Assignment
11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

Question

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.
Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Solution

Symbol	Value	Description
$x(n)$	$(x(0)+nd)\times u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1^{st} term of the A.P
d	d	Common difference
$u(n)$	unit step function	$u(n) = 0$ $(n<0)$ $u(n)=1$ $(n\geq 0)$

TABLE 0
n^{th} term of an A.P

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{2}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0)+nd)u(n)z^{-n} \tag{3}$$

$$X(z) = \sum_{n=0}^{\infty}(x(0)+nd)u(n)z^{-n} \tag{4}$$

$$X(z) = \left(x(0)\sum_{n=0}^{\infty} z^{-n}\right)+\left(d\sum_{n=0}^{\infty} nz^{-n}\right) \tag{5}$$

$$X(z) = x(0)z(z-1)^{-1}+z(z-1)^{-2} \tag{6}$$

$$a = \frac{p}{2}(2x(0)+(p-1)d) \tag{7}$$

$$b = \frac{q}{2}(2x(0)+(q-1)d) \tag{8}$$

$$c = \frac{r}{2}(2x(0)+(r-1)d) \tag{9}$$

Symbol	Value	Description
$x(n)$	$\frac{n}{2}(2a+(n-1)d)$	Sum of n terms of an A.P
n	p,q,r	n^{th} term of the sequence
a	$x(0)$	first term of the sequence
d	$x(n+2)-2x(n+1)+x(n)$	Common difference

TABLE 0
Variable description

Back substituting values into the term $\frac{a}{p}(q-r)$ it can be rewritten as $\frac{p}{2}\times\frac{1}{p}(q-r)(2x+(p-1)d)$
On further simplification it can be rewritten as

$$\frac{(q-r)}{2}(2x(0)-d+pd) \tag{10}$$

Assuming $2x(0)-d$ as a constant k

$$\frac{a}{p}(q-r) = \frac{(q-r)}{2}(k+pd) \tag{11}$$

$$\frac{(q-r)}{2}(k+pd) = \frac{kq+pqd-rk-prd}{2} \tag{12}$$

$$\frac{(r-p)}{2}(k+qd) = \frac{kr+qrd-pk-pqd}{2} \tag{13}$$

$$\frac{(p-q)}{2}(k+rd) = \frac{kp+prd-qk-qrd}{2} \tag{14}$$

Upon on addition of (12),(13) and(14) the total sum adds up to 0.