

AI1110 Assignment 2

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QUESTION 13

Question:

Evaluate:

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx \quad (1)$$

Solution:

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx \quad (2)$$

$$= \int_0^{\pi} \frac{(\pi - x) \tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} dx \quad (3)$$

$$\Rightarrow I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx - I \quad (4)$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx \quad (5)$$

$$= \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} dx \quad (6)$$

$$= \pi \int_0^{\pi} \tan x (\sec x - \tan x) dx \quad (7)$$

$$= \pi \int_0^{\pi} (\tan x \sec x - \tan^2 x) dx \quad (8)$$

$$= \pi [\sec x - \tan x + x]_0^{\pi} \quad (9)$$

$$= \pi(-1 + \pi - 1) \quad (10)$$

$$\Rightarrow 2I = \pi(\pi - 2) \quad (11)$$

$$\Rightarrow I = \frac{\pi(\pi - 2)}{2} \quad (12)$$