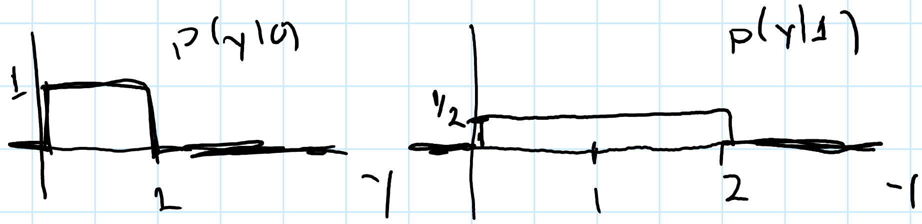
HW #1: Solutions

Monday, March 12, 2018 9:16 PM

11.2-1. a)



Likelihood ratio:

undefined for $y \notin [0,2]$

So, for threshold η:

$$\boxed{\begin{aligned} \delta(y) &= \begin{cases} 0, & 1 \le y \le 2 \\ 1, & 0 \le y < 1 \end{cases} \quad (\eta > 1/2) \\ \delta(y) &= 1, \quad 0 \le y \le 2 \quad (\eta < 1/2) \end{aligned}}$$

b) For $\eta > 1/2$:

$$P_F = Pr(y > 1 \mid H=0) = 0$$

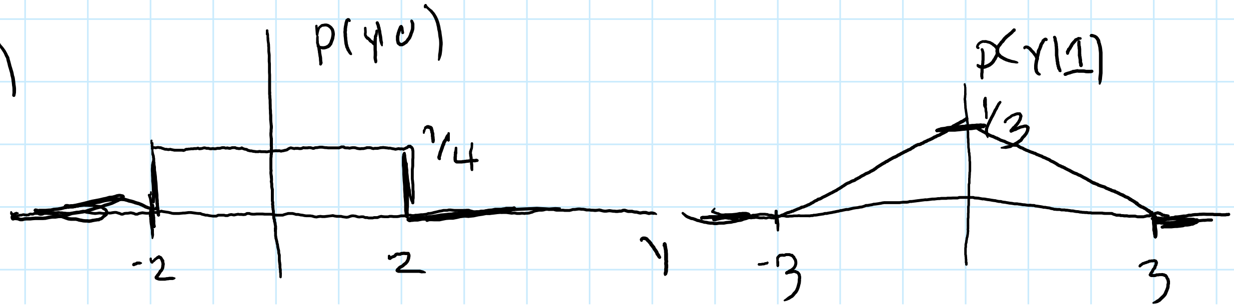
$$P_D = Pr(y > 1 \mid H=1) = 1/2$$

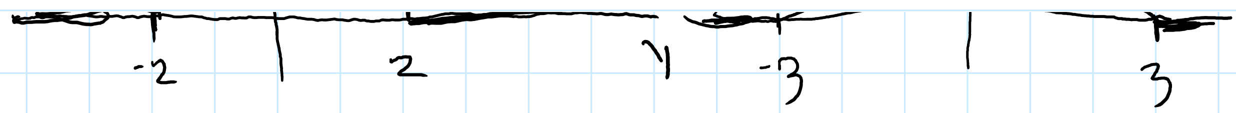
For $\eta < 1/2$:

$$P_F = 1$$

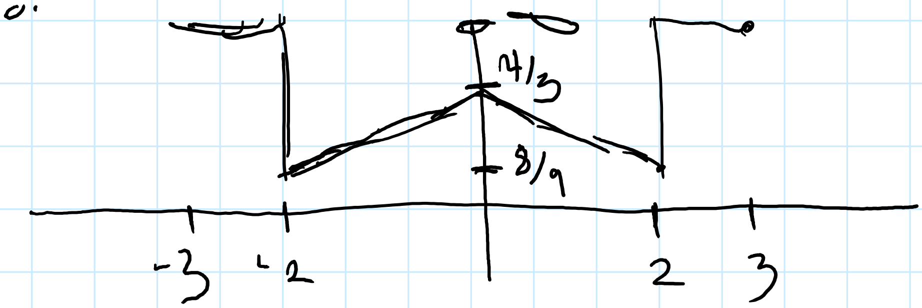
$$P_D = 1$$

11.2-2: a)





Likelihood ratio:



a) If $v = 1/4$:

$$\delta(y) = 1, \quad -3 \le y \le 3$$

$v = 2$:

$$\delta(y) = \begin{cases} 0, & -2 \le y \le 2 \\ 1, & 2 < |y| \le 3 \end{cases}$$

$v = 1$:

$$\delta(y) = \begin{cases} 0, & 1/3\, |y| \le 2 \\ 1, & 2 < |y| \le 3,\ |y| < 1/3 \end{cases}$$

$$4/3 - x \cdot 4/9 = 1$$
$$x = 4/9 \cdot \frac{3}{4}$$
$$= 1/3$$

b) $v = 1$: $P_F = 1$, $P_D = 1$

$v = 2$: $P_F = 0$, $P_D = 1/9 \cdot 3 = 1/3$

$v = 3$: $P_F = 1/4 \cdot 2/3 = 1/6$

$$P_D = 1/9 \cdot 3 + 2\int_0^{1/3} (1/3 - y/9)\, dy$$

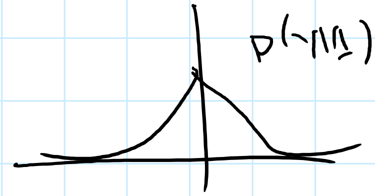
$$= 1/3 + 2\left(1/9 - \frac{1}{9}\int_0^{1/3} y/9\, dy\right)$$

$$= 1/3 + \frac{2}{9} - \frac{1}{9}\left|\frac{y^2}{18}\right|_0^{1/3} = \frac{5}{9} - \frac{1}{9} \cdot \frac{1/9}{18}$$

$$= \frac{5}{9} - \frac{1}{1458} \approx \boxed{0.554}$$

$= \frac{5}{9} - \frac{1}{1458} = \boxed{0.554}$

11.2-6: a)



b) $\ell(r) = \frac{\sqrt{2\pi}}{2} \exp\left(-|r| + \frac{1}{2} r^2\right)$

$L(r) = \log\left(\frac{\sqrt{2\pi}}{2}\right) + \frac{1}{2} r^2 - |r|$

c) LRT:

$$\log\left(\frac{\sqrt{2\pi}}{2}\right) + \frac{1}{2} r^2 - |r| > \gamma$$

$$\frac{1}{2}|r|^2 - |r| > \gamma + \log\left(\frac{2}{\sqrt{2\pi}}\right)$$

$$\boxed{|r|\left(\tfrac{1}{2} - |r|\right) > \gamma + \log\left(\frac{2}{\sqrt{2\pi}}\right)}$$

11.2-13: LRT:

$$\frac{\frac{1}{(2\pi)^{n/2}|R_1|} \exp\left(-\frac{1}{2} x^T R_1^{-1} x\right)}{\frac{1}{(2\pi)^{n/2}|R_0|} \exp\left(-\frac{1}{2} x^T R_0^{-1} x\right)} \gtrless \gamma$$

$$\Rightarrow \log\left(\frac{|R_1|}{|R_0|}\right) - \frac{1}{2} x^T \left(R_1^{-1} - R_0^{-1}\right) x \gtrless \eta$$

$$\Rightarrow \log\left(\left|R_0^{-1/2} R_1 R_0^{-1/2}\right|\right) - \frac{1}{2} x^T \left(R_1^{-1} - R_0^{-1}\right) x \gtrless \eta$$

$$\Rightarrow \log(|S|) - \frac{1}{2} x^T R_0^{-1/2} R_0^{1/2} \left(R_1^{-1} - R_0^{-1}\right) R_0^{1/2} R_0^{-1/2} x \gtrless \eta$$

$$\Rightarrow \log(|S|) - \frac{1}{2} x^T R_0^{-1/2} R_0^{1/2} (R_1^{-1} - R_0^{-1}) R_0^{1/2} R_0^{-1/2} x \gtrless M$$

$$\Rightarrow \boxed{\log(|S|) - \frac{1}{2} x^T R_0^{-1/2} (S^{-1} - I) R_0^{-1/2} x \gtrless M}$$