Assignment 14

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Question : Show that the weighted sample spectrum $S_c(\omega) = \frac{1}{2T}\left|\int_{-T}^{T} c(t)x(t)e^{-j\omega t}\right|^2$ of process $x(t)$ is the Fourier Transformation of the function

$$R_c(\tau) = \int_{-T+|\tau/2|}^{T-|\tau/2|} c\left(t+\frac{\tau}{2}\right)c\left(t-\frac{\tau}{2}\right)x\left(t+\frac{\tau}{2}\right)x\left(t-\frac{\tau}{2}\right)\,dt \tag{1}$$

Solution :
The Function

$$X_c(\omega) = \int_{-T}^{T} c(t)x(t)e^{-j\omega t}\,dt \tag{2}$$

is the Fourier Transformation of the product $c(t)x_T(t)$; where $x_T(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases}$
Hence , the function $2T.S_T(\omega) = |X_c(\omega)|^2$ is the Fourier Transformation of

$$f(t) = c(t)x_T(t).c(-t)x_T(-t) \tag{3}$$

$$= \int_{-T+|\tau/2|}^{T-|\tau/2|} c\left(t+\frac{\tau}{2}\right)c\left(t-\frac{\tau}{2}\right)x\left(t+\frac{\tau}{2}\right)x\left(t-\frac{\tau}{2}\right)\,dt \tag{4}$$