# Assignment

# 11.9.2 - 11

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## Question

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

## Solution

| Symbol | Value | Description |
|---|---|---|
| $x(n)$ | $(x(0)+nd)u(n)$ | $n^{th}$ term of an A.P |
| $x(0)$ | $x(0)$ | $1^{st}$ term of the A.P |
| $d$ | $d$ | Common difference |
| $u(n)$ | unit step function | $u(n)=0 \quad (n<0)$ $u(n)=1 \ (n\geq 0)$ |
| $y(n)$ | $\sum_{k=0}^{n} x(k)$ | Sum of n terms of an AP |
| $a$ | $y(p-1)$ | Sum of first p terms of the AP |
| $b$ | $y(q-1)$ | Sum of first q terms of the AP |
| $c$ | $y(r-1)$ | Sum of first r terms of the AP |

TABLE 0
Variable description

$$x(n) \overset{Z}{\longleftrightarrow} X(z) \tag{1}$$

$$X(z)=\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{2}$$

$$X(z)=\sum_{n=-\infty}^{\infty} (x(0)+nd)u(n)z^{-n} \tag{3}$$

$$u(n) \overset{Z}{\longleftrightarrow} U(z)=\frac{1}{1-z^{-1}}, |z|>1 \tag{4}$$

$$X(z)=\frac{x(0)}{1-z^{-1}}+\frac{dz^{-1}}{(1-z^{-1})^2} \tag{5}$$

$$y(n) \overset{Z}{\longleftrightarrow} Y(z) \tag{6}$$

$$Y(z)=\sum_{n=-\infty}^{\infty} y(n)z^{-n} \tag{7}$$

$$y(n)=x(n)*u(n) \tag{8}$$

$$Y(z)=X(z)\,U(z) \tag{9}$$

$$Y(z)=\left(\frac{x(0)}{1-z^{-1}}+\frac{dz^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right), |z|>1 \tag{10}$$

By performing Z transform on Y($z$) using contour integration we get,

$$y(n)=x(0)(n+1)u(n)+d\left(\frac{n(n+1)}{2}\right)u(n) \tag{11}$$

$$y(n)=\frac{n+1}{2}(2x(0)+nd)u(n) \tag{12}$$

$$a=\frac{p}{2}(2x(0)+(p-1)d) \tag{13}$$

$$b=\frac{q}{2}(2x(0)+(q-1)d) \tag{14}$$

$$c=\frac{r}{2}(2x(0)+(r-1)d) \tag{15}$$

Back substituting values into the term $\frac{a}{p}(q-r)$ it can be rewritten as $\left(\frac{p}{2}\right)\left(\frac{1}{p}(q-r)(2x(0)+(p-1)d)\right)$

On further simplification it can be rewritten as

$$\frac{(q-r)}{2}(2x(0)-d+pd) \tag{16}$$

Assuming $2x(0)-d$ as a constant $k$

$$\frac{a}{p}(q-r)=\frac{(q-r)}{2}(k+pd) \tag{17}$$

$$\frac{(q-r)}{2}(k+pd)=\frac{kq+pqd-kr-prd}{2} \tag{18}$$

$$\frac{(r-p)}{2}(k+qd)=\frac{kr+qrd-kp-pqd}{2} \tag{19}$$

$$\frac{(p-q)}{2}(k+rd)=\frac{kp+prd-kq-qrd}{2} \tag{20}$$

Upon on addition of (29), (30) and (31) the total sum adds up to 0.

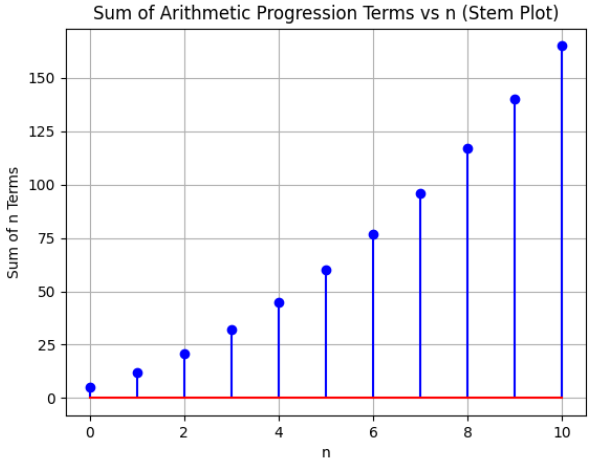


Fig. 0. Plot of x(n) *vs* n

| | |
|---|---|
| $x(0)$ | 5 |
| $d$ | 2 |
| $p$ | 8 |
| $q$ | 10 |
| $r$ | 4 |
| $a$ | 96 |
| $b$ | 140 |
| $c$ | 32 |

TABLE 0
VERIFIED VALUES