$$= \frac{(\lambda_1 e^{-\lambda_1})^k (\lambda_2 e^{-\lambda_2})^{n-k} \cdot n!}{k! \; (n-k)! \; (\lambda_1+\lambda_2)^n \; e^{-(\lambda_1+\lambda_2)}}$$

$$= \binom{n}{k} \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \left(1 - \frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{n-k} \Longrightarrow \text{Binomial distribution}$$

(c) find the minimum mean squared error estimator of X.

Here $\theta = \lambda_1$

$$\hat{\theta}_{MMSE} = \underline{E[\theta / Y = y]}$$

$$= E[P(X/Y)]$$

$$= E\left[\binom{n}{k}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \left(1 - \frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{n-k}\right]$$

Expected value of binomial distribution with parameters n and $\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)$ is

$$= \left(n \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$$

This is the MMSE the λ_1