Department of Statistics
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Course: M.Sc. (Statistics)
Practical Sheet: ST-206(C)

Topic 1: Model sampling from bivariate distributions.

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1. Generate a random sample of size 1000 from bivariate Binomial distribution with the parameters $(n, p_1, p_2) = (10, \frac{1}{5}, \frac{2}{5})$. Hence obtain the following based on the sample data.
   (a) Obtain the plot of probability mass function of the above trinomial distribution along with the marginal probability mass function.
   (b) Obtain a three dimensional (3D) graph of the sample probabilities along with the marginal sample estimate of probabilities.
   (c) Calculate sample mean and variance covariance matrix. Compare these sample quantities with the mean and variance covariance matrix of the distribution.
   (d) Obtain a sample distribution of $X|Y = 6$ and $Y|X = 2$.
   (e) Calculate the mean and sample mean of $X|Y = 6$ and $Y|X = 2$.

2. Generate a random sample of size 1000 from bivariate Poisson distribution with the parameters $(\lambda_1, \lambda_2, \lambda_3)$, where $\lambda_3 < min(\lambda_1, \lambda_2)$. Hence obtain the following based on the sample data.
   (a) Obtain a three dimensional (3D) graph of the sample probabilities along with the marginal sample estimate of probabilities.
   (b) Calculate sample mean and variance covariance matrix.
   (c) Obtain a sample distribution of $X|Y = 4$ and $Y|X = 5$.
   (d) Calculate the mean and sample mean of $X|Y = 6$ and $Y|X = 2$.

3. Sketch the probability density function of bivariate normal distribution with following parameters simultaneously using subplot command and symbolic variables.
   (a) $(\mu_1 = 0, \mu_2 = 0, \sigma_1 = 1, \sigma_2 = 1, \rho = 0)$
   (b) $(\mu_1 = 0, \mu_2 = 0, \sigma_1 = 1, \sigma_2 = 1, \rho = -0.8)$
   (c) $(\mu_1 = 0, \mu_2 = 0, \sigma_1 = 1, \sigma_2 = 1, \rho = 0.8)$
   (d) $(\mu_1 = 0, \mu_2 = 0, \sigma_1 = 2, \sigma_2 = 3, \rho = 0)$
   (e) $(\mu_1 = 0, \mu_2 = 0, \sigma_1 = 2, \sigma_2 = 3, \rho = -0.6)$
   (f) $(\mu_1 = 0, \mu_2 = 0, \sigma_1 = 2, \sigma_2 = 3, \rho = 0.6)$

   Also obtain contours of the above probability density function and observe shape for various values of ρ.

4. Generate a random sample of size 2000 from bivariate Exponential distribution with the parameters $(\lambda_1, \lambda_2, \lambda_3)$. Hence obtain the following based on the sample data.
   (a) Calculate sample mean and variance covariance matrix.
   (b) Obtain the scatter plot of sample observations from individual random variables.

Topic 2: Independence of random variables and Distribution of linear combination of Multivariate normal variates.

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1. Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (-3,1,4)'$ and $\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Obtain variance covariance matrix between the following and check whether they are independently distributed or not.
   (a) $(X_1 + X_2)/2$ and X_3.
   (b) $(X_1 - X_2)$ and $(X_1 + X_2)$.
   (c) X_2 and $(-5/2X_1 + X_2 - X_3)$.
   (d) $\begin{pmatrix} X_1 + X_2 \\ 3X_1 - X_2 \end{pmatrix}$ and $\begin{pmatrix} X_1 \\ X_1 - X_2 \\ X_1 + 4X_2 - X_3 \end{pmatrix}$.

2. Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (2,-3,1)'$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$.Obtain the parameters of the distribution of $3X_1 - 2X_2 + X_3$.

3. Let $\underline{X} \sim N_4(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (4,3,2,1)'$ and $\Sigma = \begin{pmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{pmatrix}$.Partition $\underline{X}$ as $\underline{X}^{(1)} = (X_1, X_2)'$ and $\underline{X}^{(2)} = (X_3, X_4)'$. Let $A = (1,2)$ and $B = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ and consider the linear combination of $A\underline{X}^{(1)}$ and $B\underline{X}^{(2)}$. Find the following terms
   (a) $E(A\underline{X}^{(1)}), E(B\underline{X}^{(2)})$.
   (b) $cov(A\underline{X}^{(1)}), cov(B\underline{X}^{(2)})$.
   (c) $cov(\underline{X}^{(1)}, \underline{X}^{(2)}), cov(A\underline{X}^{(1)}, B\underline{X}^{(2)})$.
   (d) Obtain $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$ and $\Sigma_{22.1} = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$.

4. Solve the above example with A and B as $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$.

5. Let $\underline{X} \sim N_5(\underline{\mu}, \Sigma)$ with $\underline{\mu} = (2,4,-1,3,0)'$ and $\Sigma = \begin{pmatrix} 4 & -1 & 1/2 & -1/2 & 0 \\ -1 & 3 & 1 & 1 & 0 \\ 1/2 & 1 & 6 & 1 & -1 \\ -1/2 & 1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{pmatrix}$.
   Partition $\underline{X}$ as $\underline{X}^{(1)} = (X_1, X_2)'$ and $\underline{X}^{(2)} = (X_3, X_4, X_5)'$. Let $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$.
   Obtain the following terms
   (a) $E(A\underline{X}^{(1)}), E(B\underline{X}^{(2)})$. (b) $cov(A\underline{X}^{(1)}), cov(B\underline{X}^{(2)})$. (c) $cov(A\underline{X}^{(1)}, B\underline{X}^{(2)})$.

Topic 3: Model sampling from multivariate and Conditional multivariate Normal distribution and parameters estimation.

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1. Generate a random sample of size n from $N_3(\underline{0}, I_3)$.Hence obtain an unbiased estimators and MLE of mean vector and variance covariance matrix.

2. Let $\underline{X} \sim N_5(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (4.32, 14.01, 1.95, 2.17, 2.45)'$ and

$$\Sigma = \begin{pmatrix} 4.308 & 1.683 & 1.803 & 2.155 & -0.253 \\ 1.683 & 1.768 & 0.588 & 0.177 & 0.176 \\ 1.803 & 0.588 & 0.81 & 1.065 & -0.158 \\ 2.155 & 0.177 & 1.065 & 1.970 & -0.357 \\ -0.253 & 0.176 & -0.158 & -0.357 & 0.504 \end{pmatrix}.$$

   (a) Draw a random sample of size $n = 15$ from the $N_5(\underline{\mu}, \Sigma)$.

   (b) Calculate the sample mean, sample variance covariance matrix and sample correlation matrix.

   (c) Calculate correlation matrix of $\underline{X}$ and its MLE.

   (d) Obtain generalized sample variance, total sample variance and generalized sample variance of standardized variables.

3. Let $\underline{X} \sim N_6(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (9.47, 25.56, 13.25, 1.44, 27.29, 8.80)'$ and

$$\Sigma = \begin{pmatrix} 02.57 & 00.85 & 01.56 & 01.79 & 01.33 & 00.42 \\ 00.85 & 37.00 & 03.34 & 13.47 & 07.59 & 00.52 \\ 01.56 & 03.34 & 08.44 & 05.77 & 02.00 & 00.50 \\ 01.79 & 13.47 & 05.77 & 34.01 & 10.50 & 01.77 \\ 01.33 & 07.59 & 02.00 & 10.50 & 23.01 & 03.43 \\ 00.42 & 00.52 & 00.50 & 01.77 & 03.43 & 04.59 \end{pmatrix}$$

   (a) Partition $\underline{X} = (\underline{X}^{(1)}, \underline{X}^{(2)})$ as $\underline{X}^{(1)} = (X_2, X_4, X_6)', \underline{X}^{(2)} = (X_1, X_3, X_5)'$.Obtain $E(\underline{X}^{(1)}|\underline{X}^{(2)} = \underline{x}^{(2)})$ and $cov(\underline{X}^{(1)}|\underline{X}^{(2)} = \underline{x}^{(2)})$ with $\underline{x}^{(2)} = (2.15, 2.50, 3.50)'$.

   (b) Generate a random sample of size n from the distribution of $\underline{X}^{(1)}|\underline{X}^{(2)} = \underline{x}^{(2)}$ as in above case. Hence obtain an unbiased estimator of $E(\underline{X}^{(1)}|\underline{X}^{(2)} = \underline{x}^{(2)})$ and $cov(\underline{X}^{(1)}|\underline{X}^{(2)} = \underline{x}^{(2)})$ using the sample obtained.

   (c) Draw a random sample of size n from the conditional distribution of $\underline{X}^{(2)}|\underline{X}^{(1)} = \underline{x}^{(1)}$ and estimate $E(\underline{X}^{(2)}|\underline{X}^{(1)} = \underline{x}^{(1)})$ and $cov(\underline{X}^{(2)}|\underline{X}^{(1)} = \underline{x}^{(1)})$ where $\underline{x}^{(1)} = (4.5, 3.5, 2.5)'$. Also obtain the sample correlation matrix of $\underline{X}^{(2)}|\underline{X}^{(1)} = \underline{x}^{(1)}$.

Topic 4: Partial and multiple correlation coefficients.

1. Following is the random sample from $N_4(\underline{\mu}, \Sigma)$ obtained from an experiment. X_1 = Head length of first son, X_2 = Head breadth of first son, X_3 = Head length of second son, X_4 = Head breadth of second son.

Sample No.	X_1	X_2	X_3	X_4	Sample No.	X_1	X_2	X_3	X_4
1	191	155	179	145	14	190	159	195	157
2	195	149	201	152	15	188	151	187	158
3	181	148	185	149	16	163	137	161	130
4	183	153	188	149	17	195	155	183	158
5	176	144	171	142	18	186	153	173	148
6	208	157	192	152	19	181	145	182	148
7	189	150	190	149	20	175	140	165	137
8	197	159	189	152	21	192	154	185	152
9	188	152	197	159	22	174	143	178	147
10	192	150	187	151	23	176	139	176	143
11	179	158	176	148	24	197	167	200	158
12	183	147	174	147	25	190	163	187	150
13	174	150	185	152	26	195	159	168	139

   (a) Estimate the parameters of $N_4(\underline{\mu}, \Sigma)$.

   (b) Calculate the sample correlation coefficient matrix (R) by using MLE of Σ.

   (c) Calculate the MLE of multiple correlation coefficient of X_1 on $X_2X_3X_4$ and partial correlation coefficient of X_1, X_2 after eliminating effect of X_3 and X_4.

   (d) Obtain MLE of $\rho_{34.12}, \rho_{2.134}, \rho_{3.124}$.

2. Obtain a random sample of size $n = 30$ from $N_5(\underline{\mu} = \underline{0}, \Sigma)$ where

$$\Sigma = \begin{pmatrix} 4.308 & 1.683 & 1.803 & 2.155 & -0.253 \\ 1.683 & 1.768 & 0.588 & 0.177 & 0.176 \\ 1.803 & 0.588 & 0.81 & 1.065 & -0.158 \\ 2.155 & 0.177 & 1.065 & 1.970 & -0.357 \\ -0.253 & 0.176 & -0.158 & -0.357 & 0.504 \end{pmatrix}$$

   Obtain MLE of (a) Correlation coefficient matrix. (b) $\rho_{1.2345}, \rho_{2.1345}, \rho_{3.1245}, \rho_{12.345}, \rho_{45.123}$.

3. $\underline{X} = (X_1, X_2, X_3)' \sim N_3(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (5, 2, 0)'$ and $\Sigma = \begin{pmatrix} 10 & 1 & -1 \\ 1 & 7 & 3 \\ -1 & 3 & 2 \end{pmatrix}$. Determine the following

   (a) MLE of multiple correlation coefficient of (a) X_1 on X_2, X_3 (b) X_2 on X_1, X_3.

   (b) MLE of partial correlation coefficient of X_1, X_2 after eliminating effect of X_3.

Topic 5: Application of Hotelling T^2.

1. Let $\underline{X} \sim N_5(\underline{\mu}, \Sigma)$ where $\underline{\mu} = (4.32, 14.01, 1.95, 2.17, 2.45)'$ and

$$\Sigma = \begin{pmatrix} 4.308 & 1.683 & 1.803 & 2.155 & -0.253 \\ 1.683 & 1.768 & 0.588 & 0.177 & 0.176 \\ 1.803 & 0.588 & 0.81 & 1.065 & -0.158 \\ 2.155 & 0.177 & 1.065 & 1.970 & -0.357 \\ -0.253 & 0.176 & -0.158 & -0.357 & 0.504 \end{pmatrix}.$$

   (a) Draw a random sample of size 20 from the $N_5(\underline{\mu}, \Sigma)$.

   (b) Based on sample find MLE and unbiased estimates of μ, Σ.

   (c) Test the hypothesis $H_0 : \underline{\mu} = \underline{\mu}_0$ where $\underline{\mu}_0 = (5, 15, 2, 3, 2)'$.

   (d) Test the hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$. (Test of symmetry)

   (e) Test $H_0 : (\mu_3 + \mu_4) - (\mu_1 + \mu_2) = (\mu_1 + \mu_3) - (\mu_2 + \mu_4) = (\mu_1 + \mu_4) - (\mu_2 + \mu_3) = 0$.

2. A random sample of size 30 from $N_4(\underline{\mu}, \Sigma)$ distribution was obtained. The MLE of μ and Σ are given as:

$$\underline{\mu} = (6.833, 7.033, 5.967, 4.91)' \text{ and } \Sigma = \begin{pmatrix} 0.606 & 0.262 & 0.060 & 0.161 \\ 0.262 & 0.637 & 0.173 & 0.143 \\ 0.060 & 0.173 & 0.810 & 0.029 \\ 0.161 & 0.0143 & 0.029 & 0.306 \end{pmatrix}$$

   (a) Test the hypothesis $H_0 : \underline{\mu} = \underline{\mu}_0$ where $\underline{\mu}_0 = (6, 7, 5, 4)'$.

   (b) Test the hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$. (Test of symmetry)

3. In a study of human aging singer administered a battery of projective psychological test to a sample of elderly men $N_1 = 11$ men had a psychiatric diagnosis of a senile quantity, while $N_2 = 35$ men did not have that diagnosis. Five projective tests were selected for analysis of differences between the two diagnostic groups. The sample mean vectors $(\underline{\mu}_A, \underline{\mu}_B)$ of covariance matrix Σ are as below

Test	Group mean	
	Senile quantity ($\underline{\mu}_A$)	No senile quantity ($\underline{\mu}_B$)
Sentence completion	2.46	1.51
Emotional projection	4.46	2.26
Family scene	3.09	1.87
Problem situation	2.36	1.43
Thematic apperception test	3.27	2.00

$$S_{pooled} = \begin{pmatrix} 1.0789 & 0.3659 & 0.4910 & 0.3061 & 0.6054 \\ 0.3659 & 2.1685 & 0.7898 & 0.6892 & 0.9690 \\ 0.4910 & 0.7898 & 1.6466 & 0.5307 & 0.9256 \\ 0.3061 & 0.6892 & 0.5307 & 0.5708 & 0.5207 \\ 0.6054 & 0.9690 & 0.9256 & 0.5207 & 1.0950 \end{pmatrix}$$

   (a) Obtain Mahalanobis distance.

   (b) Test the hypothesis of equality of mean vectors of the two diagnosis populations.

Topic 6: Classification and Discriminant analysis.

1. Suppose we have two populations $N_5(\underline{\mu}^{(1)}, \Sigma)$ and $N_5(\underline{\mu}^{(2)}, \Sigma)$ where $\underline{\mu}^{(1)} = (4.32, 14.01, 1.95, 2.17, 2.45)', \underline{\mu}^{(2)} = (5.57, 11.38, 2.56, 1.98, 3.21)'$ and

$$\Sigma = \begin{pmatrix} 4.308 & 1.683 & 1.803 & 2.155 & -0.253 \\ 1.683 & 1.768 & 0.588 & 0.177 & 0.176 \\ 1.803 & 0.588 & 0.81 & 1.065 & -0.158 \\ 2.155 & 0.177 & 1.065 & 1.970 & -0.357 \\ -0.253 & 0.176 & -0.158 & -0.357 & 0.504 \end{pmatrix}.$$

   (a) Find the best discriminant function and classify the following observation $\underline{x} = (2.7565, 15.3698, 2.0145, 2.2134, 3.1254)'$.

   (b) Test whether $X_1 + X_2 - 2X_3 + X_4 - X_5$ is good discriminant function.

   (c) Test whether X_2 and X_5 provide any information for discriminant purpose. (Test whether X_1, X_3 and X_4 are sufficient for discrimination between two populations).

2. Consider two independent populations π_1 and π_2 with distribution $N_5(\mu_1, \Sigma)$ and $N_5(\mu_2, \Sigma)$ respectively. A random sample of size 20 from each population produced the summary statistics as: $\widehat{\mu}_1 = (368.2, 404.6, 502.8, 470.2)', \widehat{\mu}_2 = (428.1, 369.7, 572.8, 512.5)'$ and pooled matrix of corrected SS and SP is,

$$A = \begin{pmatrix} 2819.2 & 3568.4 & 2943.4 & 2295.3 \\ 3568.4 & 7963.1 & 5303.9 & 6851.3 \\ 2943.4 & 5303.9 & 6851.3 & 4499.6 \\ 2295.3 & 4065.9 & 4499.6 & 4878.9 \end{pmatrix}$$

   Find the best discriminant function and classify the following observation $\underline{x} = (1.03, 9.65, 0.45, 13.30)'$.

3. Consider the sample data available in file located at "c:\program files\files\mtbwin\Data \Exhmvar.mtw" for fresh water and marine for Alaskan and Canada.

   (a) Find the best discriminant function and classify the following observation $X_0 = (100.78, 140)'$.

   (b) Test whether $2X_1 - 3X_2$ is good discriminant function.

Topic 7: Principal component and canonical correlation.

1. Generate a random sample of size 50 from$N_8(\underline{0}, \Sigma)$ where

$$\Sigma = \begin{bmatrix} 1 & & & & & & & \\ 0.846 & 1 & & & & & & \\ 0.805 & 0.881 & 1 & & & & & \\ 0.859 & 0.826 & 0.801 & 1 & & & & \\ 0.473 & 0.376 & 0.380 & 0.436 & 1 & & & \\ 0.398 & 0.326 & 0.319 & 0.329 & 0.762 & 1 & & \\ 0.301 & 0.277 & 0.237 & 0.327 & 0.730 & 0.583 & 1 & \\ 0.382 & 0.415 & 0.345 & 0.365 & 0.629 & 0.577 & 0.539 & 1 \end{bmatrix}$$

1. (a) Obtain all the principal components for population and for sample generated.
   (b) Compute the percentage variation accounted by each principal component and hence represent the information in screen plot.

2. Determine first two pairs of canonical variates between $(X_1, X_2)'$ and $(X_3, X_4)'$ for variance covariance matrix.

$$\Sigma = \begin{bmatrix} 8 & 2 & 3 & 1 \\ 2 & 5 & -1 & 3 \\ 3 & -1 & 6 & -2 \\ 1 & 3 & -2 & 7 \end{bmatrix}$$

1. The following characteristics for 14 census tracts are recorded.

Total population (Pop) Median years of schooling (School)
Total employment (Employ) Employment in health services (Health)
Median home value (Home)
The data is in c:\program files\mtbwin\data\exhmvar.mtw. Perform principal components analysis and draw appropriate conclusions.

1. Calculate canonical correlations. Determine pair of canonical variates between $(X_1, X_2)'$and$(X_3, X_4, X_5)'$ for the variance covariance matrix

$$\Sigma = \begin{bmatrix} 4308 & 1.683 & 1.803 & 2.155 & -0.253 \\ 1.683 & 1.768 & 0.588 & 0.177 & 0.176 \\ 1.803 & 0.588 & 0.81 & 1.065 & -0.158 \\ 2.155 & 0.177 & 1.065 & 1.970 & -0.357 \\ -0.253 & 0.176 & -0.158 & -0.357 & 0.504 \end{bmatrix}$$

Topic 8: MANOVA

1. Consider the samples of size $n_1 = 3,\ n_2 = 2\ and\ n_3 = 3$ from $N_2\left(\underline{\mu}^{(i)}, \Sigma\right),\ i = 1,2,3$ as given below:

$$G_1 : \begin{bmatrix} 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 9 \\ 2 \end{bmatrix} G_2 : \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} G_3 : \begin{bmatrix} 3 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

2. Obtain the matrix of sum of squares and cross products for treatments $\left(\underline{\tau}_1, \underline{\tau}_2, \underline{\tau}_3\right)$ and residuals (B and W).

3. Verify that $B + W = T$, where T is matrix of ss and sc correlation to total.

4. Test the hypothesis $H_0 : \underline{\mu}^{(1)} = \underline{\mu}^{(2)} = \underline{\mu}^{(3)}$.

5. The Wisconsin Department of health and social services collected data on 4-variables for 3-groups as (private, non-profit and government) where X_1, X_2, X_3, X_4 are

$$X_1 = \text{cost of nursing labor} X_3 = \text{cost of plant operation and maintain labor}$$

$$X_2 = \text{cost of dietary labor} X_4 = \text{cost of housekeeping and laundry}$$

The summary statistic for each group is given:

Group	No. of Observations	Sample Mean	Sample Covariance Matrix
Private	275	(2.066,0.480,0.082,0.360)'	$S_1 = \begin{bmatrix} 0.291 & -0.001 & 0.002 & 0.01 \\ -0.001 & 0.011 & 0.000 & 0.003 \\ 0.002 & 0.000 & 0.001 & 0.000 \\ 0.01 & 0.003 & 0.000 & 0.01 \end{bmatrix}$
Non-profit	138	(2.167,0.596,0.124,0.418)'	$S_2 = \begin{bmatrix} 0.561 & 0.011 & 0.001 & 0.037 \\ 0.011 & 0.025 & 0.004 & 0.007 \\ 0.001 & 0.004 & 0.005 & 0.002 \\ 0.037 & 0.007 & 0.002 & 0.079 \end{bmatrix}$
Government	107	(2.273,0.521,0.125,0.383)'	$S_3 = \begin{bmatrix} 0.261 & 0.030 & 0.003 & 0.018 \\ 0.030 & 0.017 & 0.000 & 0.006 \\ 0.003 & 0.000 & 0.004 & 0.001 \\ 0.018 & 0.006 & 0.001 & 0.013 \end{bmatrix}$

Test the hypothesis $H_0 : \underline{\mu}_1 = \underline{\mu}_2 = \underline{\mu}_3$.

1. Perform MANOVA for following data.

$$T_1 : \begin{bmatrix} 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \end{bmatrix} T_2 : \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} T_3 : \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

1. Consider the observations on two responses x_1 and $x_2$as follows

$$\text{Factor 1} \quad \begin{array}{ll} \text{Level 1} & \begin{bmatrix} 6 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix} \\ \text{Level 2} & \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ -5 \end{bmatrix} \\ \text{Level 3} & \begin{bmatrix} 8 \\ 12 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \end{bmatrix} \\ \text{Level 4} & \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ -6 \end{bmatrix} \end{array}$$

2. Verify the following.

$$\text{SCP}_{\text{tot}} = \text{SCP}_{\text{mean}} + \text{SCP}_{\text{FA}} + \text{SCP}_{\text{RES}}$$

3. Construct one way MANOVA table using information in (a).

4. Evaluate Wilks Λ, Λ^*.

5. Conduct the test for $H_0 : \underline{\tau}_1 = \underline{\tau}_2 = \underline{\tau}_3$.