

Assignment 9

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Question : Suppose the Conditional distribution of x given $y = n$ is binomial with parameters n and p_1 . Further, Y is a binomial random variable with parameters M and p_2 . Show that the distribution of x is also binomial. Find its parameters.

Solution :

Using the symbols in their Standard definition and given as in question ,

$$P(X = k|Y = n) = \binom{n}{k} p_1^k q_1^{n-k}; k = 1, 2, 3, \dots, n \quad (1)$$

$$\begin{aligned} E(e^{j\omega k}|Y = n) &= \sum_{k=0}^n e^{j\omega k} P(X = k|Y = n) \quad (2) \\ &= (p_1 e^{j\omega} + q_1)^n \quad (3) \end{aligned}$$

Also we know that ,

$$\phi_x(\omega) = E[e^{j\omega X}] \quad (4)$$

$$= E(E[e^{j\omega x}|Y = n]) \quad (5)$$

$$= \sum_{n=0}^M E(E[e^{j\omega x}|Y = n]) P(Y = n) \quad (6)$$

$$= \sum_{n=0}^{\infty} (p_1 e^{j\omega} + q_1)^n \binom{M}{n} p_2^n q_2^{M-n} \quad (7)$$

$$= \sum_{n=0}^M \binom{M}{n} (p_2(p_1 e^{j\omega} + q_1))^n q_2^{M-n} \quad (8)$$

$$= (p_1 p_2 e^{j\omega} + q_1 p_2 + q_2)^M \quad (9)$$

But

$$(1 - p_1 p_2) = 1 - (1 - q_1)(1 - q_2) \quad (10)$$

$$= q_1 p_2 + q_2 \quad (11)$$

Hence $\phi_\omega = (p e^{j\omega} + q)^m$ where $p = p_1 p_2$

Thus $\boxed{X \sim \text{Binomial}(M, p_1 p_2)}$