Assignment 12

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Question : Suppose that the time between arrvals of patients in a dentist's office constitutes samples of a random variable X with density $\theta e^{-\theta x}U(x)$. The 40th patient arrived 4 hours after the first. Find the 0.95 confidence interval of the mean arrival time $\eta = \dfrac{1}{\theta}$

Solution :

The time of arrival of the 40th patient is the sum $x_1 + x_2 + ... + x_n$ of $n = 39$ RVs with exponential distribution.

We can estimate the mean $\eta = \dfrac{1}{\theta}$ of x in terms of its sample mean $X = \dfrac{240}{39} = 6.15$ minutes using Normal Approximation method.

Taking $\lambda = \eta$ and $z_{0.975}/\sqrt{39} = 0.315$

$$P\left\{ \frac{\overline{x}}{1.315} < \eta < \frac{\overline{x}}{0.685} \right\} = 0.95 \quad (1)$$

$$\implies 4.68 minutes < \eta < 8.98 minutes \quad (2)$$