Assignment 10

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Question : Suppose the Conditional distribution of x given $y = n$ is binomial with parameters n and p_1. Further, Y is a binomial random variable with parameters M and p_2. Show that the distribution of x is also binomial. Find its parameters.

Solution :
Using the symbols in their Standard definition and given as in question ,

$$P(X = k|Y = n) = \binom{n}{k} {p_1}^k {q_1}^{n-k}; k = 1, 2, 3, ...n \tag{1}$$

$$E(e^{j\omega k}|Y = n) = \sum_{k=0}^{n} e^{j\omega k} P(X = k|Y = n) \tag{2}$$

$$= (p_1 e^{j\omega} + q_1)^n \tag{3}$$

Also we know that ,

$$\phi_x(\omega) = E[e^{j\omega X}] \tag{4}$$

$$= E(E[e^{j\omega x}|Y = n]) \tag{5}$$

$$= \sum_{n=0}^{M} E(E[e^{j\omega x}|Y = n])P(Y = n) \tag{6}$$

$$= \sum_{n=0}^{\infty} (p_1 e^{j\omega} + q_1)^n \binom{M}{n} {p_2}^n {q_2}^{M-n} \tag{7}$$

$$= \sum_{n=0}^{M} \binom{M}{n} (p_2(p_1 e^{j\omega} + q_1))^n {q_2}^{M-n} \tag{8}$$

$$= (p_1 p_2 e^{j\omega} + q_1 p_2 + q_2)^M \tag{9}$$

But

$$(1 - p_1 p_2) = 1 - (1 - q_1)(1 - q_2) \tag{10}$$

$$= q_1 p_2 + q_2 \tag{11}$$

Hence $\phi_\omega = (pe^{j\omega} + q)^m$ where $p = p_1 p_2$
Thus $\boxed{X \backsim Binomial(M, p_1 p_2)}$