

Anant 24-25 Recruitment Test

Attitude Determination and Controls Subsystem (ADCS)

January 12th, 2024

Instructions

All general instructions are provided in the Google Classroom. Make sure you refer to them for submission details. Failure to adhere to the given rules may result in disqualification.

1. This paper relies more on comprehension and general aptitude than it does in depth knowledge of the subject. You are encouraged to use the internet to solve this paper.
2. Any changes to the paper will be posted in Google Classroom and an updated file will be shared. Please keep an eye out.

Rule 1 : A student must prioritize developing an understanding of the topics being tested and demonstrating general aptitude, as evaluated during the subsequent interview where their solutions will be the main topic of discussion.

Rule 2 : A student must not collaborate with peers unless such collaboration is necessary to better achieve the goals outlined in Rule 1.

Rule 3 : A student must prioritize their physical and mental well-being, avoiding undue stress, all-nighters, or harmful study practices, unless doing so would conflict with Rules 1 or 2.

If you have any questions regarding the content of the paper, feel free to reach out to any of the undersigned for help.

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All the best!

Question 1: The Hunt for the Cosmic Chronic

Scene opens with Rick drunkenly scribbling on an old map while Morty nervously watches from the side.

Rick: “Morty! MOR-TY! Look at this piece of interstellar gold! Found it in some dusty ass drawer in my lab. Says here it leads to the best f***ing weed on Earth—some primo cosmic kush, Morty. None of that legal dispensary crap. We’re talking divine-grade ganja, Morty. D-I-V-I-N-E! Some cosmic entity mapped this out, pointing to $(3182.655, 2000.94, 5135.159)km$ back on January 2, 1920, at 1420 VLAT—don’t look at me like that, Morty, it’s UTC+10:00, you little idiot.”

Morty: “Wait, wait, what are you even talking about, Rick? Why does the map matter? Can’t we just, like, drive there?”

Rick: “Oh, yeah, sure, genius. Just f***ing drive there! Except this hill? Yeah, turns out it’s in the middle of nowhere, off-grid. And the f***ing coordinates? They’re in the Earth-Centered Earth-Fixed Frame. That’s right, Morty, we’re dealing with an Earth-Centric frame. Your midget-ass brain doesn’t even know what that is, does it?! ECEF is, uh... you know... fixed relative to the rotating Earth. It’s got an origin at Earth’s center of mass—big f***ing deal, right? You need to use this to figure out the latitude, longitude, and altitude of this hill. Oh, and while you’re at it, tell me what physical location on Earth this is! Chop-chop, Morty!”

(a) Locate the hill using the ECEF coordinates $(3182.655, 2000.94, 5135.159)km$. Find the latitude, longitude, and altitude. Also, determine the physical location (i.e., name of the place) on Earth.

(A few hours pass, Morty calculates furiously, then Rick comes back with a teleporter.)

Rick: “Good news, Morty! We don’t need to hoof it. We’ve got this sh***y divine teleporter that came with the map. Bad news? The lazy celestial a**hole that made it didn’t bother accounting for Earth’s rotation. So guess what? The m*****f***** teleporter only accepts coordinates in the Earth-Centered Inertial Frame. Yeah, that’s right, the ECI Frame, Morty. You know what ECI is? I’ll tell you what it f***ing is. It’s a frame fixed to the stars, unaffected by Earth’s spinning—like my tolerance for your stupidity. And, conveniently, ECI and ECEF align perfectly every day at 1200 GMT. That means the Prime Meridian and equator line up exactly at noon GMT like some kind of cosmic clockwork.”

Morty: “So, uh, what do you want me to do, Rick?”

Rick: “What do I f***ing want? I want you to convert the hill’s coordinates from ECEF to ECI for right f***ing now! Use today’s time, Morty, and don’t screw it up! We’ve got some intergalactic dankness waiting, and I’m not waiting around for your f***-ups.”

(b) Convert the hill’s position into the Earth-Centered Inertial Frame for today’s date and time. Use the assumption that ECEF and ECI align perfectly at 1200 GMT daily.

(Morty does the conversion, but when they teleport, they end up in deep space.)

Rick: “Oh, for the love of fu***** ch****t Morty! You worthless bag of human meat! What did you do? I told you to convert the coordinates to ECI, but you used Earth’s current position, didn’t you? Didn’t you?! Morty, you can’t just convert the coordinates like that! The Earth back then—yeah, you know, 1920, when the map was made—was in a different f***ing position in space! The ECI frame is anchored to the stars, Morty, not the planet spinning on its tiny-ass axis today.”

Morty: “I-I thought it wouldn’t matter, Rick! I mean, how different could it—”

Rick: “Shut up, Morty! I’ll explain, but only because I’m surrounded by idiots. The Earth’s position back then needs to be parameterized in its orbit around the Sun. Picture this, Morty: the Earth’s elliptical orbit, where the origin is at the center of the Sun. The z -axis is perpendicular to the orbital plane, pointing north, and the x -axis is toward the Sun at perihelion. Got it?”

Morty: “N-not really...”

Rick: “F*** you*! I’m not done! I’ll calculate Earth’s position back in 1920 and its position now, using celestial mechanics way above your comprehension level. Once I’ve got the Earth’s orientation in 1920, you’ll re-convert those coordinates to ECI from back then and input them into the teleporter! Otherwise, we’re gonna be floating in space until you die, and trust me, I’ve got better things to do!”

Rick: “And hurry up, Morty! If you f*** this up one more time, I’m dumping you on a random planet and smoking the whole stash myself!”

(c) Parameterize Earth’s position in its orbit for January 2nd, 1920 at 1420 VLAT (UTC+10:00). Re-convert the hill’s coordinates to the ECI frame of 1920 and input them into the teleporter.

Question 2 - F*** Physics, We're Smoking Stars!

(Rick and Morty are still floating in space after Morty's last screw-up. Rick is furiously tinkering with the teleporter, while Morty nervously looks at the flashing “ERROR: COORDINATE MISMATCH” on the screen.)

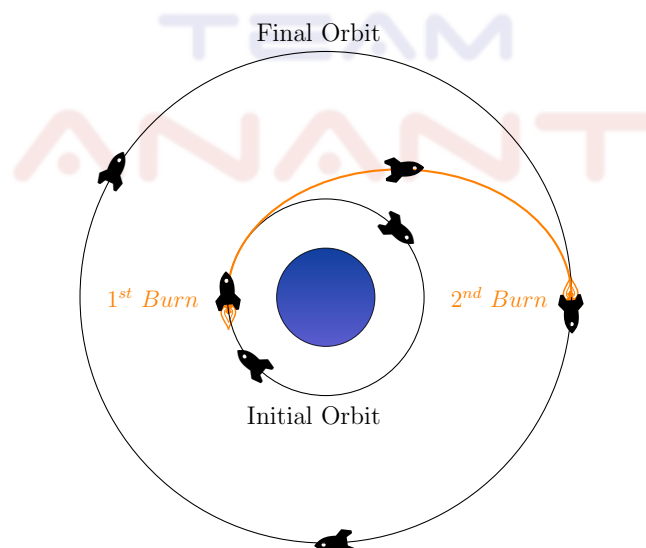
Rick: “Morty, I'm about two nanoseconds away from strapping you to this teleporter and using you as a human gravity anchor! Do you have any idea how badly you f***ed up?! We're stuck out here in the cosmic a**h*** of the universe, Morty, and guess what? YOU are gonna fix it!”

Morty: “B-but Rick, I don't even know what I did wrong! I just punched in the numbers like you said!”

Rick: “F***ing exactly! That's the problem, Morty—you're all punching and no thinking. Fine. You want a way back? You're doing orbital dynamics homework now, you little dipsh***. Here's your first problem!”

Rick: “Okay, listen up, Morty. There's this planet over there in a nice little circular orbit, and we're stuck in another circular orbit. Now, to get to that planet, we need to pull some fancy orbital maneuvering—real rocket scientist s***, Morty. No teleporters, no shortcuts, no bulls** magic m*****f*****. I want you to figure out the two burns we need to get there.”

(a) Figure out how much ΔV we need for both burns for going from an orbital altitude of 200km to 2000km.

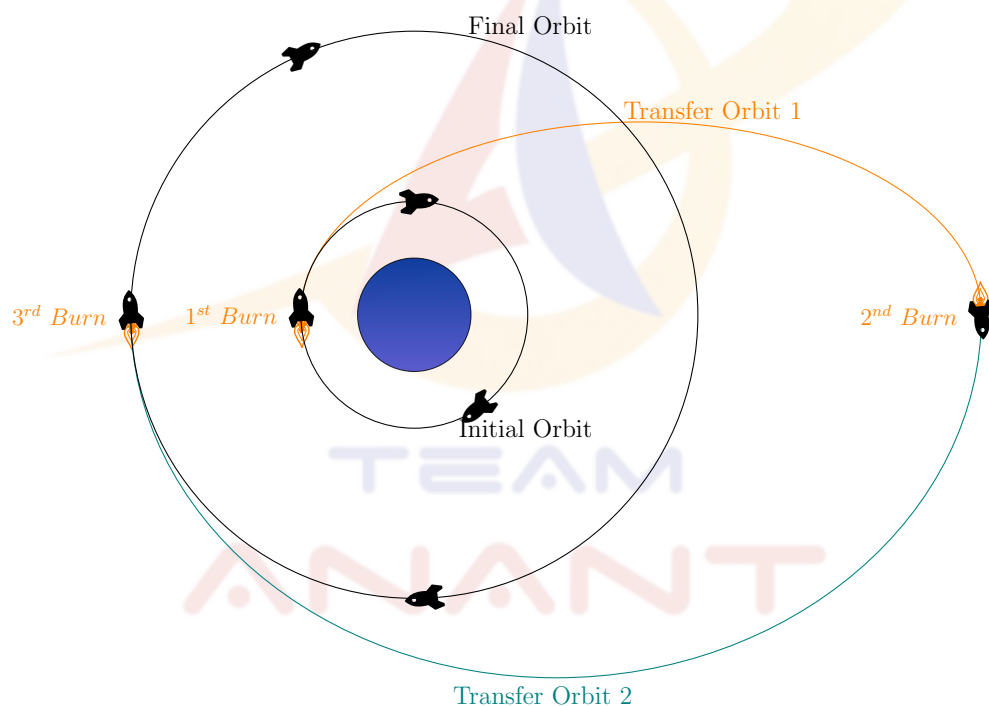


Rick: “Here's a little visual aid, Morty, since your brain can't handle more than two variables at once. Now get to work!”

Rick: “Alright, Morty, you figured out the baby orbit. Congrats, you’re officially smarter than a rock—but only barely. Here’s the kicker: sometimes, going way out of your way can actually save fuel. That’s right, Morty, instead of just hopping straight to the target, we’re gonna overachieve and swing out to a huge-ass intermediate orbit before dropping back down to the target. Think of it like a galactic victory lap, Morty—except it’s not for fun, it’s for fuel efficiency!”

Morty: “Wait, Rick, why would that even work? Doesn’t that waste more fuel?”

Rick: “Oh my god, Morty, why don’t you just tattoo ‘I’m stupid’ on your forehead and save me the trouble?! Look, sometimes going farther out gives you a better angle or less energy loss. I’m not here to explain the f***ing why—just calculate the burns! One to leave the starting orbit, one to enter the outer orbit, and one to finish up at the target. Here’s your hint, Morty! Three burns, three orbits. Go do the math while I sit here regretting my entire existence.”



(b) Figure out how much ΔV we need for three burns in this new transfer for going from an orbital altitude of 200km to 2000km. How is it different from the transfer in part (a) and why is it used?

(Morty finishes scribbling, types in some coordinates, and then slams a button. The ship alarms blare and Morty looks down at the button he pushed. It's bright red and marked **DO NOT PRESS**)

Morty: "Rick, what the hell?! What's happening now?!"

Rick: "Oh, great, Morty, just great! We're in another f***ed-up universe! You know how gravity works with $F = G \frac{m_1 m_2}{r^2}$? Yeah, well, not here, Morty! In this sh***y corner of the cosmos, gravity scales with r^{-3} . Imagine that, Morty! The universe decided, 'Hey, let's screw with physics just to piss Rick off!'"

Morty: "R-rick, how does that even work? What happens to the orbits?"

Rick: "F*** me, Morty, why don't you figure it out?! You can't just slap a little minus-two exponent on the force law and expect Kepler's laws to hold up. You need to re-derive everything—everything! What kind of orbits do we get with $F \propto r^{-3}$, Morty? Are they stable? Are they spirals? Do they explode like your last homework attempt? Go figure it out before I rip my hair out!"

(c) Figure out how this changes the orbits and the maneuvers required to transfer between them.

Rick: "If you fail at this, Morty, I swear to God, I'm teleporting you to a universe where you are the gravitational constant. Let's see how you like being orbited by rocks for eternity, you useless little s**!"

TEAM
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Question 3: Lost in Space, Screwed by Science (and Worm Goo)

(Morty is floating in space, clutching the half-broken GPS tracker that Rick gave him before they were separated. A flickering hologram of Rick is berating him for poking a dangerous glowing space worm.)

Morty: “Oh man, Rick! I-I don’t know where I am! One second we were fine, and then the worm sucked us into its dimension, and now I’m floating in space! HELP!”

Rick (Hologram): “First off, Morty, I TOLD YOU NOT TO TOUCH THE WORM! But nooo, you couldn’t resist. And now you’re stranded in deep space, tumbling around like a f***ing cosmic piñata. Great job, Morty. Just great.”

Morty: “I-I thought the worm was harmless, Rick! It was just sitting there glowing! You didn’t tell me it could explode into a vortex!”

Rick: “You’re gonna have to use that busted GPS I left you. It’s not much, but it’s all you’ve got. Alright, Morty, let’s start with something simple. The GPS tracker I gave you has a pinging system. It sends a signal to satellites, and they bounce it back. Basic f***ing physics. I gave you the coordinates of three satellites before you decided to turn a worm into a dimension grenade. Here they are:”

Satellite A: (-3141, -5926, 5358)

Satellite B: (-9793, 2384, -6264)

Satellite C: (3383, -2795, -0288)

Rick: “But here’s the thing: there’s a fourth satellite that I didn’t give you the coordinates for because—let’s be honest—you don’t deserve them. You’re gonna have to figure out its position. Lucky for you, I made sure none of the satellites are in the same octant. Oh, and for the missing satellite, symmetry is key—its x, y, z coordinates are probably all the same.”

Morty: “B-but Rick, how do I even know where I am?!”

Rick: “Oh my god, Morty, are you even listening? I gave you ping times for all four satellites! Here they are:”

$$t_A = 23.90987365 \text{ ms}, \quad t_B = 51.57336220 \text{ ms}, \quad t_C = 21.79227717 \text{ ms}, \quad t_D = 14.96656189 \text{ ms}.$$

(a) Use all of the given data to figure out two things: (i) where Morty is and (ii) where the fourth satellite is.

(After managing to calculate his position and the missing satellite's coordinates, Morty accidentally triggers the GPS's emergency protocol. The tracker sparks violently and blurts out: **“WORM CONTAMINATION DETECTED! INITIATING SELF-CLEANSE!”**)

Morty: “AHHHHH! Rick! The GPS is going crazy!”

Rick (Hologram): “Oh, brilliant, Morty. You managed to infect the GPS with interdimensional worm goo. That's a new low, even for you. Now it's purging itself, which means it's gonna wipe out half its functionality. Lucky for you, it's leaving just enough to save your ass. But here's the catch—you're only getting data from three satellites now. You'll have to make do!”

(The GPS fires a hidden rocket, sending Morty hurtling uncontrollably through space. It spits out two sets of pings before finally breaking down completely.)

Rick: “Okay, Morty, here's what you've got. The GPS managed to record pings from three satellites before it totally crapped out. Here's the data:”

$$\begin{aligned}A_1 &= 28.21941442 \text{ ms}, & A_2 &= 32.56521822 \text{ ms}, \\B_1 &= 37.40539321 \text{ ms}, & B_2 &= 37.48077648 \text{ ms}, \\C_1 &= 21.88477974 \text{ ms}, & C_2 &= 35.47581482 \text{ ms}.\end{aligned}$$

Rick: “Oh, and the timestamps? The first set is at $t_1 = 0$ s, and the second set is at $t_2 = 5000$ s. That's 5000 seconds, Morty, because worm goo screws with everything. Now, using just these three satellites, calculate your direction vector and velocity. Good luck, dumbass!”

(b) Figure out the velocity vector using the distance data given.

(After tumbling through space for what feels like an eternity, Morty finally crash-lands into Earth's orbit. He breathes a sigh of relief, but the GPS splits into two pieces—a sun sensor and a magnetic field sensor.)

Morty: “Oh man, Rick, the GPS is broken! What am I supposed to do now?!”

Rick (Hologram): “Oh my god, Morty, you're so useless! You've got a sun sensor, a magnetic field sensor, and a radio! Tape the sensors to your chest like the panicked idiot you are, and figure out your orientation relative to Earth. Then, send the data to ground control so they can save your sorry ass.”

(c) Figure out how to use the sun and magnetic sensors to figure out your orientation relative to Earth and include a sketch.

Question 4: The Final Puff

(Morty is finally in orbit around Earth, after a chaotic adventure filled with worm goo, broken GPS systems, and intergalactic stupidity. His GPS is down to its last feature—a distance tracker—and Rick’s hologram flickers into life.)

Morty: “Rick! I see Earth, but I don’t see the weed! I thought you said this was a treasure map or something! Where is it?!”

Rick (Hologram): “Oh my god, Morty. I didn’t just say it was a treasure map—I meant it! The weed is right there, Morty! Floating in orbit, just like you. But, surprise surprise, your dumb ass is too incompetent to find it. Lucky for you, I left you one last device that can measure its distance. So quit whining and use it!”

Morty: “I-I see it, Rick! It’s right there, floating in front of me! What do I do now?”

Rick: “What do you do now? You find the weed, Morty! Use the first n positions to estimate its true location. That’s right, Morty, this is math class now. Here’s the deal: you’ve got the weed’s positions at different times, like so:”

$$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$$

Rick: “Use this data to estimate where the weed actually is. And no, Morty, you can’t just write it down manually every time. That’s f***ing tedious, and data storage isn’t free! Find a better way to estimate the true position based on the last position and the new measurement. Figure it out!”

Rick: “Alright, Morty, let’s break this down. First, assume the weed is right in front of you and not moving. Use the first n positions to estimate where it is. If you don’t know how to do that, Morty, maybe just float there and cry about it.”

(a) Estimate the weed’s true position: Use the first n positions to calculate the weed’s actual location

Rick: “Obviously, noting down every single position like a human Excel sheet isn’t gonna cut it. What you need is a formula, Morty—something that updates the estimate dynamically. Use the k^{th} estimation and the $(k + 1)^{th}$ measurement to estimate the true position. Less writing, more science, Morty!”

(b) Derive a formula that uses the k^{th} estimation and the $(k + 1)^{th}$ measurement to refine the weed’s position.

Rick: “Now, let’s get specific. If these are the weed’s measurements over the first 5 seconds:”

$\{(-4.135, 2.81), (-4.859, -1.178), (-3.52, 3.551), (-4.543, 2.088), (-4.185, 2.736)$
 $(-4.993, 0.269), (-4.455, 2.27), (-1.566, 4.748), (-4.843, -1.243), (-4.848, 1.222)\}$

Rick: “Estimate the true position of the weed and calculate its distance from you. Don’t screw it up, Morty! That weed isn’t gonna smoke itself!”

(c) Use the provided measurements to estimate the weed’s true distance from Morty.

(Morty stares at the weed, and suddenly it starts drifting through space.)

Morty: “Rick! The weed is moving! It’s getting away!”

Rick: “Of course it’s moving, Morty! Did you think it was just gonna sit there waiting for you to catch it? The weed’s rotation is governed by some cosmic stoner physics, Morty. It’s drifting at an angular velocity of $\omega = \frac{2\pi}{14} \text{ rad/s}$. You know what that means? You’ve got to predict its next move! Using the first n measurements, estimate where the weed will be at $t = t_k$. And hurry up, Morty—I’m not waiting all day for you to figure this out.”

(d) Given the weed’s angular velocity $\omega = \frac{2\pi}{14} \text{ rad/s}$, estimate where it will be at $t = t_k$

Rick: “Alright, Morty, let’s get real. If these are the weed’s measurements over the first 5 seconds:”

$\{(-4.349, 2.468), (-4.789, 1.438), (-4.898, 1.003), (-4.741, 1.588), (-4.92, 0.893),$
 $(-4.92, -0.893), (-3.471, -3.598), (1.373, -4.808), (2.428, -4.371), (4.315, -2.526)\}$

Rick: “Where’s the weed gonna be at $t = 5.5 \text{ s}$, Morty? Get your act together and figure it out!”

(e) Predict the weed’s location at $t = 5.5 \text{ s}$ based on the provided measurements

(Morty finally calculates the weed’s position and snags it just before it drifts out of reach. He breathes a sigh of relief as Rick’s hologram flickers back on.)

Morty: “I-I did it, Rick! I got the weed! What do I do now?”

Rick: “What do you do now?! What do you THINK, Morty? You light it up and pass it to me! Oh wait, you can’t, because you’re still floating in space and I’m back on Earth! Guess I’ll

just have to enjoy the stash without you. See you when you get back, Morty—if you don't screw that up too!"

(Morty groans as the screen fades to black, with the sound of Rick laughing maniacally in the background.)

