CONCEPTS OF TESTING OF HYPOTHESIS

In our day-to-day life, we see different commercials advertisements in television, newspapers, magazines, etc. such as

(i) The refrigerator of certain brand saves up to 20% electric bill,
(ii) The motorcycle of certain brand gives 60 km/liter mileage,
(iii) A detergent of certain brand produces the cleanest wash

Now, the question may arise in our mind “can such types of claims be verified statistically?” Fortunately, in many cases the answer is “yes”.

- The technique of testing such type of claims or statements or assumptions is known as testing of hypothesis.
- The truth or falsity of a claim or statement is never known unless we examine the entire population.
- But practically it is not possible in mostly situations so we take a random sample from the population under study and use the information contained in this sample to take the decision whether a claim is true or false.

- In hypothesis testing problems first of all we should being identifying the claim or statement or assumption or hypothesis to be tested and write it in the words.
- Once the claim has been identified then we write it in symbolic form if possible.

As in the above examples,

(i) Customer of motorcycle may write the claim or postulate the hypothesis
"the motorcycle of certain brand gives the average mileage 60 km/liter."
Here, we are concerning the average mileage of the motorcycle so let μ represents the average mileage then our hypothesis becomes $\mu = 60$ km /liter.

(ii) Similarly, the businessman of banana may write the statement or postulate the hypothesis
"the average weight of a banana of Kerala is greater than 200 gm."
So our hypothesis becomes $\mu > 200$ gm.

Simple and Composite Hypotheses

Simple Hypothesis: If a hypothesis specifies only one value or exact value of the population parameter
Example: $\mu = 60$ km/liter is

Composite Hypothesis: If a hypothesis specifies not just one value but a range of values that the population parameter may assume.
Example: $\mu > 200$ gm

Null and Alternative Hypotheses

Null Hypothesis: The hypothesis which we wish to test.
Alternative Hypothesis: The hypothesis which complements to the null hypothesis.

The thump rule is that the statement containing equality is the null hypothesis. That is, the hypothesis which contains symbols $=$ or $\leq$ or $\geq$ is taken as null hypothesis and the hypothesis which does not contain equality i.e. contains $\neq$ or $<$ or $>$ is taken as alternative hypothesis. The null hypothesis is denoted by H_0 and alternative hypothesis is denoted by H_1 or H_A.

In our second example of banana, the claim is $\mu > 200$ gm and its complement is $\mu \leq 200$ gm. Since complement $\mu \leq 200$ gm contains equality sign so we take complement as a null hypothesis and claim $\mu > 200$ gm as an alternative hypothesis, that is,

$$H_0: \mu \leq 200 \text{ gm and } H_1: \mu > 200 \text{ gm}$$

The alternative hypothesis has two types:

(i) Two-sided (tailed) alternative hypothesis
(ii) One-sided (tailed) alternative hypothesis

Two-sided (tailed) alternative hypothesis

If the alternative hypothesis gives the alternate of null hypothesis in both directions (less than and greater than) of the value of parameter specified in null hypothesis then it is known as two-sided alternative hypothesis.
Example: H1: $\theta \neq 60$ then it is a two-sided alternative hypothesis because its means that the value of parameter θ is greater than or less than 60.

One-sided (tailed) alternative hypothesis

If it gives an alternate only in one direction(less than or greater than) only then it is known as one-sided alternative hypothesis.
Example: H1: $\theta > 60$ then it is a right-sided alternative hypothesis
H1: $\theta < 60$ then it is a left-sided alternative hypothesis

Homework (Questions page 9, Answers page 23)

A company manufactures car tyres. Company claims that the average life of its tyres is 50000 miles. To test the claim of the company, formulate the null and alternative hypotheses.

Businessman of orange formulates different hypotheses about the average weight of the orange which are given below:
(i) H0: $= 100$ (ii) H1: >100 (iii) H0: ≤ 100 (iv) H1: $\neq 100$
Categorize the above cases into simple and composite hypotheses.

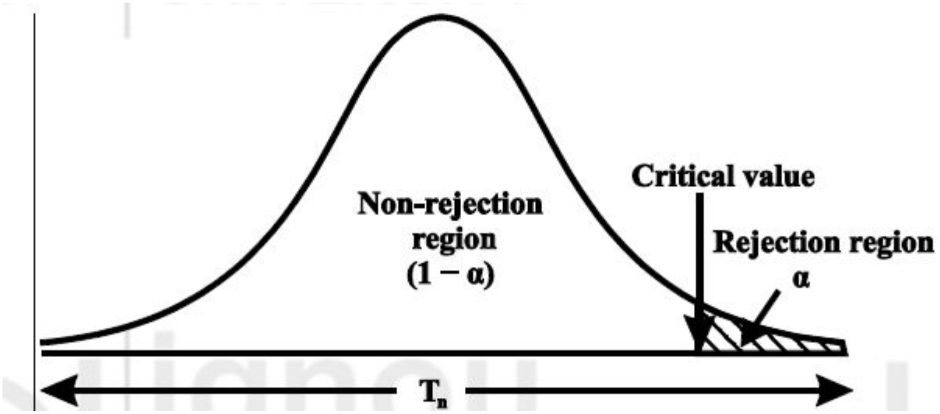
CRITICAL REGION

A region in the sample space in which if the calculated value of the test statistic lies, we reject the null hypothesis then it is called critical region or rejection region.

The region of rejection is called critical region. It has a pre-fixed area generally denoted by alpha α

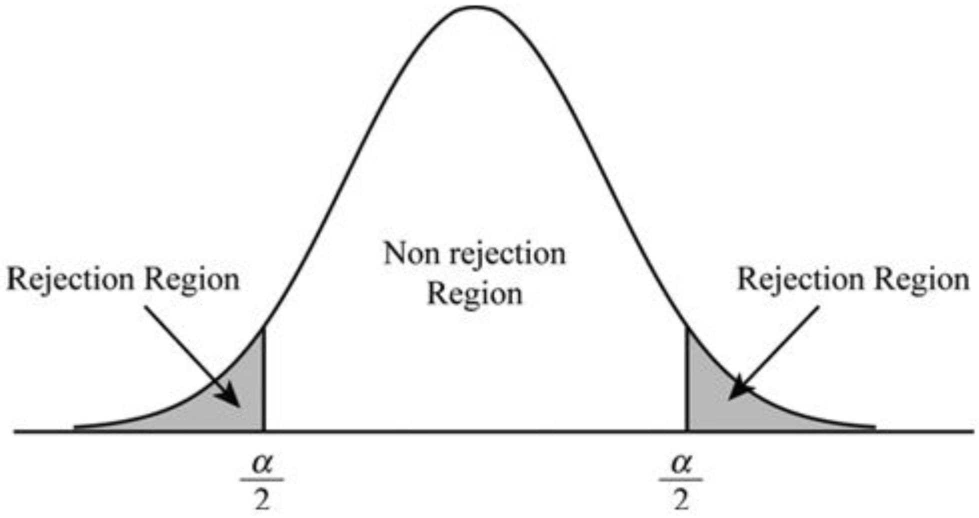
The rejection (critical) region lies in one-tail or two-tails on the probability curve of sampling distribution of the test statistic its depends upon the alternative hypothesis. Therefore, three cases arise:

Case I: If the alternative hypothesis is right-sided such as H1: θ1 > θ2 then the entire critical or rejection region of size α lies on right tail of the probability curve of sampling distribution of the test statistic



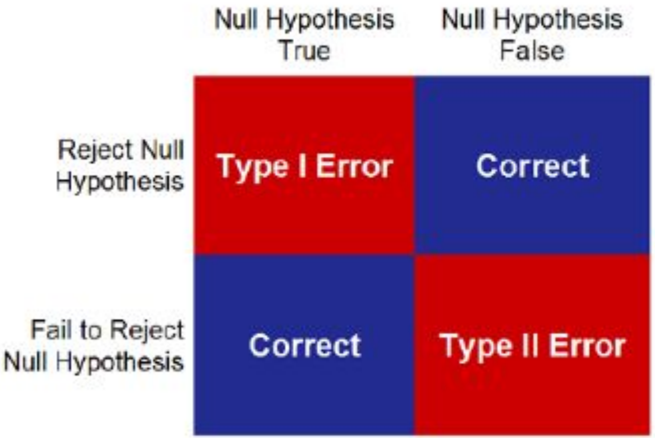
Case II: If the alternative hypothesis is left-sided such as H1: θ1 < θ2 then the entire critical or rejection region of size α lies on left tail of the probability curve of sampling distribution of the test statistic.

Case III: If the alternative hypothesis is two sided such as H1: θ1 ≠ θ2 then critical or rejection regions of size α/2 lies on both tails of the probability curve of sampling distribution of the test statistic.



In hypothesis testing, a critical value is a point on the test distribution that is compared to the test statistic to determine whether to reject the null hypothesis.

TYPE-I AND TYPE-II ERRORS



H0: The patient is a malaria patient
H1: The patient is not a malaria patient

Type-I Error:
The decision relating to rejection of null hypothesis H0 when it is true is called type-I error. The probability of committing the type-I error is called size of test, denoted by α and is given by
α = P [Reject H0 when H0 is true] = P [Reject H0 / H0 is true]
The (1- α) is the probability of correct decision and

Type-II Error:
Beta = P[Do not reject H0 when H0 is false]
The (1 - beta) is the probability of correct decision and also known as “power of the test”.

LEVEL OF SIGNIFICANCE

The probability of type-I error is known as level of significance of a test. It is also called the size of the test or size of critical region, denoted by α. Generally, it is pre-fixed as 5% or 1% level (α = 0.05 or 0.01).

Homework If probability of type-I error is 0.05 then what is the level of significance?

ONE-TAILED AND TWO-TAILED TESTS

Null Hypothesis	Alterative Hypothesis	Types of Critical Region / Test
$H_0 : \theta = \theta_0$	$H_1 : \theta \neq \theta_0$	Two-tailed test having critical regions under both tails.
$H_0 : \theta \leq \theta_0$	$H_1 : \theta > \theta_0$	Right-tailed test having critical region under right tail only.
$H_0 : \theta \geq \theta_0$	$H_1 : \theta < \theta_0$	Left- tailed test having critical region under left tail only.

Homework (pg 17, solution 24)

The test whether one-tailed or two-tailed depends on

(i) Null hypothesis (H_0) (ii) Alternative hypothesis (H_1)

(iii) Neither H_0 nor H_1 (iv) Both H_0 and H_1

GENERAL PROCEDURE OF TESTING A HYPOTHESIS

Step I: Setup null hypothesis H0 and alternative hypothesis H1.
Step II: Decide the level of significance (), at which we want to test our hypothesis.
Generally, it is taken as 5% or 1% (α = 0.05 or 0.01).
Step III: The third step is to choose an appropriate test statistic under H0 for testing the null hypothesis as given below:

$$\text{Test statistic} = \frac{\text{Statistic} - \text{Value of the parameter under } H_0}{\text{Standard error of statistic}}$$

Step IV: In testing of hypothesis ultimately we have to reach at a conclusion.

(i) If calculated value of test statistic lies in rejection region at level of significance then we reject null hypothesis.
(ii) If calculated value of test statistic lies in non-rejection region at level of significance then we do not reject null hypothesis.

CONCEPT OF p-VALUE

"Could the null hypothesis also be rejected at values of α smaller than 0.01?"

The answer is "yes" and we can compute the smallest level of significance (α) at which a null hypothesis can be rejected. This smallest level of significance (α) is known as "p-value".

To take the decision about the null hypothesis based on p-value, the p-value is compared with level of significance (α) and if p-value is equal or less than then we reject the null hypothesis and if the p-value is greater than we do not reject the null hypothesis.

Note: when sample size is large then test statistic follows the normal distribution as the parent population is normal or non-normal so we do not require any assumption of the form of the parent population for large sample size but when sample size is small ($n < 30$) then for applying parametric test we must require the assumption that the population is normal

We will cover following topics:

1. How to judge a given situation whether we should go for large sample test or not.
2. Applying the Z-test for testing the hypothesis about the population mean and difference of two population means.
3. Applying the Z-test for testing the hypothesis about the population proportion and difference of two population proportions.
4. Applying the Z-test for testing the hypothesis about the population variance and two population variances.

Large Sample Tests

As a thumb rule, a sample of size n is treated as a large sample only if it contains more than 30 units (or observations, $n > 30$). And we know that, for large sample ($n > 30$), one statistical fact is that almost all sampling distributions of the statistic(s) are closely approximated by the normal distribution. Therefore, the test statistic, which is a function of sample observations based on $n > 30$, could be assumed follow the normal distribution approximately (or exactly).

Z-test for testing the hypothesis about the population mean

For testing the null hypothesis, the test statistic Z is given by

$$Z = \frac{\bar{X} - E(\bar{X})}{SE(\bar{X})}$$

Case I: When σ^2 is known

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0, 1)$$

Case II: When σ^2 is unknown

$$Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim N(0,1)$$

Example: A light bulb company claims that the 100-watt light bulb it sells has an average life of 1200 hours with a standard deviation of 100 hours. For testing the claim 50 new bulbs were selected randomly and allowed to burn out. The average lifetime of these bulbs was found to be 1180 hours. Is the company's claim is true at 5% level of significance?

Solution: Here, we are given that
Specified value of population mean = mu_0 = 1200 hours,
Population standard deviation = σ = 100 hours,
Sample size = n = 50
Sample mean = X_bar = 1180 hours.

H_0 : mu_0 = 1200 (average life of a bulb is 1200 hours
H_1 : mu_0 != 1200 (average life of a bulb is not 1200 hours)

Thus, for testing the null hypothesis the test statistic is given by

$$Z = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$= \frac{1180 - 1200}{100 / \sqrt{50}} = \frac{-20}{14.14} = -1.41$$

The critical (tabulated) values for two-tailed test at 5% level of significance are $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$

Since calculated value of test statistic Z (= –1.41) is greater than critical value (= − 1.96) and less than the critical value (= 1.96), that means it lies in non rejection region, so we do not reject the null hypothesis.Since the null hypothesis is the claim so we support the claim at 5% level of significance.

TESTING OF HYPOTHESIS FOR DIFFERENCE OF TWO POPULATION MEANS USING Z-TEST

Example 3: In two samples of women from Punjab and Tamilnadu, the mean height of 1000 and 2000 women are 67.6 and 68.0 inches respectively. If population standard deviation of Punjab and Tamilnadu are same and equal to 5.5 inches then, can the mean heights of Punjab and Tamil Nadu women be regarded as same at 1% level of significance?