

Assignment

GATE-EE-50

EE23BTECH11034 - Prabhat Kukunuri

I. QUESTION

The Fourier transform $X(j\omega)$ of the signal

$x(t) = \frac{t}{(1+t^2)^2}$ is _____.

- (A) $\frac{\pi}{2j}\omega e^{-|\omega|}$
- (B) $\frac{\pi}{2}\omega e^{-|\omega|}$
- (C) $\frac{\pi}{2j}e^{-|\omega|}$
- (D) $\frac{\pi}{2}e^{-|\omega|}$

Solution:

Symbol	Value	Description
$x(t)$	$\frac{t}{(1+t^2)^2}$	Signal
$X(\omega)$	$\int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt$	Fourier transform of $x(t)$

TABLE 0
VARIABLE DESCRIPTION

Let $x(t)$ be a signal such that,

$$x(t) \xleftrightarrow{\text{FT}} X(\omega) \quad (1)$$

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (2)$$

$$\frac{d}{d\omega} X(\omega) = \int_{t=-\infty}^{\infty} x(t) (-jt) e^{-j\omega t} dt \quad (3)$$

$$j \frac{d}{d\omega} X(\omega) = \int_{t=-\infty}^{\infty} tx(t) e^{-j\omega t} dt \quad (4)$$

$$tx(t) \xleftrightarrow{\text{FT}} j \frac{d}{d\omega} X(\omega) \quad (5)$$

This is known as the "Differentiation in frequency domain property".

From inverse Fourier transform we get,

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \quad (6)$$

Replacing t by $-t$ and multiplying 2π on both sides we get,

$$2\pi x(-t) = \int_{-\infty}^{\infty} X(\omega) e^{-j\omega t} d\omega \quad (7)$$

$$X(t) \xleftrightarrow{\text{FT}} 2\pi x(-\omega) \quad (8)$$

This is called the "Duality property of Fourier transform".

The Fourier transform of the form $x(t)=e^{-a|t|}$ is

$$x(t) \xleftrightarrow{\text{FT}} X(\omega) \quad (9)$$

$$X(\omega) = \frac{2a}{a^2 + \omega^2} \quad (10)$$

Consider,

$$x(t) = e^{-|t|} \quad (11)$$

$$X(\omega) = \frac{2}{1 + \omega^2} \quad (12)$$

By using differentiation property in frequency domain,

$$tx(t) \xleftrightarrow{\text{FT}} j \frac{d}{d\omega} X(\omega) \quad (13)$$

$$tx(t) \xleftrightarrow{\text{FT}} j \left[\frac{d}{d\omega} \left(\frac{2}{1 + \omega^2} \right) \right] \quad (14)$$

$$te^{-|t|} \xleftrightarrow{\text{FT}} \frac{-4j\omega}{(1 + \omega^2)^2} \quad (15)$$

Applying duality property,

$$\frac{-4jt}{(1 + t^2)^2} \xleftrightarrow{\text{FT}} 2\pi(-\omega) e^{-|\omega|} \quad (16)$$

$$\frac{t}{(1 + t^2)^2} \xleftrightarrow{\text{FT}} \frac{-2\pi\omega e^{-|\omega|}}{-4j} \quad (17)$$

$$\frac{t}{(1 + t^2)^2} \xleftrightarrow{\text{FT}} \frac{\pi}{2j} \omega e^{-|\omega|} \quad (18)$$