SUBMITTED BY:	Prachi Aggarwal
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TOPIC:	Practical File

INDEX

Q1) Solution of first order differential equation.

xy’ = (y/x)^3+y

```
In[5]:= eq1 = DSolve[x * y'[x] == (y[x] / x) ^ 3 + y[x], y[x], x]
```

Out[5]= $\left\{\left\{y[x] \to -\frac{x^{3/2}}{\sqrt{2 + x\, c_1}}\right\}, \left\{y[x] \to \frac{x^{3/2}}{\sqrt{2 + x\, c_1}}\right\}\right\}$

```
In[6]:= gs = y[x] /. eq1
```

Out[6]= $\left\{-\frac{x^{3/2}}{\sqrt{2 + x\, c_1}}, \frac{x^{3/2}}{\sqrt{2 + x\, c_1}}\right\}$

xy’ = x tan(y/x)+y

```
In[7]:= eq1 = DSolve[x * y'[x] == x * Tan[y[x] / x] + y[x], y[x], x]
```

Solve : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete

Out[7]= $\{\{y[x] \to x\, \mathrm{ArcSin}[e^{c_1} x]\}\}$

```
In[8]:= gs = y[x] /. eq1
```

Out[8]= $\{x\, \mathrm{ArcSin}[e^{c_1} x]\}$

y’ = e^(x-y)+x^2e^(-y)

```
In[9]:= eq1 = DSolve[y'[x] == Exp[x - y[x]] + (x ^ 2) * Exp[-y[x]], y[x], x]
```

Solve : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for com

Out[9]= $\left\{\left\{y[x] \to \mathrm{Log}\left[e^x + \frac{x^3}{3} + c_1\right]\right\}\right\}$

```
In[10]:= gs = y[x] /. eq1
```

Out[10]= $\left\{\mathrm{Log}\left[e^x + \frac{x^3}{3} + c_1\right]\right\}$

first

y’ + 3x^2 y = x^2 where y(0) = 2

```
In[11]:= eq1 = DSolve[y'[x] + 3 * (x^2) * y[x] == x^2, y[x], x]
```

Out[11]= $\left\{\left\{y[x] \to \frac{1}{3} + e^{-x^3} c_1\right\}\right\}$

```
In[12]:= eqSolve = DSolve[{y'[x] + 3 * (x^2) * y[x] == x^2, y[0] == 2}, y[x], x]
```

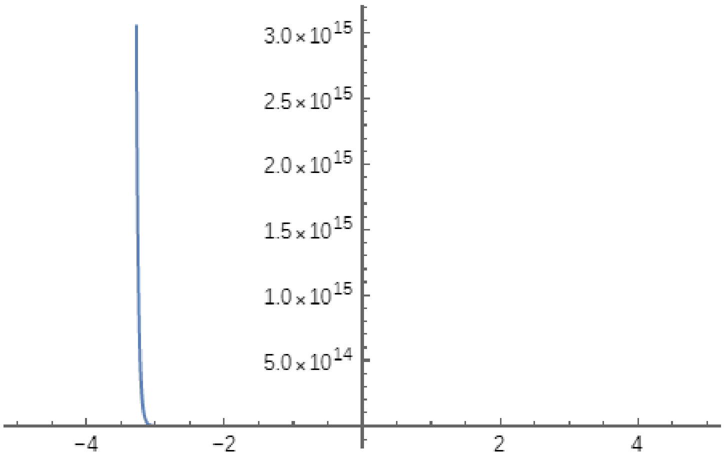
Out[12]= $\left\{\left\{y[x] \to \frac{1}{3} e^{-x^3} \left(5 + e^{x^3}\right)\right\}\right\}$

```
In[13]:= gs = y[x] /. eqSolve
```

Out[13]= $\left\{\frac{1}{3} e^{-x^3} \left(5 + e^{x^3}\right)\right\}$

```
In[14]:= Plot[gs, {x, -5, 5}]
```

Out[14]=



4xyy’ = y^2 +1, where y(1)=1

```
In[18]:= eq1 = DSolve[4 * x * y[x] * y'[x] == y[x]^2 + 1, y[x], x]
```

Out[18]= $\left\{\left\{y[x] \to -\sqrt{-1 + e^{2 c_1} \sqrt{x}}\right\}, \left\{y[x] \to \sqrt{-1 + e^{2 c_1} \sqrt{x}}\right\}\right\}$

```
In[19]:= eqSolve = DSolve[{4 * x * y[x] * y'[x] == y[x]^2 + 1, y[1] == 1}, y[x], x]
```

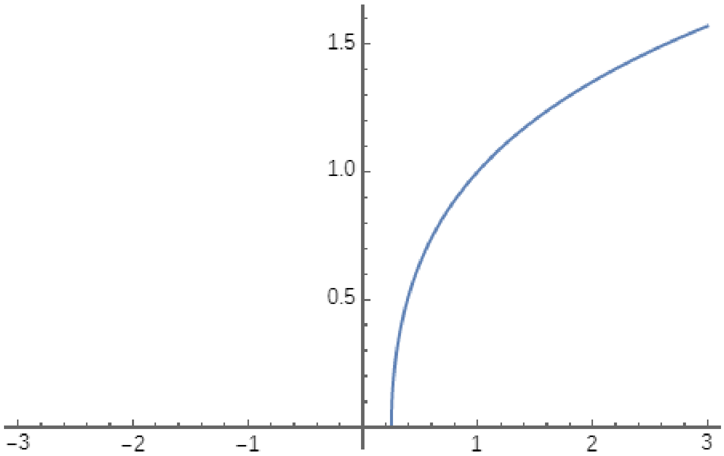
Out[19]= $\left\{\left\{y[x] \to \sqrt{-1+2\sqrt{x}}\right\}\right\}$

In[20]:= `gs = y[x] /. eqSolve`

Out[20]= $\left\{\sqrt{-1+2\sqrt{x}}\right\}$

In[21]:= `Plot[gs, {x, -3, 3}]`

Out[21]=



x^2y' + xy = y^3/x, where y(1)=1

In[22]:= eq1 = DSolve[x^2 * y'[x] + x * y[x] == y[x]^3/x, y[x], x]

Out[22]= $\left\{\left\{y[x] \to -\frac{\sqrt{2}\, x}{\sqrt{1+2 x^4 \mathbb{C}_1}}\right\}, \left\{y[x] \to \frac{\sqrt{2}\, x}{\sqrt{1+2 x^4 \mathbb{C}_1}}\right\}\right\}$

In[23]:= eqSolve = DSolve[{x^2 * y'[x] + x * y[x] == y[x]^3/x, y[1] == 1}, y[x], x]

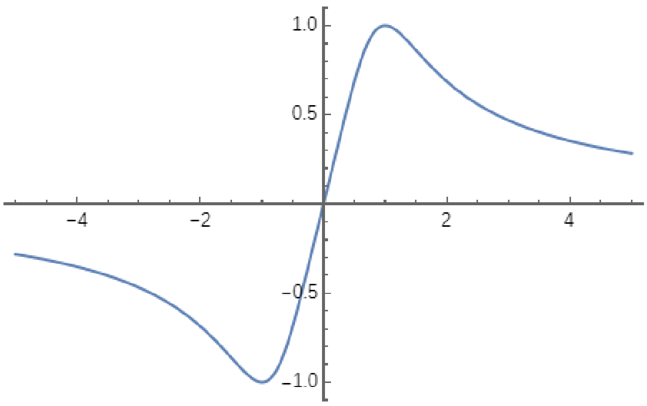
DSolve : For some branches of the general solution, the given boundary conditions lead to an empty solution.

Out[23]= $\left\{\left\{y[x] \to \frac{\sqrt{2}\, x}{\sqrt{1+x^4}}\right\}\right\}$

In[24]:= gs = y[x] /. eqSolve

Out[24]= $\left\{\frac{\sqrt{2}\, x}{\sqrt{1+x^4}}\right\}$

In[25]:= Plot[gs, {x, -5, 5}]

Out[25]=


Q2) Plotting of second order solution family of differential equation.

y'' – 6y' + 5y == 0 where y'(0)=1 and y(0)=0

In[26]:= `eq1 = DSolve[{y''[x] - 6 * y'[x] + 5 * y[x] == 0}, y[x], x]`

Out[26]= $\{\{y[x] \to e^{x} c_1 + e^{5x} c_2\}\}$

In[29]:= `eqSolve = DSolve[{y''[x] - 6 * y'[x] + 5 * y[x] == 0, y'[0] == 1, y[0] == 0}, y[x], x]`

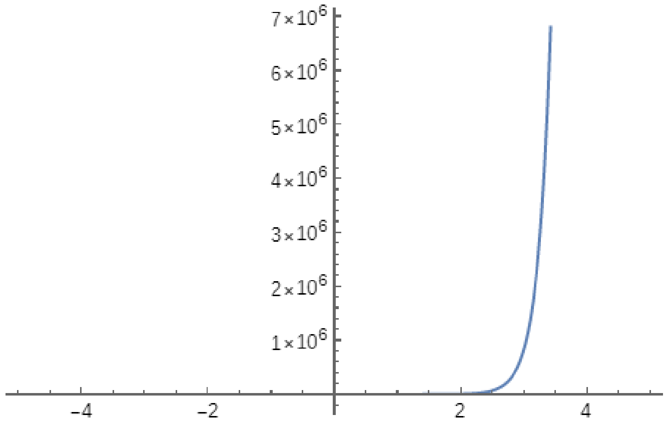
Out[29]= $\left\{\left\{y[x] \to \frac{1}{4} e^{x} \left(-1 + e^{4x}\right)\right\}\right\}$

In[30]:= `gs = y[x] /. eqSolve`

Out[30]= $\left\{\frac{1}{4} e^{x} \left(-1 + e^{4x}\right)\right\}$

In[31]:= `Plot[gs, {x, -5, 5}]`

Out[31]=



y'' + 2y' – 25y = 0, y(0)=3, y'(1)=3

In[1]:= `eq1 = DSolve[{y''[x] + 2 * y'[x] - 25 * y[x] == 0}, y[x], x]`

Out[1]= $\{\{y[x] \to e^{(-1-\sqrt{26})x} c_1 + e^{(-1+\sqrt{26})x} c_2\}\}$

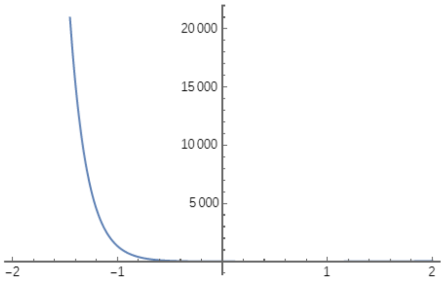
In[2]:= `eqsolve = DSolve[{y''[x] + 2 * y'[x] - 25 * y[x] == 0, y'[1] == 3, y[0] == 3}, y[x], x]`

Out[2]= $$\left\{\left\{y[x] \to \frac{3\left(e^{(-1+\sqrt{26})x} + \sqrt{26}\, e^{(-1+\sqrt{26})x} - e^{1+\sqrt{26}+(-1-\sqrt{26})x} - e^{2\sqrt{26}+(-1-\sqrt{26})x} + \sqrt{26}\, e^{2\sqrt{26}+(-1-\sqrt{26})x} + e^{1+\sqrt{26}+(-1+\sqrt{26})x}\right)}{1+\sqrt{26} - e^{2\sqrt{26}} + \sqrt{26}\, e^{2\sqrt{26}}}\right\}\right\}$$

In[3]:= `gs = y[x] /. eqsolve`

Out[3]= $$\left\{\frac{3\left(e^{(-1+\sqrt{26})x} + \sqrt{26}\, e^{(-1+\sqrt{26})x} - e^{1+\sqrt{26}+(-1-\sqrt{26})x} - e^{2\sqrt{26}+(-1-\sqrt{26})x} + \sqrt{26}\, e^{2\sqrt{26}+(-1-\sqrt{26})x} + e^{1+\sqrt{26}+(-1+\sqrt{26})x}\right)}{1+\sqrt{26} - e^{2\sqrt{26}} + \sqrt{26}\, e^{2\sqrt{26}}}\right\}$$

In[4]:= `Plot[gs, {x, -2, 2}]`

Out[4]=


y’’-y’+y=0

In[5]:= `eq1 = DSolve[{y''[x] - y'[x] + y[x] == 0}, y[x], x]`

Out[5]= $\left\{\left\{y[x] \to e^{x/2} c_1 \operatorname{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + e^{x/2} c_2 \operatorname{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]\right\}\right\}$

In[7]:= `esolve = y[x] /. eq1[[1]] /. {C[1] → 2, C[2] → 3}`

Out[7]= $2\, e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + 3\, e^{x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]$

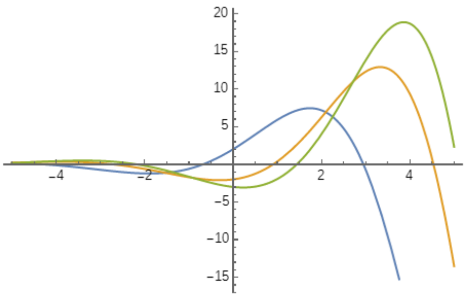
In[9]:= `esolve2 = y[x] /. eq1[[1]] /. {C[1] → -2, C[2] → 2}`

Out[9]= $-2\, e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + 2\, e^{x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]$

In[10]:= `esolve3 = y[x] /. eq1[[1]] /. {C[1] → -3, C[2] → 1}`

Out[10]= $-3\, e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + e^{x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]$

In[12]:= `Plot[{esolve, esolve2, esolve3}, {x, -5, 5}]`

Out[12]=


y’’+y==4

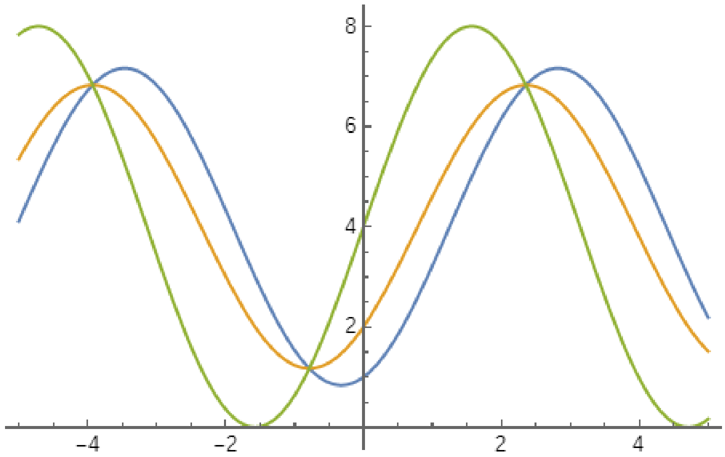
```
In[16]:= eq1 = DSolve[{y''[x] + y[x] == 4}, y[x], x]
Out[16]= {{y[x] → 4 + C1 Cos[x] + C2 Sin[x]}}

In[17]:= eqsol = y[x] /. eq1[[1]] /. {C[1] → -3, C[2] → 1}
Out[17]= 4 - 3 Cos[x] + Sin[x]

In[18]:= eqsol2 = y[x] /. eq1[[1]] /. {C[1] → -2, C[2] → 2}
Out[18]= 4 - 2 Cos[x] + 2 Sin[x]

In[19]:= eqsol3 = y[x] /. eq1[[1]] /. {C[1] → 0, C[2] → 4}
Out[19]= 4 + 4 Sin[x]

In[20]:= Plot[{eqsol, eqsol2, eqsol3}, {x, -5, 5}]
Out[20]=
```



y''-2y-6=0 where y'(0)=2 and y(0)=1

In[21]:= eq1 = DSolve[{y''[x] - 2 * y[x] - 6 == 0}, y[x], x]

Out[21]= $\{\{y[x] \to -3 + e^{\sqrt{2}x} c_1 + e^{-\sqrt{2}x} c_2\}\}$

In[23]:= eSS = DSolve[{y''[x] - 2 * y[x] - 6 == 0, y'[0] == 2, y[0] == 1}, y[x], x]

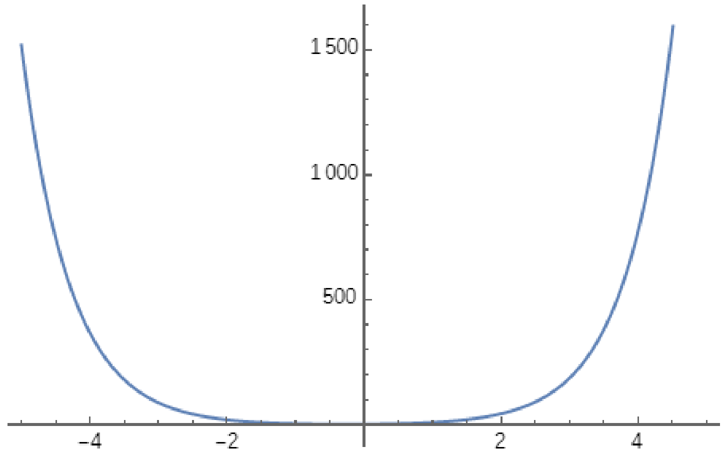
Out[23]= $\left\{\left\{y[x] \to \frac{1}{2} e^{-\sqrt{2}x} \left(4 - \sqrt{2} - 6 e^{\sqrt{2}x} + 4 e^{2\sqrt{2}x} + \sqrt{2} e^{2\sqrt{2}x}\right)\right\}\right\}$

In[24]:= gs = y[x] /. eSS

Out[24]= $\left\{\frac{1}{2} e^{-\sqrt{2}x} \left(4 - \sqrt{2} - 6 e^{\sqrt{2}x} + 4 e^{2\sqrt{2}x} + \sqrt{2} e^{2\sqrt{2}x}\right)\right\}$

In[26]:= Plot[gs, {x, -5, 5}]

Out[26]=



2y''+2y'-y=0

In[27]:= `eq1 = DSolve[{2*y''[x] + 2*y'[x] - y[x] == 0}, y[x], x]`

Out[27]= $\left\{\left\{y[x] \to e^{\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x} \mathbb{c}_1 + e^{\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)x} \mathbb{c}_2\right\}\right\}$

In[28]:= `eqsol = y[x] /. eq1[[1]] /. {C[1] → -1, C[2] → 3}`

Out[28]= $-e^{\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x} + 3\, e^{\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)x}$

In[29]:= `eqsol2 = y[x] /. eq1[[1]] /. {C[1] → -4, C[2] → 2}`

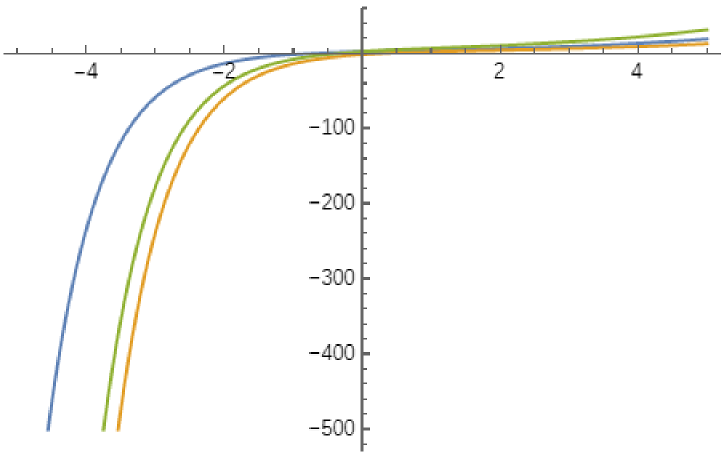
Out[29]= $-4\, e^{\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x} + 2\, e^{\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)x}$

In[30]:= `eqsol3 = y[x] /. eq1[[1]] /. {C[1] → -3, C[2] → 5}`

Out[30]= $-3\, e^{\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x} + 5\, e^{\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)x}$

In[31]:= `Plot[{eqsol, eqsol2, eqsol3}, {x, -5, 5}]`

Out[31]=



Q3) Plotting of third order solution family of differential equation.

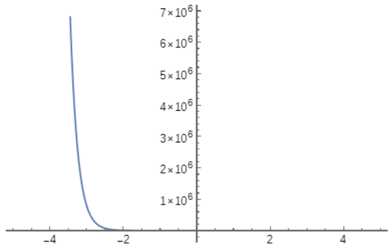
y'''+5y''+2y'+10y=0 where y''(0)=10, y'(0)=3,y(0)=0

```
In[32]:= eq1 = DSolve[{y'''[x] + 5 * y''[x] + 2 * y'[x] + y'[x] + 10 * y[x] == 0}, y[x], x]
Out[32]= {{y[x] → e^(x (-4.81...)) c1 + e^(x (-0.0957... - 1.44... i)) c2 + e^(x (-0.0957... + 1.44... i)) c3}}

In[33]:= eqsol = DSolve[{y'''[x] + 5 * y''[x] + 2 * y'[x] + y'[x] + 10 * y[x] == 0, y''[0] == 10, y'[0] == 3, y[0] == 0}, y[x], x]
Out[33]= {{y[x] →
 - (10 e^(x (-0.0957... - 1.44... i)) (-4.81...) - 10 e^(x (-0.0957... + 1.44... i)) (-4.81...) - 3 e^(x (-0.0957... - 1.44... i)) (-4.81...)^2 + 3 e^(x (-0.0957... + 1.44..

In[34]:= sol = y[x] /. eqsol
Out[34]= {- (10 e^(x (-0.0957... - 1.44... i)) (-4.81...) - 10 e^(x (-0.0957... + 1.44... i)) (-4.81...) - 3 e^(x (-0.0957... - 1.44... i)) (-4.81...)^2 + 3 e^(x (-0.0957... + 1.44... i))

In[35]:= Plot[sol, {x, -5, 5}]
Out[35]=
```



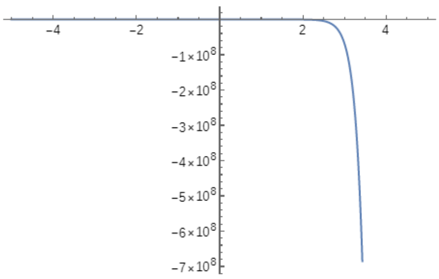
y'''-9y''+15y'+25y=0 where y''(0)=15, y'(0)=10,y(0)=5.

```
In[36]:= eq1 = DSolve[{y'''[x] - 9 * y''[x] + 15 * y'[x] + 25 * y[x] == 0}, y[x], x]
Out[36]= {{y[x] → e^(-x) c1 + e^(5 x) c2 + e^(5 x) x c3}}

In[37]:= esol = DSolve[{y'''[x] - 9 * y''[x] + 15 * y'[x] + 25 * y[x] == 0, y''[0] == 15, y'[0] == 10, y[0] == 5}, y[x], x]
Out[37]= {{y[x] → -5/9 e^(-x) (-2 - 7 e^(6 x) + 15 e^(6 x) x)}}

In[38]:= gs = y[x] /. esol
Out[38]= {-5/9 e^(-x) (-2 - 7 e^(6 x) + 15 e^(6 x) x)}

In[39]:= Plot[gs, {x, -5, 5}]
Out[39]=
```



y'''-y''-4y'-2y=0 where y''(0)=10, y'(0)=5,y(0)=10

In[40]:= eq1 = DSolve[{y'''[x] - y''[x] - 4 * y'[x] - 2 * y[x] == 0}, y[x], x]

Out[40]= $\{\{y[x] \to e^{(1-\sqrt{3})x} c_1 + e^{(1+\sqrt{3})x} c_2 + e^{-x} c_3\}\}$

In[41]:= sol = DSolve[{y'''[x] - y''[x] - 4 * y'[x] - 2 * y[x] == 0, y''[0] == 10, y'[0] == 5, y[0] == 10}, y[x], x]

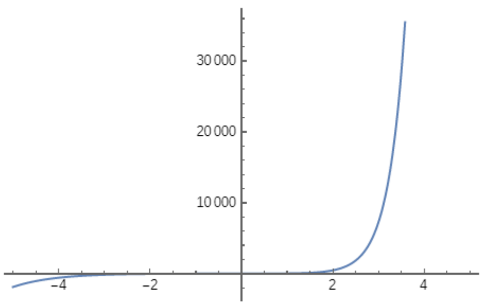
Out[41]= $\left\{\left\{y[x] \to \frac{5 e^{-x}\left(-16 - 8\sqrt{3} + 21 e^{x+(1-\sqrt{3})x} + 12\sqrt{3} e^{x+(1-\sqrt{3})x} + 3 e^{x+(1+\sqrt{3})x}\right)}{2(2+\sqrt{3})}\right\}\right\}$

In[42]:= gs = y[x] /. sol

Out[42]= $\left\{\frac{5 e^{-x}\left(-16 - 8\sqrt{3} + 21 e^{x+(1-\sqrt{3})x} + 12\sqrt{3} e^{x+(1-\sqrt{3})x} + 3 e^{x+(1+\sqrt{3})x}\right)}{2(2+\sqrt{3})}\right\}$

In[43]:= Plot[gs, {x, -5, 5}]

Out[43]=



y'''+4y''+5y'+20y=0

In[44]:= eq1 = DSolve[{y'''[x] + 4 * y''[x] + 5 * y'[x] + 20 * y[x] == 0}, y[x], x]

Out[44]= $\{\{y[x] \to e^{-4x} c_3 + c_1 \text{Cos}[\sqrt{5}\, x] + c_2 \text{Sin}[\sqrt{5}\, x]\}\}$

In[45]:= sol = y[x] /. eq1

Out[45]= $\{e^{-4x} c_3 + c_1 \text{Cos}[\sqrt{5}\, x] + c_2 \text{Sin}[\sqrt{5}\, x]\}$

In[46]:= gs = Evaluate[sol /. {C[1] → 1, C[2] → 2, C[3] → 3}]

Out[46]= $\{3 e^{-4x} + \text{Cos}[\sqrt{5}\, x] + 2 \text{Sin}[\sqrt{5}\, x]\}$

In[47]:= gs2 = Evaluate[sol /. {C[1] → 2, C[2] → 4, C[3] → 6}]

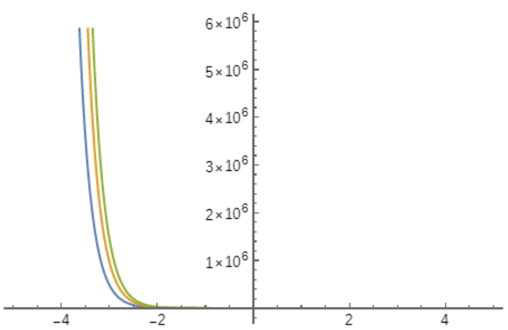
Out[47]= $\{6 e^{-4x} + 2 \text{Cos}[\sqrt{5}\, x] + 4 \text{Sin}[\sqrt{5}\, x]\}$

In[48]:= gs3 = Evaluate[sol /. {C[1] → 3, C[2] → 6, C[3] → 9}]

Out[48]= $\{9 e^{-4x} + 3 \text{Cos}[\sqrt{5}\, x] + 6 \text{Sin}[\sqrt{5}\, x]\}$

In[49]:= Plot[{gs, gs2, gs3}, {x, -5, 5}]

Out[49]=



y'''-13y''+19y'+33y=0

In[50]:= `eq1 = DSolve[{y'''[x] - 13 * y''[x] + 19 * y'[x] + 33 * y[x] == 0}, y[x], x]`

Out[50]= $\{\{y[x] \to e^{-x} c_1 + e^{3x} c_2 + e^{11x} c_3\}\}$

In[51]:= `sol = y[x] /. eq1`

Out[51]= $\{e^{-x} c_1 + e^{3x} c_2 + e^{11x} c_3\}$

In[52]:= `gs = Evaluate[sol /. {C[1] → -6, C[2] → 0, C[3] → 8}]`

Out[52]= $\{-6\, e^{-x} + 8\, e^{11x}\}$

In[53]:= `gs2 = Evaluate[sol /. {C[1] → -4, C[2] → 3, C[3] → 0}]`

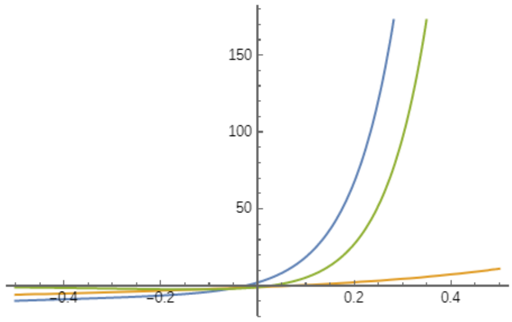
`gs3 = Evaluate[sol /. {C[1] → 0, C[2] → -5, C[3] → 4}]`

Out[53]= $\{-4\, e^{-x} + 3\, e^{3x}\}$

Out[54]= $\{-5\, e^{3x} + 4\, e^{11x}\}$

In[56]:= `Plot[{gs, gs2, gs3}, {x, -0.5, 0.5}]`

Out[56]=



y'''+3y''+6y'+18y=0

In[58]:= `eq1 = DSolve[{ y'''[x] + 3 * y''[x] + 6 * y'[x] + 18 * y[x] == 0, y[0] == a, y'[0] == 0, y''[0] == 0}, y[x], x]`

Out[58]= $\{\{y[x] \to \frac{1}{5} a\, e^{-3x} (2 + 3\, e^{3x} \text{Cos}[\sqrt{6}\, x] + \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}\}$

In[59]:= `sol = Evaluate[y[x] /. eq1 /. {{a → 2}, {a → 4}, {a → 6}}]`

Out[59]= $\{\{\frac{2}{5} e^{-3x} (2 + 3\, e^{3x} \text{Cos}[\sqrt{6}\, x] + \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}, \{\frac{4}{5} e^{-3x} (2 + 3\, e^{3x} \text{Cos}[\sqrt{6}\, x] + \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}, \{\frac{6}{5} e^{-3x} (2 + 3\, e^{3x} \text{Cos}[\sqrt{6}\, x] + \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}\}$

In[60]:= `gs = DSolve[{ y'''[x] + 3 * y''[x] + 6 * y'[x] + 18 * y[x] == 0, y[0] == 0, y'[0] == a, y''[0] == 0}, y[x], x]`

Out[60]= $\{\{y[x] \to \frac{a\, \text{Sin}[\sqrt{6}\, x]}{\sqrt{6}}\}\}$

In[61]:= `sol2 = Evaluate[y[x] /. gs /. {{a → 2}, {a → 4}, {a → 6}}]`

Out[61]= $\{\{\sqrt{\frac{2}{3}}\, \text{Sin}[\sqrt{6}\, x]\}, \{2 \sqrt{\frac{2}{3}}\, \text{Sin}[\sqrt{6}\, x]\}, \{\sqrt{6}\, \text{Sin}[\sqrt{6}\, x]\}\}$

In[62]:= `gs2 = DSolve[{ y'''[x] + 3 * y''[x] + 6 * y'[x] + 18 * y[x] == 0, y[0] == 0, y'[0] == 0, y''[0] == a}, y[x], x]`

Out[62]= $\{\{y[x] \to -\frac{1}{30} a\, e^{-3x} (-2 + 2\, e^{3x} \text{Cos}[\sqrt{6}\, x] - \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}\}$

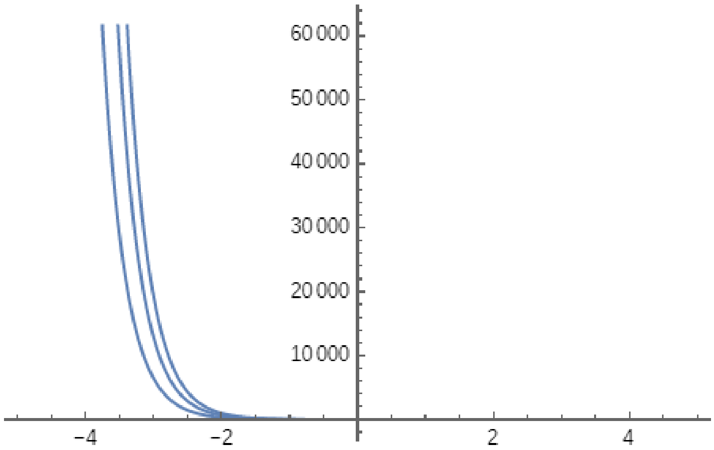
In[63]:= `sol3 = Evaluate[y[x] /. gs2 /. {{a → 2}, {a → 4}, {a → 6}}]`

Out[63]= $\{\{-\frac{1}{15} e^{-3x} (-2 + 2\, e^{3x} \text{Cos}[\sqrt{6}\, x] - \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}, \{-\frac{2}{15} e^{-3x} (-2 + 2\, e^{3x} \text{Cos}[\sqrt{6}\, x] - \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}, \{-\frac{1}{5} e^{-3x} (-2 + 2\, e^{3x} \text{Cos}[\sqrt{6}\, x] - \sqrt{6}\, e^{3x} \text{Sin}[\sqrt{6}\, x])\}\}$

```
Sin[√6 x])}}
```

In[64]:=
```
Plot[{sol, gs, gs2}, {x, -5, 5}]
```

Out[64]=



Q4) Solution of differential equation by variation of parameter method.

y’’-3y’+2y=e^(3x)

prac4a.nb File Edit Format Insert Evaluation View Help

In[65]:= `eq1 = {y''[x] - 3 * y'[x] + 2 * y[x] == 0}`

Out[65]= $\{2\,y[x] - 3\,y'[x] + y''[x] == 0\}$

In[66]:= `r[x] = Exp[3 * x]`

Out[66]= e^{3x}

In[67]:= `eq2 = DSolve[eq1, y[x], x]`

Out[67]= $\{\{y[x] \to e^{x} c_1 + e^{2x} c_2\}\}$

In[68]:= `yh[x] = y[x] /. eq2`

Out[68]= $\{e^{x} c_1 + e^{2x} c_2\}$

In[69]:= `Print["THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=", y[x] /. eq2[[1]]]`

THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=$e^{x} c_1 + e^{2x} c_2$

In[70]:= `y1[x] = Exp[x]`

Out[70]= e^{x}

In[71]:= `y2[x] = Exp[2 * x]`

Out[71]= e^{2x}

In[72]:= `W = Wronskian[{y1[x], y2[x]}, x]`

Out[72]= e^{3x}

In[74]:= `Print["THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS\n W(y1,y2)=", W]`

THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS
W(y1,y2)=e^{3x}

In[75]:= `u[x] = Integrate[-y2[x] * r[x] / W, x]`

Out[75]= $-\frac{e^{2x}}{2}$

In[75]:= `u[x] = Integrate[-y2[x] * r[x] / W, x]`

Out[75]= $-\frac{e^{2x}}{2}$

In[76]:= `v[x] = Integrate[y1[x] * r[x] / W, x]`

Out[76]= e^{x}

In[77]:= `yp[x] = u[x] * y1[x] + v[x] * y2[x]`

Out[77]= $\frac{e^{3x}}{2}$

In[78]:= `Print["THE PARTICULAR INTEGRAL OF NON-HOMOGENEOUS EQUATION IS yp[x]=", yp[x]]`

THE PARTICULAR INTEGRAL OF NON-HOMOGENEOUS EQUATION IS yp[x]=$\frac{e^{3x}}{2}$

In[79]:= `Print["THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS\n y[x]=", Sol = yh[x] + yp[x]]`

THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS
y[x]=$\left\{\frac{e^{3x}}{2} + e^{x} c_1 + e^{2x} c_2\right\}$

y’’=y= 2x^2-x-3

```
In[80]:= eq1 = {y''[x] - y[x] == 0}
Out[80]= {-y[x] + y''[x] == 0}

In[81]:= r[x] = 2 * x ^ 2 - x - 3
Out[81]= -3 - x + 2 x^2

In[82]:= eq2 = DSolve[eq1, y[x], x]
Out[82]= {{y[x] → e^x c1 + e^-x c2}}

In[87]:= yh[x] = y[x] /. eq2
Out[87]= {e^x c1 + e^-x c2}

In[88]:= Print["THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=",
  y[x] /. eq2[[1]]]
THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=e^x c1 + e^-x c2

In[84]:= y1[x] = Exp[x]
y2[x] = Exp[-x]
Out[84]= e^x

Out[85]= e^-x

In[86]:= W = Wronskian[{y1[x], y2[x]}, x]
Out[86]= -2

In[89]:= Print["THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS\n W(y1,y2)=", W]
THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS
 W(y1,y2)=-2

In[90]:= u[x] = Integrate[-y2[x] * r[x] / W, x]
```


```
 W(y1,y2)=-2
In[90]:= u[x] = Integrate[-y2[x] * r[x] / W, x]
```


```
Out[90]= -1/2 e^-x x (3 + 2 x)

In[91]:= v[x] = Integrate[y1[x] * r[x] / W, x]
Out[91]= 1/2 e^x (-2 + 5 x - 2 x^2)

In[92]:= yp[x] = u[x] * y1[x] + v[x] * y2[x]
Out[92]= -1/2 x (3 + 2 x) + 1/2 (-2 + 5 x - 2 x^2)

In[93]:= Print["THE PARTICULAR INTEGRAL OF NON-HOMOGENEOUS EQUATION IS yp[x]=", yp[x]]
THE PARTICULAR INTEGRAL OF NON-HOMOGENEOUS EQUATION IS yp[x]=-1/2 x (3 + 2 x) + 1/2 (-2 + 5 x - 2 x^2)

In[94]:= Print["THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS\n y[x]=", Sol = yh[x] + yp[x]]
THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS
 y[x]={-1/2 x (3 + 2 x) + 1/2 (-2 + 5 x - 2 x^2) + e^x c1 + e^-x c2}
```

y''+y=tan(x)

```
In[95]:= eq1 = {y''[x] + y[x] == 0}
Out[95]= {y[x] + y''[x] == 0}

In[96]:= r[x] = Tan[x]
Out[96]= Tan[x]

In[97]:= eq2 = DSolve[eq1, y[x], x]
Out[97]= {{y[x] → c1 Cos[x] + c2 Sin[x]}}

In[98]:= yh[x] = y[x] /. eq2
Out[98]= {c1 Cos[x] + c2 Sin[x]}

In[99]:= Print["THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=",
          y[x] /. eq2[[1]]]
         THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=c1 Cos[x] + c2 Sin[x]

In[100]:= y1[x] = Cos[x]
          y2[x] = Sin[x]
Out[100]= Cos[x]

Out[101]= Sin[x]

In[102]:= W = Wronskian[{y1[x], y2[x]}, x]
Out[102]= 1

In[103]:= Print["THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS\n W(y1,y2)=", W]
          THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS
           W(y1,y2)=1

In[104]:= u[x] = Integrate[-y2[x] * r[x] / W, x]
Out[104]= Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x]


In[107]:= v[x] = Integrate[y1[x] * r[x] / W, x]
Out[107]= -Cos[x]


Out[106]= -1/2 e^(-i x) (1 + e^(2 i x))

In[108]:= yp[x] = u[x] * y1[x] + v[x] * y2[x]
Out[108]= -Cos[x] Sin[x] + Cos[x] (Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x])

In[109]:= Print["THE PARTICULAR INTEGRAL OF NON-HOMOGENEOUS EQUATION IS yp[x]=", yp[x]]
          THE PARTICULAR INTEGRAL OF NON-HOMOGENEOUS EQUATION IS yp[x]=-Cos[x] Sin[x] + Cos[x] (Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x])

In[110]:= Print["THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS\n y[x]=", Sol = yh[x] + yp[x]]
          THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS
           y[x]={c1 Cos[x] + c2 Sin[x] - Cos[x] Sin[x] + Cos[x] (Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x])}
```

y’’+y==cosx-sinx

In[113]:= `eq1 = {y''[x] + y[x] == 0}`

Out[113]= $\{y[x] + y''[x] == 0\}$

In[114]:= `r[x] = Cos[x] - Sin[x]`

Out[114]= $Cos[x] - Sin[x]$

In[115]:= `eq2 = DSolve[eq1, y[x], x]`

Out[115]= $\{\{y[x] \to c_1 Cos[x] + c_2 Sin[x]\}\}$

In[116]:= `yh[x] = y[x] /. eq2`

Out[116]= $\{c_1 Cos[x] + c_2 Sin[x]\}$

In[117]:= `Print["THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=", y[x] /. eq2[[1]]]`

THE GENERAL SOLUTION OF THE CORRESPONDING HOMOGENEOUS DIFFERENTIAL EQUATION IS yh=$c_1 Cos[x] + c_2 Sin[x]$

In[118]:= `y1[x] = Cos[x]`

Out[118]= $Cos[x]$

In[120]:= `y2[x] = Sin[x]`

Out[120]= $Sin[x]$

In[121]:= `W = Wronskian[{y1[x], y2[x]}, x]`

Out[121]= 1

In[122]:= `Print["THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS\n W(y1,y2)=", W]`

THE WRONSKIAN OF Y1 AND Y2 SOLUTION IS

W(y1,y2)=1

In[123]:= `u[x] = Integrate[-y2[x] * r[x] / W, x]`

Out[123]= $\frac{x}{2} + \frac{Cos[x]^2}{2} - \frac{1}{4} Sin[2 x]$

In[123]:= `u[x] = Integrate[-y2[x] * r[x] / W, x]`

Out[123]= $\frac{x}{2} + \frac{Cos[x]^2}{2} - \frac{1}{4} Sin[2 x]$

In[124]:= `v[x] = Integrate[y1[x] * r[x] / W, x]`

Out[124]= $\frac{x}{2} + \frac{Cos[x]^2}{2} + \frac{1}{4} Sin[2 x]$

In[125]:= `yp[x] = u[x] * y1[x] + v[x] * y2[x]`

Out[125]= $Cos[x] \left(\frac{x}{2} + \frac{Cos[x]^2}{2} - \frac{1}{4} Sin[2 x]\right) + Sin[x] \left(\frac{x}{2} + \frac{Cos[x]^2}{2} + \frac{1}{4} Sin[2 x]\right)$

In[126]:= `Print["THE PARTICULAR INTEGRAL OF NON-HOMOG` Copy to clipboard. `yp[x]=", yp[x]]`

THE PARTICULAR INTEGRAL OF NON-HOMOGENEOUS EQUATION IS yp[x]=$Cos[x] \left(\frac{x}{2} + \frac{Cos[x]^2}{2} - \frac{1}{4} Sin[2 x]\right) + Sin[x] \left(\frac{x}{2} + \frac{Cos[x]^2}{2} + \frac{1}{4} Sin[2 x]\right)$

In[127]:= `Print["THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS\n y[x]=", Sol = yh[x] + yp[x]]`

THE GENERAL SOLUTIONO FTHE GIVEN NON-HOMOGENEOUS DIFFERNTIAL EQUATION IS

y[x]=$\left\{c_1 Cos[x] + c_2 Sin[x] + Cos[x] \left(\frac{x}{2} + \frac{Cos[x]^2}{2} - \frac{1}{4} Sin[2 x]\right) + Sin[x] \left(\frac{x}{2} + \frac{Cos[x]^2}{2} + \frac{1}{4} Sin[2 x]\right)\right\}$

Q5) Solution of system of ordinary differential equations.

dy/dx = y+z

dz/dx=2y+z

In[128]:= `e1 = {y'[x] == y[x] + z[x], z'[x] == 2 * y[x] + z[x]}`

Out[128]= $\{y'[x] == y[x] + z[x],\ z'[x] == 2\, y[x] + z[x]\}$

In[131]:= `sol1 = DSolve[e1, {y[x], z[x]}, x]`

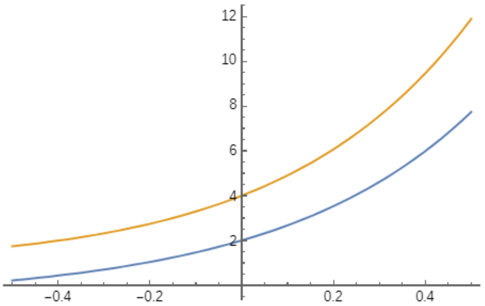
Out[131]= $\left\{\left\{y[x] \to \frac{1}{2} e^{x-\sqrt{2}x}\left(1+e^{2\sqrt{2}x}\right) c_1 + \frac{e^{x-\sqrt{2}x}\left(-1+e^{2\sqrt{2}x}\right) c_2}{2\sqrt{2}},\ z[x] \to \frac{e^{x-\sqrt{2}x}\left(-1+e^{2\sqrt{2}x}\right) c_1}{\sqrt{2}} + \frac{1}{2} e^{x-\sqrt{2}x}\left(1+e^{2\sqrt{2}x}\right) c_2\right\}\right\}$

In[132]:= `sol2 = {y[x], z[x]} /. sol1[[1]] /. {C[1] → 2, C[2] → 4}`

Out[132]= $\left\{\sqrt{2}\, e^{x-\sqrt{2}x}\left(-1+e^{2\sqrt{2}x}\right) + e^{x-\sqrt{2}x}\left(1+e^{2\sqrt{2}x}\right),\ \sqrt{2}\, e^{x-\sqrt{2}x}\left(-1+e^{2\sqrt{2}x}\right) + 2\, e^{x-\sqrt{2}x}\left(1+e^{2\sqrt{2}x}\right)\right\}$

In[133]:= `Plot[sol2, {x, -0.5, 0.5}]`

Out[133]=



axes ▼ | image size ▼ | add fill | background ▼ | more...

dy/dx = y+2z

dz/dx=4y-z

In[134]:= `eq2 = {y'[x] == y[x] + 2 * z[x], z'[x] == 4 * y[x] - z[x]}`

Out[134]= $\{y'[x] == y[x] + 2 z[x],\ z'[x] == 4 y[x] - z[x]\}$

In[135]:= `sol = DSolve[eq2, {y[x], z[x]}, x]`

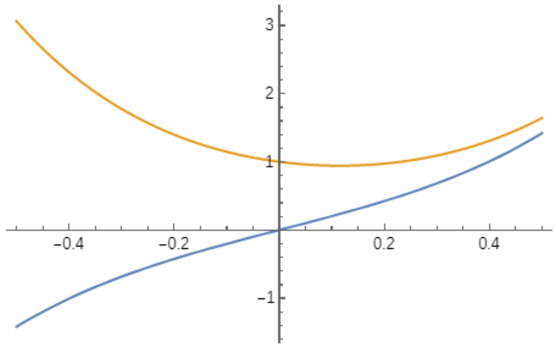
Out[135]= $\left\{\left\{y[x] \to \frac{1}{3} e^{-3x}\left(1 + 2 e^{6x}\right) c_1 + \frac{1}{3} e^{-3x}\left(-1 + e^{6x}\right) c_2,\ z[x] \to \frac{2}{3} e^{-3x}\left(-1 + e^{6x}\right) c_1 + \frac{1}{3} e^{-3x}\left(2 + e^{6x}\right) c_2\right\}\right\}$

In[136]:= `gs = {y[x], z[x]} /. sol[[1]] /. {C[1] → 0, C[2] → 1}`

Out[136]= $\left\{\frac{1}{3} e^{-3x}\left(-1 + e^{6x}\right),\ \frac{1}{3} e^{-3x}\left(2 + e^{6x}\right)\right\}$

In[137]:= `Plot[gs, {x, -0.5, 0.5}]`

Out[137]=



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dy/dx=4y+3z

dz/dx=y+z

In[138]:= `e1 = {y'[x] == 4 * y[x] + 3 * z[x], z'[x] == y[x] + z[x]}`

Out[138]= $\{y'[x] == 4 y[x] + 3 z[x],\ z'[x] == y[x] + z[x]\}$

In[139]:= `sol1 = DSolve[e1, {y[x], z[x]}, x]`

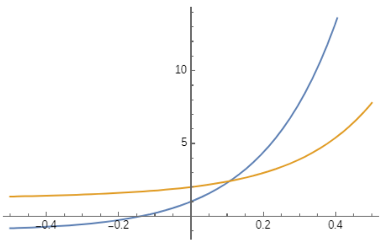
Out[139]= $\left\{\left\{y[x] \to \frac{1}{14} e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(7 - \sqrt{21} + 7 e^{\sqrt{21}x} + \sqrt{21} e^{\sqrt{21}x}\right) c_1 + \sqrt{\frac{3}{7}} e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(-1 + e^{\sqrt{21}x}\right) c_2,\ z[x] \to \frac{e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(-1 + e^{\sqrt{21}x}\right) c_1}{\sqrt{21}} - \frac{1}{14} e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(-7 - \sqrt{21} - 7 e^{\sqrt{21}x} + \sqrt{21} e^{\sqrt{21}x}\right) c_2\right\}\right\}$

In[140]:= `sol2 = {y[x], z[x]} /. sol1[[1]] /. {C[1] → 1, C[2] → 2}`

Out[140]= $\left\{2\sqrt{\frac{3}{7}} e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(-1 + e^{\sqrt{21}x}\right) + \frac{1}{14} e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(7 - \sqrt{21} + 7 e^{\sqrt{21}x} + \sqrt{21} e^{\sqrt{21}x}\right),\ \frac{e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(-1 + e^{\sqrt{21}x}\right)}{\sqrt{21}} - \frac{1}{7} e^{\frac{5x}{2} - \frac{\sqrt{21}x}{2}}\left(-7 - \sqrt{21} - 7 e^{\sqrt{21}x} + \sqrt{21} e^{\sqrt{21}x}\right)\right\}$

In[141]:= `Plot[sol2, {x, -0.5, 0.5}]`

Out[141]=



axes ▾ | image size ▾ | add fill | background ▾ | more...

dy/dx=6z

dz/dx=2y+3z

In[142]:= `e1 = {y'[x] == 6 * z[x], z'[x] == 2 * y[x] + 3 * z[x]}`

Out[142]= `{y′[x] == 6 z[x], z′[x] == 2 y[x] + 3 z[x]}`

In[143]:= `sol1 = DSolve[e1, {y[x], z[x]}, x]`

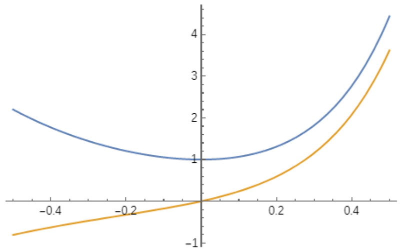
Out[143]= $\left\{\left\{y[x] \to -\frac{1}{38} e^{\frac{3x}{2}-\frac{\sqrt{57}\,x}{2}} \left(-19-\sqrt{57}-19 e^{\sqrt{57}\,x}+\sqrt{57}\, e^{\sqrt{57}\,x}\right) c_1 + 2\sqrt{\frac{3}{19}}\, e^{\frac{3x}{2}-\frac{\sqrt{57}\,x}{2}} \left(-1+e^{\sqrt{57}\,x}\right) c_2,\ z[x] \to \frac{2 e^{\frac{3x}{2}-\frac{\sqrt{57}\,x}{2}} \left(-1+e^{\sqrt{57}\,x}\right) c_1}{\sqrt{57}} + \frac{1}{38} e^{\frac{3x}{2}-\frac{\sqrt{57}\,x}{2}} \left(19-\sqrt{57}+19 e^{\sqrt{57}\,x}+\sqrt{57}\, e^{\sqrt{57}\,x}\right) c_2\right\}\right\}$

In[144]:= `sol2 = {y[x], z[x]} /. sol1[[1]] /. {C[1] → 1, C[2] → 0}`

Out[144]= $\left\{-\frac{1}{38} e^{\frac{3x}{2}-\frac{\sqrt{57}\,x}{2}} \left(-19-\sqrt{57}-19 e^{\sqrt{57}\,x}+\sqrt{57}\, e^{\sqrt{57}\,x}\right),\ \frac{2 e^{\frac{3x}{2}-\frac{\sqrt{57}\,x}{2}} \left(-1+e^{\sqrt{57}\,x}\right)}{\sqrt{57}}\right\}$

In[145]:= `Plot[sol2, {x, -0.5, 0.5}]`

Out[145]=



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dy/dx=y+2z

dz/dx=3y+2z

In[146]:= `e1 = {y'[x] == y[x] + 2 * z[x], z'[x] == 3 * y[x] + 2 * z[x]}`

Out[146]= `{y′[x] == y[x] + 2 z[x], z′[x] == 3 y[x] + 2 z[x]}`

In[147]:= `sol1 = DSolve[e1, {y[x], z[x]}, x]`

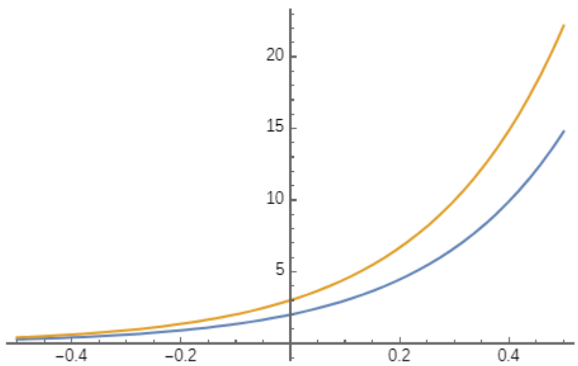
Out[147]= $\left\{\left\{y[x] \to \frac{1}{5} e^{-x} \left(3+2 e^{5x}\right) c_1 + \frac{2}{5} e^{-x} \left(-1+e^{5x}\right) c_2,\ z[x] \to \frac{3}{5} e^{-x} \left(-1+e^{5x}\right) c_1 + \frac{1}{5} e^{-x} \left(2+3 e^{5x}\right) c_2\right\}\right\}$

In[148]:= `sol2 = {y[x], z[x]} /. sol1[[1]] /. {C[1] → 2, C[2] → 3}`

Out[148]= $\left\{\frac{6}{5} e^{-x} \left(-1+e^{5x}\right) + \frac{2}{5} e^{-x} \left(3+2 e^{5x}\right),\ \frac{6}{5} e^{-x} \left(-1+e^{5x}\right) + \frac{3}{5} e^{-x} \left(2+3 e^{5x}\right)\right\}$

In[150]:= `Plot[sol2, {x, -0.5, 0.5}]`

Out[150]=



axes ▾ | image size ▾ | add fill | background ▾ | more...

dy/dx=y-z

dz/dx=-y+z

```
In[151]:= e1 = {y'[x] == y[x] - z[x], z'[x] == -y[x] + z[x]}
Out[151]= {y'[x] == y[x] - z[x], z'[x] == -y[x] + z[x]}

In[152]:= sol1 = DSolve[e1, {y[x], z[x]}, x]
```

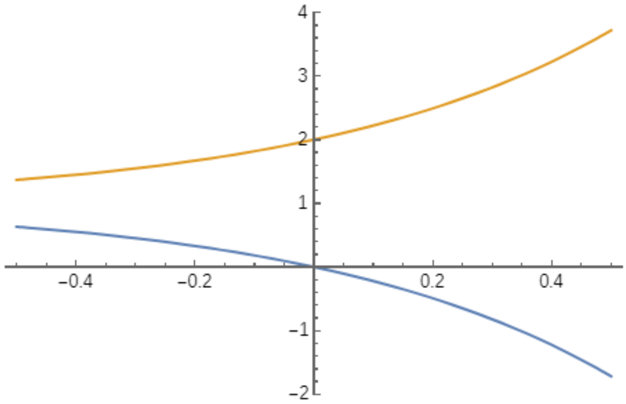
Out[152]= $\left\{\left\{y[x] \to \frac{1}{2}\left(1+e^{2x}\right) c_1 + \frac{1}{2}\left(1-e^{2x}\right) c_2,\ z[x] \to \frac{1}{2}\left(1-e^{2x}\right) c_1 + \frac{1}{2}\left(1+e^{2x}\right) c_2\right\}\right\}$

```
In[153]:= sol2 = {y[x], z[x]} /. sol1[[1]] /. {C[1] → 0, C[2] → 2}
```

Out[153]= $\{1-e^{2x},\ 1+e^{2x}\}$

```
In[154]:= Plot[sol2, {x, -0.5, 0.5}]
```

Out[154]=



axes ▼ | image size ▼ | add fill | background ▼ | more

Q6) Solution of Cauchy problem for first order partial differential equation.

z= u(x,y)

p=du/dx=D[u[x,y],x]

q=du/dy=D[u[x,y],y]

-3p+2q+z=0 with u(x,y)=sinx

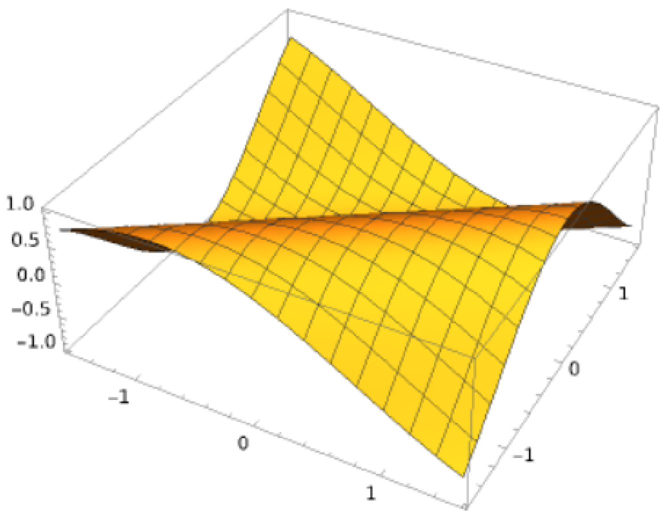
```
In[155]:= eq1 = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0
Out[155]= 2 u^(0,1)[x, y] + 3 u^(1,0)[x, y] == 0

In[157]:= sol = DSolve[{eq1, u[x, 0] == Sin[x]}, u, {x, y}]
```

Out[157]= $\left\{\left\{u \to \text{Function}\left[\{x, y\}, -\text{Sin}\left[\frac{3}{2}\left(-\frac{2x}{3}+y\right)\right]\right]\right\}\right\}$

```
In[159]:= Plot3D[u[x, y] /. sol, {x, -Pi / 2, Pi / 2}, {y, -Pi / 2, Pi / 2}]
```

Out[159]=



-yp+xq=0 with u(0,y)=e^-y^2

```
In[160]:= eq1 = y * D[u[x, y], x] + x * D[u[x, y], y] == 0
```

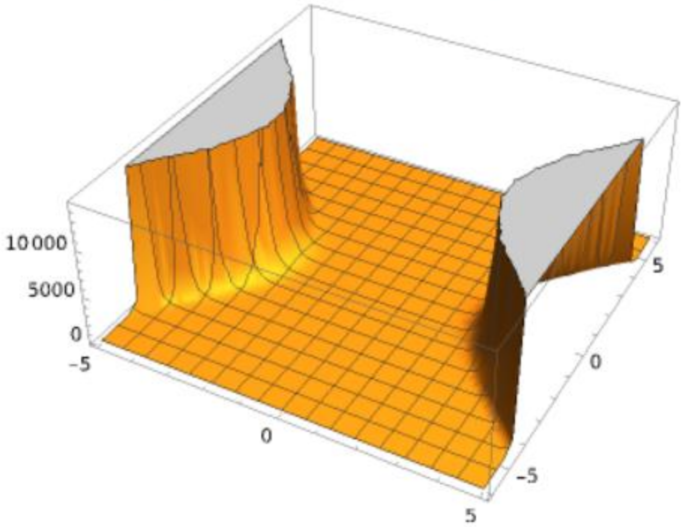
Out[160]= $x\, u^{(0,1)}[x, y] + y\, u^{(1,0)}[x, y] == 0$

```
In[161]:= sol = DSolve[{eq1, u[0, y] == Exp[-y^2]}, u, {x, y}]
```

Out[161]= $\{\{u \to \text{Function}[\{x, y\}, e^{x^2-y^2}]\}\}$

```
In[162]:= Plot3D[u[x, y] /. sol, {x, -5, 5}, {y, -6, 6}]
```

Out[162]=



-xp+yq=2xy with u=2 on y=x^2

```
In[163]:= eq1 = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y
```

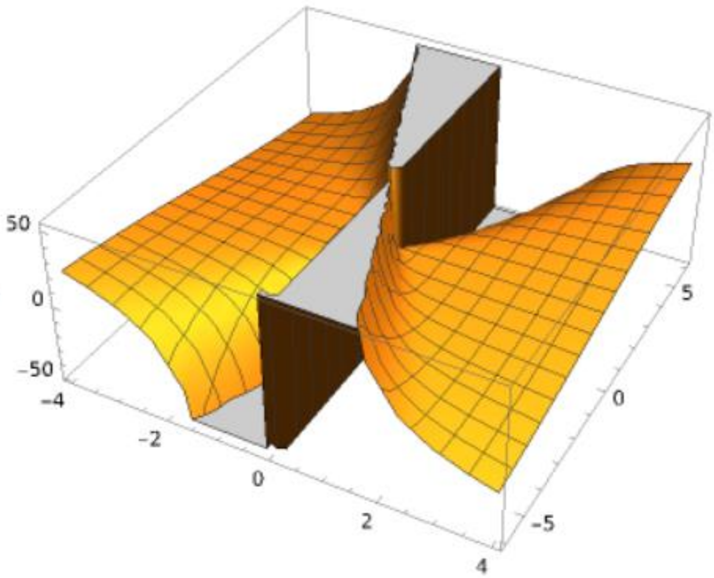
Out[163]= $y\,u^{(0,1)}[x, y] + x\,u^{(1,0)}[x, y] == 2\,x\,y$

```
In[164]:= sol = DSolve[{eq1, u[x, x^2] == 2}, u, {x, y}]
```

Out[164]= $\left\{\left\{u \to \text{Function}\left[\{x, y\}, \frac{2\,x^3 + x^4\,y - y^3}{x^3}\right]\right\}\right\}$

```
In[165]:= Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -6, 6}]
```

Out[165]=



-p+xq=(y-1/2x^2)^2 with u(0,y)=e^y

In[166]:= `eq1 = D[u[x, y], x] + x * D[u[x, y], y] == (y - 1 / 2 x ^ 2) ^ 2`

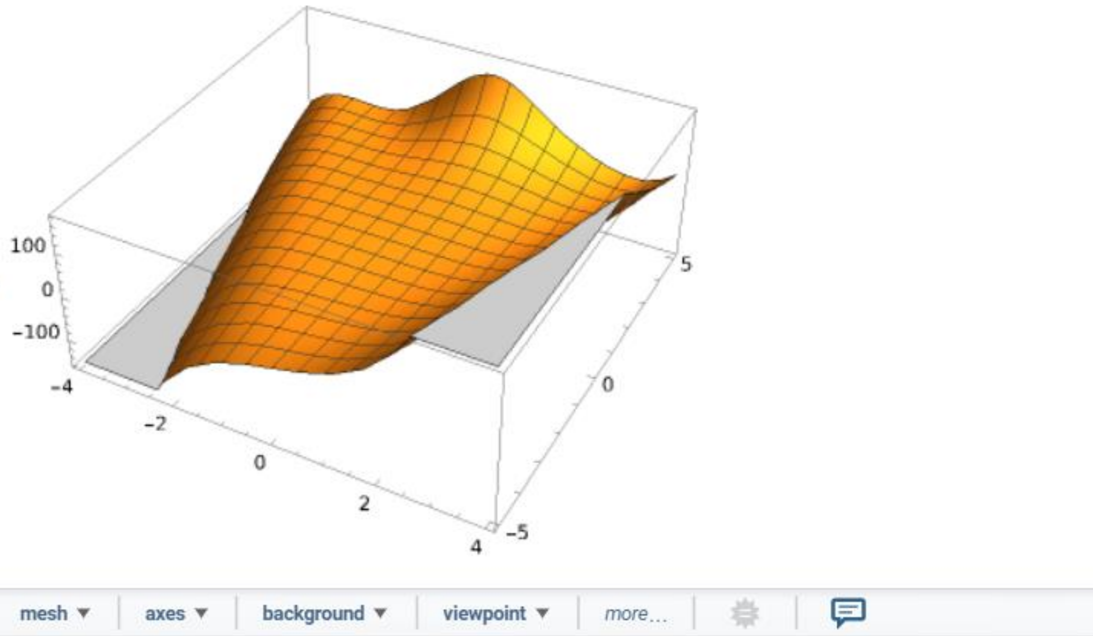
Out[166]= $x\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == \left(-\frac{x^2}{2} + y\right)^2$

In[167]:= `sol = DSolve[{eq1, u[0, y] == Exp[y]}, u, {x, y}]`

Out[167]= $\left\{\left\{u \to \text{Function}\left[\{x, y\}, \frac{1}{4} e^{-\frac{x^2}{2}} \left(4\, e^{y} + e^{\frac{x^2}{2}} x^5 - 4\, e^{\frac{x^2}{2}} x^3 y + 4\, e^{\frac{x^2}{2}} x\, y^2\right)\right]\right\}\right\}$

In[168]:= `Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5}]`

Out[168]=



Q7) Plotting the characteristics for the first order partial differential equations.

P+2xq=2xu with Cauchy data u[x,0]=x^2

```
In[169]:= eq1 = D[u[x, y], x] + 2 * x * D[u[x, y], y] == 2 * x * u[x, y]
```

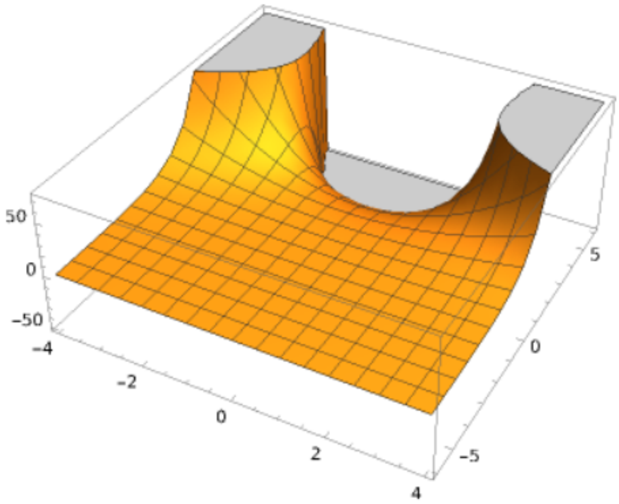
Out[169]= $2\,x\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 2\,x\,u[x, y]$

```
In[170]:= sol = DSolve[{eq1, u[x, 0] == x^2}, u, {x, y}]
```

Out[170]= $\{\{u \to \text{Function}[\{x, y\}, -e^{y}\,(-x^2 + y)]\}\}$

```
In[171]:= Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -6, 6}]
```

Out[171]=



xp+yq=u+1 where Cauchy data u[x,y]=x^2 on y=x^2

```
In[175]:= eq1 = x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1
```

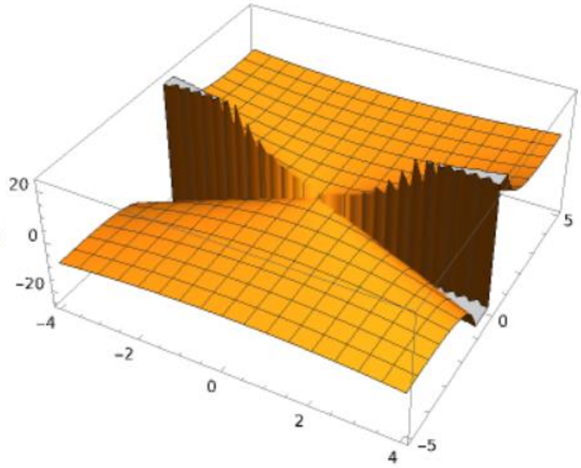
Out[175]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 1 + u[x, y]$

```
In[176]:= sol = DSolve[{eq1, u[x, x^2] == x^2}, u, {x, y}]
```

Out[176]= $\left\{\left\{u \to \text{Function}\left[\{x, y\}, \frac{x^2 - y + y^2}{y}\right]\right\}\right\}$

```
In[177]:= Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5}]
```

Out[177]=



2xyp+q(x^2+y^2)=0 with cauchy data u=e^x/x-y on x+y=1

```
In[178]:= eq1 = 2 * x * y * D[u[x, y], x] + (x^2 + y^2) * D[u[x, y], y] == 0
```

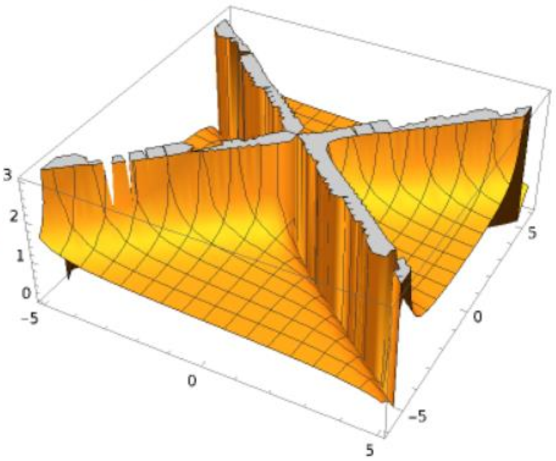
Out[178]= $(x^2 + y^2)\, u^{(0,1)}[x, y] + 2\, x\, y\, u^{(1,0)}[x, y] == 0$

```
In[179]:= sol = DSolve[{eq1, u[x, 1 - x] == Exp[x / (2 * x - 1)]}, u, {x, y}]
```

Out[179]= $\left\{\left\{u \to \text{Function}\left[\{x, y\}, e^{\frac{1}{\left(2 - x + \frac{y^2}{x}\right)\left(-1 + \frac{2}{2 - x + \frac{y^2}{x}}\right)}}\right]\right\}\right\}$

```
In[180]:= Plot3D[u[x, y] /. sol, {x, -5, 5}, {y, -6, 6}]
```

Out[180]=

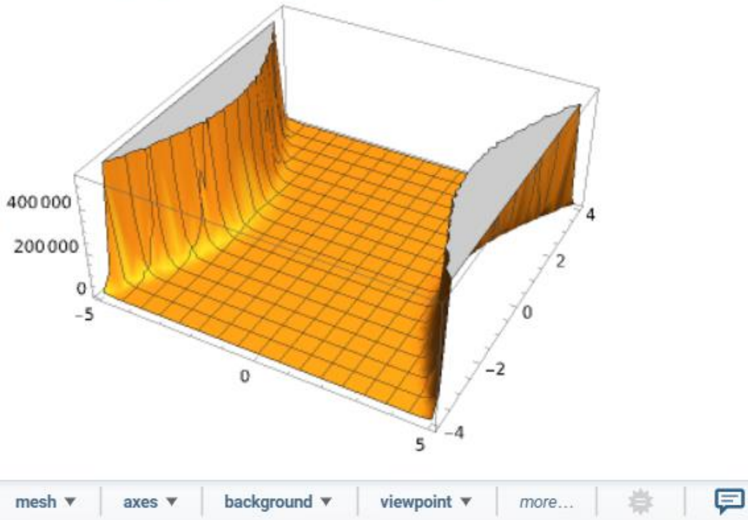


-yp+xq==0 with u(0,y)=e^-y^2

```
In[181]:= eq1 = y * D[u[x, y], x] + x * D[u[x, y], y] == 0
```
Out[181]= $x\,u^{(0,1)}[x, y] + y\,u^{(1,0)}[x, y] == 0$

```
In[182]:= sol = DSolve[{eq1, u[0, y] == Exp[-y^2]}, u, {x, y}]
```
Out[182]= $\{\{u \to \text{Function}[\{x, y\}, e^{x^2-y^2}]\}\}$

```
In[183]:= Plot3D[u[x, y] /. sol, {x, -5, 5}, {y, -4, 4}]
```
Out[183]=

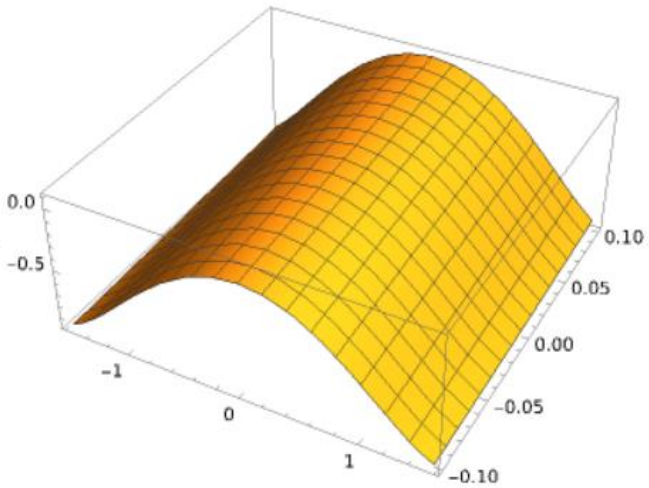


p+xq==0 where u(0,y)=siny

```
In[184]:= eq1 = D[u[x, y], x] + x * D[u[x, y], y] == 0
```
Out[184]= $x\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

```
In[185]:= sol = DSolve[{eq1, u[0, y] == Sin[y]}, u, {x, y}]
```
Out[185]= $\{\{u \to \text{Function}[\{x, y\}, -\text{Sin}[\frac{x^2}{2} - y]]\}\}$

```
In[186]:= Plot3D[u[x, y] /. sol, {x, -Pi/2, Pi/2}, {y, -0.1, 0.1}]
```
Out[186]=



Q8)Plot the integral surfaces of the first order partial differential equations with initial data.

up+q=1 where x(s,0)=x0 and y(s,0)=y0 , u(s,0)-u0(s)

Ans))))))))

QUESTION 1 : FIND THE INTEGRAL SURFACE OF THE EQUATION

u * Subscript[u, x] + Subscript[u, y] = 1, and initial data x (s, 0) = x0 (s), y (s, 0) = y0, u (s, 0) = u0 (s)

solution : – The Characteristic Equation is

dx / u = dy / 1 = du / 1 = dt, = > Then

x (s, t) = t ^ 2 + tu0 (s) + x0 (s)

y (s, t) = t + y0 (s)

u (s, t) = u0 (s)

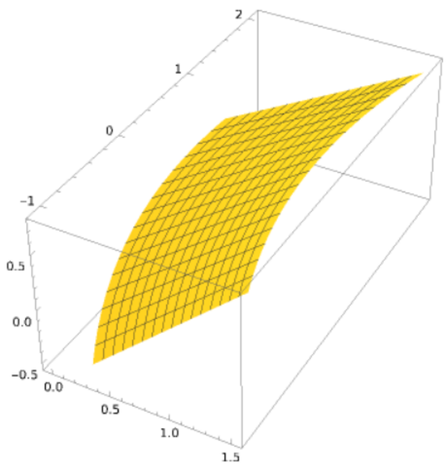
(a) x (s, 0) = s, y (s, 0) = 2 s, u (s, 0) = s

Then x = 1 / 2 t ^ 2 + ts, y = t + 2 s

u = t + s.

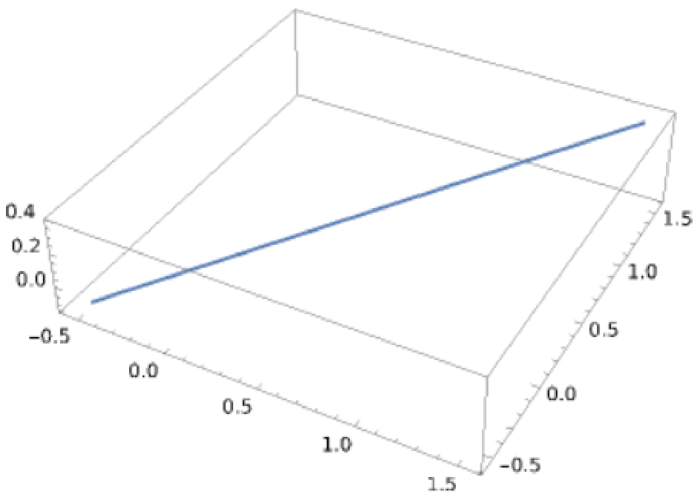
```
In[187]:= f = ParametricPlot3D[{s + ((t^2 + s * t) / 4), t + s, (2 t + s) / 4}, {s, 0, 1}, {t, -1, 1}, PlotPoints → 10]
```

Out[187]=



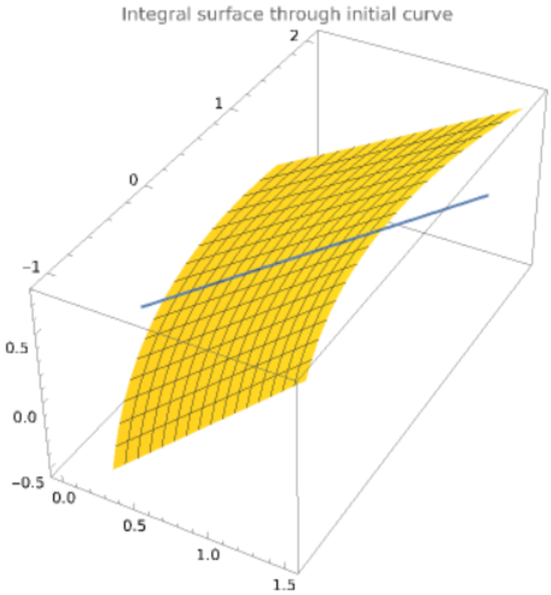

```
In[188]:= f2 = ParametricPlot3D[{s, s, s / 4}, {s, -0.5, 1.5}]
```

Out[188]=



```
In[190]:= Show[f, f2, PlotLabel → "Integral surface through initial curve"]
```

Out[190]=



up+q=1 and x(s,0)=x0, y(s,0)=y0(s), u(s,0)=u0(s).

solution : - The Characteristic Equation is

dx / u = dy / 1 = du / 1 = dt, = > Then

x (s, t) = t ^ 2, tu0 (s) + x0 (s),

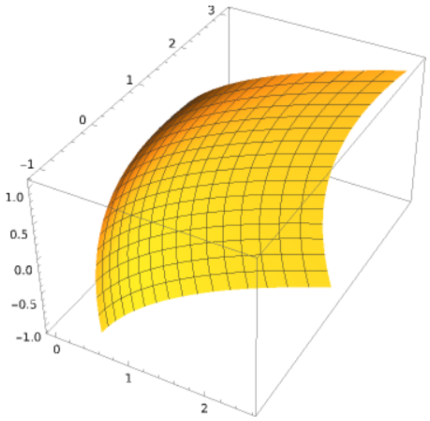
y (s, t) = t + y0 (s)

u (s, t) = t + u0 (s).

(a) x (s, 0) = 2 s ^ 2, y (s, 0) = 2 s, u (s, 0) = s, then x = 1 / 2 t ^ 2 + 2 s ^ 2, y = t + 2 s, u = t

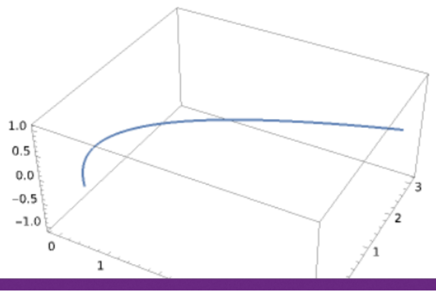
```
In[191]:= f = ParametricPlot3D[{(1/2) * t^2 + 2 * s^2, t + 2 s, t}, {s, 0, 1}, {t, -1, 1}, PlotPoints → 10]
```

Out[191]=



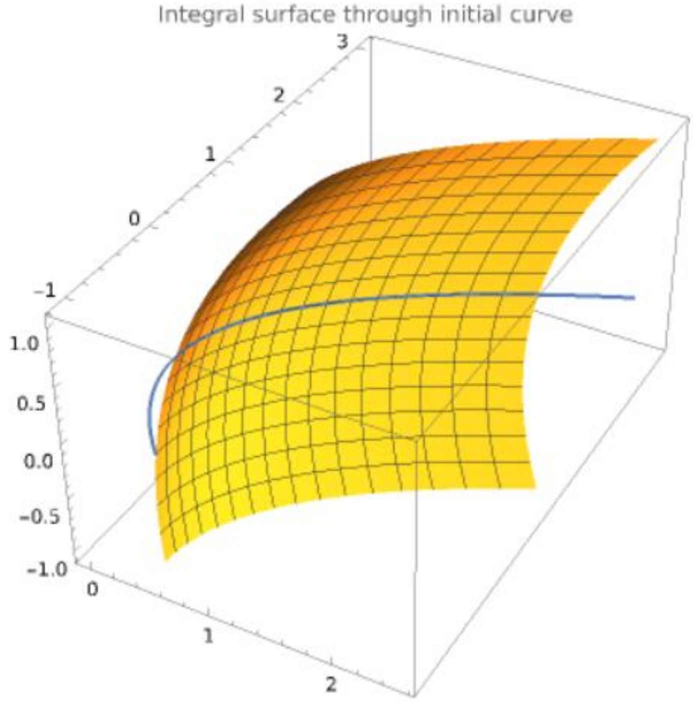

```
In[192]:= f2 = ParametricPlot3D[{2 s^2, 2 s, 0}, {s, -0.5, 1.5}]
```

Out[192]=



```
In[193]:= Show[f, f2, PlotLabel → "Integral surface through initial curve"]
```

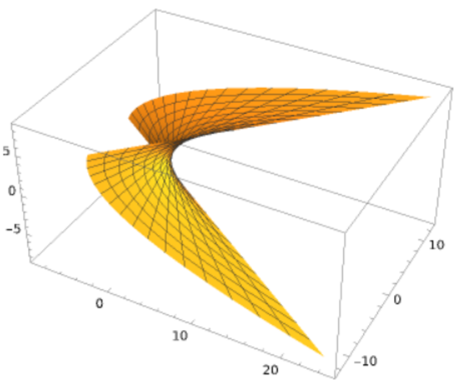
Out[193]=



up+q=1 and x(s,0)=x0, y(s,0)=y0(s), u(s,0)=u0(s).

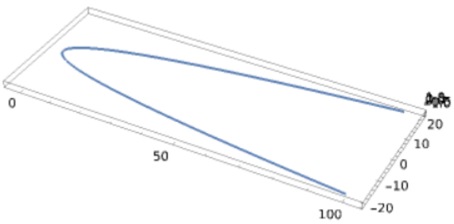
```
In[194]:= f = ParametricPlot3D[{(t^2)/2+t*s, t+2s, t+s}, {s, -4, 4}, {t, -4, 4}, PlotPoints → 10]
```

Out[194]=



```
In[195]:= f2 = ParametricPlot3D[{s^2, 2s, 0}, {s, -10, 10}]
```

Out[195]=



```
In[196]:= Show[f, f2, PlotLabel → "Integral surface through initial curve"]
```

Out[196]=