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HW # 5

Problem 12.6-19

$X \sim P(\lambda_1)$ (Poisson)

$N \sim P(\lambda_2)$

$Y = X + N$

(a) find the distribution of Y

$$X = \frac{\lambda_1^k e^{-\lambda_1}}{k!}$$

$$N = \frac{\lambda_2^k e^{-\lambda_2}}{k!}$$

Then;

$Y \sim P(\lambda)$ where $\lambda = \lambda_1 + \lambda_2$

$$Y = \frac{(\lambda_1 + \lambda_2)^k e^{-(\lambda_1+\lambda_2)}}{k!}$$

(b) find the conditional pmf for X given Y

$$P(X|Y) = \frac{P(X \text{ and } Y)}{P(Y)}$$

$$P(X=k \mid Y=n) = \frac{P(X=k,\ Y=n)}{P(Y=n)} = \frac{P(X=k)\ P(\overset{Y}{X+N}=n)}{P(Y=n)}$$

$$= \frac{P(X=k)\ P(N=n-k)}{P(Y=n)}$$

$$= \left(\frac{\lambda_1^k e^{-\lambda_1}}{k!}\right)\left(\frac{\lambda_2^{(n-k)} e^{-\lambda_2}}{(n-k)!}\right) \div \frac{(\lambda_1+\lambda_2)^n}{n!} e^{-(\lambda_1+\lambda_2)}$$