

(2)

① Compute the mean and mean squared error for your MMSE estimator. Is the estimate unbiased?

$$\text{Mean of } \hat{\theta}_{\text{MMSE}} : E\left[n \frac{\lambda_1}{\lambda_1 + \lambda_2}\right]$$

$$\frac{n \lambda_1}{\lambda_1 + \lambda_2}$$

Mean squared error:

$$\begin{aligned} & E[(\hat{\theta} - \theta)]^2 \\ &= E\left[\left(\frac{n \lambda_1}{\lambda_1 + \lambda_2}\right) - \lambda_1\right]^2 \\ &= E[\hat{\theta}]^2 + E[\theta]^2 - 2E[\theta \hat{\theta}] \\ &= E\left[\frac{n \lambda_1}{\lambda_1 + \lambda_2}\right]^2 + E[\lambda_1]^2 - 2E\left[\frac{n \lambda_1}{\lambda_1 + \lambda_2} \cdot \lambda_1\right] \\ &= \left(\frac{n \lambda_1}{\lambda_1 + \lambda_2}\right)^2 + \lambda_1^2 - \frac{2n \lambda_1^2}{\lambda_1 + \lambda_2} \end{aligned}$$

This is unbiased estimator because
 $E[\hat{\theta}] \neq \lambda_1$