Assignment
11.9.2 - 11

EE23BTECH11034 - Prabhat Kukunuri

Question

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.
Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Solution

Symbol	Value	Description
$x(n)$	$(x(0)+nd)\times u(n)$	n^{th} term of an A.P
$x(0)$	$x(0)$	1^{st} term of the A.P
d	d	Common difference
$u(n)$	unit step function	$u(n) = 0 \ (n<0)$ $u(n)=1 \ (n\geq 0)$
$y(n)$	$\sum_{k=0}^{n} x(k)$	Sum of n terms of an AP
a	$y(p-1)$	Sum of p terms of the AP
b	$y(q-1)$	Sum of q terms of the AP
c	$y(r-1)$	Sum of r terms of the AP

TABLE 0
Variable description

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{1}$$

$$X(z)=\sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \tag{2}$$

$$X(z)=\sum_{n=-\infty}^{\infty} (x(0)+nd)\, u(n)\, z^{-n} \tag{3}$$

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} U(z)=\frac{1}{1-z^{-1}}, |z|>1 \tag{4}$$

$$X(z)=\frac{x(0)}{1-z^{-1}}+\frac{dz^{-1}}{(1-z^{-1})^2} \tag{5}$$

$$y(n) \overset{\mathcal{Z}}{\longleftrightarrow} Y(z) \tag{6}$$

$$Y(z)=\sum_{n=-\infty}^{\infty} y(n)\, z^{-n} \tag{7}$$

$$y(n)=x(n)*u(n) \tag{8}$$

$$Y(z)=X(z)\,U(z) \tag{9}$$

$$Y(z)=\left(\frac{x(0)}{1-z^{-1}}+\frac{dz^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{10}$$

$$n^2 u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}+z^{-2}}{(1-z^{-1})^3} \tag{11}$$

By performing inverse Z transform on Y(z)

$$y(n)=x(0)(n+1)\,u(n)+d\left(\frac{n(n+1)}{2}\right)u(n) \tag{12}$$

$$y(n)=\frac{n+1}{2}(2x(0)+nd) \tag{13}$$

$$a=\frac{p}{2}(2x(0)+(p-1)d) \tag{14}$$

$$b=\frac{q}{2}(2x(0)+(q-1)d) \tag{15}$$

$$c=\frac{r}{2}(2x(0)+(r-1)d) \tag{16}$$

Back substituting values into the term $\frac{a}{p}(q-r)$ it can be rewritten as $\frac{p}{2}\times\frac{1}{p}(q-r)(2x+(p-1)d)$

On further simplification it can be rewritten as

$$\frac{(q-r)}{2}(2x(0)-d+pd) \tag{17}$$

Assuming $2x(0)-d$ as a constant k

$$\frac{a}{p}(q-r)=\frac{(q-r)}{2}(k+pd) \tag{18}$$

$$\frac{(q-r)}{2}(k+pd)=\frac{kq+pqd-rk-prd}{2} \tag{19}$$

$$\frac{(r-p)}{2}(k+qd)=\frac{kr+qrd-pk-pqd}{2} \tag{20}$$

$$\frac{(p-q)}{2}(k+rd)=\frac{kp+prd-qk-qrd}{2} \tag{21}$$

Upon on addition of (19),(20) and(21) the total sum adds up to 0.