Anant 24-25 Recruitment Test

Attitude Determination and Controls Subsystem (ADCS)

January 12th, 2024

Instructions

All general instructions are provided in the Google Classroom. Make sure you refer to them for submission details. Failure to adhere to the given rules may result in disqualification.

1. This paper relies more on comprehension and general aptitude than it does in depth knowledge of the subject. You are encouraged to use the internet to solve this paper.

2. Any changes to the paper will be posted in Google Classroom and an updated file will be shared. Please keep an eye out.

Rule 1 : A student must prioritise developing an understanding of the topics being tested and demonstrating general aptitude, as evaluated during the subsequent interview where their solutions will be the main topic of discussion.

Rule 2 : A student must not collaborate with peers unless such collaboration is necessary to better achieve the goals outlined in Rule 1.

Rule 3 : A student must prioritise their physical and mental well-being, avoiding undue stress, all-nighters, or harmful study practices, unless doing so would conflict with Rules 1 or 2.

This paper is the formal version, however, if you are looking for a fun and entertaining story to read alongside the test, you can find a story mode paper here.

If you have any questions regarding the content of the paper, feel free to reach out to any of the undersigned for help.

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All the best!

Question 1: Coordinate transformations and orbital mechanics

(a) Using the provided Earth-Centered Earth-Fixed (ECEF) coordinates of

$$(3182.655, 2000.94, 5135.159) \ km$$

calculate the corresponding latitude, longitude, and altitude of the point on Earth's surface. Additionally, determine the physical location (name of the place) on Earth corresponding to these coordinates.

(b) Convert the position from the Earth-Centered Earth-Fixed (ECEF) frame to the Earth-Centered Inertial (ECI) frame for today's date and time (mention the time and date you used). Assume that the ECEF and ECI frames align perfectly at 1200 GMT daily. [**Caution:** The assumptions in these questions overly simplify reality. They may not necessarily match other resources.]

(c) For the date January 3, 1920, at 1420 VLAT (UTC+10:00), parameterize the Earth's position in an elliptical orbit with:

   (i) The origin is the center of the orbital ellipse.

   (ii) The z-axis perpendicular to the orbital plane and the North pole lying in the positive z-half.

   (iii) The x-axis pointing toward the aphelion from the origin.

   Re-convert the current ECEF coordinates to the ECI frame of January 3, 1920, at 1420 VLAT (UTC+10:00).

Question 2: Orbital mechanics and gravitational anomalies

(a) Calculate the ΔV required for an orbit transfer maneuver as depicted in the image below to move a spacecraft from a circular orbit of 200 km altitude to a circular orbit of 2000 km altitude. Assume a central body with a radius of 6371 km and a standard gravitational parameter $\mu = 3.986 \times 10^{14}\,\mathrm{m^3/s^2}$.
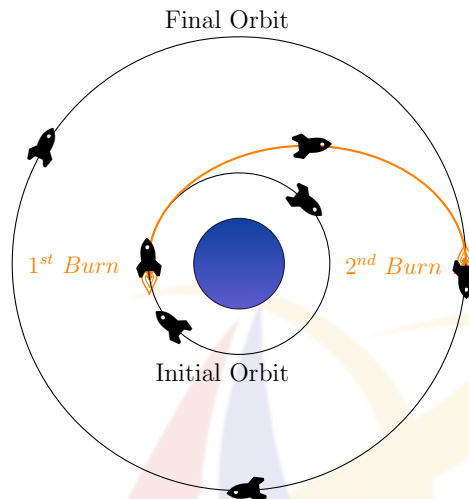
(b) Consider a two-stage transfer where the spacecraft first transitions to an intermediate high-altitude orbit before reaching the final circular orbit at 2800 km altitude. Ensure that the total ΔV remains minimum. Mention the value of the intermediate radius you used in your computations and the reason you chose this value. Calculate the total ΔV for this three-burn maneuver. Compare the result with the ΔV from part (a). Explain the circumstances under which this method might be preferred over the maneuver in part (a).
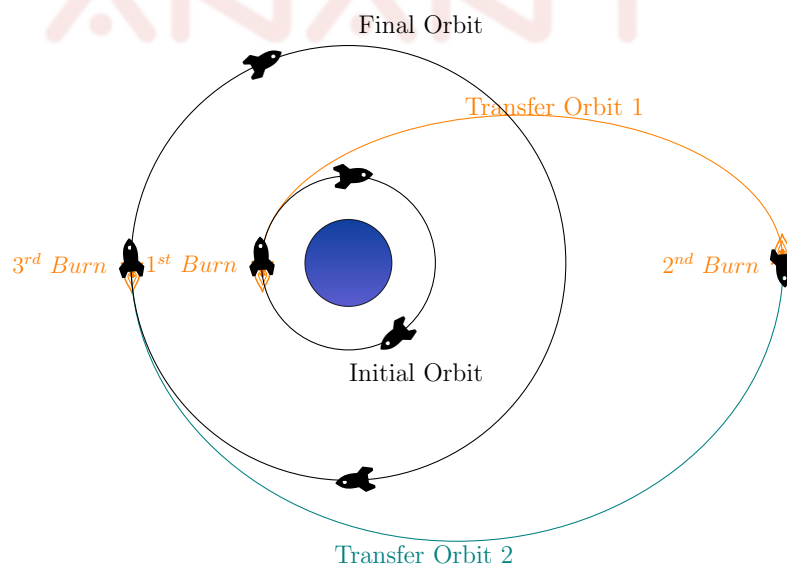
(c) In an alternate universe, the gravitational force is given by $F = G\frac{m_1 m_2}{r^3}$. Derive the resulting orbital characteristics under this new force law. Determine the shape and stability of the orbits and calculate how the maneuvers required for orbital transfers would change compared to the standard r^{-2} gravitational law.

Note: Assume standard units for mass, distance, and time throughout. Provide all derivations and necessary equations in your solutions.

Question 3: Positioning, velocity, and orientation in space

(a) A spacecraft in deep space is equipped with a GPS-like system that communicates with satellites. The coordinates of three satellites are provided:

$$\text{Satellite A: } (-3141, -5926, 5358)\,\text{km}$$
$$\text{Satellite B: } (-9793, 2384, -6264)\,\text{km}$$
$$\text{Satellite C: } (3383, -2795, -0288)\,\text{km}$$

The fourth satellite's coordinates are unknown but follow the symmetry rule: $x = y = z$. The signal travel times to the spacecraft from the four satellites are as follows:

$$t_A = 23.90987365\,\text{ms}, \quad t_B = 51.57336220\,\text{ms}, \quad t_C = 21.79227717\,\text{ms}, \quad t_D = 14.96656189\,\text{ms}. \tag{1}$$

Using this data, determine:

   (i) The coordinates of the fourth satellite. (Note that the none of the coordinates are greater than 1000 km)

   (ii) The spacecraft's position in Cartesian coordinates.

(b) After a system malfunction, the spacecraft can only receive signals from three satellites. The following ping times are recorded for two separate instances at timestamps $t_1 = 0\,\text{s}$ and $t_2 = 5000\,\text{s}$:

$$A_1 = 28.21941442\,\text{ms}, \qquad A_2 = 32.56521822\,\text{ms},$$
$$B_1 = 37.40539321\,\text{ms}, \qquad B_2 = 37.48077648\,\text{ms},$$
$$C_1 = 21.88477974\,\text{ms}, \qquad C_2 = 35.47581482\,\text{ms}.$$

Using this data, calculate all the possible spacecraft's velocity vector.

(c) The spacecraft's GPS system is damaged, leaving only a sun sensor, a magnetic field sensor. Determine how to use these sensors to calculate the spacecraft's orientation relative to Earth. Include a sketch in your solution showing how the sensors' data can provide the required orientation information.

Question 4: Position estimation and prediction

(a) A target object is observed at various positions over time. The first n observed positions are:

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n).$$

Using this data, estimate the target object's true position.

(b) Derive a formula that dynamically updates the estimated position of the target using the kth estimation and the $(k+1)^{th}$ measurement. Explain how the formula refines the estimate with each new measurement.

(c) Using the provided measurements below, estimate the true position of the target and calculate its distance from the observer:

$$\{(-4.135, 2.81), \quad (-4.859, -1.178), \quad (-3.52, 3.551), \quad (-4.543, 2.088), \quad (-4.185, 2.736),$$
$$(-4.993, 0.269), \quad (-4.455, 2.27), \quad (-1.566, 4.748), \quad (-4.843, -1.243), \quad (-4.848, 1.222)\}.$$

(d) The target object is observed to drift with an angular velocity ω. Using the first n measurements, estimate the target's position at a future time $t = t_k$ and the value of ω. Show your calculations clearly.

(e) Using the following measurements over the first 5 seconds:

$$\{(-4.349, 2.468), \quad (-4.789, 1.438), \quad (-4.898, 1.003), \quad (-4.741, 1.588), \quad (-4.92, 0.893),$$
$$(-4.92, -0.893), \quad (-3.471, -3.598), \quad (1.373, -4.808), \quad (2.428, -4.371), \quad (4.315, -2.526)\},$$

predict the target's position at $t = 5.5\,\text{s}$ based on the observed drift and angular velocity. Provide your prediction and explain your approach.

Bonus

We have multiple Easter Eggs hidden in the paper. List down your three favourite ones. :)