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Logical Development of Vogel's ApproximationMethod (LD-VAM): An Approach to Find BasicFeasible Solution of Transportation Problem

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1 INTRODUCTION

The Transportation Problem is the special class of Linear Programming Problem in the field of Applied Mathematics and also in Operation Research. There are some existing algorithms to solve the Transportation Problem such as Northwest Corner Rule (NWC), Least Cost Method (LCM), andVogel's Approximation Method (VAM) etc. where Vogel's Approximation Method (VAM) is known as more efficient algorithm to find the basic feasible solution. But VAM hassome limitations which are discussed in **(2.2)** and we fixed this problem and developed a new Algorithm in (2.3). To solve transportation problem first a fall we have to formulate original problem as a linear programming problem (LPP) for m sources and n destinations.

Each i^{th} source has a capacity of supply amount that is s_i and d_j be the demand amount of each j^{th} destination and c_{ij} be the cost of i^{th} source to j^{th} destination. The ultimate goal of solving the Transportation Problem is to find the amount of product x_{ij} which will be transfer from i^{th} source to j^{th} destination so that total transportation cost will be minimized. The Linear Programming Problem (LPP) representing the Transportation problem for m sources and n destinations are generally given as:

Minimize: $$Z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$$

Subject to:

$$\sum_{j=1}^{n}x_{ij}\le s_i \quad , \qquad for\ i=1,2,3..................m$$

$$\sum_{i=1}^{m}x_{ij}\ge d_j \quad , \qquad for\ j=1,2,3...................n$$

$$x_{ij}\ge 0, \qquad for\ all\ i,j$$

To applying transportation algorithm for solving TP we have to make a mathematical model for this LPP.

2. METHODOLOGY:

Initially we mention that there are some existing methods for solving transportation problem such as Northwest Corner Rule (NWC), Least Cost Method (LCM), and Vogel's Approximation Method (VAM) etc. In this section we discuss about Vogel's Approximation Method (VAM) and our proposed method Logical Development of Vogel's Approximation Method (LD-VAM).

2.1 EXISTING ALGORITHM FOR VOGEL'S APPROXIMATION METHOD (VAM):

The Vogel's Approximation Method (VAM) is an iterative procedure for computing a basic feasible solution of a transportation problem. This method is better than other two methods *i.e.* North West

Corner Rule (NWC) and Least cost Method (LCM), because the basic feasible solution obtained
The Vogel's Approximation Method (VAM) is an iterative procedure for computing a basic feasible solution of a transportation problem. This method is better than other two methods *i.e.* North West Corner Rule (NWC) and Least cost Method (LCM), because the basic feasible solution obtained.

Step-1: Indentify the boxes having minimum and next to minimum transportation cost in each row and write the difference (Penalty) along the side of the table against the corresponding row.

Step-2: Indentify the boxes having minimum and next to minimum transportation cost in each column and write the difference (Penalty) along the side of the table against the corresponding column. If minimum cost appears in two or more times in a row or column then select this same cost as a minimum and next to minimum cost and penalty will be zero.

Step-3:

- **a.** Identify the row and column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least in the selected row or column. Adjust the supply and demand and cross out the satisfied row or column. If a row and column are satisfying simultaneously, only one of them is crossed out and the remaining row or column is assigned a zero supply or demand.
- **b.** If two or more penalty costs have same largest magnitude, then select any one of them (or select most top row or extreme left column).

Step-4:

- **a.** If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.
- **b.** If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.
- **c.** If all out rows or column have (remaining) zero supply or demand, the zero basic variables are determined by the Least-Cost uncrossed Method. Stop.
- **d.** Otherwise, go to Step-1.

2.2 FINDING LIMITATIONS OF VOGEL'S APPROXIMATION METHOD (VAM):

In VAM approach, Penalties are determined by the difference of two minimum costs of each row and each column. The highest penalty indicates that among the two minimum costs, one is the lowest and another is too high. In VAM algorithm select such row or column which contains largest penalty to ensure the least cost in current iteration and avoiding the chance appear of taking larger cost in next iteration. If the highest penalty cost is in two or more rows or columns, the VAM algorithm selects any one of them (or selects the most top row or extreme left column) [(2.) Step-3(b)]. But it is not necessarily true that the largest penalty always ensures the lowest cost because the difference of two pairs of numbers can be equal when one of the pair is smaller than another pair. As an example, the difference of 10 and 5 and difference of 7 and 2 are equal but lowest numbers exist in second pair. For that reason, we can say that in VAM algorithm, if the lower pair does not appear in the topmost or extreme left position then lowest cost will not be selected in the current iteration so that total transportation cost may not be minimized. For that case VAM may not be given the lowest feasible solution.

2.2 PROPOSED ALGORITHM FOR LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD(VAM) :

From the above discussion of the limitations of Vogel's Approximation Method (VAM) we fixed this problem and proposed an improved algorithm which is named by ¯*Logical Development of Vogel's Approximation Method (LD-VAM)*‖. In this algorithm, we resolved the above discussed problem of VAM *i.e.,* when largest penalty appear in two or more rows or columns then select that row or column among them which contains the least cost and give the maximum possible allocation. The algorithm of LD-VAM is given below: Set s_i be supply amount of the i^{th} source and d_j be the amount of demand of j^{th} destination and c_{ij} be the unit transportation cost of i^{th} source to

j^{th} destination.

Step-1: Check: $if\ s_i < 0\ and\ d_j < 0$ then Stop.

Step-2: $if \sum_i s_i > \sum_j d_j\ or\ if \sum_i s_i < \sum_j d_j$ then balance the transportation problem adding dummy demand or supply.

Step-3:

a. Identify the smallest and next to smallest cost of each row and column and calculate the difference between them which is called by penalty. Set p_i be the row penalty and be the column penalty.

$$p_i = |c_{ih} - c_{ik}| \quad and \quad p_j = |c_{hj} - c_{kj}|$$

b. If the smallest cost appears two or more times in a rowor column then select this same cost as a smallest and next to smallest cost and penalty will be zero.

$$p_i = |c_{ih} - c_{ik}| = 0 \quad and \quad p_j = |c_{hj} - c_{kj}| = 0$$

Step-4: Select $\max(p_i, p_j)$. Select lowest cost of that rowor column which has largest penalty and allocate maximum possible amount x_{ij} *i.e,* $\min(s_i, d_j)$. if the lowest cost appears in two or more cells in that row or column then choose the extreme left or most top lowest cost cell.

Step-5: If tie occurs in the largest penalties in some rows or columns then select that row or column whichcontains the least cost among them.

Step-6:

Adjust the supply and demand and cross out the satisfied row or column. If row and column are satisfied simultaneously then cross out one of them and set zero supply or demand in remaining row or column.

Step-7:

a. If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.

b. If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.

c. If all uncrossed out rows or column have (remaining) zero supply or demand, determine the zero basicvariables by the Least-Cost Method. Stop.

d. Otherwise go to Step-3.

3. NUMERICAL ILLUSTRATION:

Consider some special types of Transportation Problems where highest penalty appears in two or more rows or columns and solve these using Vogel's Approximation Method (VAM) and the proposed method Logical Development of Vogel's Approximation Method (LD-VAM) and compare these results with optimal solution for measuring accuracy.

3.1 EXAMPLE-1:

Consider a Mathematical Model of a Transportation Problem below.

Table-1.1:

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	10	8	9	5	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	

Now we find the feasible solution of this problem using **LD- VAM** and **VAM** respectively in below:

3.1.1 SOLUTION OF EXAMPLE-1 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD- VAM):

Costs are indicated in the top corner and allocations areindicated in the bottom left corner.

Iteration-1:

In Iteration-1, the largest penalty appears three times in S1, S2, S3 rows, but lowest cost appears in (S3, D2) cell. As per **LD-VAM** algorithm allocate 40 amount commodities in (S3, D2) cell.

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	3
S2	7	9	8	10	4	80	3
S3	9	3 40	7	10	6	30	3
S4	11	4	8	3	9	90	1
Demand	60		100	50	90		
Column Penalty	2	1	1	2	2		

Iteration-2:

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	4
S2	7	9	8	10	4	80	3
S3	9	3 40	7	10	6	30	1
S4	11	4	8	3 50	9	40	5
Demand	60		100		90		
Column Penalty	2		1	2	2		

Iteration-3:

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	1

S2	7	9	8	10	4 **80**		**3**
S3	9	3 **40**	7	10	6	**30**	1
S4	11	4	8	3 **50**	9	**40**	1
Demand	**60**		**100**		**10**		
Column Penalty	2		1		2		

Iteration-4

Source	**Destination**					**Supply**	**Row Penalty**
	D1	**D2**	**D3**	**D4**	**D5**		
S1	10	8	9	5	13	**100**	1
S2	7	9	8	10	4 **80**		
S3	9	3 **40**	7	10	6 **10**	**20**	1
S4	11	4	8	3 **50**	9	**40**	1
Demand	**60**		**100**				
Column Penalty	1		1		**3**		

Iteration-5

Source	D1	D2	D3	D4	D5	Supply	Row Penalty
S1	10	8	9	5	13	**100**	1
S2	7	9	8	10	4 **80**		
S3	9	3 **40**	7	10	6 **10**	**20**	2
S4	11	4	8 **40**	3 **50**	9		**3**
Demand	**60**		**60**				
Column Penalty	1		1				

Iteration-6

Source	D1	D2	D3	D4	D5	Supply	Row Penalty
S1	10	8	9	5	13	**100**	1
S2	7	9	8	10	4 **80**		
S3	9	3 **40**	7 **20**	10	6 **10**		**2**
S4	11	4	8 **40**	3 **50**	9		
Demand	**60**		**40**				
Column Penalty	1		**2**				

Iteration-7

Source	D1	D2	D3	D4	D5	Supply	Row Penalty
S1	10 **60**	8	9 **40**	5	13		
S2	7	9	8	10	4 **80**		
S3	9	3 **40**	7 **20**	10	6 **10**		
S4	11	4	8 **40**	3 **50**	9		
Demand	**0**						
Column Penalty							

In Iteration-6, only one row has remains with positive supply and demand amount then as per **LD-VAM** algorithm allocates these by Least Cost Method. The final feasible solution table isgiven below:

Table-1.2

Total Transportation Cost (Using LD-VAM):
(10×60) +(9×40) +(4×80) +(3×40) +(7×20) +(6×10) +(8×40) +(3×50) =2070

3.1.2 SOLUTION OF EXAMPLE-1 USING VOGEL'S APPROXIMATION METHOD (VAM):

Costs are indicated in right-top corner and allocations are indicated in the bottom-left corner.

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	10 **60**	8	9 **40**	5	13	**100**
S2	7	9	8	10	4 **80**	**80**
S3	9	3 **40**	7 **20**	10	6 **10**	**70**
S4	11	4	8 **40**	3 **50**	9	**90**
Demand	**60**	**40**	**100**	**50**	**90**	

Table-1.3:

Source	Destination					Supply	Row Penalty					
	D1	D2	D3	D4	D5							
S1	10 **50**	8	9	5 **50**	13	**100**	**3**	1	1	1	1	1
S2	7	9	8	10	4 **80**	**80**	3	3	**3**			
S3	9 **10**	3	7 **50**	10	6 **10**	**70**	3	3	1	1	2	**2**
S4	11	4 **40**	8 **50**	3	9	**90**	1	**4**	1	1	**3**	
Demand	**60**	**40**	**100**	**50**	**90**							
Column Penalty	2	1	1	2	2							
	2	1	1		2							
	2		1		2							
	1		1		**3**							
	1		1									
	1		2									

Total transportation cost (using VAM):
(10*50) + (5*50) +(4*80) +(9*10) +(7*50) +(6*10) +(4*40) +(8*80) = 2130

Observation:

We observed that VAM provides feasible solution for Example-1 is 2130 and LD-VAM provides 2070 which is lower than VAM.

3.2 **Example-2:**

Consider a Mathematical Model of a Transportation Problem inbelow:

Source	Destination				Supply
	D1	D2	D3	D4	
S1	7	5	9	11	**30**
S2	4	3	8	6	**25**
S3	3	8	10	5	**20**
S4	2	6	7	3	**15**
Demand	**30**	**30**	**20**	**10**	

Table-2.1

Now we find the feasible solution of this problem using **LD-VAM** and **VAM** respectively in below

3.2.1 SOLUTION OF EXAMPLE-2 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD- VAM):

Costs are indicated in right-top corner and allocations areindicated in the bottom-left corner.

Table-2.1:

Source	Destination				Supply
	D1	D2	D3	D4	
S1	7 **5**	5 **5**	9 **20**	11	**30**
S2	4	3 **25**	8	6	**25**
S3	3 **20**	8	10	5	**20**
S4	2 **5**	6	7	3 **10**	**15**
Demand	**30**	**30**	**20**	**10**	

Total Transportation Cost (Using LD-VAM):

$$(7 \times 5) + (5\times5) + (9\times 20) + (3\times 25) + (3\times 20) + (2\times 5) + (3\times 10) = 415$$

3.2.2 SOLUTION OF EXAMPLE-2 USING VOGEL'S APROXIMATION METHOD (VAM):

Costs are indicated in right-top corner and allocations areindicated in the bottom-left corner.

Table-2.2 :

Source	Destination				Supply
	D1	D2	D3	D4	
S1	7	5 **30**	9	11	**30**
S2	4 **25**	3 **0**	8	6	**25**
S3	3 **5**	8	10 **5**	5 **10**	**20**
S4	2	6	7 **15**	3	**15**
Demand	**30**	**30**	**20**	**10**	

Total Transportation Cost (Using VAM):

$$(5\times 30) + (4\times 25) + (3\times 0) + (3\times5) + (10\times5) + (5\times10) + (7\times15) = 470$$

Observation:

We observed that VAM provides feasible solution for Example-2 is 470 and LD-VAM provides 415 whichis lower than VAM.

3.3 **EXAMPLE-3:**

Consider a Mathematical Model of a Transportation Problem inbelow:

Table-3.1

3.3.1 SOLUTION OF EXAMPLE-3 USING LOGICAL DEVELOPMENT OF

Source	Destination						Supply
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
S1	12	4	13	18	9	2	**120**
S2	9	16	10	7	15	11	**80**
S3	4	9	10	8	9	7	**50**
S4	9	3	12	6	4	5	**90**
S5	7	11	5	18	2	7	**100**
S6	16	8	4	5	1	10	**60**
Demand	**75**	**85**	**140**	**40**	**95**	**65**	

VOGEL'S APPROXIMATION METHOD (LD- VAM):

Costs are indicated in right-top corner and allocations areindicated in bottom-left corner.

Table-3.2

Source	Destination						Supply
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
S1	12	4 **55**	13	18	9	2 **65**	**120**
S2	9 **25**	16	10 **40**	7 **15**	15	11	**80**
S3	4 **50**	9	10	8	9	7	**50**
S4	9	3 **30**	12	6 **25**	4 **35**	5	**90**
S5	7	11	5 **100**	18	2	7	**100**
S6	16	8	4	5	1 **60**	10	**60**
Demand	**75**	**85**	**140**	**40**	**95**	**65**	

Total transportation Cost:

$$(4\times 55)+(2\times 65)+(9\times 25)+(10\times 40)+(7\times 15)+(4\times 50)+(3\times 30)+(6\times 25)+(4\times 35)+(5\times 100)+(1\times 60)=2220$$

3.4 EXAMPLE-4:

Consider a Mathematical Model of a Transportation Problem inbelow:

Table-4.1

3.4.1 SOLUTION OF EXAMPLE-4 USING LOGICAL DEVELOPMENT OF

Source	Destination				Supply
	D1	D2	D3	D4	
S1	6	1	9	3	**70**
S2	11	5	2	8	**55**
S3	10	12	4	7	**90**
Demand	**85**	**35**	**50**	**45**	

VOGEL'S APPROXIMATION METHOD (LD- VAM):

Costs are indicating in right-top corner and allocations areindicating in bottom-left corner.

Table-4.2

Source	Destination				Supply
	D1	D2	D3	D4	
S1	6	1 **35**	9	3 **35**	**70**
S2	11 **5**	5	2 **50**	8	**55**
S3	10 **80**	12	4	7 **10**	**90**
Demand	**85**	**35**	**50**	**45**	

Total transportation Cost:

$$(1\times35)+(3\times35)+(11\times5)+(2\times50)+(10\times80)+(7\times10)=1165$$

3.5 EXAMPLE-5:

Consider a Mathematical Model of a Transportation Problem inbelow:

Table-5.1:

Source	Destinations							Supply
	D1	D2	D3	D4	D5	D6	D7	
S1	12	7	3	8	10	6	6	**60**
S2	6	9	7	12	8	12	4	**80**
S3	10	12	8	4	9	9	3	**70**

S4	8	5	11	6	7	9	3	**100**
S5	7	6	8	11	9	5	6	**90**
Demand	**20**	**30**	**40**	**70**	**60**	**80**	**100**	

3.5.1 SOLUTION OF EXAMPLE-4 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD- VAM):

Costs are indicated in right-top corner and allocations areindicated in the bottom-left corner.

Table-5.2:

Source	**Destinations**							**Supply**
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	
S1	12	7 **20**	3 **40**	8 **0**	10	6	6	**60**
S2	6 **20**	9 **0**	7	12	8 **60**	12	4 **0**	**80**
S3	10	12	8	4 **70**	9	9	3	**70**
S4	8	5	11	6	7	9	3 **100**	**100**
S5	7	6 **10**	8	11	9	5 **80**	6	**90**
Demand	**20**	**30**	**40**	**70**	**60**	**80**	**100**	

Total transportation Cost:

$(7\times 20) + (3\times 40) + (8\times 0) + (6\times 20) + (9\times 0) + (8\times 60) + (4\times 0) + (4\times 70) + (3\times 100) + (6\times 10) + (5\times 80) = 1900.$

4. Result analysis

In above two examples and some other examples, we observed that feasible solutions using Logical Development of Vogel's Approximation Method (LD-VAM) rare lower than Vogel's Approximation Method (VAM) where some of these areexactly equal to the optimal solution and some are very closeto optimal solution. The comparison table of these solutions is given below:

Table-6

Transportation Problem	**Problem Size**	**Methods**		
		LD-VAM	**VAM**	**Optimal Solution**
Example-1	4×5	**2070**	2130	2070
Example-2	4×4	**415**	470	410
Example-3	6×6	**2220**	2310	2170
Example-4	3×4	**1165**	1220	1160
Example-5	5×7	**1900**	1930	1900

5. CONCLUSION

In this paper we find a limitation of Vogel’s Approximation Method (VAM) and developed an improved algorithm by resolving this limitation named “Logical Development of Vogel’s Approximation Method (LD-VAM)” for solving Transportation Problem. From the above examples and other transportation problems, LD-VAM provides the less feasible solution than VAM which are very close to optimal solution and sometimes equal to optimal solution.

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