Assignment 8

PRABHAV SINGH (BT21BTECH11004)

Question : Show that for any random variable X
$[E(|X|^{k-1})]^{\frac{1}{k-1}} \leq [E(|X|^k)]^{\frac{1}{k}}$

Solution :
Consider

$$\beta_k = E|X|^k < \infty \tag{1}$$

and the random variable

$$Y = a|X|^{\frac{k-1}{2}} + |X|^{\frac{k+1}{2}} \tag{2}$$

$$\implies E(Y^2) = a^2\beta_{k-1} + 2a\beta_k + \beta_{k+1} \geq 0 \tag{3}$$

Clearly the discriminant of the quaderatic equation must be negative for this case.

$$\implies {\beta_k}^2 \leq \beta_{k-1}\beta_{k+1} \tag{4}$$

$$\implies {\beta_k}^{2k} \leq {\beta_{k-1}}^k{\beta_{k+1}}^k \tag{5}$$

This gives ${\beta_1}^2 \leq \beta_0\beta_2$, ${\beta_2}^4 \leq {\beta_1}^2{\beta_3}^2$, ${\beta_3}^6 \leq {\beta_2}^3{\beta_4}^3$ .... ${\beta_{n-1}}^{2(n-1)} \leq {\beta_{n-2}}^{n-1}{\beta_n}^{n-1}$

Also $\beta_0 = 1$ and multiplying successively

$$\implies {\beta_1}^2 \leq \beta_2, {\beta_2}^3 \leq {\beta_3}^2, {\beta_3}^4 \leq {\beta_4}^3 ...., \tag{6}$$

$$\implies {\beta_{k-1}}^k \leq {\beta_k}^{k-1} \tag{7}$$

$$\implies {\beta_{k-1}}^{1/k-1} \leq {\beta_k}^{1/k} \tag{8}$$

$$\implies \boxed{[E(|X|^{k-1})]^{\frac{1}{k-1}} \leq [E(|X|^k)]^{\frac{1}{k}}} \tag{9}$$