Anant 24-25 Recruitment Test

Attitude Determination and Controls Subsystem (ADCS)

January 12th, 2024

Instructions

All general instructions are provided in the Google Classroom. Make sure you refer to them for submission details. Failure to adhere to the given rules may result in disqualification.

1. This paper relies more on comprehension and general aptitude than it does in depth knowledge of the subject. You are encouraged to use the internet to solve this paper.
2. Any changes to the paper will be posted in Google Classroom and an updated file will be shared. Please keep an eye out.

Rule 1 : A student must prioritize developing an understanding of the topics being tested and demonstrating general aptitude, as evaluated during the subsequent interview where their solutions will be the main topic of discussion.

Rule 2 : A student must not collaborate with peers unless such collaboration is necessary to better achieve the goals outlined in Rule 1.

Rule 3 : A student must prioritize their physical and mental well-being, avoiding undue stress, all-nighters, or harmful study practices, unless doing so would conflict with Rules 1 or 2.

If you have any questions regarding the content of the paper, feel free to reach out to any of the undersigned for help.

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All the best!

Question 1: The Hunt for the Cosmic Chronic

Scene opens with Rick drunkenly scribbling on an old map while Morty nervously watches from the side.

Rick: "Morty! MOR-TY! Look at this piece of interstellar gold! Found it in some dusty ass drawer in my lab. Says here it leads to the best f***ing weed on Earth—some primo cosmic kush, Morty. None of that legal dispensary crap. We're talking divine-grade ganja, Morty. D-I-V-I-N-E! Some cosmic entity mapped this out, pointing to $(3182.655, 2000.94, 5135.159)km$ back on January 2, 1920, at 1420 VLAT—don't look at me like that, Morty, it's UTC+10:00, you little idiot."

Morty: "Wait, wait, what are you even talking about, Rick? Why does the map matter? Can't we just, like, drive there?"

Rick: "Oh, yeah, sure, genius. Just f***ing drive there! Except this hill? Yeah, turns out it's in the middle of nowhere, off-grid. And the f***ing coordinates? They're in the Earth-Centered Earth-Fixed Frame. That's right, Morty, we're dealing with an Earth-Centric frame. Your midget-ass brain doesn't even know what that is, does it?! ECEF is, uh... you know... fixed relative to the rotating Earth. It's got an origin at Earth's center of mass—big f***ing deal, right? You need to use this to figure out the latitude, longitude, and altitude of this hill. Oh, and while you're at it, tell me what physical location on Earth this is! Chop-chop, Morty!"

(a) Locate the hill using the ECEF coordinates $(3182.655, 2000.94, 5135.159)km$. Find the latitude, longitude, and altitude. Also, determine the physical location (i.e., name of the place) on Earth.

(A few hours pass, Morty calculates furiously, then Rick comes back with a teleporter.)

Rick: "Good news, Morty! We don't need to hoof it. We've got this sh***y divine teleporter that came with the map. Bad news? The lazy celestial a**hole that made it didn't bother accounting for Earth's rotation. So guess what? The m*****f****** teleporter only accepts coordinates in the Earth-Centered Inertial Frame. Yeah, that's right, the ECI Frame, Morty. You know what ECI is? I'll tell you what it f***ing is. It's a frame fixed to the stars, unaffected by Earth's spinning—like my tolerance for your stupidity. And, conveniently, ECI and ECEF align perfectly every day at 1200 GMT. That means the Prime Meridian and equator line up exactly at noon GMT like some kind of cosmic clockwork."

Morty: "So, uh, what do you want me to do, Rick?"

Rick: "What do I f***ing want? I want you to convert the hill's coordinates from ECEF to ECI for right f***ing now! Use today's time, Morty, and don't screw it up! We've got some intergalactic dankness waiting, and I'm not waiting around for your f***-ups."

(b) Convert the hill's position intot he Earth-Centered Inertial Frame for today's date and time. Use the assumption that ECEF and ECI align perfectly at 1200 GMT daily.

(Morty does the conversion, but when they teleport, they end up in deep space.)

Rick: "Oh, for the love of fu***** ch***t Morty! You worthless bag of human meat! What did you do? I told you to convert the coordinates to ECI, but you used Earth's current position, didn't you? Didn't you?! Morty, you can't just convert the coordinates like that! The Earth back then—yeah, you know, 1920, when the map was made—was in a different f***ing position in space! The ECI frame is anchored to the stars, Morty, not the planet spinning on its tiny-ass axis today."

Morty: "I-I thought it wouldn't matter, Rick! I mean, how different could it—"

Rick: "Shut up, Morty! I'll explain, but only because I'm surrounded by idiots. The Earth's position back then needs to be parameterized in its orbit around the Sun. Picture this, Morty: the Earth's elliptical orbit, where the origin is at the center of the Sun. The z-axis is perpendicular to the orbital plane, pointing north, and the x-axis is toward the Sun at perihelion. Got it?"

Morty: "N-not really..."

Rick: "F*** you*! I'm not done! I'll calculate Earth's position back in 1920 and its position now, using celestial mechanics way above your comprehension level. Once I've got the Earth's orientation in 1920, you'll re-convert those coordinates to ECI from back then and input them into the teleporter! Otherwise, we're gonna be floating in space until you die, and trust me, I've got better things to do!"

Rick: "And hurry up, Morty! If you f*** this up one more time, I'm dumping you on a random planet and smoking the whole stash myself!"

(c) Parameterize Earth's position in its orbit for January 2nd, 1920 at 1420 VLAT (UTC+10:00). Re-convert the hill's coordiantes to the ECI frame of 1920 and input them into the teleporter.

Question 2 - F*** Physics, We're Smoking Stars!

(Rick and Morty are still floating in space after Morty's last screw-up. Rick is furiously tinkering with the teleporter, while Morty nervously looks at the flashing "ERROR: COORDINATE MISMATCH" on the screen.)

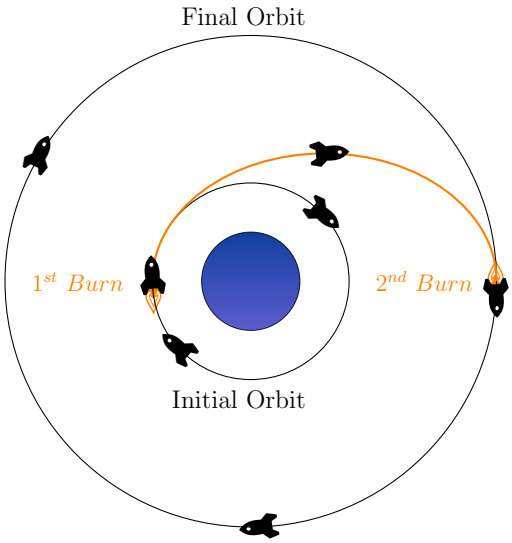
Rick: "Morty, I'm about two nanoseconds away from strapping you to this teleporter and using you as a human gravity anchor! Do you have any idea how badly you f***ed up?! We're stuck out here in the cosmic a**h*** of the universe, Morty, and guess what? YOU are gonna fix it!"

Morty: "B-but Rick, I don't even know what I did wrong! I just punched in the numbers like you said!"

Rick: "F***ing exactly! That's the problem, Morty—you're all punching and no thinking. Fine. You want a way back? You're doing orbital dynamics homework now, you little dipsh***. Here's your first problem!"

Rick: "Okay, listen up, Morty. There's this planet over there in a nice little circular orbit, and we're stuck in another circular orbit. Now, to get to that planet, we need to pull some fancy orbital maneuvering—real rocket scientist s***, Morty. No teleporters, no shortcuts, no bulls** magic m*****f*****. I want you to figure out the two burns we need to get there."

(a) Figure out how much ΔV we need for both burns for going from an orbital altitude of 200km to 2000km.

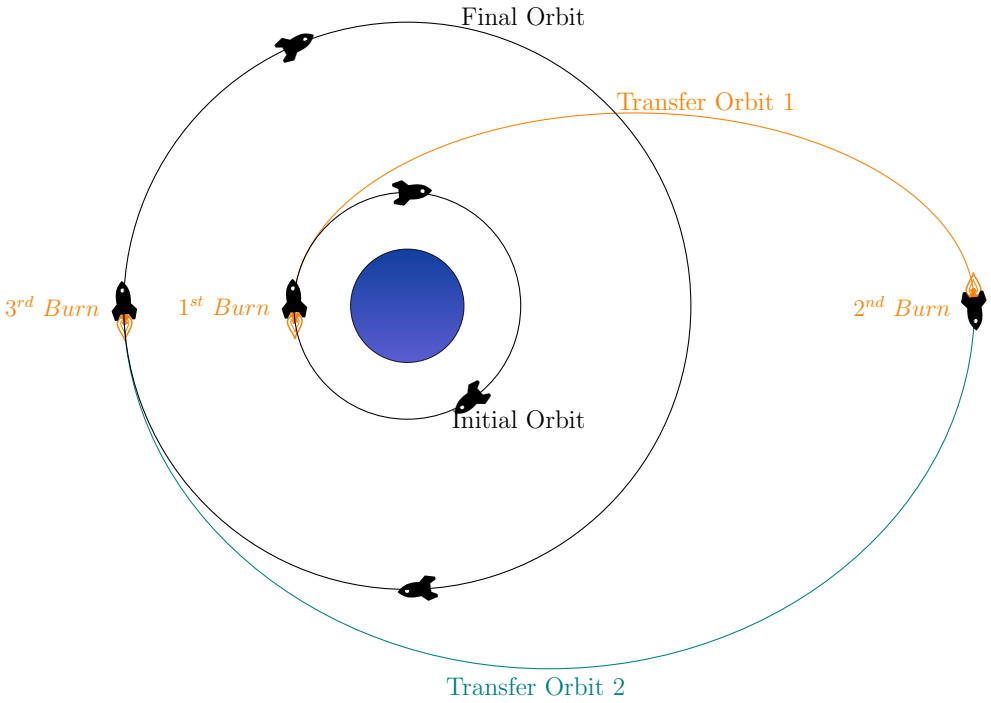


Rick: "Here's a little visual aid, Morty, since your brain can't handle more than two variables at once. Now get to work!"

Rick: "Alright, Morty, you figured out the baby orbit. Congrats, you're officially smarter than a rock—but only barely. Here's the kicker: sometimes, going way out of your way can actually save fuel. That's right, Morty, instead of just hopping straight to the target, we're gonna overachieve and swing out to a huge-ass intermediate orbit before dropping back down to the target. Think of it like a galactic victory lap, Morty—except it's not for fun, it's for fuel efficiency!"

Morty: "Wait, Rick, why would that even work? Doesn't that waste more fuel?"

Rick: "Oh my god, Morty, why don't you just tattoo 'I'm stupid' on your forehead and save me the trouble?! Look, sometimes going farther out gives you a better angle or less energy loss. I'm not here to explain the f***ing why—just calculate the burns! One to leave the starting orbit, one to enter the outer orbit, and one to finish up at the target. Here's your hint, Morty! Three burns, three orbits. Go do the math while I sit here regretting my entire existence."



(*b*) Figure out how much ΔV we need for three burns in this new transfer for going from an orbital altitude of 200km to 2000km. How is it different from the transfer in part (*a*) and why is it used?

(Morty finishes scribbling, types in some coordinates, and then slams a button. The ship alarms blare and Morty looks down at the button he pushed. It's bright red and marked DO NOT PRESS)

Morty: "Rick, what the hell?! What's happening now?!"

Rick: "Oh, great, Morty, just great! We're in another f***ed-up universe! You know how gravity works with $F = G\frac{m_1 m_2}{r^2}$? Yeah, well, not here, Morty! In this sh***y corner of the cosmos, gravity scales with r^{-3}. Imagine that, Morty! The universe decided, 'Hey, let's screw with physics just to piss Rick off!'"

Morty: "R-rick, how does that even work? What happens to the orbits?"

Rick: "F*** me, Morty, why don't you figure it out?! You can't just slap a little minus-two exponent on the force law and expect Kepler's laws to hold up. You need to re-derive everything—everything! What kind of orbits do we get with $F \propto r^{-3}$, Morty? Are they stable? Are they spirals? Do they explode like your last homework attempt? Go figure it out before I rip my hair out!"

(*c*) Figure out how this changes the orbits and the maneuvers required to transfer between them.

Rick: "If you fail at this, Morty, I swear to God, I'm teleporting you to a universe where you are the gravitational constant. Let's see how you like being orbited by rocks for eternity, you useless little s**!"

Question 3 - Trying to get back Home

Morty has now unfortuantely gotten separated from Rick in his travel through space. Before he got lost though, Rick gave him a GPS tracker. Although it's a little messed up because of his previous adventures, Morty is able to ping satellites and see the amount of time it takes for the signal to go from Morty to the satellite and back.
Rick however, is a little f*****. He's only given Morty 3 of the four satellite positions. They are:

$$A = \{-3141, -5926, 5358\}$$
$$B = \{-9793, 2384, -6264\}$$
$$C = \{3383, -2795, -288\}$$

(Note that the coordinates defined are in an absolute frame which we ignore for simplicity.)

(a) Morty's first job is to figure out where he is and where the fourth satellite is. He knows Rick is enough of an a**h*** to make sure that none of the satellites are in the same octant. He also knows that the x, y and z coordinates of the satellite are probably the same. The ping times off of the four satellites are:

$$t_A = 23.90987365ms$$
$$t_B = 51.57336220ms$$
$$t_C = 21.79227717ms$$
$$t_D = 14.96656189ms$$

(b) Congratulations on finding the fourth satellite, unfortunately Morty pressed the button labelled don't press, setting off a hidden rocket inside the GPS has suddenly gone off, sending Morty tumbling through the universe. Before the GPS dies, Morty gets two sets of pings off the satellites.

$$A_1 = 28.21941442ms \qquad A_2 = 32.56521822ms$$
$$B_1 = 37.40539321ms \qquad B_2 = 37.48077648ms$$
$$C_1 = 21.88477974ms \qquad C_2 = 35.47581482ms$$
$$D_1 = 7.603382343ms \qquad D_2 = 21.38749482ms$$

According to the clock, the first ping is at $t_1 = 0$ seconds, and the second is at $t_2 = 5000$ seconds (Rick is a b****). Find the direction vector and the velocity of Morty as he hurtles through space trying not to c*** his pants.

(c) Now, we'll hop into the theoretical world for a bit. Given the four satellites, the starting location and ending location, as well as the velocity that Morty is moving at, write out expressions that describe how much time a ping to a particular satellite might take. Assume that the velocity is in the order of $10^7 m/s$

(d) Morty has now landed in the orbit of Earth. His GPS splits into two pieces, a sun-sensor and a magnetic field sensor. Rick being Rick has also included a radio for Morty's use. However, for it to work, Morty needs to figure out his relative orientation to the planet. He bravely tapes the sensors to his chest pointing out, and he resigns himself to this horrible fate. Draw an approximate diagram with Morty, and show how he'd aptly describe his orienation to ground control.

Question 4 - Bob's Cat

His cat has suddenly joined him. Measurements are taken every 0.5 seconds.

(a) Given the first n positions $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$, estimate the true position of the cat.

(b) Obviously, noting down every position and finding the exact position every time is unfeasible (because data, duh). Give us a new expression that uses the $(n-1)_{th}$ estimation and the n_{th} measurement to estimate the true position of the cat.

(c) If the measurements for the first 5 seconds are:

$$1, 2, 3, 4, 5$$