Assignment 10

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LATEX

Outline

Question

Suppose the Conditional distribution of x given $y = n$ is binomial with parameters n and p_1. Further, Y is a binomial random variable with parameters M and p_2. Show that the distribution of x is also binomial. Find its parameters.

Solution

Using the symbols in their Standard definition and given as in question ,
$P(X = k|Y = n) = \binom{n}{k} p_1{}^k q_1{}^{n-k}; k = 1, 2, 3, ...n$

$$E(e^{j\omega k}|Y = n) = \sum_{k=0}^{n} e^{j\omega k} P(X = k|Y = n) \tag{1}$$

$$= (p_1 e^{j\omega} + q_1)^n \tag{2}$$

Also we know that ,

$$\phi_x(\omega) = E[e^{j\omega X}] \tag{3}$$

$$= E(E[e^{j\omega x}|Y = n]) \tag{4}$$

$$= \sum_{n=0}^{M} E(E[e^{j\omega x}|Y = n])P(Y = n) \tag{5}$$

$$= \sum_{n=0}^{\infty} (p_1 e^{j\omega} + q_1)^n \binom{M}{n} p_2{}^n q_2{}^{M-n} \tag{6}$$

$$= \sum_{n=0}^{M} \binom{M}{n} (p_2(p_1 e^{j\omega} + q_1))^n q_2{}^{M-n} \tag{7}$$

$$= (p_1 p_2 e^{j\omega} + q_1 p_2 + q_2)^M \tag{8}$$

But

$$(1 - p_1 p_2) = 1 - (1 - q_1)(1 - q_2) \tag{9}$$

$$= q_1 p_2 + q_2 \tag{10}$$

Hence $\phi_{\omega} = (p e^{j\omega} + q)^m$ where $p = p_1 p_2$

Thus $\boxed{X \curvearrowleft Binomial(M, p_1 p_2)}$