$$= \frac{\lambda^{\sum_{i=1}^{n} x_i}}{} e^{-n\lambda - a\lambda} \cdot \frac{(n+a)^{t-1}}{\Gamma(t+1)}$$

$$= \lambda^{t} e^{-\lambda(n+a)} \frac{(n+a)^{t-1}}{\Gamma(t+1)}$$

(d) Show that the conditional mean (Bayes estimate) of $\lambda$ is

$$\hat{\lambda} = \frac{t+1}{n+a}$$

$$\hat{\lambda} = E\left[\lambda^{t} e^{-\lambda(n+a)} \frac{(n+a)^{t-1}}{\Gamma(t+1)}\right]$$

$$= E\left[\underbrace{\lambda^{t} e^{-\lambda/\frac{1}{(n+a)}} \frac{(n+a)^{t-1}}{\Gamma(t+1)}}_{\Gamma \text{ distribution with parameters}}\right]$$

$$\alpha' = (t+1)$$

$$\beta' = \frac{1}{n+a}$$

The mean of $\Gamma$ distribution is $= \alpha\beta$

$$\Rightarrow \hat{\lambda} = \alpha'\beta' = \frac{(t+1)}{n+a}$$