Assignment

GATE-EE-50

EE23BTECH11034 - Prabhat Kukunuri

I. Question

The discrete-time Fourier transform of a signal x $[n]$ is $X(\Omega) = (1 + \cos\Omega)\, e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that

$$x_p[n] = x[n], \text{for n= 0, 1, 2} \tag{1}$$

$$= 0, \text{for n= 3, 4} \tag{2}$$

Note that $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient a_3 is ____________ (Round off to 3 decimal places).

Solution:

Symbol	Value	Description
$X(\Omega)$	$(1+\cos\Omega)\, e^{-j\Omega}$	Frequency function
Ω	ωF_s	angular frequency
ω	$\omega \in (-\pi, \pi)$	radian frequency
F_s	$1Hz$	Sampling frequency
$X(\omega)$	$\sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$	D.T.F.T
$x(n)$	$x(n)$	Signal
a_k	$\frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi}{N}kn}$	Fourier coefficient
N	5	Period of the signal

TABLE 0
VARIABLE DESCRIPTION

Using Euler's form of representation of complex numbers,

$$e^{j\Omega} = \cos\Omega + j\sin\Omega \tag{3}$$

$X(\Omega)$ can be expressed as,

$$X(\Omega) = \frac{1}{2} + e^{-j\Omega} + \frac{e^{-j2\Omega}}{2} \tag{4}$$

As sampling frequency is $1Hz$ $(\omega = \Omega)$ from DTFT(discrete time fourier transform) we get,

$$X(\Omega) = \sum_{n=0}^{n=2} x(n)\, e^{-j\Omega n}, \Omega \in (-\pi, \pi) \tag{5}$$

$$\implies \sum_{n=0}^{n=2} x(n)\, e^{-j\Omega n} = \frac{1}{2} + e^{-j\Omega} + \frac{e^{-j2\Omega}}{2} \tag{6}$$

On comparing coefficients we get,

$$x(n) = \begin{cases} \frac{1}{2} & \text{if n=0} \\ 1 & \text{if n=1} \\ \frac{1}{2} & \text{if n=2} \\ 0 & \text{if n} \neq \{0, 1, 2\} \end{cases} \tag{7}$$

$$x_p(n) = \left[\frac{1}{2}, 1, \frac{1}{2}, 0, 0\right] \text{with period, N=5} \tag{8}$$

$$a_3 = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-\frac{j6\pi}{5}n} \tag{9}$$

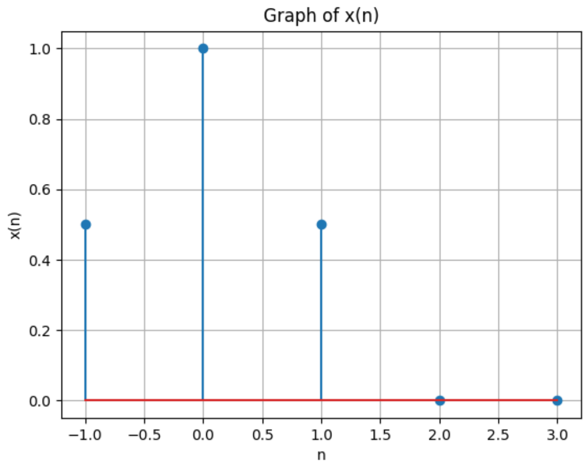

Fig. 0. Plot of x(n) *vs* n

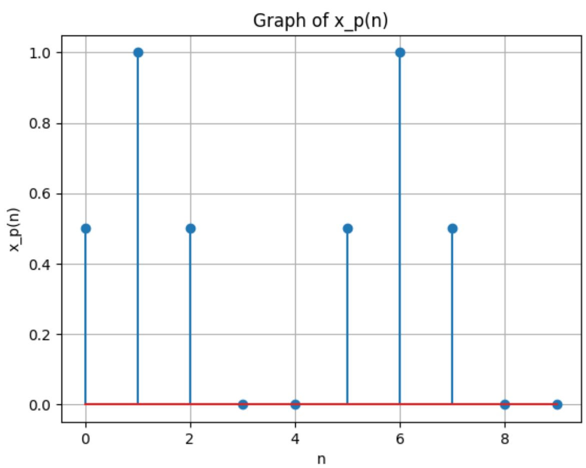

Fig. 0. Plot of $x_p(n)$ *vs* n

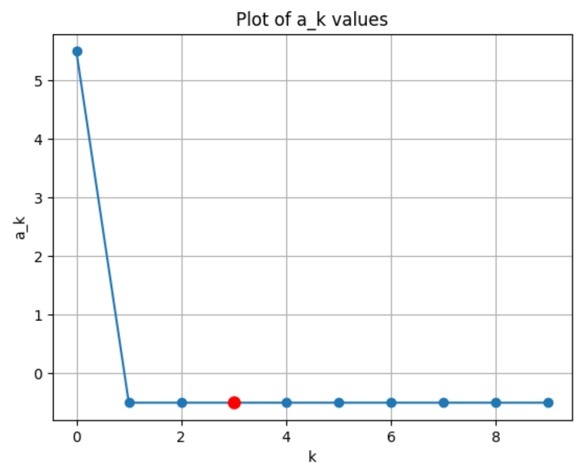

Fig. 0. Plot of $x_p(n)$ *vs* n