Assignment 10

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Question : Therandom variable X has a Student t distribution $t(n)$. Show that
$E\{X^2\} = \frac{n}{n-2}$

Solution :

We know that $X^2 = \frac{ny^2}{z}$, where y is $N(0,1)$ and z is $X^2(n)$.
Hence $E\{y^2\} = 1$.
Also

$$E\{\frac{1}{Z}\} = \frac{1}{2^{n/2}\Gamma(n/2)} \int_0^\infty z^{\frac{n}{z-2}} e^{\frac{-z}{2}} dz = \frac{2^{(\frac{n}{2}-1)}\Gamma(n/2-1)}{2^{\frac{n}{2}}\Gamma(n/2-1)} \tag{1}$$

From this and independence of y and z ,it follows that

$$E\{X^2\} = n.E\{y^2\}.E\{\tfrac{1}{z}\} = \frac{n}{n-2}$$