Real Analysis Class Notes

Prabhu Ranjan Mahapatra

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Intro to Real analysis

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1 Definations

1.1 Function

Consider two sets A and B, whose elements may be any objects. whatsover, and suppose that with each element x of A there is associated ,in some manner, an element of B, which we denote by $f(x)$, then f is said to be a function from A to B (or mapping from A to B).

The set A is called the *domain* of f, and elements $f(x)$ are called the values of f, the set of all values of f is called the *range* of f.

My interpretation:

$$f : A \to B \ := \ \forall x \in A, \exists! \ f(x) \in B \ni x \ R \ f(x)$$

$$\Rightarrow A \to Domain \qquad B \to Codomain \qquad \{f(x) \mid x \in A\} \to Range$$

1.2 Inverse Function

Let, A and B be two sets and f be a mapping of A into B.
$f : A \to B$
$E \subseteq A \qquad f(E) := \{f(x) \mid x \in A\}$
we know, $f(A) \subseteq B$
if $f(A) = B$, we say that f is *onto*.
$F \subseteq B \qquad f^{-1}(F) := \{x \in A \mid f(x) \in F\}$
$F = \{y\} \subseteq B$

$\Rightarrow f^{-1}(\{y\}) = \{x \in A \mid f(x) = y\}$
if, for each $y \in B,\ f^{-1}(y)$ consists of at most one element of A, then f is said to be $1-1$ mapping of A into B
This may also be expressed as follows:
f is $1-1$ mapping of A into B provided that $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2,\ x_1, x_2 \in A$

My interpretation:

$f : A \to B \Rightarrow E \subseteq A \Rightarrow f(E) := \{f(x) \mid x \in A\}$

& $\Rightarrow F \subseteq B \Rightarrow f^{-1}(F) := \{x \in A \mid f(x) \in F\}$

- f is onto $\iff \forall\ y \in B, \exists\ x \in A \ni y = f(x) \iff \{f(x) \mid x \in A\} = B$
- f is one-one $\iff \forall y \in B, |f^{-1}(y)| \leq 1 \iff [\forall x_1, x_2 \in A, x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)]$

1.3 Equivalent:

if there exists a 1-1 mapping of A onto B, we say that A and B, can be put in 1-1 correspondence or A and B have same cardinal number or briefly that, A and B are equivalent and write $A \sim B$

My interpretation:

$A \sim B := \exists f : A \to B \ni \forall y \in B, f^{-1}(\{y\}) = 1$ (f is both one-one and onto)

1.4 Finite,Infinite,Countable,Uncountable,At most countable

For any positive integer n, let J_n be the set whose elements are the integers $1, 2, ...n$
Let, J be the set consisting of all positive integers.
for any set A, we say:

i) A is finite if $A \sim J_n$ for some n or $A = \phi$.

ii) A is infinite if A is not finite.

iii) A is countable if $A \sim J$.

iv) A is uncountable if A is neither finite nor countable.

v) A is at most countable if A is finite or countable.

My interpretation:

i) A is finite if $|A| \in \mathbb{N} \cup \{0\}$ else infinite.

ii) if $A \sim \mathbb{N} \iff A$ is countable

iii) A is at most countable if A is countable or finite else uncountable

```python
def what_is(A):
    if len(A).is_integer():
        return "finite and at most countable"
    elif len(A)=len(N):
        return "infinite, countable and at most countable"
    else:
        return "infinite and uncountable"
```