AI1110 Assignment 2

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QUESTION 13

Question:

Evaluate:

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} \, \mathrm{d}x \tag{1}$$

Solution:

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} \, \mathrm{d}x \tag{2}$$

$$\implies I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} \, \mathrm{d}x \tag{3}$$

$$\implies I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} \, \mathrm{d}x - I \tag{4}$$

$$\implies 2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} \, \mathrm{d}x \tag{5}$$

$$= \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} \, \mathrm{d}x \tag{6}$$

$$= \pi \int_0^{\pi} \tan x(\sec x - \tan x) \, \mathrm{d}x \tag{7}$$

$$= \pi \int_0^{\pi} (\tan x \sec x - \tan^2 x) \, \mathrm{d}x \tag{8}$$

$$= \pi \left[\sec x - \tan x + x\right]_0^{\pi} \tag{9}$$

$$= \pi(-1 + \pi - 1) \tag{10}$$

$$\implies 2I = \pi(\pi - 2) \tag{11}$$

$$\implies I = \frac{\pi(\pi - 2)}{2} \tag{12}$$