Discrete Mathematics ~ PrabhuBikash

- It includes subjects like combinatorics.
- The motive is to know about finite set.
- How we know counting?
- Ideas will flow from abstract to concrete and vice versa.

problem:

we are given a sea of finite sets $\{A_n\}_{n=1}^{\infty}$ & WISH to count the number of elements of each A_n
Let, $f(n)$ denote the number of elements of A_n, $\forall$ integer $n \geq 1$
i.e. $f(n) = |A_n|$, $\forall$ integer $n \geq 1$
The problem is to determine $f(n)$

Example:

A)

1. For each positive integer $n \geq 1$, let $f(n)$ denote the number of subsets of $[n] = \{1, 2, \ldots, n\}$
   solution:
   Here, $f(n) = 2^n$, $\forall$ integer $n \geq 1$

My Solution

$$\begin{aligned}\sum_{r=0}^{n} {}^nC_r &= {}^nC_0 + {}^nC_1 + \cdots + {}^nC_{n-1} + {}^nC_n \\ &= {}^nC_0 \times 1 + {}^nC_1 \times 1 + \cdots + {}^nC_{n-1} \times 1 + {}^nC_n \times 1 \\ &= (1+1)^n\end{aligned}$$

2. For each positive integer $n \geq 1$, let $f(n)$ denote the number of n-tuples consists of zero or one.
   $f(n) = |\{(x_1, x_2, \ldots, x_n) | \forall i \in [n], x_i \in \{0, 1\}\}|$
   solution :
   Here, $f(n) = 2^n$

My Solution

for each position we have two options 0 or 1
$\Rightarrow f(n) = 2 \times 2 \times 2 \times \cdots \times 2$ (n times)

B)Derangement Problem:

1. Suppose there are n many letter and n envelops. Put a letter into an envelop.
   Let $f(n)$ denote the number of ways that one letter put into one envelop so that no letters receives it's own envelop.
   solution:
   Here, $f(n) = n! \sum_{k=0}^{n} (-1)^k \frac{1}{k!}, \forall$ integer $n \geq 1$

C)Matrix

Definition

An $n \times n$ matrix over $\mathbb{C}$ is a function $f(n) : [n] \times [n] \to \mathbb{C}$
i.e. $f : \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\} \to \mathbb{C}$

$$\begin{array}{llll} f(1,1) = a_{11} & f(1,2) = a_{12} & \cdots & f(1,n) = a_{1,n} \\ f(2,1) = a_{21} & f(2,2) = a_{22} & \cdots & f(2,n) = a_{2,n} \\ \vdots & \vdots & & \vdots \\ f(n,1) = a_{n1} & f(n,2) = a_{n2} & \cdots & f(n,n) = a_{n,n} \end{array}$$

in short, $[a_{ij}]_{i,j \in [n]} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,n} \\ a_{21} & a_{22} & \cdots & a_{2,n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{n,n} \end{bmatrix}$

1. Let, $f(n)$ denote the number of $n \times n$ matrix over $\mathbb{C}$ with zero one entries.
   solution:
   Here, $f(n) = 2^{n^2}$

   count the number
   a) Each row consists of 3 ones.
   b) Each row and column consists of 3 ones.

My Solution

a)
if $n < 3 : f(n) = 0$
if $n \geq 3 : f(n) = {}^{n}C_3{}^{n}$ (possible combination in one row into number of rows)
as number of combination is zero if $n < 3$ it is reasonable to let ${}^{n<3}C_3 = 0$
$\Rightarrow f(n) = {}^{n}C_3{}^{n}$
generalizing this idea for k ones: $f(n, k) = {}^{n}C_k{}^{n}$

b)

D)Recurrence

Definition

A recurrence for $f(n)$ may be given in terms of $f(1)$ or $f(2)$ or $\cdots$ or $f(n-1)$

1. Let, $f(x)$ denote the number of subsets of $[n]$ that don't contain integers.
   $f(1) = 2 \qquad f(2) = 3 \qquad f(3) = 5$
   <u>claim</u>:$f(n) = f(n-1) + f(n-2), \forall$ integer $n \geq 3$
   <u>solution</u>:
   Here,$f(n) = \frac{1}{\sqrt{5}}(\tau^{n+2} - \frac{1}{\tau^{n+2}}), \forall$ integer $n \geq 3$
   where, $\tau = \frac{1+\sqrt{5}}{2}$