Assignment
GATE-EE-50

EE23BTECH11034 - Prabhat Kukunuri

I. Question

The discrete-time Fourier transform of a signal x $[n]$ is $X(\Omega) = (1 + \cos\Omega)\, e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that

$$x_p[n] = x[n], \text{for n= 0, 1, 2} \tag{1}$$
$$= 0, \text{for n= 3, 4} \tag{2}$$

Note that $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient a_3 is ________ (Round off to 3 decimal places).

Solution:

Using Euler's form of representation of complex numbers,

$$e^{j\Omega} = \cos\Omega + j\sin\Omega \tag{3}$$

$X(\Omega)$ can be expressed as,

$$X(\Omega) = \frac{1}{2} + e^{-j\Omega} + \frac{e^{-j2\Omega}}{2} \tag{4}$$

As sampling frequency is $1Hz$ $(\omega = \Omega)$ from DTFT(discrete time fourier transform) we get,

$$X(\Omega) = \sum_{n=0}^{n=2} x(n)\, e^{-j\Omega n}, \Omega \in (-\pi, \pi) \tag{5}$$

$$\implies \sum_{n=0}^{n=2} x(n)\, e^{-j\Omega n} = \frac{1}{2} + e^{-j\Omega} + \frac{e^{-j2\Omega}}{2} \tag{6}$$

On comparing coefficients we get,

x(n)=$\left\{\underset{\uparrow}{\tfrac{1}{2}}, 1, \tfrac{1}{2}\right\}$

$$x_p(n) = \left\{\frac{1}{2}, 1, \frac{1}{2}, 0, 0\right\} \text{with period, N=5} \tag{7}$$

$$X(3) = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-\frac{j6\pi}{5}n} \tag{8}$$

$$|X(3)| = 0.038 \tag{9}$$

Symbol	Value	Description
$X(\Omega)$	$(1 + \cos\Omega)\, e^{-j\Omega}$	Frequency function
Ω	ωF_s	angular frequency
ω	$\omega \in (-\pi, \pi)$	radian frequency
F_s	$1Hz$	Sampling frequency
$X(\omega)$	$\sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$	D.T.F.T
$x(n)$	$x(n)$	Signal
$X(k)$	$\frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi}{N}kn}$	Fourier coefficient
N	5	Period of the signal

TABLE 0
VARIABLE DESCRIPTION

X(k)	Real part	Imaginary part	$\lvert X(k)\rvert$
X(0)	0.400	0	0.400
X(1)	0.081	-0.249	0.262
X(2)	-0.031	-0.225	0.038
X(3)	-0.031	0.225	0.038
X(4)	0.081	0.249	0.262

TABLE 0
VALUES OF X(K)

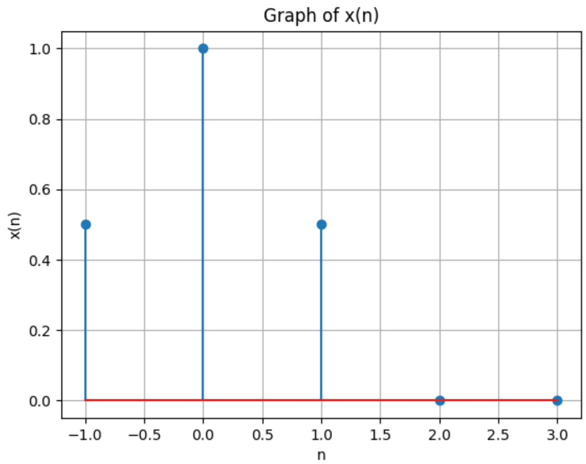

Fig. 0. Plot of x(n) *vs* n

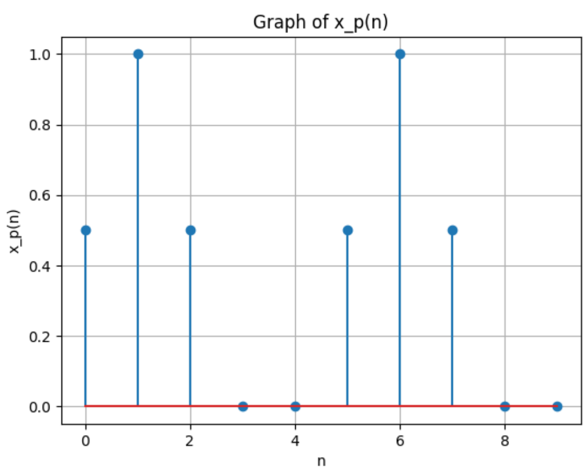

Fig. 0. Plot of $x_p(n)$ *vs* n

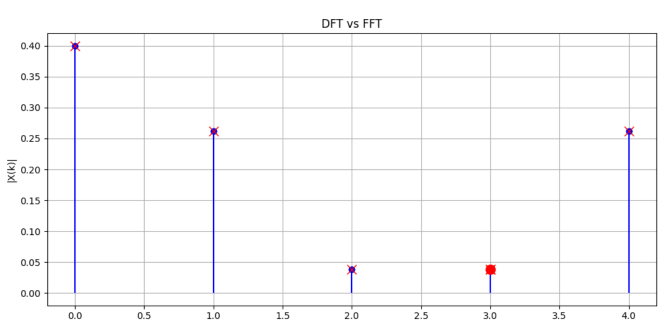

Fig. 0. Plot of $X(k)$ *vs* k