Assignment
11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

Question

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.
Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Solution

Symbol	Value	Description
$x(n)$	$(x(0)+nd)u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1^{st} term of the A.P
d	d	Common difference
$y(n)$	$x(n) * u(n)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of first p terms of the AP
b	$y(q-1)$	Sum of first q terms of the AP
c	$y(r-1)$	Sum of first r terms of the AP

TABLE 0
VARIABLE DESCRIPTION

$$y(n)=\frac{n+1}{2}(2x(0)+nd)u(n) \tag{1}$$

Using y(n),

$$a=\frac{p}{2}(2x(0)+(p-1)d) \tag{2}$$
$$b=\frac{q}{2}(2x(0)+(q-1)d) \tag{3}$$
$$c=\frac{r}{2}(2x(0)+(r-1)d) \tag{4}$$

The equations (2),(3) and (4) can be represented using an matrix equation,

$$\begin{pmatrix} p & \frac{p(p-1)}{2} \\ q & \frac{q(q-1)}{2} \\ r & \frac{r(r-1)}{2} \end{pmatrix}\begin{pmatrix} x(0) \\ d \end{pmatrix}=\begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{5}$$

Using an augmented matrix to represent the matrix equation,

$$\rightarrow\begin{pmatrix} p & \frac{p(p-1)}{2} & a \\ q & \frac{q(q-1)}{2} & b \\ r & \frac{r(r-1)}{2} & c \end{pmatrix} \tag{6}$$

$$\xrightarrow[R_1=\frac{R_1}{p},R_2=\frac{R_2}{q}]{R_3=\frac{R_3}{r}}\begin{pmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 1 & \frac{q-1}{2} & \frac{b}{q} \\ 1 & \frac{r-1}{2} & \frac{c}{r} \end{pmatrix} \tag{7}$$

$$\xrightarrow[R_2=R_2-R_1]{R_3=R_3-R_1}\begin{pmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 0 & \frac{q-p}{2} & \frac{b}{q}-\frac{a}{p} \\ 0 & \frac{r-p}{2} & \frac{c}{r}-\frac{a}{p} \end{pmatrix} \tag{8}$$

$$\xrightarrow{R_2=\frac{R_2}{\frac{q-p}{2}}} \begin{pmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 0 & 1 & \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p} \\ 0 & \frac{r-p}{2} & \frac{c}{r}-\frac{a}{p} \end{pmatrix} \tag{9}$$

$$\xrightarrow[R_1=R_1-\frac{p-1}{2}R_2]{R_3=R_3-\frac{r-p}{2}R_2} \begin{pmatrix} 1 & 0 & \frac{a}{p}-\frac{\left(\frac{b}{q}-\frac{a}{p}\right)(p-1)}{q-p} \\ 0 & 1 & \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p} \\ 0 & 0 & \left(\frac{c}{r}-\frac{a}{p}\right)-\frac{\left(\frac{b}{q}-\frac{a}{p}\right)(r-p)}{q-p} \end{pmatrix} \tag{10}$$

$$\implies \begin{pmatrix} 1 & 0 & \frac{aq(q-1)-bp(p-1)}{pq(q-p)} \\ 0 & 1 & \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p} \\ 0 & 0 & \frac{\frac{a}{p}(r-q)+\frac{b}{q}(p-r)+\frac{c}{r}(q-p)}{q-p} \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} x(0) \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{aq(q-1)-bp(p-1)}{pq(q-p)} \\ \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p} \\ \frac{\frac{a}{p}(r-q)+\frac{b}{q}(p-r)+\frac{c}{r}(q-p)}{q-p} \end{pmatrix} \tag{12}$$

$$\frac{\frac{a}{p}(r-q)+\frac{b}{q}(p-r)+\frac{c}{r}(q-p)}{q-p} = 0 \tag{13}$$

For the equations to be consistent,

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q) = 0 \tag{14}$$

$$d = \left(\frac{b}{q}-\frac{a}{p}\right)\frac{2}{q-p} \tag{15}$$

$$x(0) = \frac{aq(q-1)-bp(p-1)}{pq(q-p)} \tag{16}$$

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{17}$$

$$X(z) = \frac{aq(q-1)-bp(p-1)}{pq(q-p)(1-z^{-1})} + \frac{2\left(\frac{b}{q}-\frac{a}{p}\right)z^{-1}}{(q-p)(1-z^{-1})^2} \tag{18}$$

$$R.O.C\ (|z|>1) \tag{19}$$

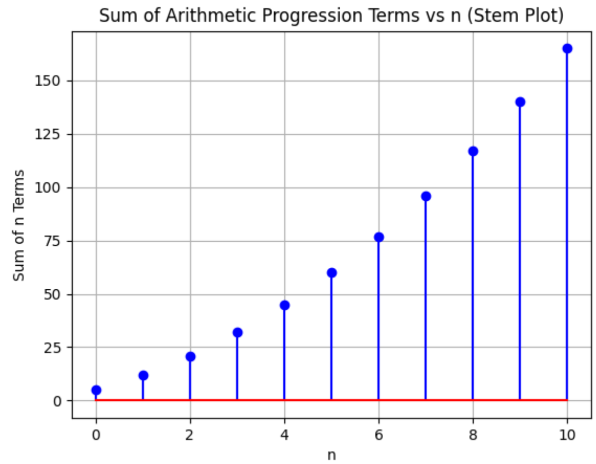

Fig. 0. Plot of x(n) *vs* n

$x(0)$	5
d	2
p	8
q	10
r	4
a	96
b	140
c	32

TABLE 0
Verified Values