Answers to ADCS Subsystem Test

Team Anant Recruitment Test

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2. Reaching for the stars.

This question is essentially the concept behind a Hohmann transfer orbit[1].

We make the following assumptions in order to actually carry out this maneuver:

- Velocity changes to the aircraft occur instantaneously. - The assumption is valid since the burn time of the rocket will likely be much shorter than the period of the orbit[2].

Using these assumptions, and overall just using the conservation of energy, we can get the following formulae. (Ref. Figure 1)

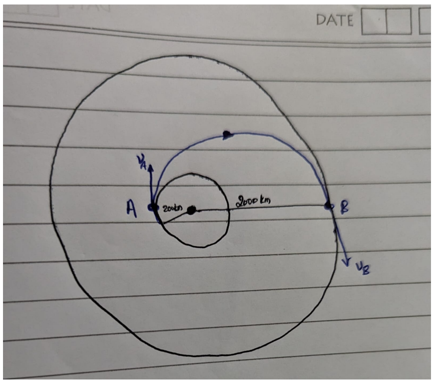

Figure 1:

Starting from conservation of energy of a small body m in elliptical orbit around a larger body M:

$$T.E = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$$

Now solving for v:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Also using the formulas $v_{orbit} = \sqrt{\frac{GM}{r}}$ and $a = \frac{r_1+r_2}{2}$ for orbit at a particular height, we can write the change in velocity at point A as:

$$\Delta v_A = \sqrt{GM\left(\frac{2}{r_1} - \frac{2}{r_1 + r_2}\right)} - \sqrt{\frac{GM}{r_1}}$$

$$\Rightarrow \Delta v_1 = \sqrt{\frac{GM}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right)$$

[1]Thanks to Dev in my SPARKLE project for telling me the name of this process lol

[2]https://dspace.mit.edu/bitstream/handle/1721.1/60691/16-07-fall-2004/contents/lecture-notes/d30.pdf

Now that we have the equation for Δv_1, we can plug in our numbers:

$$r_1 = 200 \times 10^3 + 6.378 \times 10^3 = 206.378 \times 10^6 m$$
$$r_2 = 2000 \times 10^3 + 6.378 \times 10^3 = 2006.378 \times 10^6 m$$
$$M = 5.972 \times 10^{24} kg$$

Giving us the equation:

$$\Delta v_1 = \sqrt{\frac{6.67 \times 10^{-11} \times 5.972 \times 10^{24}}{206.378 \times 10^6}} \left(\sqrt{\frac{2 \times 2006.378 \times 10^{24}}{(2006.378 + 206.378) \times 10^3}} - 1 \right)$$
$$\Rightarrow \Delta v_1 = 481.631 m/s$$

Bonus Question:

To calculate the braking Δv_2, we use:

$$\Delta v_2 = \sqrt{\frac{GM}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}} \right)$$
$$\Rightarrow \Delta v_2 = 789.257 m/s$$

4. Lost In The Gaps

1. **Given the first n measurements $x_1, x_2, \cdots, x_n$ estimate the true distance to the car.**

   Quite simply, we can use the mean:

   $$\bar{x} = \frac{\sum_{n=0}^{n=N} x_n}{N}$$

   However it may be wise to apply a filter and exclude the upper and lower 5% of all the readings obtained to reduce fluctuations. This will be discussed later.

2. **It is unfeasible to store data for long periods of time. Rewrite the obtained expression in (1) to use the estimate after $(n-1)^{th}$ measurement (say x) and the n^{th} measurement to obtain the new estimate.**

   Taking the previous mean to be $\bar{x}_{N-1}$, the current mean to be $\bar{x}_N$ and the incoming measurement to be x_N:

   $$\bar{x}_N = \frac{(\bar{x}_{N-1} \times N) + x_N}{N}$$

3. **Given the measurements, estimate the distance.**

   We will organize the measurements in descending order and apply a filter to remove the top and bottom 5%, as shown in the image below.

Raw Data	In Descending Order			
19.67	21.56		Top 5%	21.5465
21.53	21.53		Bottom 5%	18.6735
19.24	20.88			
20.04	20.45			
19.71	20.04			
20.88	19.95			
19.95	19.71			
18.21	19.67			
21.56	19.24			
20.45	18.21			

$$\bar{x}_N = 20.33m$$

4. **Given the first n measurements, estimate the (n + 1)th measurement. (You may use the result obtained in (2)).**

   To do this, we will first try to find the initial position and then use $x = vt$ to update subsequent positions.

$$x_n = x_{n-1} + vt$$

$$\sum_{i=0}^{i=N} x_i = x_1 + (x_1 + vt) + (x_1 + 2vt) + \cdots + (x_1 + (n-1)vt)$$

$$\sum_{i=0}^{i=N} x_i = nx_1 + \frac{(n-1)(n)}{2}vt$$

$$x_1 = \frac{\sum_{i=0}^{i=N} x_i - \frac{(n-1)(n)}{2}vt}{n}$$

   The final part of the above equation gives us a consistently updatable and relaible way to measure the original distance x_1, we can now write:

$$x_{n+1} = x_1 + nvt$$