In[ ]:=
```
<< xAct`xPlain`;
<< xAct`xTensor`;
<< xAct`xPerm`;
<< xAct`xTras`;
<< xAct`xPand`;
```

```
------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

CopyRight (C) 2003-2020, Jose M. Martin-Garcia, under the General Public License.

Connecting to external MinGW executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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------------------------------------------------------------

Package xAct`xPlain` version 0.0.0-developer, {2025, 3, 14}

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------------------------------------------------------------

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  it under certain conditions. See the General Public License for details.

------------------------------------------------------------
```

RunProcess: Program basename not found.  Check Environment["PATH"].

StringDelete: String or list of strings expected at position 1 in StringDelete[$Failed[StandardOutput],
].

StringJoin: String expected at position 2 in system–tests– <> StringDelete[$Failed[StandardOutput],
].

```
------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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------------------------------------------------------------

------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

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------------------------------------------------------------
```

```
Connecting to external MinGW executable...

Connection established.

------------------------------------------------------------

Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.

** Variable $PrePrint assigned value ScreenDollarIndices

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

CopyRight (C) 2006-2020, J. M. Martin-Garcia,
  D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.

** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

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  and Jose M. Martin-Garcia, under the General Public License.

------------------------------------------------------------

Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}

CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.

------------------------------------------------------------

Package xAct`xTras`  version 1.4.2, {2014, 10, 30}

CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.

** Variable $CovDFormat changed from Postfix to Prefix

** Option CurvatureRelations of DefCovD changed from False to True

------------------------------------------------------------

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------------------------------------------------------------

------------------------------------------------------------

Package xAct`xPand`  version 0.4.4, {2025, 4, 1}

CopyRight (C) 2012-2025, Cyril Pitrou,
  Xavier Roy and Obinna Umeh under the General Public License.

------------------------------------------------------------
```

```
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  it under certain conditions. See the General Public License for details.
------------------------------------------------------------
```

In[◦]:=
```
ClearAll["Global`*"]
DefManifold[M4, 4, IndexRange[{c, z}]];
DefMetric[-1, G[-c, -d], CD, SymCovDQ → True];
```

```
** DefManifold: Defining manifold M4.
** DefVBundle: Defining vbundle TangentM4.
** DefTensor: Defining symmetric metric tensor G[-c, -d].
** DefTensor: Defining antisymmetric tensor epsilonG[-c, -d, -e, -f].
** DefTensor: Defining tetrametric TetraG[-c, -d, -e, -f].
** DefTensor: Defining tetrametric TetraG†[-c, -d, -e, -f].
** DefCovD: Defining covariant derivative CD[-c].
** DefTensor: Defining vanishing torsion tensor TorsionCD[c, -d, -e].
** DefTensor: Defining symmetric Christoffel tensor ChristoffelCD[c, -d, -e].
** DefTensor: Defining Riemann tensor RiemannCD[-c, -d, -e, -f].
** DefTensor: Defining symmetric Ricci tensor RicciCD[-c, -d].
** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
** DefTensor: Defining Ricci scalar RicciScalarCD[].
** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
** DefTensor: Defining symmetric Einstein tensor EinsteinCD[-c, -d].
** DefTensor: Defining Weyl tensor WeylCD[-c, -d, -e, -f].
** DefTensor: Defining symmetric TFRicci tensor TFRicciCD[-c, -d].
** DefTensor: Defining Kretschmann scalar KretschmannCD[].
** DefCovD:  Computing RiemannToWeylRules for dim 4
** DefCovD:  Computing RicciToTFRicci for dim 4
** DefCovD:  Computing RicciToEinsteinRules for dim 4
** DefTensor: Defining symmetrized Riemann tensor SymRiemannCD[-c, -d, -e, -f].
** DefTensor: Defining symmetric Schouten tensor SchoutenCD[-c, -d].
** DefTensor: Defining symmetric cosmological Schouten tensor SchoutenCCCD[LI[_], -c, -d].
** DefTensor: Defining symmetric cosmological Einstein tensor EinsteinCCCD[LI[_], -c, -d].
** DefCovD: Defining covariant derivative CD[-c]. to be symmetrizable
** DefTensor: Defining weight +2 density DetG[]. Determinant.
** DefParameter: Defining parameter PerturbationParameterG.
** DefTensor: Defining tensor PerturbationG[LI[order], -c, -d].
```

In[•]:=
```
DefTensor[B[-c], M4]
DefConstantSymbol[E]
DefConstantSymbol[σ]
DefConstantSymbol[ν]
DefConstantSymbol[μ]

DefConstantSymbol[ϕ0, PrintAs → "ϕ0" ]

DefConstantSymbol[γ]
DefConstantSymbol[β]
DefConstantSymbol[α]

DefTensor[φ[], M4]
DefTensor[L[], M4]
```

```
** DefTensor: Defining tensor B[-c].
** DefConstantSymbol: Defining constant symbol E.
** DefConstantSymbol: Defining constant symbol σ.
** DefConstantSymbol: Defining constant symbol ν.
** DefConstantSymbol: Defining constant symbol μ.
** DefConstantSymbol: Defining constant symbol ϕ0.
** DefConstantSymbol: Defining constant symbol γ.
** DefConstantSymbol: Defining constant symbol β.
** DefConstantSymbol: Defining constant symbol α.
** DefTensor: Defining tensor φ[].
** DefTensor: Defining tensor L[].
```

In[•]:=
```
$PrePrint = ScreenDollarIndices;
```

In[•]:=
```
RicciScalarCD[]
φ[]
ϕ0
```

Out[•]=

R[∇]

Out[•]=

$\varphi$

Out[•]=

$\phi_0$

In[•]:=
```
SetSlicing[G, Nn, Hh, cd, {"|", "∂"}, "FLFlat"]
```

```
** DefTensor: Defining tensor Nn[z$21823].
** DefTensor: Defining symmetric metric tensor Hh[-z$21823, -z$21824].
** DefTensor: Defining antisymmetric tensor epsilonHh[-c, -d, -e].
** DefTensor: Defining tetrametric TetraHh[-c, -d, -e, -f].
** DefTensor: Defining tetrametric TetraHh†[-c, -d, -e, -f].
** DefCovD: Defining covariant derivative cd[-z$21823].
** DefTensor: Defining vanishing torsion tensor Torsioncd[c, -d, -e].
```

```
** DefTensor: Defining symmetric Christoffel tensor Christoffelcd[c, -d, -e].
** DefTensor: Defining Riemann tensor Riemanncd[-c, -d, -e, -f].
** DefTensor: Defining symmetric Ricci tensor Riccicd[-c, -d].
** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
** DefTensor: Defining Ricci scalar RicciScalarcd[].
** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
** DefTensor: Defining symmetric Einstein tensor Einsteincd[-c, -d].
** DefTensor: Defining vanishing Weyl tensor Weylcd[-c, -d, -e, -f].
** DefTensor: Defining symmetric TFRicci tensor TFRiccicd[-c, -d].
** DefTensor: Defining Kretschmann scalar Kretschmanncd[].
** DefCovD:  Computing RiemannToWeylRules for dim 3
** DefCovD:  Computing RicciToTFRicci for dim 3
** DefCovD:  Computing RicciToEinsteinRules for dim 3
** DefTensor: Defining symmetrized Riemann tensor SymRiemanncd[-c, -d, -e, -f].
** DefTensor: Defining symmetric Schouten tensor Schoutencd[-c, -d].
** DefTensor: Defining symmetric cosmological Schouten tensor SchoutenCCcd[LI[_], -c, -d].
** DefTensor: Defining symmetric cosmological Einstein tensor EinsteinCCcd[LI[_], -c, -d].
** DefTensor: Defining weight +2 density DetHh[]. Determinant.
** DefTensor: Defining extrinsic curvature tensor ExtrinsicKHh[c, d]. Associated to vector Nn
** DefTensor: Defining acceleration vector AccelerationNn[c]. Associated to vector Nn
** DefInertHead: Defining projector inert-head ProjectorHh.
** DefParameter: Defining parameter PerturbationParameterHh.
** DefTensor: Defining tensor PerturbationHh[LI[order], -c, -d].
   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for Hh.
   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for Hh.
   Rules {1, 2} have been declared as UpValues for Nn.
   Rules {1, 2, 3, 4} have been declared as UpValues for Nn.
   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for Nn.
** DefTensor: Defining tensor
 𝒩0Hh[LI[xAct`xPand`Private`p$23385], LI[xAct`xPand`Private`q$23385]].
** DefTensor: Defining tensor 𝒩iHh[c].
** DefTensor: Defining tensor 𝒩Hh[-c].
** DefTensor: Defining tensor d𝒩Hh[LI[order], -c].
** DefTensor: Defining tensor
 aHh[LI[xAct`xPand`Private`p$23404], LI[xAct`xPand`Private`q$23404]].
** DefTensor: Defining tensor
 HHh[LI[xAct`xPand`Private`p$23410], LI[xAct`xPand`Private`q$23410]].
** DefTensor: Defining symmetric metric tensor GaHh2[-z$23417, -z$23418].
** DefTensor: Defining inverse metric tensor InvGaHh2[z$23417, z$23418]. Metric is frozen!
** DefTensor: Defining antisymmetric tensor epsilonGaHh2[-c, -d, -e, -f].
```

```
** DefTensor: Defining tetrametric TetraGaHh2[-c, -d, -e, -f].

** DefTensor: Defining tetrametric TetraGaHh2†[-c, -d, -e, -f].

** DefCovD: Defining covariant derivative CDaHh2[-z$23417].

** DefTensor: Defining vanishing torsion tensor TorsionCDaHh2[c, -d, -e].

** DefTensor: Defining symmetric Christoffel tensor ChristoffelCDaHh2[c, -d, -e].

** DefTensor: Defining Riemann tensor RiemannDownCDaHh2[-c, -d, -e, -f].

** DefTensor: Defining Riemann tensor
 RiemannCDaHh2[-c, -d, -e, f]. Antisymmetric only in the first pair.

** DefTensor: Defining symmetric Ricci tensor RicciCDaHh2[-c, -d].

** DefCovD:  Contractions of Riemann automatically replaced by Ricci.

** DefTensor: Defining Ricci scalar RicciScalarCDaHh2[].

** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.

** DefTensor: Defining symmetric Einstein tensor EinsteinCDaHh2[-c, -d].

** MakeRule: Potential problems moving indices on the LHS.

** DefTensor: Defining Weyl tensor WeylCDaHh2[-c, -d, -e, -f].

** DefTensor: Defining symmetric TFRicci tensor TFRicciCDaHh2[-c, -d].

** DefTensor: Defining Kretschmann scalar KretschmannCDaHh2[].

** DefCovD:  Computing RiemannToWeylRules for dim 4

** DefCovD:  Computing RicciToEinsteinRules for dim 4

** DefTensor: Defining symmetrized Riemann tensor SymRiemannCDaHh2[-c, -d, -e, -f].

** DefTensor: Defining symmetric Schouten tensor SchoutenCDaHh2[-c, -d].

** DefTensor: Defining
 symmetric cosmological Schouten tensor SchoutenCCCDaHh2[LI[_], -c, -d].

** DefTensor: Defining
 symmetric cosmological Einstein tensor EinsteinCCCDaHh2[LI[_], -c, -d].

** MakeRule: Potential problems moving indices on the LHS.

** MakeRule: Potential problems moving indices on the LHS.

** DefTensor: Defining weight +2 density DetGaHh2[]. Determinant.

** DefParameter: Defining parameter PerturbationParameterGaHh2.

** DefTensor: Defining tensor PerturbationGaHh2[LI[order], -c, -d].

   Rules {1, 2, 3, 4, 5, 6, 7, 8} have been declared as UpValues for G.

   Rules {1} have been declared as UpValues for G.

** DefTensor: Defining tensor ConnectionHh[-z$21823, -z$21824, -z$21825].

** DefTensor: Defining tensor CSHh[-z$21823, -z$21824, -z$21825].

** DefTensor: Defining tensor ntHh[z$21823, z$21824].

** DefTensor: Defining tensor avHh[-z$21823].

   Rules {1, 2, 3, 4} have been declared as UpValues for avHh.

** DefTensor: Defining tensor
 KHh[LI[xAct`xPand`Private`p$24291], LI[xAct`xPand`Private`q$24291], -z$21823, -z$21824].
```

In[∘]:=
```
$Metrics
```
Out[∘]=

$\{G, Hh, GaHh2\}$

In[∘]:=
```
ConformalRules[G, GaHh2]
```
Out[∘]=

$$\left\{ G^{\underline{\underline{cd}}} \to \frac{[G\,a^2]^{cd}}{a^2},\; G_{\underline{\underline{cd}}} \to a^2\, i[G\,a^2]^{cd},\; \tilde{\tilde{G}} \to \frac{[\tilde{\tilde{G\,a^2}}]}{a^8} \right\}$$

In[∘]:=
```
LRule = MakeRule[{L[], (α*ϕ0^2 + β*(φ[])^2 + γ*ϕ0*φ[])*RicciScalarCD[]},
  {MetricOn → All, ContractMetrics → True}]
AutomaticRules[L, LRule];
```
Out[∘]=

$$\{\text{HoldPattern}[L] :\to \text{Module}[\{\}, \alpha\,\phi_0{}^2\,R[\nabla] + \gamma\,\phi_0\,R[\nabla]\,\varphi + \beta\,R[\nabla]\,\varphi^2]\}$$

Rules {1} have been declared as DownValues for L.

In[∘]:=
```
JL = Sqrt[-DetG[]] L[]
```
Out[∘]=

$$\sqrt{-\tilde{\tilde{G}}}\,\left(\alpha\,\phi_0{}^2\,R[\nabla] + \gamma\,\phi_0\,R[\nabla]\,\varphi + \beta\,R[\nabla]\,\varphi^2\right)$$

In[∘]:=
```
DefConstantSymbol[M] (*M = M_p/\sqrt{2}*)
DefScalarFunction[A]
InterpretAsField = MakeRule[{
    aHh[xAct`xTensor`LI[0],
    xAct`xTensor`LI[0]],
    Evaluate[(1/Sqrt[A[φ[]]])]}, MetricOn → All, ContractMetrics → True]
```
** DefConstantSymbol: Defining constant symbol M.

** DefScalarFunction: Defining scalar function A.

Out[∘]=

$$\left\{\text{HoldPattern}[a] :\to \text{Module}\left[\{\}, \frac{1}{\sqrt{A[\varphi]}}\right]\right\}$$

In[∘]:=
```
JL = Conformal[G, GaHh2][JL]
JL = JL /. InterpretAsField;
```
** DefTensor: Defining tensor ChristoffelCDCDaHh2[c, -d, -e].

Out[∘]=

$$\frac{\alpha\,\phi_0{}^2\,R[\nabla]\,\sqrt{-\tilde{\tilde{G}}\,(a)^8}}{(a)^2} + \frac{\gamma\,\phi_0\,R[\nabla]\,\sqrt{-\tilde{\tilde{G}}\,(a)^8}\,\varphi}{(a)^2} + \frac{\beta\,R[\nabla]\,\sqrt{-\tilde{\tilde{G}}\,(a)^8}\,\varphi^2}{(a)^2} - \frac{6\,\alpha\,\phi_0{}^2\,\sqrt{-\tilde{\tilde{G}}\,(a)^8}\,(\nabla_c\nabla^c a)}{(a)^3} - \frac{6\,\gamma\,\phi_0\,\sqrt{-\tilde{\tilde{G}}\,(a)^8}\,\varphi\,(\nabla_c\nabla^c a)}{(a)^3} - \frac{6\,\beta\,\sqrt{-\tilde{\tilde{G}}\,(a)^8}\,\varphi^2\,(\nabla_c\nabla^c a)}{(a)^3}$$

In[∘]:=
```
JL = JL ~ PowerExpand ~ A[φ[]];
```
In[∘]:=
```
A[φ_] := (α*ϕ0^2 + β*(φ)^2 + γ*ϕ0*φ) / M^2
A[φ]
```
Out[∘]=

$$\frac{\beta\,\varphi^2 + \gamma\,\varphi\,\phi_0 + \alpha\,\phi_0{}^2}{M^2}$$

In[ ]:=
```
(*JL//=(#-Sqrt[-DetG[]]*CD[a]×
    (CD[-a][φ[]]Evaluate@(#~Coefficient~CD[-a][CD[a][φ[]]])/Sqrt[-DetG[]]))&*)
JL //= ToCanonical;
```

In[ ]:=
```
JL //= FullSimplify
```

Out[ ]=

$$\left(M^2\sqrt{-\tilde{\tilde{G}}}\left(2\left(\alpha\,\phi_0^2+\varphi\,(\gamma\,\phi_0+\beta\,\varphi)\right)\right)\left(\alpha\,\phi_0^2\,R[\nabla]+R[\nabla]\,\varphi\,(\gamma\,\phi_0+\beta\,\varphi)+3\,(\gamma\,\phi_0+2\,\beta\,\varphi)\,(\nabla_c\nabla^c\varphi)\right)-3\left(-4\,\alpha\,\beta\,\phi_0^2+3\,\gamma^2\,\phi_0^2+8\,\beta\,\gamma\,\phi_0\,\varphi+8\,\beta^2\,\varphi^2\right)(\nabla_c\varphi)\,(\nabla^c\varphi)\right)\Big/\left(2\left(\alpha\,\phi_0^2+\varphi\,(\gamma\,\phi_0+\beta\,\varphi)\right)^2\right)$$

In[ ]:=
```
IndexCoefficient[JL, RicciScalarCD[] //= FullSimplify
  (*PROVES THAT THIS CONFORMAL TRANSFORMATION MAKES IT EINSTEIN HILBERT*)
```

Set: Tag IndexCoefficient in

$$\text{IndexCoefficient}\left[\frac{M^2\,\alpha\,\phi_0^2\sqrt{-\tilde{\tilde{G}}}\,R[\nabla]}{\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2}+\frac{M^2\,\gamma\,\phi_0\sqrt{-\ll 1\gg}\,R[\nabla]\,\varphi}{\alpha\ll 26\gg^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2}+\frac{\ll 1\gg}{\ll 1\gg}+\ll 20\gg+\frac{\ll 1\gg}{\ll 1\gg^2}+\frac{6\,M^2\,\beta^2\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\,(\nabla_{z\$27172}\varphi)\,(\nabla^{z\$27172}\varphi)}{\left(\alpha\ll 26\gg^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2},\ll 1\gg\right]$$

is Protected.

Out[ ]=

$$M^2\sqrt{-\tilde{\tilde{G}}}$$

In[ ]:=
```
JL = JL~PowerExpand~φ[]
```

Out[ ]=

$$\frac{M^2\,\alpha\,\phi_0^2\sqrt{-\tilde{\tilde{G}}}\,R[\nabla]}{\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2}+\frac{M^2\,\gamma\,\phi_0\sqrt{-\tilde{\tilde{G}}}\,R[\nabla]\,\varphi}{\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2}+\frac{M^2\,\beta\sqrt{-\tilde{\tilde{G}}}\,R[\nabla]\,\varphi^2}{\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2}+$$

$$\frac{3\,M^2\,\alpha\,\gamma\,\phi_0^3\sqrt{-\tilde{\tilde{G}}}\,(\nabla_c\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}+\frac{6\,M^2\,\alpha\,\beta\,\phi_0^2\sqrt{-\tilde{\tilde{G}}}\,\varphi\,(\nabla_c\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}+\frac{3\,M^2\,\gamma^2\,\phi_0^2\sqrt{-\tilde{\tilde{G}}}\,\varphi\,(\nabla_c\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}+$$

$$\frac{9\,M^2\,\beta\,\gamma\,\phi_0\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\,(\nabla_c\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}+\frac{6\,M^2\,\beta^2\sqrt{-\tilde{\tilde{G}}}\,\varphi^3\,(\nabla_c\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}-\frac{9\,M^2\,\alpha\,\gamma^2\,\phi_0^4\sqrt{-\tilde{\tilde{G}}}\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{2\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^3}-$$

$$\frac{18\,M^2\,\alpha\,\beta\,\gamma\,\phi_0^3\sqrt{-\tilde{\tilde{G}}}\,\varphi\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^3}-\frac{9\,M^2\,\gamma^3\,\phi_0^3\sqrt{-\tilde{\tilde{G}}}\,\varphi\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{2\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^3}-$$

$$\frac{18\,M^2\,\alpha\,\beta^2\,\phi_0^2\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^3}-\frac{45\,M^2\,\beta\,\gamma^2\,\phi_0^2\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{2\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^3}-$$

$$\frac{36\,M^2\,\beta^2\,\gamma\,\phi_0\sqrt{-\tilde{\tilde{G}}}\,\varphi^3\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^3}-\frac{18\,M^2\,\beta^3\sqrt{-\tilde{\tilde{G}}}\,\varphi^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^3}+$$

$$\frac{6\,M^2\,\alpha\,\beta\,\phi_0^2\sqrt{-\tilde{\tilde{G}}}\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}+\frac{6\,M^2\,\beta\,\gamma\,\phi_0\sqrt{-\tilde{\tilde{G}}}\,\varphi\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}+\frac{6\,M^2\,\beta^2\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0^2+\gamma\,\phi_0\,\varphi+\beta\,\varphi^2\right)^2}$$

In[◦]:= **JL - OC0 =**

$$\left(M^2 \sqrt{-\tilde{\tilde{G}}}\ \left(2 \left(\alpha \phi_0^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right) \left(3 (\gamma \phi_0 + 2 \beta \varphi) (\nabla_c \nabla^c \varphi)\right) - 3 \left(-4 \alpha \beta \phi_0^2 + 3 \gamma^2 \phi_0^2 + 8 \beta \gamma \phi_0 \varphi + 8 \beta^2 \varphi^2\right) (\nabla_c \varphi) (\nabla^c \varphi)\right)\right) \Big/ \left(2 \left(\alpha \phi_0^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2\right) \text{ // ExpandAll}$$

••• Set: Tag Plus in

$$\left(\ll 11 \gg + \frac{\ll 1 \gg}{\ll 1 \gg} + \frac{24 \ll 7 \gg (\ll 1 \gg \varphi)}{\ll 1 \gg} + \frac{24 M^2 \beta^2 \sqrt{-\ll 92 \gg}\ \varphi^2 (\nabla_c \varphi)(\nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \ll 26 \gg^3 \varphi + 4 \ll 3 \gg \ll 1 \gg + \ll 1 \gg + 4 \beta \gamma \ll 26 \gg \varphi^3 + 2 \beta^2 \varphi^4}\right) + \left(\frac{M^2 \alpha \phi_0^2 \sqrt{-\tilde{\tilde{G}}}\ R[\nabla]}{\alpha \phi_0^2 + \gamma \phi_0 \varphi + \beta \varphi^2} + \frac{M^2 \gamma \ll 26 \gg \sqrt{-\ll 92 \gg}\ R[\nabla] \varphi}{\alpha \ll 26 \gg^2 + \gamma \ll 26 \gg \varphi + \beta \varphi^2} + \frac{\ll 1 \gg}{\ll 1 \gg} + \frac{\ll 1 \gg}{\ll 1 \gg} + \ll 18 \gg + \frac{\ll 1 \gg}{\ll 1 \gg^2} + \frac{6 \ll 7 \gg (\nabla^{\ll 7 \gg} \varphi)}{(\ll 1 \gg)^2} + \frac{6 M^2 \beta^2 \sqrt{-\ll 92 \gg}\ \varphi^2 (\nabla_{z\$27172} \varphi)(\nabla^{z\$27172} \varphi)}{(\alpha \ll 26 \gg^2 + \gamma \ll 26 \gg \varphi + \beta \varphi^2)^2}\right) \text{ is Protected.}$$

Out[◦]=

$$\frac{6 M^2 \alpha \gamma \phi_0^3 \sqrt{-\tilde{\tilde{G}}}\ (\nabla_c \nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} +$$

$$\frac{12 M^2 \alpha \beta \phi_0^2 \sqrt{-\tilde{\tilde{G}}}\ \varphi (\nabla_c \nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} +$$

$$\frac{6 M^2 \gamma^2 \phi_0^2 \sqrt{-\tilde{\tilde{G}}}\ \varphi (\nabla_c \nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} +$$

$$\frac{18 M^2 \beta \gamma \phi_0 \sqrt{-\tilde{\tilde{G}}}\ \varphi^2 (\nabla_c \nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} +$$

$$\frac{12 M^2 \beta^2 \sqrt{-\tilde{\tilde{G}}}\ \varphi^3 (\nabla_c \nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} +$$

$$\frac{12 M^2 \alpha \beta \phi_0^2 \sqrt{-\tilde{\tilde{G}}}\ (\nabla_c \varphi) (\nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} -$$

$$\frac{9 M^2 \gamma^2 \phi_0^2 \sqrt{-\tilde{\tilde{G}}}\ (\nabla_c \varphi) (\nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} -$$

$$\frac{24 M^2 \beta \gamma \phi_0 \sqrt{-\tilde{\tilde{G}}}\ \varphi (\nabla_c \varphi) (\nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4} -$$

$$\frac{24 M^2 \beta^2 \sqrt{-\tilde{\tilde{G}}}\ \varphi^2 (\nabla_c \varphi) (\nabla^c \varphi)}{2 \alpha^2 \phi_0^4 + 4 \alpha \gamma \phi_0^3 \varphi + 4 \alpha \beta \phi_0^2 \varphi^2 + 2 \gamma^2 \phi_0^2 \varphi^2 + 4 \beta \gamma \phi_0 \varphi^3 + 2 \beta^2 \varphi^4}$$

In[∘]:= 
$$\text{OC} = \left(M^2 \sqrt{-\tilde{\tilde{G}}} \left(2 \left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right) \left(3 (\gamma \phi_0 + 2 \beta \varphi) \left(\nabla_c \nabla^c \varphi\right)\right) - 3 \left(-4 \alpha \beta \phi_0{}^2 + 3 \gamma^2 \phi_0{}^2 + 8 \beta \gamma \phi_0 \varphi + 8 \beta^2 \varphi^2\right) (\nabla_c \varphi) \left(\nabla^c \varphi\right)\right)\right) \Big/ \left(2 \left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2\right)$$

(*COEFFICIENT OF THE \VARPHI DERIVATIVES (FIRST OR HIGHER)*)

Out[∘]=
$$\left(M^2 \sqrt{-\tilde{\tilde{G}}} \left(6 (\gamma \phi_0 + 2 \beta \varphi) \left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right) (\nabla_c \nabla^c \varphi) - 3 \left(-4 \alpha \beta \phi_0{}^2 + 3 \gamma^2 \phi_0{}^2 + 8 \beta \gamma \phi_0 \varphi + 8 \beta^2 \varphi^2\right) (\nabla_c \varphi) (\nabla^c \varphi)\right)\right) \Big/ \left(2 \left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2\right)$$

In[∘]:= 
$$\text{CD}[-c]\left[\frac{-3 (2 * \beta * \varphi[] + \gamma * \phi 0)}{\left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)}\right]$$
(*THIS IS THE INTEGRATION BY PARTS LEFTOVER OF EQN: OC*)

Out[∘]=
$$-3 \left(\frac{2 \beta (\nabla_c \varphi)}{\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)} - \frac{(\gamma \phi_0 + 2 \beta \varphi) (\beta \varphi (\nabla_c \varphi) + (\gamma \phi_0 + \beta \varphi) (\nabla_c \varphi))}{\left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2}\right)$$

In[∘]:= 
```
DefTensor[Term1[], M4]
DefTensor[Term2[], M4]
DefTensor[Tot1[], M4]
DefTensor[Tot2[], M4]
```

** DefTensor: Defining tensor Term1[].

** DefTensor: Defining tensor Term2[].

** DefTensor: Defining tensor Tot1[].

** DefTensor: Defining tensor Tot2[].

In[∘]:= 
$$\text{Term1}[] = -3 \left(\frac{2 \beta (\nabla_c \varphi)}{\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)} - \frac{(\gamma \phi_0 + 2 \beta \varphi) (\beta \varphi (\nabla_c \varphi) + (\gamma \phi_0 + \beta \varphi) (\nabla_c \varphi))}{\left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2}\right) \text{CD}[c][\varphi[]]$$

$$\text{Term2nt} = \frac{-3}{2} \left(\frac{\phi 0\text{^}2 * (\gamma\text{^}2 - 4 \alpha * \beta) + 2 (2 \beta * \varphi[] + \gamma * \phi 0)\text{^}2}{\left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2}\right) //$$
ExpandNumerator (*SAME AS SECOND TERM in OC*)

Out[∘]=
$$-3 \left(\frac{2 \beta (\nabla_c \varphi)}{\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)} - \frac{(\gamma \phi_0 + 2 \beta \varphi) (\beta \varphi (\nabla_c \varphi) + (\gamma \phi_0 + \beta \varphi) (\nabla_c \varphi))}{\left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2}\right) (\nabla^c \varphi)$$

Out[∘]=
$$\frac{12 \alpha \beta \phi_0{}^2 - 9 \gamma^2 \phi_0{}^2 - 24 \beta \gamma \phi_0 \varphi - 24 \beta^2 \varphi^2}{2 \left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2}$$

In[∘]:= 
$$\text{Term2}[] = \text{Term2nt} * \text{CD}[-c][\varphi[]] \times \text{CD}[c][\varphi[]]$$

Out[∘]=
$$\frac{\left(12 \alpha \beta \phi_0{}^2 - 9 \gamma^2 \phi_0{}^2 - 24 \beta \gamma \phi_0 \varphi - 24 \beta^2 \varphi^2\right) (\nabla_c \varphi) (\nabla^c \varphi)}{2 \left(\alpha \phi_0{}^2 + \varphi (\gamma \phi_0 + \beta \varphi)\right)^2}$$

In[ ]:=
```
Tot1[] = M^2 * (Term1[] + Term2[]) //
  FullSimplify (*The extra term in Eqn (37a) is correct!!!*)
```

Out[ ]=

$$-\frac{3\,M^2\,(\gamma\,\phi_0+2\,\beta\,\varphi)^2\,(\nabla_c\,\varphi)\,(\nabla^c\,\varphi)}{2\,\left(\alpha\,\phi_0^2+\varphi\,(\gamma\,\phi_0+\beta\,\varphi)\right)^2}$$

In[ ]:=
```
BRule = MakeRule[
  {B[c], 1/2 CD[c][Log[(E - 12 β) * (φ[])^2 + (σ - 12 γ) * ϕ0 * (φ[]) + (ν - 12 α) * ϕ0^2]]},
  {MetricOn → All, ContractMetrics → True}];
AutomaticRules[B, BRule];
B[c]
```

Rules {1} have been declared as DownValues for B.

Out[ ]=

$$-\frac{6\,\gamma\,\phi_0\,(\nabla^c\,\varphi)}{(-12\,\alpha+\nu)\,\phi_0^2+(-12\,\gamma+\sigma)\,\phi_0\,\varphi+(-12\,\beta+\mathrm{E})\,\varphi^2}+\frac{\sigma\,\phi_0\,(\nabla^c\,\varphi)}{2\,\left((-12\,\alpha+\nu)\,\phi_0^2+(-12\,\gamma+\sigma)\,\phi_0\,\varphi+(-12\,\beta+\mathrm{E})\,\varphi^2\right)}-\frac{12\,\beta\,\varphi\,(\nabla^c\,\varphi)}{(-12\,\alpha+\nu)\,\phi_0^2+(-12\,\gamma+\sigma)\,\phi_0\,\varphi+(-12\,\beta+\mathrm{E})\,\varphi^2}+\frac{\mathrm{E}\,\varphi\,(\nabla^c\,\varphi)}{(-12\,\alpha+\nu)\,\phi_0^2+(-12\,\gamma+\sigma)\,\phi_0\,\varphi+(-12\,\beta+\mathrm{E})\,\varphi^2}$$

In[ ]:=
```
DefTensor[L2[], M4]
DefTensor[T2correct[], M4]
DefTensor[Tot01[], M4]
```

** DefTensor: Defining tensor L2[].

** DefTensor: Defining tensor T2correct[].

** DefTensor: Defining tensor Tot01[].

In[∘]:=
```
L2Rule = MakeRule[{L2[], (α*ϕ0^2 + β*(φ[])^2 + γ*ϕ0*φ[])*( - 6*B[c]×B[-c]) +
    6*CD[-c][(α*ϕ0^2 + β*(φ[])^2 + γ*ϕ0*φ[])]×B[c] +
    E/2 (CD[-c][φ[]] - (φ[] + σ/E*ϕ0) B[-c]) × (CD[c][φ[]] - φ[]*B[c]) +
    (ν*ϕ0^2)/2 B[c]×B[-c]}, {MetricOn → All, ContractMetrics → True}];
AutomaticRules[L2, L2Rule];
(* !!!! WRONG SIGN USED HERE< CHECK *)
Tot01[] = CollectTensors[L2[]]
```

Rules {1} have been declared as DownValues for L2.

CollectTensors: There are denominators with a sum inside TensorWrappers. Things might not have been fully collected.

Out[∘]=

$$\frac{1}{2}\,\mathrm{E}\,(\nabla_c\varphi)\,(\nabla^c\varphi)-\frac{(12\alpha-\nu)\,(-12\gamma+\sigma)^2\,\phi_0^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{8\left((-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2\right)^2}-$$
$$\frac{(12\gamma-\sigma)\left(576\alpha\beta+144\gamma^2-48\alpha\mathrm{E}-48\beta\nu+4\mathrm{E}\nu-24\gamma\sigma+\sigma^2\right)\phi_0^3\,\varphi\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{8\left((-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2\right)^2}-$$
$$\frac{(12\beta-\mathrm{E})\left(576\alpha\beta+720\gamma^2-48\alpha\mathrm{E}-48\beta\nu+4\mathrm{E}\nu-120\gamma\sigma+5\sigma^2\right)\phi_0^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{8\left((-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2\right)^2}-$$
$$\frac{(-12\beta+\mathrm{E})^2\,(12\gamma-\sigma)\,\phi_0\,\varphi^3\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left((-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2\right)^2}-$$
$$\frac{(12\beta-\mathrm{E})^3\,\varphi^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{2\left((-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2\right)^2}-$$
$$\frac{(-12\gamma+\sigma)^2\,\phi_0^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{4\left((-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2\right)}-$$
$$\frac{(12\beta-\mathrm{E})\,(12\gamma-\sigma)\,\phi_0\,\varphi\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2}-$$
$$\frac{(-12\beta+\mathrm{E})^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(-12\alpha+\nu)\,\phi_0^2+(-12\gamma+\sigma)\,\phi_0\,\varphi+(-12\beta+\mathrm{E})\,\varphi^2}$$

In[∘]:=
```
(*Define substitutions for ε,σ,ν in terms of k*)
paramRules = {E → (k + 12) β, σ → (k + 12) γ, ν → (k + 12) α};
```

In[∘]:=
```
Clear[Lnograv]
```

In[•]:=
```
Tot01[] // Together // FullSimplify
T2correct[] = Tot01[] /. paramRules // FullSimplify
Lnograv[φ_] := 4/5 * IndexCoefficient[Tot01[], CD[-c][φ[]] × CD[c][φ[]]] // Together
(*This is the coeff of ((∇_c φ) (∇^c φ))/2 *)
Lnograv[φ] //= FullSimplify
```

Out[•]=

$$\frac{\left(\left(48\,\alpha\,\mathrm{E}-4\,\mathrm{E}\,\nu+(-12\,\gamma+\sigma)^{2}\right)\phi_{0}^{2}+48\,\beta\,(12\,\gamma-\sigma)\,\phi_{0}\,\varphi-48\,\beta\,(-12\,\beta+\mathrm{E})\,\varphi^{2}\right)(\nabla_{c}\varphi)\,(\nabla^{c}\varphi)}{96\,\alpha\,\phi_{0}^{2}-8\left(\nu\,\phi_{0}^{2}+\varphi\,(-12\,\gamma\,\phi_{0}+\sigma\,\phi_{0}-12\,\beta\,\varphi+\mathrm{E}\,\varphi)\right)}$$

Out[•]=

$$\frac{\left(4\,(12+k)\,\alpha\,\beta\,\phi_{0}^{2}-k\,\gamma^{2}\,\phi_{0}^{2}+48\,\beta\,\varphi\,(\gamma\,\phi_{0}+\beta\,\varphi)\right)(\nabla_{c}\varphi)\,(\nabla^{c}\varphi)}{8\left(\alpha\,\phi_{0}^{2}+\varphi\,(\gamma\,\phi_{0}+\beta\,\varphi)\right)}$$

Out[•]=

$$\frac{\left(48\,\alpha\,\mathrm{E}-4\,\mathrm{E}\,\nu+(-12\,\gamma+\sigma)^{2}\right)\phi_{0}^{2}+48\,\beta\,(12\,\gamma-\sigma)\,\phi_{0}\,\varphi-48\,\beta\,(-12\,\beta+\mathrm{E})\,\varphi^{2}}{48\,\alpha\,\phi_{0}^{2}-4\left(\nu\,\phi_{0}^{2}+\varphi\,(-12\,\gamma\,\phi_{0}+\sigma\,\phi_{0}-12\,\beta\,\varphi+\mathrm{E}\,\varphi)\right)}$$

In[•]:=
```
Tot1[]
Lgravonly[φ_] := 4/5 * IndexCoefficient[Tot1[], CD[-c][φ[]] × CD[c][φ[]]]
(*This is the coeff of ((∇_c φ) (∇^c φ))/2 in the *)
Lgravonly[φ] //= Factor
```

Out[•]=

$$-\frac{3\,M^{2}\,(\gamma\,\phi_{0}+2\,\beta\,\varphi)^{2}\,(\nabla_{c}\varphi)\,(\nabla^{c}\varphi)}{2\left(\alpha\,\phi_{0}^{2}+\varphi\,(\gamma\,\phi_{0}+\beta\,\varphi)\right)^{2}}$$

Out[•]=

$$-\frac{3\,M^{2}\,(\gamma\,\phi_{0}+2\,\beta\,\varphi)^{2}}{\left(\alpha\,\phi_{0}^{2}+\gamma\,\phi_{0}\,\varphi+\beta\,\varphi^{2}\right)^{2}}$$

In[•]:=
```
Clear[Lnograv0, Lnograv1]
```

In[•]:=
```
Lnograv0[φ_] :=
 4/5 * IndexCoefficient[Tot01[], CD[-c][φ[]] × CD[c][φ[]]] /. paramRules // FullSimplify
Lnograv0[φ]
Lnograv1[φ_] :=
 4/5 * IndexCoefficient[T2correct[], CD[-c][φ[]] × CD[c][φ[]]] // FullSimplify
Lnograv1[φ]
```

Out[•]=

$$12\,\beta+\frac{k\left(4\,\alpha\,\beta-\gamma^{2}\right)\phi_{0}^{2}}{4\left(\alpha\,\phi_{0}^{2}+\varphi\,(\gamma\,\phi_{0}+\beta\,\varphi)\right)}$$

Out[•]=

$$12\,\beta+\frac{k\left(4\,\alpha\,\beta-\gamma^{2}\right)\phi_{0}^{2}}{4\left(\alpha\,\phi_{0}^{2}+\varphi\,(\gamma\,\phi_{0}+\beta\,\varphi)\right)}$$

In[∘]:= 

```
f[φ_] := (4 (12 + k) α β ϕ0^2 − k γ^2 ϕ0^2 + 48 β φ (γ ϕ0 + β φ)) / (8 (α ϕ0^2 + φ (γ ϕ0 + β φ))) //= FullSimplify
f[φ]
```

$$f[\varphi\_] := \frac{\left(4\ (12+k)\ \alpha\,\beta\,\phi_0{}^2 - k\,\gamma^2\,\phi_0{}^2 + 48\,\beta\,\varphi\ (\gamma\,\phi_0 + \beta\,\varphi)\right)}{8\ \left(\alpha\,\phi_0{}^2 + \varphi\ (\gamma\,\phi_0 + \beta\,\varphi)\right)}\ \text{//= FullSimplify}$$

··· Set: Tag Times in $\frac{4\,(12+k)\,\alpha\,\beta\,\phi_0{}^2 - k\,\gamma^2\,\phi_0{}^2 + 48\,\beta\,\varphi\,(\beta\,\varphi + \gamma\,\phi_0)}{8\,(\alpha\,\phi_0{}^2 + \varphi\,(\beta\,\varphi + \gamma\,\phi_0))}$ is Protected.

Out[∘]=

$$\frac{4\ (12+k)\ \alpha\,\beta\,\phi_0{}^2 - k\,\gamma^2\,\phi_0{}^2 + 48\,\beta\,\varphi\ (\beta\,\varphi + \gamma\,\phi_0)}{8\ \left(\beta\,\varphi^2 + \phi_0\ (\gamma\,\varphi + \alpha\,\phi_0)\right)}$$

In[∘]:=

$$\text{Lconfnograv}[\varphi\_] := \text{M\^{}2}\ \frac{(\text{Lnograv0}[\varphi])}{\left(\alpha\,\phi_0{}^2 + \gamma\,\phi_0\,\varphi + \beta\,\varphi^2\right)}$$

```
Lconfnograv[φ] // FullSimplify
```

Out[∘]=

$$\frac{M^2\ \left(4\ (12+k)\ \alpha\,\beta\,\phi_0{}^2 - k\,\gamma^2\,\phi_0{}^2 + 48\,\beta\,\varphi\ (\gamma\,\phi_0 + \beta\,\varphi)\right)}{4\ \left(\alpha\,\phi_0{}^2 + \varphi\ (\gamma\,\phi_0 + \beta\,\varphi)\right)^2}$$

In[∘]:=

```
Ltot[φ_] := +Lgravonly[φ] + Lconfnograv[φ]
Ltot[φ] // Together // FullSimplify
```

Out[∘]=

$$\frac{(12+k)\ M^2\ \left(4\,\alpha\,\beta - \gamma^2\right)\ \phi_0{}^2}{4\ \left(\alpha\,\phi_0{}^2 + \varphi\ (\gamma\,\phi_0 + \beta\,\varphi)\right)^2}$$

---

# Starobinsky Check

In[∘]:=

```
DefTensor[B2[c], M4]
```

** DefTensor: Defining tensor B2[c].

In[∘]:=

```
B2Rule = MakeRule[{B2[c], 1/2 CD[c][Log[(β) * (φ[])^2 + (α) * ϕ0^2]]},
    {MetricOn → All, ContractMetrics → True}];
AutomaticRules[B2, B2Rule]
B2[c]
B2[-c]
```

Rules {1} have been declared as DownValues for B2.

Out[∘]=

$$\frac{\beta\,\varphi\ (\nabla^c\,\varphi)}{\alpha\,\phi_0{}^2 + \beta\,\varphi^2}$$

Out[∘]=

$$\frac{\beta\,\varphi\ (\nabla_c\,\varphi)}{\alpha\,\phi_0{}^2 + \beta\,\varphi^2}$$

In[∘]:=

```
DefTensor[LS[], M4]
```

** DefTensor: Defining tensor LS[].

In[ ]:=
```
LSRule =
 MakeRule[{LS[], (α*ϕ0^2 + β*(φ[])^2) * (RicciScalarCD[] - 6*B2[c]×B2[-c]) +
    6*CD[-c][(α*ϕ0^2 + β*(φ[])^2)]×B2[c]},
  {MetricOn → All, ContractMetrics → True}]
AutomaticRules[LS, LSRule]
(* !!!! WRONG SIGN USED HERE< CHECK *)
CollectTensors[LS[]]
```

Out[ ]=

$$\left\{\text{HoldPattern}[\text{LS}] :\to \text{Module}\left[\{c\},\ \alpha\,\phi_0{}^2\,R[\nabla] + \beta\,R[\nabla]\,\varphi^2 - \frac{6\,\alpha\,\beta^2\,\phi_0{}^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2+\beta\,\varphi^2\right)^2} - \frac{6\,\beta^3\,\varphi^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2+\beta\,\varphi^2\right)^2} + \frac{12\,\beta^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\alpha\,\phi_0{}^2+\beta\,\varphi^2}\right]\right\}$$

Rules {1} have been declared as DownValues for LS.

CollectTensors: There are denominators with a sum inside TensorWrappers. Things might not have been fully collected.

Out[ ]=

$$\alpha\,\phi_0{}^2\,R[\nabla] + \beta\,R[\nabla]\,\varphi^2 - \frac{6\,\alpha\,\beta^2\,\phi_0{}^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2+\beta\,\varphi^2\right)^2} - \frac{6\,\beta^3\,\varphi^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2+\beta\,\varphi^2\right)^2} + \frac{12\,\beta^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\alpha\,\phi_0{}^2+\beta\,\varphi^2}$$

In[ ]:=
```
JLS = Sqrt[-DetG[]] LS[]
```

Out[ ]=

$$\sqrt{-\tilde{\tilde{G}}}\left(\alpha\,\phi_0{}^2\,R[\nabla] + \beta\,R[\nabla]\,\varphi^2 - \frac{6\,\alpha\,\beta^2\,\phi_0{}^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2+\beta\,\varphi^2\right)^2} - \frac{6\,\beta^3\,\varphi^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2+\beta\,\varphi^2\right)^2} + \frac{12\,\beta^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\alpha\,\phi_0{}^2+\beta\,\varphi^2}\right)$$

In[ ]:=
```
DefConstantSymbol[M] (*M = M_p/\sqrt{2}*)
DefScalarFunction[A2]
InterpretAsField = MakeRule[{
    aHh[xAct`xTensor`LI[0],
    xAct`xTensor`LI[0]],
    Evaluate[(1/Sqrt[A2[φ[]]])]}, MetricOn → All, ContractMetrics → True]
```

ValidateSymbol: Symbol M is already used as a constant-symbol.

** DefScalarFunction: Defining scalar function A2.

Out[ ]=

$$\left\{\text{HoldPattern}[a] :\to \text{Module}\left[\{\},\ \frac{1}{\sqrt{\text{A2}[\varphi]}}\right]\right\}$$

In[ ]:=
```
JLS = Conformal[G, GaHh2][JLS]
JLS = JLS /. InterpretAsField;
```

Out[ ]=

$$\frac{\alpha\,\phi_0^2\,R[\nabla]\,\sqrt{-\tilde{\tilde{G}}}\,(a)^8}{(a)^2}+\frac{\beta\,R[\nabla]\,\sqrt{-\tilde{\tilde{G}}}\,(a)^8\,\varphi^2}{(a)^2}-\frac{6\,\alpha\,\phi_0^2\,\sqrt{-\tilde{\tilde{G}}}\,(a)^8\,(\nabla_c\nabla^c a)}{(a)^3}-\frac{6\,\beta\,\sqrt{-\tilde{\tilde{G}}}\,(a)^8\,\varphi^2\,(\nabla_c\nabla^c a)}{(a)^3}-\frac{6\,\alpha\,\beta^2\,\phi_0^2\,\sqrt{-\tilde{\tilde{G}}}\,(a)^8\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(a)^2\,(\alpha\,\phi_0^2+\beta\,\varphi^2)^2}-\frac{6\,\beta^3\,\sqrt{-\tilde{\tilde{G}}}\,(a)^8\,\varphi^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(a)^2\,(\alpha\,\phi_0^2+\beta\,\varphi^2)^2}+\frac{12\,\beta^2\,\sqrt{-\tilde{\tilde{G}}}\,(a)^8\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(a)^2\,(\alpha\,\phi_0^2+\beta\,\varphi^2)}$$

In[ ]:=
```
JLS = JLS ~ PowerExpand ~ A2[φ[]];
```

In[ ]:=
```
A2[φ_] := (α * ϕ0^2 + β * (φ)^2) / M^2
A2[φ]
```

Out[ ]=

$$\frac{\beta\,\varphi^2+\alpha\,\phi_0^2}{M^2}$$

In[ ]:=
```
JLS //= (# - Sqrt[-DetG[]] * CD[a] ×
      (CD[-a][φ[]] × Evaluate@(#~Coefficient~CD[-a][CD[a][φ[]]]) / Sqrt[-DetG[]])) &
JLS //= ToCanonical;
```

Out[ ]=

$$\frac{M^2\,\alpha\,\phi_0^2\,\sqrt{-\tilde{\tilde{G}}}\,R[\nabla]}{\alpha\,\phi_0^2+\beta\,\varphi^2}+\frac{M^2\,\beta\,\sqrt{-\tilde{\tilde{G}}}\,R[\nabla]\,\varphi^2}{\alpha\,\phi_0^2+\beta\,\varphi^2}-\frac{6\,M^2\,\alpha\,\beta^2\,\phi_0^2\,\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(\alpha\,\phi_0^2+\beta\,\varphi^2)^3}-\frac{6\,M^2\,\beta^3\,\sqrt{-\tilde{\tilde{G}}}\,\varphi^4\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(\alpha\,\phi_0^2+\beta\,\varphi^2)^3}+\frac{12\,M^2\,\beta^2\,\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(\alpha\,\phi_0^2+\beta\,\varphi^2)^2}-\frac{3\,\alpha\,\phi_0^2\,\sqrt{-\tilde{\tilde{G}}}\left(\frac{6\,\beta^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{M^4\left(\frac{\alpha\,\phi_0^2+\beta\,\varphi^2}{M^2}\right)^{5/2}}-\frac{\frac{2\,\beta\,\varphi\,(\nabla_c\nabla^c\varphi)}{M^2}+\frac{2\,\beta\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{M^2}}{\left(\frac{\alpha\,\phi_0^2+\beta\,\varphi^2}{M^2}\right)^{3/2}}\right)}{\sqrt{\frac{\alpha\,\phi_0^2+\beta\,\varphi^2}{M^2}}}-\frac{3\,\beta\,\sqrt{-\tilde{\tilde{G}}}\,\varphi^2\left(\frac{6\,\beta^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{M^4\left(\frac{\alpha\,\phi_0^2+\beta\,\varphi^2}{M^2}\right)^{5/2}}-\frac{\frac{2\,\beta\,\varphi\,(\nabla_c\nabla^c\varphi)}{M^2}+\frac{2\,\beta\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{M^2}}{\left(\frac{\alpha\,\phi_0^2+\beta\,\varphi^2}{M^2}\right)^{3/2}}\right)}{\sqrt{\frac{\alpha\,\phi_0^2+\beta\,\varphi^2}{M^2}}}$$

In[ ]:=
```
JLS //= FullSimplify
```

Out[ ]=

$$\frac{M^2\,\sqrt{-\tilde{\tilde{G}}}\,\left((\alpha\,\phi_0^2+\beta\,\varphi^2)\,(\alpha\,\phi_0^2\,R[\nabla]+\beta\,R[\nabla]\,\varphi^2+6\,\beta\,\varphi\,(\nabla_c\nabla^c\varphi))+6\,\beta\,(\alpha\,\phi_0^2-\beta\,\varphi^2)\,(\nabla_c\varphi)\,(\nabla^c\varphi)\right)}{(\alpha\,\phi_0^2+\beta\,\varphi^2)^2}$$

In[∘]:= 
$$OC2 = \frac{\left(\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)\,\left(6\,\beta\,\varphi\,\left(\nabla_c\nabla^c\varphi\right)\right) + 6\,\beta\,\left(\alpha\,\phi_0{}^2 - \beta\,\varphi^2\right)\,\left(\nabla_c\varphi\right)\,\left(\nabla^c\varphi\right)\right)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)^2} \;//=\; \mathrm{ToCanonical}$$

Set: Tag Times in $\frac{6\,\beta\,\varphi\,(\alpha\,\phi_0{}^2 + \beta\,\varphi^2)\,(\nabla_c\nabla^c\varphi) + 6\,\beta\,(\alpha\,\phi_0{}^2 - \beta\,\varphi^2)\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{(\alpha\,\phi_0{}^2 + \beta\,\varphi^2)^2}$ is Protected.

Out[∘]=
$$\frac{6\,\alpha\,\beta\,\phi_0{}^2\,\varphi\,(\nabla_c\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)^2} + \frac{6\,\beta^2\,\varphi^3\,(\nabla_c\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)^2} + \frac{6\,\alpha\,\beta\,\phi_0{}^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)^2} - \frac{6\,\beta^2\,\varphi^2\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)^2}$$

In[∘]:=
```
DefTensor[Terms1[], M4]
DefTensor[Terms2[], M4]
DefTensor[Tots[], M4]
```

ValidateSymbol: Symbol Times is Protected.

ValidateSymbol: Symbol Times is Protected.

```
** DefTensor: Defining tensor Tots[].
```

In[∘]:=
$$\mathrm{Terms1}[\,] = -\mathrm{CD}[c]\left[\frac{6\,\beta\,\varphi}{\alpha\,\phi_0{}^2 + \beta\,\varphi^2}\right] \times \mathrm{CD}[-c][\varphi[\,]] \;//\; \mathrm{FullSimplify}$$

Out[∘]=
$$\frac{6\,\beta\,\left(-\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)^2}$$

In[∘]:=
$$\mathrm{Terms2}[\,] = \frac{6\,\beta\,\left(\alpha\,\phi_0{}^2 - \beta\,\varphi^2\right)\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)\hat{}\,2} \;//\; \mathrm{FullSimplify}$$

Out[∘]=
$$\frac{6\,\beta\,\left(\alpha\,\phi_0{}^2 - \beta\,\varphi^2\right)\,(\nabla_c\varphi)\,(\nabla^c\varphi)}{\left(\alpha\,\phi_0{}^2 + \beta\,\varphi^2\right)^2}$$

In[∘]:=
```
Tots[] = Terms1[] + Terms2[] // FullSimplify
```

Out[∘]=
```
0
```