CSC 6580 –Design and Analysis of Algorithms

Final Exam

Time: 10:15am—12:15pm. Your solution may be hand-written and scanned. But please upload a single PDF file in Canvas. You are given 15 minutes to upload your exam. **You must upload by 12:30pm**. For every minute delay after 12:30pm, you will loose 1 point.

Total points: 100

Question 1 (3+4+4+4=15 points)

(i) Write an algorithm based on Depth-First Search to determine if an undirected graph is connected.

(ii) Write an algorithm based on Depth-First Search to determine the number of connected components in an undirected graph.

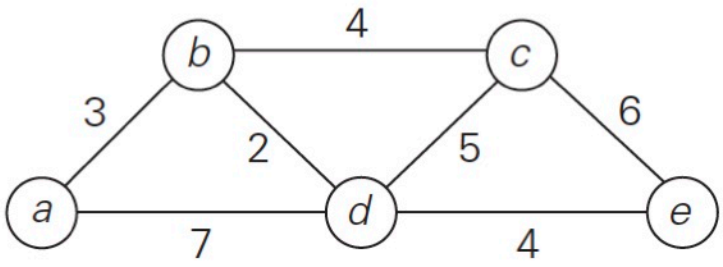
(iii) Write an algorithm based on Depth-First Search to determine if a graph has any cycle. Analyze its time complexity using big-O notation.

(iv) For a connected undirected graph G=(V, E), write an O(|V|)-time algorithm to determine if the graph has any cycle.

Question 2 (5+10=15 points)

(i) Write a linear-time algorithm for determining single-source shortest paths when all edges have an equal weight (also called *cost*)in a connected undirected graph.

(ii) Using Dijkstra's single-source shortest path algorithm, determine the shortest paths to every node from node *a* in the following graph. The label on each edge indicates its weight (also called *cost*). You must show each step of the algorithm using text and figures. Finally, write down every shortest path from node *a* and write its total cost.



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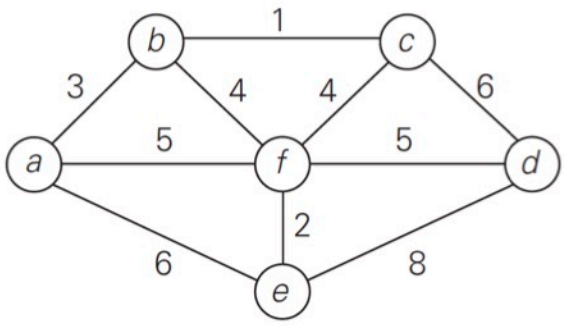
Question 3 (8+7=15 points)

(i) Using Prim's Algorithm, create a minimum spanning tree rooted at node *a* from the following graph. The label on each edge indicates its weight (also called *cost*). You must show each step of the algorithm using text and figures.



(ii) Create a minimum spanning from the above graph using Kruskal's algorithm. You must show each step of the algorithm using text and figures.

Question 4 (3+4+13=20 points)

Consider *the maximum subarray problem* that determines a subarray whose sum of the values is maximum among all subarrays in an array that may have both positive and negative numbers.

(i) Using *Divide and Conquer* method, write an algorithm to determine the maximum subarray.

(ii) Analyze the time complexity of the above *Divide and Conquer* algorithm using **Master Theorem**.

(iii) Derive the optimal substructure property of the maximum subarray problem. Write an O(n) time algorithm using dynamic programming to find the sum of the maximum subarray, where n is the size of the array.

Question 5 (15 points)

Consider a binary search tree that consists of the keys **71**, **49**, **56**, and **33** that have frequencies of searches 1500, 1000, 2000, and 500, respectively with no unsuccessful search. Construct an optimal binary search tree of these four keys using dynamic programming. You must show the required calculation and fill up the used tables and then finally draw the tree.

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Question 6 (5+10+2+3=20 points)

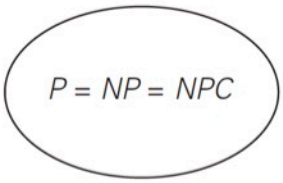
(i) The HALF-CLIQUE problem asks if a graph G=(V,E) has a clique whose size is |V|/2. The HALF-CLIQUE problem is NP-Complete. Based on this knowledge, prove that the CLIQUE problem (that asks if the graph has a clique of size k) is NP-Complete.

(ii) The QUARTER-CLIQUE problem asks if a graph G=(V,E) has a clique whose size is |V|/4. Based on the knowledge that HALF-CLIQUE is NP-Complete, prove that QUARTER-CLIQUE is NP-Complete.

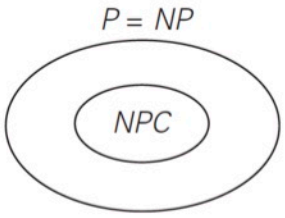
(iii) Alice has reduced problem A to the 3-SAT problem to prove the NP-hardness of A. As 3-SAT is NP-Complete, Alice claims that A is NP-Hard. Explain if Alice is correct or wrong and justify your answer.

(iv) Based on today's definitions of P, NP, and NPC (the class of NP-complete problems), which of the following diagrams contradict the current state of our knowledge?

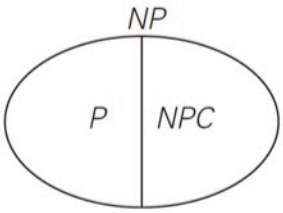
a.



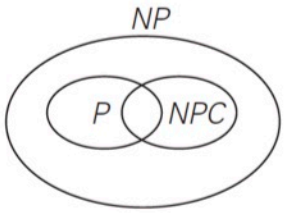
b.



c.



d.



e.