

GATE-EE-Q14

EE23BTECH11015 - DHANUSH V NAYAK*

Question: Consider a unity-gain negative feedback system consisting of the plant $G(s)$ and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

(GATE EE 2023)

Solution:

Parameter	Description	Value
K_p	Proportional Gain	3
K_i	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$e(t)$	Error Input	$r(t) - y(t)$

TABLE I
PARAMETER TABLE

From the Fig. 1:

$$E(s) = U(s) - Y(s) \quad (1)$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \quad (2)$$

$$Y(s) = G(s)W(s) \quad (3)$$

Some results:

$$tx(t) \xleftrightarrow{\mathcal{L}} -\frac{dX(s)}{ds} \quad (4)$$

$$e^{-at}x(t) \xleftrightarrow{\mathcal{L}} X(s+a) \quad (5)$$

By using (4) and (5):

$$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+1}, \operatorname{Re}(s) > -1 \quad (6)$$

$$te^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)^2}, \operatorname{Re}(s) > -1 \quad (7)$$

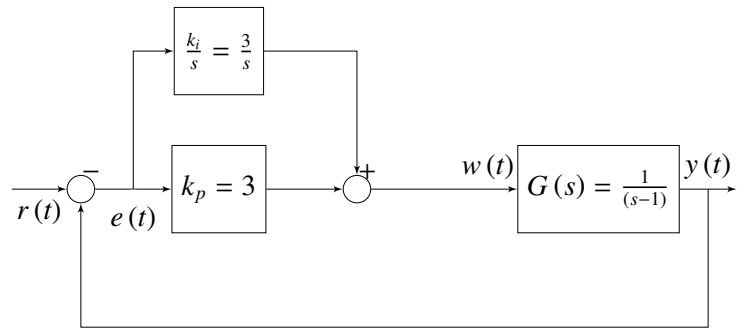


Fig. 1. Block Diagram of System

1) Plant Output:

From (1), (2) and (3):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, \operatorname{Re}(s) > -1 \quad (8)$$

Final Value Theorem:

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s) \quad (9)$$

Using (9) on Y(s):

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) \quad (10)$$

$$= 1 \quad (11)$$

Taking partial fraction of (8):

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \quad (12)$$

Using (6) and (7):

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \quad (13)$$

2) Controller Output:

From (2)

$$W(s) = \frac{3}{s} + \frac{1}{s^2} - Y(s) \left(3 + \frac{1}{s} \right) \quad (14)$$

Substituting (8)

$$W(s) = \frac{(s-1)(3s+1)}{s(s+1)^2}, \text{Re}(s) > -1 \quad (15)$$

Using (9) on $W(s)$

$$\lim_{t \rightarrow \infty} w(t) = \lim_{s \rightarrow 0} sW(s) \quad (16)$$

$$= -1 \quad (17)$$

Taking partial fraction of equation(15) :

$$W(s) = -\frac{1}{s} - \frac{4}{(s+1)^2} + \frac{4}{s+1} \quad (18)$$

Using equations (6) and (7) and taking inverse lapalace transform:

$$w(t) = -u(t) - 4te^{-t}u(t) + 4e^{-t}u(t) \quad (19)$$

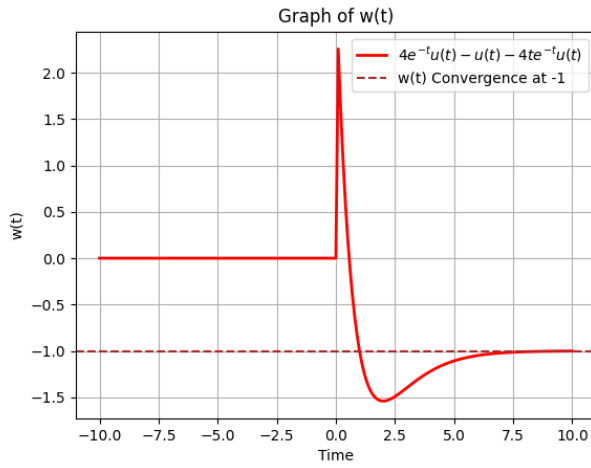


Fig. 2. $w(t)$ converges at -1.

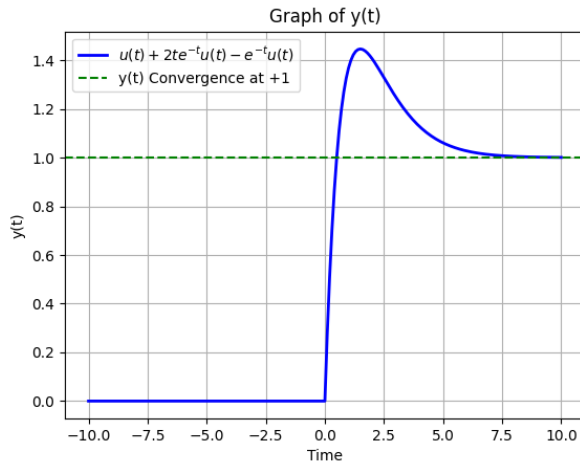


Fig. 3. $y(t)$ converges at +1