Assignment 9

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Outline

Question

Suppose the Conditional distribution of x given $y = n$ is binomial with parameters n and p_1. Further, Y is a binomial random variable with parameters M and p_2. Show that the distribution of x is also binomial. Find its parameters.

Solution

Using the symbols in their Standard definition and given as in question , $P(X = k|Y = n) = \binom{n}{k} {p_1}^k {q_1}^{n-k}; k = 1, 2, 3, ...n$

$$\begin{aligned} E(e^{j\omega k}|Y = n) &= \sum_{k=0}^{n} e^{j\omega k} P(X = k|Y = n) && (1) \\ &= (p_1 e^{j\omega} + q_1)^n && (2) \end{aligned}$$

Also we know that ,

$$\begin{aligned} \phi_x(\omega) &= E[e^{j\omega X}] && (3) \\ &= E(E[e^{j\omega x}|Y = n]) && (4) \\ &= \sum_{n=0}^{M} E(E[e^{j\omega x}|Y = n])P(Y = n) && (5) \end{aligned}$$

$$= \sum_{n=0}^{\infty} (p_1 e^{j\omega} + q_1)^n \binom{M}{n} {p_2}^n {q_2}^{M-n} \tag{6}$$

$$= \sum_{n=0}^{M} \binom{M}{n} (p_2(p_1 e^{j\omega} + q_1))^n {q_2}^{M-n} \tag{7}$$

$$= (p_1 p_2 e^{j\omega} + q_1 p_2 + q_2)^M \tag{8}$$

But

$$(1 - p_1 p_2) = 1 - (1 - q_1)(1 - q_2) \tag{9}$$

$$= q_1 p_2 + q_2 \tag{10}$$

Hence $\phi_\omega = (pe^{j\omega} + q)^m$ where $p = p_1 p_2$

Thus $\boxed{X \backsim Binomial(M, p_1 p_2)}$