12.6.21

Problem 21 (1)–(d) $X_1, X_2 \dots X_n$ iid samp from Poisson distribution.

$$f_{x|\lambda}(x|\lambda) = \frac{\lambda^x}{x!} e^{-\lambda} \qquad x \in Z^+ \quad \lambda \geq 0$$

prior on λ is exponential.

$$f_\lambda(\lambda) = a e^{-a\lambda} \qquad \lambda \geq 0 \quad ; a > 0$$

(a) Show that $t = \sum_{i=1}^n x_i$ is sufficient for λ

$$p(x^n|\lambda) = \prod_{i=1}^n \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$= e^{-n\lambda} \frac{\lambda^{\sum_{i=1}^n x_i}}{\prod_{i=1}^n x_i!}$$

$$= \underbrace{\frac{\lambda}{\prod_{i=1}^n x_i!}}_{h(x)} \cdot \underbrace{e^{-n\lambda} \lambda^{\sum_{i=1}^n x_i}}_{g(T(x), \lambda)}$$

$h(x)$ This does not depend on λ

This depends on λ through $\sum_{i=1}^n x_i$ t

$\Rightarrow t$ is sufficient for λ

(b) Show that the marginal density for X is

$$f_x(x) = \frac{a}{\prod_{i=1}^n x_i!} \frac{\Gamma(t+1)}{(n+a)^{t-1}}$$