

Assignment

11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

QUESTION

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

SOLUTION

Symbol	Value	Description
$x(n)$	$(x(0) + nd)u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1 st term of the A.P
d	d	Common difference
$y(n)$	$x(n) * u(n)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of first p terms of the AP
b	$y(q-1)$	Sum of first q terms of the AP
c	$y(r-1)$	Sum of first r terms of the AP

TABLE 0
VARIABLE DESCRIPTION

Using $y(n)$,

$$a = \frac{p}{2} (2x(0) + (p-1)d) \quad (2)$$

$$b = \frac{q}{2} (2x(0) + (q-1)d) \quad (3)$$

$$c = \frac{r}{2} (2x(0) + (r-1)d) \quad (4)$$

The equations (2),(3) and (4) can be represented as,

$$\begin{bmatrix} x(0) \\ d \end{bmatrix} = \begin{bmatrix} p & \frac{p(p-1)}{2} \\ q & \frac{q(q-1)}{2} \\ r & \frac{r(r-1)}{2} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (5)$$

$$\xrightarrow[R_1=\frac{R_1}{p}, R_2=\frac{R_2}{q}]{R_3=\frac{R_3}{r}} \begin{bmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 1 & \frac{q-1}{2} & \frac{b}{q} \\ 1 & \frac{r-1}{2} & \frac{c}{r} \end{bmatrix} \quad (6)$$

$$\xrightarrow[R_2=R_2-R_1]{R_3=R_3-R_1} \begin{bmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 0 & \frac{q-p}{2} & \frac{b}{q} - \frac{a}{p} \\ 0 & \frac{r-p}{2} & \frac{c}{r} - \frac{a}{p} \end{bmatrix} \quad (7)$$

$$\xrightarrow{R_2=\frac{R_2}{\frac{q-p}{2}}} \begin{bmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 0 & 1 & \left(\frac{b}{q} - \frac{a}{p}\right) \frac{2}{q-p} \\ 0 & \frac{r-p}{2} & \frac{c}{r} - \frac{a}{p} \end{bmatrix} \quad (8)$$

$$\xrightarrow[R_1=R_1-\frac{p-1}{2}R_2]{R_3=R_3-\frac{r-p}{2}R_2} \begin{bmatrix} 1 & 0 & \frac{a}{p} - \frac{(\frac{b}{q} - \frac{a}{p})(p-1)}{q-p} \\ 0 & 1 & \left(\frac{b}{q} - \frac{a}{p}\right) \frac{2}{q-p} \\ 0 & 0 & \left(\frac{c}{r} - \frac{a}{p}\right) - \frac{(\frac{b}{q} - \frac{a}{p})(r-p)}{q-p} \end{bmatrix} \quad (9)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{aq(q-1)-bp(p-1)}{pq(q-p)} \\ 0 & 1 & \left(\frac{b}{q} - \frac{a}{p}\right) \frac{2}{q-p} \\ 0 & 0 & \frac{\frac{a}{p}(r-q) + \frac{b}{q}(p-r) + \frac{c}{r}(q-p)}{q-p} \end{bmatrix} \quad (10)$$

Effectively the third row of the matrix can be written as,

$$0 + 0 = \frac{\frac{a}{p}(r-q) + \frac{b}{q}(p-r) + \frac{c}{r}(q-p)}{q-p} \quad (11)$$

For the equations to be consistent,

$$y(n) = \frac{n+1}{2} (2x(0) + nd) u(n) \quad (1) \quad \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0 \quad (12)$$

Effectively the second row of the matrix can be written as,

$$d = \left(\frac{b}{q} - \frac{a}{p} \right) \frac{2}{q-p} \quad (13)$$

Effectively the first row of the matrix can be written as,

$$x(0) = \frac{aq(q-1) - bp(p-1)}{pq(q-p)} \quad (14)$$

$$x(n) \xleftrightarrow{z} X(z) \quad (15)$$

$$X(z) = \frac{aq(q-1) - bp(p-1)}{pq(q-p)(1-z^{-1})} x(0) + \frac{2\left(\frac{b}{q} - \frac{a}{p}\right)z^{-1}}{(q-p)(1-z^{-1})^2} d \quad (16)$$

$$R.O.C (|z| > 1) \quad (17)$$

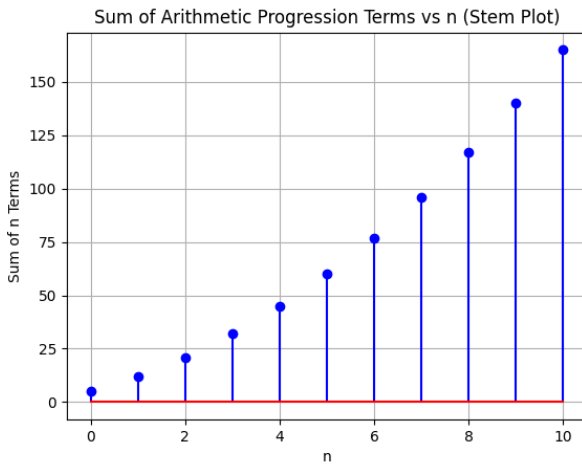


Fig. 0. Plot of $x(n)$ vs n

$x(0)$	5
d	2
p	8
q	10
r	4
a	96
b	140
c	32

TABLE 0
VERIFIED VALUES