

Assignment 8

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Question

Show that for any random variable X

$$[E(|X|^{k-1})]^{\frac{1}{k-1}} \leq [E(|X|^k)]^{\frac{1}{k}}$$

Solution

Consider

$$\beta_k = E|X|^k < \infty \quad (1)$$

and the random variable

$$Y = a|X|^{\frac{k-1}{2}} + |X|^{\frac{k+1}{2}} \quad (2)$$

$$\implies E(Y^2) = a^2\beta_{k-1} + 2a\beta_k + \beta_{k+1} \geq 0 \quad (3)$$

Clearly the discriminant of the quadratic equation must be negative for this case,

$$\implies \beta_k^2 \leq \beta_{k-1}\beta_{k+1} \quad (4)$$

$$\implies \beta_k^{2k} \leq \beta_{k-1}^k \beta_{k+1}^k \quad (5)$$

This gives $\beta_1^2 \leq \beta_0 \beta_2$, $\beta_2^4 \leq \beta_1^2 \beta_3^2$, $\beta_3^6 \leq \beta_2^3 \beta_4^3$
 $\beta_{n-1}^{2(n-1)} \leq \beta_{n-2}^{n-1} \beta_n^{n-1}$

Also $\beta_0 = 1$ and multiplying successively

$$\Rightarrow \beta_1^2 \leq \beta_2, \beta_2^3 \leq \beta_3^2, \beta_3^4 \leq \beta_4^3 \dots\dots\dots, \quad (6)$$

$$\Rightarrow \beta_{k-1}^k \leq \beta_k^{k-1} \quad (7)$$

$$\Rightarrow \beta_{k-1}^{1/k-1} \leq \beta_k^{1/k} \quad (8)$$

$$\Rightarrow \boxed{[E(|X|^{k-1})]^{\frac{1}{k-1}} \leq [E(|X|^k)]^{\frac{1}{k}}} \quad (9)$$