Assignment
11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

Question

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.
Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Solution

Symbol	Value	Description
$x(n)$	$(x(0)+nd)u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1^{st} term of the A.P
d	d	Common difference
$y(n)$	$x(n)*u(n)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of first p terms of the AP
b	$y(q-1)$	Sum of first q terms of the AP
c	$y(r-1)$	Sum of first r terms of the AP

TABLE 0
Variable description

$$y(n)=\frac{n+1}{2}(2x(0)+nd)u(n) \tag{1}$$

Using y(n),

$$a=\frac{p}{2}(2x(0)+(p-1)d) \tag{2}$$
$$b=\frac{q}{2}(2x(0)+(q-1)d) \tag{3}$$
$$c=\frac{r}{2}(2x(0)+(r-1)d) \tag{4}$$

which can be represented as,

$$p.x(0)+\frac{p(p-1)}{2}.d+a.(-1)=0 \tag{5}$$
$$q.x(0)+\frac{q(q-1)}{2}.d+b.(-1)=0 \tag{6}$$
$$r.x(0)+\frac{r(r-1)}{2}.d+c.(-1)=0 \tag{7}$$

resulting in the matrix equation,

$$\begin{pmatrix} p & \frac{p(p-1)}{2} & a \\ q & \frac{q(q-1)}{2} & b \\ r & \frac{r(r-1)}{2} & c \end{pmatrix}\mathbf{x}=0 \tag{8}$$

where,

$$\mathbf{x}=\begin{pmatrix} x(0) \\ d \\ -1 \end{pmatrix} \tag{9}$$

solving the equations (2),(3) and (4) by row reducing the matrix in (8),

$$\begin{pmatrix} p & \frac{p(p-1)}{2} & a \\ q & \frac{q(q-1)}{2} & b \\ r & \frac{r(r-1)}{2} & c \end{pmatrix} \xleftrightarrow[R_1 \leftarrow \frac{R_1}{p}, R_2 \leftarrow \frac{R_2}{q}]{R_3 \leftarrow \frac{R_3}{r}} \begin{pmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 1 & \frac{q-1}{2} & \frac{b}{q} \\ 1 & \frac{r-1}{2} & \frac{c}{r} \end{pmatrix} \tag{10}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 0 & \frac{q-p}{2} & \frac{b}{q} - \frac{a}{p} \\ 0 & \frac{r-p}{2} & \frac{c}{r} - \frac{a}{p} \end{pmatrix} \tag{11}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{\frac{q-p}{2}}} \begin{pmatrix} 1 & \frac{p-1}{2} & \frac{a}{p} \\ 0 & 1 & \left(\frac{b}{q} - \frac{a}{p}\right) \frac{2}{q-p} \\ 0 & \frac{r-p}{2} & \frac{c}{r} - \frac{a}{p} \end{pmatrix} \tag{12}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - \frac{p-1}{2} R_2]{R_3 \leftarrow R_3 - \frac{r-p}{2} R_2} \begin{pmatrix} 1 & 0 & \frac{a}{p} - \frac{\left(\frac{b}{q} - \frac{a}{p}\right)(p-1)}{q-p} \\ 0 & 1 & \left(\frac{b}{q} - \frac{a}{p}\right) \frac{2}{q-p} \\ 0 & 0 & \left(\frac{c}{r} - \frac{a}{p}\right) - \frac{\left(\frac{b}{q} - \frac{a}{p}\right)(r-p)}{q-p} \end{pmatrix} \tag{13}$$

$$\implies \begin{pmatrix} 1 & 0 & \frac{aq(q-1)-bp(p-1)}{pq(q-p)} \\ 0 & 1 & \left(\frac{b}{q} - \frac{a}{p}\right) \frac{2}{q-p} \\ 0 & 0 & \frac{\frac{a}{p}(r-q)+\frac{b}{q}(p-r)+\frac{c}{r}(q-p)}{q-p} \end{pmatrix} \tag{14}$$

After row reduction of matrix we get,

$$x(0) = \left(\frac{aq(q-1) - bp(p-1)}{pq(q-p)}\right) \tag{15}$$

$$d = \left(\frac{b}{q} - \frac{a}{p}\right) \frac{2}{q-p} \tag{16}$$

$$\frac{\frac{a}{p}(r-q) + \frac{b}{q}(p-r) + \frac{c}{r}(q-p)}{q-p} = 0 \tag{17}$$

$$\therefore \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0 \tag{18}$$

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{19}$$

$$X(z) = \frac{aq(q-1) - bp(p-1)}{pq(q-p)(1-z^{-1})} + \frac{2\left(\frac{b}{q} - \frac{a}{p}\right)z^{-1}}{(q-p)(1-z^{-1})^2} \tag{20}$$

$$R.O.C\ (|z| > 1) \tag{21}$$

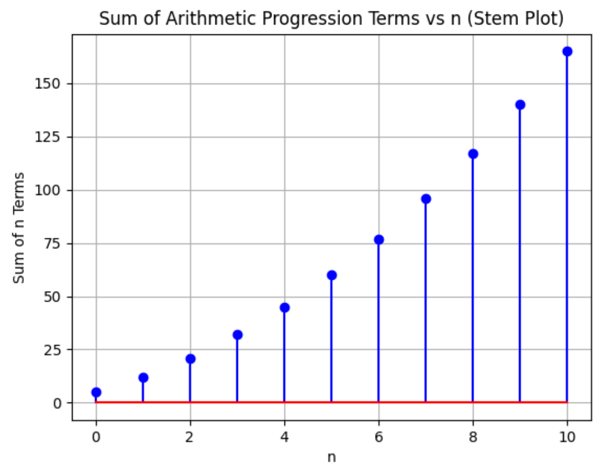

Fig. 0. Plot of x(n) *vs* n

$x(0)$	5
d	2
p	8
q	10
r	4
a	96
b	140
c	32

TABLE 0
VERIFIED VALUES