF AN A.P

Symbol	Value	Description
$x(n)$	$\frac{n}{2}(2a + (n-1)d)$	Sum of n terms of an A.P
n	p, q, r	n^{th} term of the sequence
a	$x(0)$	first term of the sequence
d	$x(n+2) - 2x(n+1) + x(n)$	Common difference

TABLE 0
VARIABLE DESCRIPTION

$$x(n) \xrightarrow{Z} X(z) \quad (1)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (2)$$

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd)u(n)z^{-n} \quad (3)$$

$$X(z) = \sum_{n=0}^{\infty} (x(0) + nd)u(n)z^{-n} \quad (4)$$

$$X(z) = \left(x(0) \sum_{n=0}^{\infty} z^{-n} \right) + \left(d \sum_{n=0}^{\infty} nz^{-n} \right) \quad (5)$$

$$X(z) = x(0)z(z-1)^{-1} + z(z-1)^{-2}; ROC = |z| > 1 \quad (6)$$

$$a = \frac{p}{2}(2x(0) + (p-1)d) \quad (7)$$

$$b = \frac{q}{2}(2x(0) + (q-1)d) \quad (8)$$

$$c = \frac{r}{2}(2x(0) + (r-1)d) \quad (9)$$

Back substituting values into the term $\frac{a}{p}(q-r)$ it

can be rewritten as $\frac{p}{2} \times \frac{1}{p}(q-r)(2x + (p-1)d)$

On further simplification it can be rewritten as

$$\frac{(q-r)}{2}(2x(0) - d + pd) \quad (10)$$

Assuming $2x(0) - d$ as a constant k

$$\frac{a}{p}(q-r) = \frac{(q-r)}{2}(k + pd) \quad (11)$$

$$\frac{(q-r)}{2}(k + pd) = \frac{kq + pqd - rk - prd}{2} \quad (12)$$

$$\frac{(r-p)}{2}(k + qd) = \frac{kr + qrd - pk - pqd}{2} \quad (13)$$

$$\frac{(p-q)}{2}(k + rd) = \frac{kp + prd - qk - qrd}{2} \quad (14)$$

Upon on addition of (12),(13) and(14) the total sum adds up to 0.