③

$$f_x(x) = \int f_{x|\lambda}(x|\lambda) \cdot f_\lambda(\lambda)$$

$$= \int \frac{\lambda^x}{x!} e^{-\lambda} \cdot a e^{-a\lambda} \, d\lambda$$

$$= \int \frac{a\lambda^x}{x!} e^{-(\lambda + a\lambda)} \, d\lambda = \frac{a}{x!} \int \lambda^x e^{-\lambda(1+a)} \, d\lambda$$

$$= \frac{a}{x!} \left(-\lambda^{x+1} \left((a+1)\lambda \right)^{-x-1} \Gamma(x+1, a\lambda + \lambda) \right)$$

$$= \frac{a}{\prod_{i=1}^{n} x_i!} \left\{ \prod_{i=1}^{n} -\lambda^{x+1} \left((a+1)\lambda \right)^{-x-1} \Gamma(x+1, a\lambda + \lambda) \right)$$

$$= \frac{a}{\prod_{i=1}^{n} x_i!} \left\{ -\lambda^{\overset{t}{\sum_{i=1}^{n} x_i} + 1} \left((a+1)\lambda \right)^{-\overset{t}{\sum_{i=1}^{n} x_i} - 1} \Gamma\left(\overset{t}{\sum_{i=1}^{n} x_i} + 1, a\lambda + \lambda \right) \right)$$

ⓒ Show that conditional (posterior) density for λ given x is ; $f_{\lambda|x}(\lambda|x) = e^{-(n+a)\lambda} \lambda^t \dfrac{(n+a)^{t-1}}{\Gamma(t+1)}$

$$f_{\lambda|x}(\lambda|x) = \frac{f_{x|\lambda}(x|\lambda) \cdot f_\lambda(\lambda)}{f_x(x)}$$

$$= \prod_{i=1}^{n} \left(\frac{\lambda^x e^{-\lambda}}{x!} \right) \left(a e^{-a\lambda} \right) \div \frac{a}{\prod_{i=1}^{n} x_i!} \frac{\Gamma(t+1)}{(n+a)^{t-1}}$$

$$= \left(\frac{\lambda^{\sum_{i=1}^{n} x_i}}{\cancel{\prod_{i=1}^{n} x_i!}} a e^{-n\lambda} \cdot e^{-a\lambda} \right) \times \frac{\cancel{\prod_{i=1}^{n} x_i!} \, (n+a)^{t-1}}{\cancel{a}\, \Gamma(t+1)}$$