

Assignment

11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

QUESTION

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

SOLUTION

Symbol	Value	Description
$x(n)$	$(x(0) + nd)u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1 st term of the A.P
d	d	Common difference
$y(n)$	$x(n) * u(n)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of first p terms of the AP
b	$y(q-1)$	Sum of first q terms of the AP
c	$y(r-1)$	Sum of first r terms of the AP

TABLE 0
VARIABLE DESCRIPTION

By performing inverse Z transform on $Y(z)$ we get,

$$y(n) = \frac{n+1}{2} (2x(0) + nd) u(n) \quad (4)$$

Using $y(n)$,

$$a = \frac{p}{2} (2x(0) + (p-1)d) \quad (5)$$

$$b = \frac{q}{2} (2x(0) + (q-1)d) \quad (6)$$

$$c = \frac{r}{2} (2x(0) + (r-1)d) \quad (7)$$

Back substituting values into the term $\frac{a}{p}(q-r)$ it can

be rewritten as $\left(\frac{p}{2}\right)\left(\frac{1}{p}(q-r)(2x(0) + (p-1)d)\right)$

On further simplification it can be rewritten as

$$\frac{(q-r)}{2} (2x(0) - d + pd) \quad (8)$$

Assuming $2x(0) - d$ as a constant k

$$\frac{a}{p}(q-r) = \frac{(q-r)}{2} (k + pd) \quad (9)$$

$$\frac{(q-r)}{2} (k + pd) = \frac{kq + pqd - kr - prd}{2} \quad (10)$$

$$\frac{(r-p)}{2} (k + qd) = \frac{kr + qrd - kp - pqd}{2} \quad (11)$$

$$\frac{(p-q)}{2} (k + rd) = \frac{kp + prd - kq - qrd}{2} \quad (12)$$

Upon on addition of (10), (11) and (12) the total sum adds up to 0.

$$y(n) \xrightarrow{Z} Y(z) \quad (1)$$

$$Y(z) = X(z) U(z) \quad (2)$$

$$Y(z) = \left(\frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \right) \left(\frac{1}{1-z^{-1}} \right), |z| > 1 \quad (3)$$

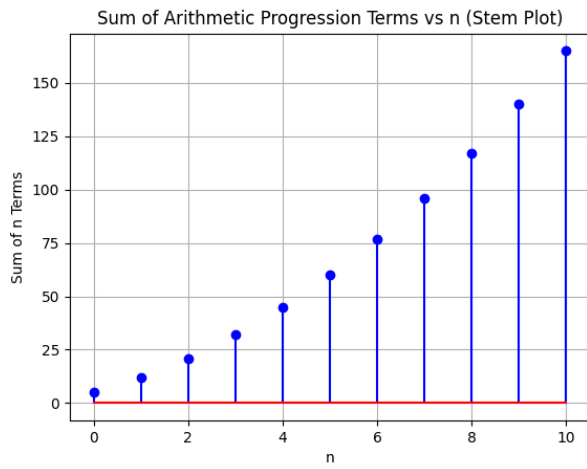


Fig. 0. Plot of $x(n)$ vs n

$x(0)$	5
d	2
p	8
q	10
r	4
a	96
b	140
c	32

TABLE 0
VERIFIED VALUES