Assignment 0

1

Question : Find error in this proof

claim

In any set of h horses, all horses are the same color.

Proof

Base case: $h = 1$

any set containing only one horse, all horses (means just that one horse) clearly of the same color.

Induction hypothesis

Lets, assume the claim is true for $h = k \geq 1$

Induction step

$\{h_1, h_2, h_3, \cdots, h_{k+1}\} \rightarrow \{h_1, h_2, h_3, \cdots, h_{k+1}\}/\{h_i\}, \{h_1, h_2, h_3, \cdots, h_{k+1}\}/\{h_j\} \qquad (i, j \leq k+1; i \neq j)$
each horse in intersection of these two sets $= \{h_1, h_2, h_3, \cdots, h_{k+1}\}/\{h_i, h_j\}$ has same color as h_i and h_j as they have been together in a set with k horses, thus h_i and h_j has same color as others and in conclusion they all have same color.

Solution

Thought process

For once Let suppose the proof is correct then using the same logic, we should be able to proof that it is indeed true for $h = 2$ from the fact that it is true for $h = 1$.
$\{h_1, h_2\} \rightarrow \{h_1\}, \{h_2\}$

But now we can clearly see we can't do so because we don't have any common element to compare and come to conclusion that all possible set share horses of same color.

conclusion

The error is in equality part of our assumption that $k \geq 1$
$k = 1 \Rightarrow k + 1 = 2$
that is the case of proving $h = 2$ from the fact that it is true for $h = 1$
which we know can't be done with the logic applied on induction step as we will not find any common element in subsets with $k = 1$ element.
we will be able to find the common element as required for the logic to follow whenever
$h > 1 \Rightarrow h \geq 2$
Thus, the base case should be $h = 2$ which is not true.
If it were then the claim must have been true.

2

Given: $A = \{x, y, z\}$ and $B = \{x, y\}$

(i) Is A a subset of B?

No

> as (both are finite sets and) A has all elements of B and an extra element z.
> Another approach can be:
> $\exists f : A \rightarrow B \ni f$ is onto
> but $\nexists f : B \rightarrow A \ni f$ is onto
> $\Rightarrow A \nsubseteq B$

(ii) Is B a subset of A?

Yes

> the reasons specified in above question are sufficient.
> Yet another argument can be:
> $B - A = \phi \Rightarrow B \subseteq A$

(iii) What is $A \cup B$?

$\{x, y, z\} = A$

> $A \cup B$ is the set of all elements that are present in either A or B or both.
> $A \cup B := \{x | x \in A \text{ or } x \in B\}$
> In other words, the set C with minimum number of elements $\ni A \subseteq C$ and $B \subseteq C$ is the set $A \cup B$

(iv) What is $A \cap B$?

$\{x, y\} = B$

> $A \cap B$ is the set of all elements that are present in both A and B.
> $A \cap B := \{x | x \in A \text{ and } x \in B\} = \{x \in A | x \in B\}$
> In other words, the set C with maximum number of elements $\ni C \subseteq A$ and $C \subseteq B$ is the set $A \cap B$

(v) What is $A \times B$?

$\{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$

> $A \times B$ is the set of ordered pairs such that first element comes from A and second from B.
> $A \times B := \{(a, b) | a \in A, b \in B\}$

(vi) What is the power set of B?

$\{\{\}, \{x\}, \{y\}, \{x, y\}\}$

> power set of B is the set of all subsets of B.
> power set of $B := \{B_i | B_i \subseteq B\}$

3

Given: A, B, C are sets with a, b, c many elements Respectively

i) How many Elements does $A \times B$ have?

ab

> The problem can be seen like this: we have two places: $[\quad|\quad]$
> the first place can be filled up in a ways and the second place can be filled up in b ways:
> $[\, a \mid b \,]$
> and a and b are independent of each other.
> thus the number of ways two places can be filled up $= a \times b$

ii) How many elements does the power set of C have?

2^c

> A subset of C can have number of elements between $0 - c$
> order doesn't matter in sets thus number subsets will be summation of all possible combinations:
> $\sum_{r=0}^{c} {}^cC_r = {}^cC_0 + {}^cC_1 + \cdots + {}^cC_{c-1} + {}^nC_c$
> $\quad = {}^cC_0 \times 1 + {}^cC_1 \times 1 + \cdots + {}^cC_{c-1} \times 1 + {}^cC_c \times 1$
> $\quad = (1+1)^c$

4

Given: $f : X \to Y$ and $g : X \times Y \to Y$

$X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$

n	$f(n)$						g	6	7	8	9	10
1	6						1	10	10	10	10	10
2	7						2	7	8	9	10	6
3	6						3	7	7	8	8	9
4	7						4	9	8	7	6	10
5	6						5	6	6	6	6	6

i) What is the value of $g(4, f(4))$?

8

> $g(4, f(4)) = g(4, 7) = 8$

ii)What is the range and domain of f ?

Domain : X ; Range: $\{6, 7\}$

> $f : X \to Y$
> $\Rightarrow X$ is called the Domain
> and Range $= \{y \in Y | y = f(x)$ for some $x \in X\}$

iii) What is the range and domain of g?

Domain : $X \times Y$ **; Range:** Y

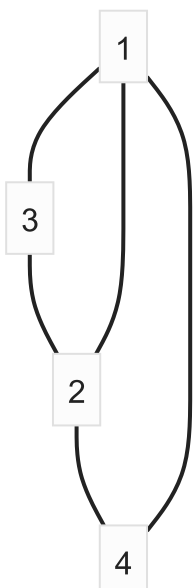
> $g : X \times Y \rightarrow Y$
> $\Rightarrow X \times Y$ is the domain
> Range $= \{6, 7, 8, 9, 10\}$

5

Given: The undirected graph $G = (V, E)$

V **, the set of nodes , is** $\{1, 2, 3, 4\}$
E **, the set of edge , is** $\{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{2, 4\}, \{1, 4\}\}$

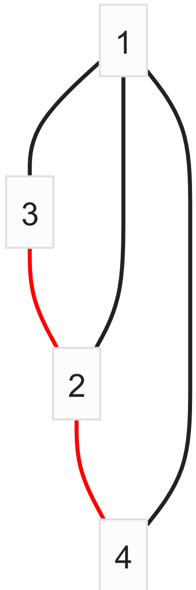
i) Draw the graph G.



ii) What are the degrees of each node?

node	1	2	3	4
degree	3	3	2	2

iii) Indicate a path from mode 3 to mode 4 on your drawing of G.


1
3
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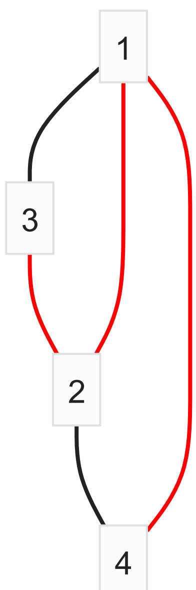
4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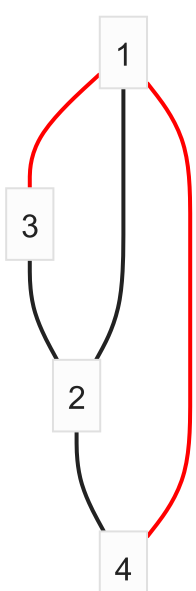
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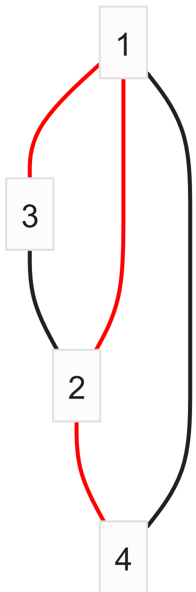

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