

Assignment

11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

QUESTION

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

SOLUTION

Symbol	Value	Description
$x(n)$	$(x_0 + nd)u(n)$	n^{th} term of an A.P
x_0	x_0	1 st term of the A.P
d	d	Common difference
$u(n)$	unit step function	$u(n) = 0 \ (n < 0)$ $u(n) = 1 \ (n \geq 0)$
$y(n)$	$\sum_{k=0}^n x(k)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of first p terms of the AP
b	$y(q-1)$	Sum of first q terms of the AP
c	$y(r-1)$	Sum of first r terms of the AP

TABLE 0
VARIABLE DESCRIPTION

$$x(n) \xleftrightarrow{Z} X(z) \quad (1)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (2)$$

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd)u(n)z^{-n} \quad (3)$$

$$u(n) \xleftrightarrow{Z} U(z) = \frac{1}{1-z^{-1}}, |z| > 1 \quad (4)$$

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (5)$$

$$y(n) \xleftrightarrow{Z} Y(z) \quad (6)$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)z^{-n} \quad (7)$$

$$y(n) = x(n) * u(n) \quad (8)$$

$$Y(z) = X(z) U(z) \quad (9)$$

$$Y(z) = \left(\frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \right) \left(\frac{1}{1-z^{-1}} \right) \quad (10)$$

By performing Z transform on Y(z) using contour integration

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \quad (11)$$

$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{x(0)z^{n-1}}{(1-z^{-1})^2} + \frac{dz^{n-2}}{(1-z^{-1})^3} \right) dz \quad (12)$$

For R_1 we can observe that the pole has been repeated twice.

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{x(0)z^{n+1}}{(z-1)^2} \right) \quad (14)$$

$$R_1 = x(0) (n+1) \lim_{z \rightarrow 1} (z^n) \quad (15)$$

$$R_1 = x(0) (n+1) \quad (16)$$

For R_2 we can observe that the pole has been repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{dz^{n+1}}{(z-1)^3} \right) \quad (17)$$

$$R_2 = \frac{d(n+1)}{2} \lim_{z \rightarrow 1} \frac{d}{dz} (z^n) \quad (18)$$

$$R_2 = \frac{d(n+1)(n)}{2} \lim_{z \rightarrow 1} (z^{n-1}) \quad (19)$$

$$R_2 = \frac{d(n)(n+1)}{2} \quad (20)$$

$$R = x(0)(n+1) + \frac{d(n)(n+1)}{2} \quad (21)$$

Finally,

$$y(n) = x(0)(n+1)u(n) + d \left(\frac{n(n+1)}{2} \right) u(n) \quad (22)$$

$$y(n) = \frac{n+1}{2} (2x(0) + nd) u(n) \quad (23)$$

$$a = \frac{p}{2} (2x(0) + (p-1)d) \quad (24)$$

$$b = \frac{q}{2} (2x(0) + (q-1)d) \quad (25)$$

$$c = \frac{r}{2} (2x(0) + (r-1)d) \quad (26)$$

Back substituting values into the term $\frac{a}{p}(q-r)$ it can

be rewritten as $\left(\frac{p}{2} \right) \left(\frac{1}{p} (q-r)(2x(0) + (p-1)d) \right)$

On further simplification it can be rewritten as

$$\frac{(q-r)}{2} (2x(0) - d + pd) \quad (27)$$

Assuming $2x(0) - d$ as a constant k

$$\frac{a}{p}(q-r) = \frac{(q-r)}{2} (k + pd) \quad (28)$$

$$\frac{(q-r)}{2} (k + pd) = \frac{kq + pqd - kr - prd}{2} \quad (29)$$

$$\frac{(r-p)}{2} (k + qd) = \frac{kr + qrd - kp - pqd}{2} \quad (30)$$

$$\frac{(p-q)}{2} (k + rd) = \frac{kp + prd - kq - qrd}{2} \quad (31)$$

Upon on addition of (29), (30) and (31) the total sum adds up to 0.

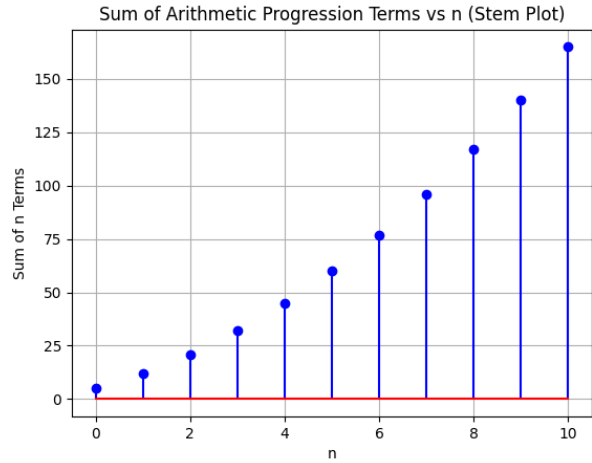


Fig. 0. Plot of $x(n)$ vs n

$x(0)$	2
d	3
p	8
q	10
r	4
a	96
b	140
c	32

TABLE 0
VERIFIED VALUES