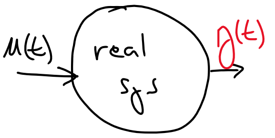
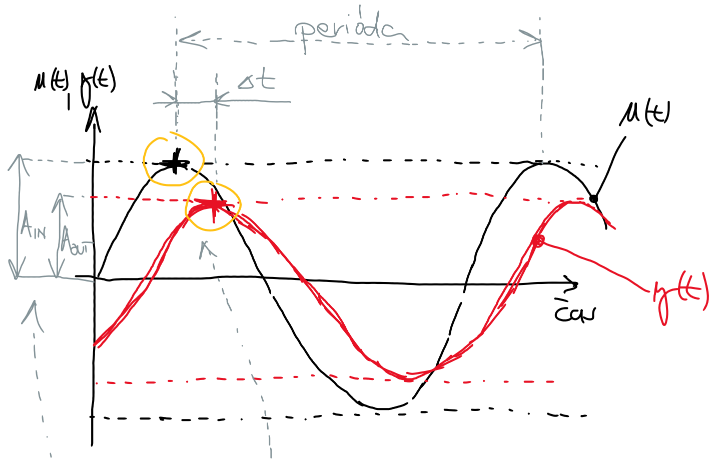
FCH





$u(t) = A_{IN} \sin(\omega t)$

... jednotková amplitúda

rad/s
$\omega = 2\pi f$

$y(t) = ?$

- iná amplitúda
- iná „fáza"

ROVNAKÁ FREKVENCIA!

model:

$$\boxed{G(s) = \frac{B(s)}{A(s)}} = \frac{B(s)}{(s+s_1)(s+s_2)\cdots(s+s_n)}$$

freku. ch.

$G(\underline{j\omega})$

$Y(s) = G(s)U(s)$

$U(s)$? $\quad u(t) = \sin(\omega t)$

$$U(s) = \frac{\omega}{s^2+\omega^2}$$

$$Y(s) = G(s)\left[\frac{\omega}{s^2+\omega^2}\right]$$

$\Downarrow$

parc. zlomky ... $\rightarrow y(t)$ ...

komplexne združené k A

$$G(s)\left[\frac{\omega}{(s+j\omega)(s-j\omega)}\right] = \underbrace{\frac{A}{s+j\omega} + \frac{A^*}{s-j\omega} + \frac{B_1}{s+s_1} + \frac{B_2}{s+s_2} + \cdots + \frac{B_n}{s+s_n}}$$

$$y(t) = Ae^{-j\omega t} + A^* e^{j\omega t} + \underbrace{B_1 e^{-s_1 t} + B_2 e^{-s_2 t} + \cdots + B_n e^{-s_n t}}$$

ak stabil. systém tak tieto výrazy $\rightarrow 0$
(aj v prípade viacnásobného pólu)

„frekvenčný" ustálený stav:

$$y_{FUS}(t) = Ae^{-j\omega t} + A^* e^{j\omega t}$$

$A, A^* = ?$

$$Y_{FUS}(s) = G(s)\left[\frac{\omega}{(s+j\omega)(s-j\omega)}\right] = \frac{A}{s+j\omega} + \frac{A^*}{s-j\omega}$$

$$G(s)\omega = A(s-j\omega) + A^*(s+j\omega)$$

$$s = j\omega \qquad G(j\omega)\,\omega = A^*(j\omega + j\omega) \qquad A^* = \frac{G(j\omega)}{2j}$$

$$s = -j\omega \qquad G(-j\omega)\,\omega = A(-j\omega - j\omega) \qquad A = \frac{G(-j\omega)}{-2j}$$

$$G(j\omega) = \underbrace{|G(j\omega)|}_{\text{veľkosť}}\, e^{j\overbrace{\phi}^{\text{uhol}}}$$

$$G(-j\omega) = |G(-j\omega)|\, e^{-j\phi} = |G(j\omega)|\, e^{-j\phi}$$

$$y_{FUS}(t) = \frac{|G(j\omega)|\, e^{-j\phi}\, e^{-j\omega t}}{-2j} + \frac{|G(j\omega)|\, e^{j\phi}\, e^{j\omega t}}{2j}$$

$$y_{FUS}(t) = |G(j\omega)| \left(\frac{-e^{-j(\omega t+\phi)} + e^{j(\omega t+\phi)}}{2j} \right)$$

$$= |G(j\omega)| \left(\frac{e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}}{2j} \right)$$

platí:

$$e^{j(\omega t+\phi)} = \cos(\omega t+\phi) + j\sin(\omega t+\phi)$$

$$e^{-j(\omega t+\phi)} = \cos(-(\omega t+\phi)) + j\sin(-(\omega t+\phi))$$

cos je párna fcn, sin nepárna...

$$= \cos(\omega t+\phi) - j\sin(\omega t+\phi)$$

$$y_{FUS}(t) = |G(j\omega)| \left(\frac{\cancel{\cos(\omega t+\phi)} + j\sin(\omega t+\phi) - (\cancel{\cos(\omega t+\phi)} - j\sin(\omega t+\phi))}{2j} \right)$$

$$y_{FUS}(t) = |G(j\omega)| \left(\frac{j\sin(\omega t+\phi) + j\sin(\omega t+\phi)}{2j} \right)$$

$$y_{FUS}(t) = \underbrace{|G(j\omega)|}_{\text{amplitúda}}\ \sin(\omega t + \underbrace{\phi}_{\text{fázové posunutie}})$$

amplitúda

fázové posunutie

amplitúda
... iná ako A_{IN}

fázové posunutie

pripomeňme: $G(j\omega) = |G(j\omega)| e^{j\phi}$

komplexné číslo

veľkosť

uhol

ak poznáme $G(s)$

zaujíma nás „frekvenčný ustálený stav" pre frekvenciu ω

poznáme teda $G(j\omega)$

$|G(j\omega)|$ ← amplitúdové zosilnenie (závisí na ω)

ϕ ← fázové posunutie (závisí na ω)

„frekvenčná" prenosová funkcia

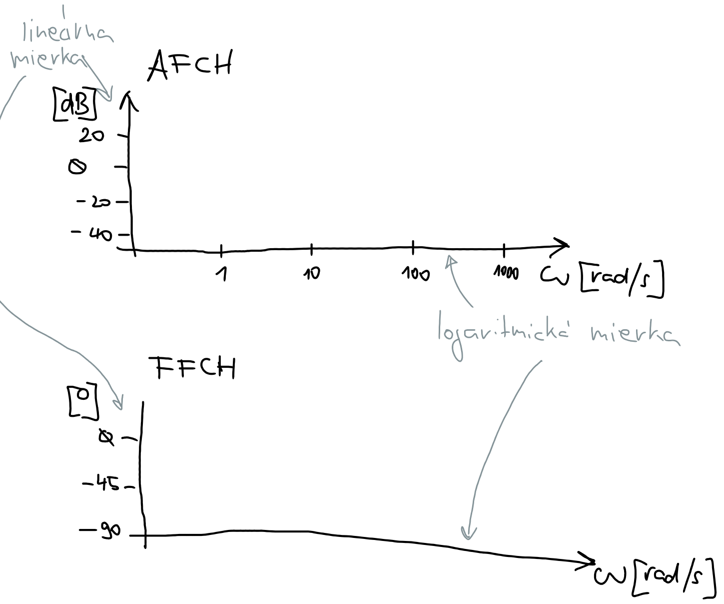
frekvenčné charakteristiky

FCH

AFCH

FFCH

Bodeho diagram



decibel [dB]

...jednotka pre zosilnenie

napríklad:

$|G(j\omega)|$ je veľkosť komplexného čísla

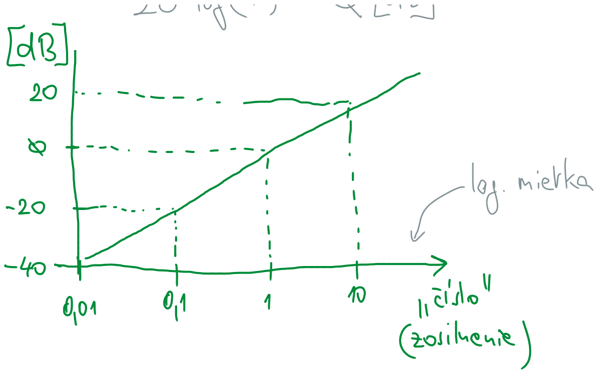
„Logaritmická veľkosť" potom je

$$20 \log(|G(j\omega)|)$$

kde log je pri základe 10

napríklad „jednotkové" zosilnenie

$$20 \log(1) = 0 \text{ [dB]}$$



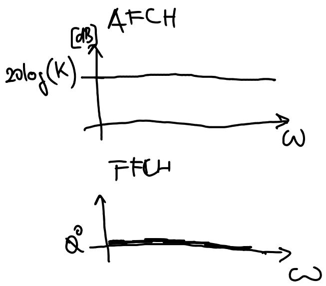
Bodeho diagram pre:

$G(s) = K$ (len zosilnenie)

$G(j\omega) = K$

$|G(j\omega)| = K$

$\phi = 0$



mimochodom:

$a + jb$

$|a+jb| = \sqrt{a^2+b^2}$

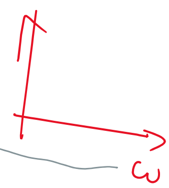
$\angle a+jb = -\,\mathrm{atan}\left(\frac{b}{a}\right)$

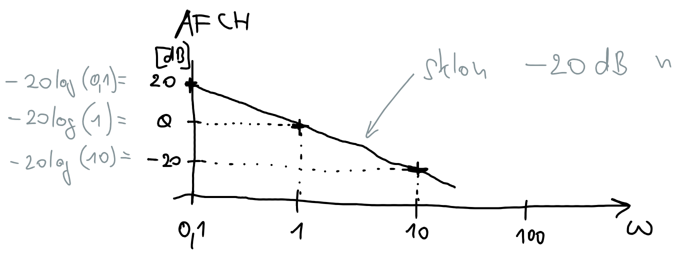
Bodeho diag. pre $G(s) = \frac{1}{s}$

$G(j\omega) = \frac{1}{j\omega}$ $\quad \frac{1}{j\omega} \cdot \frac{-j\omega}{-j\omega} = \frac{-j\omega}{\omega^2} = \frac{-j}{\omega} = 0 + j\frac{1}{\omega}$

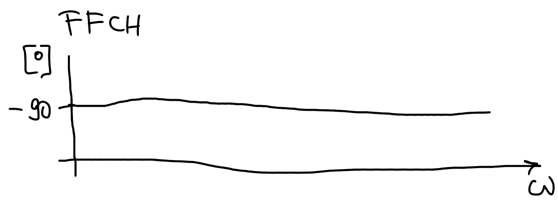
$|G(j\omega)| = \sqrt{0^2 + \frac{1}{\omega^2}} = \sqrt{\frac{1}{\omega^2}} = \frac{1}{\omega}$

$|G(j\omega)| = \frac{1}{\omega}$ $\quad 20\log\left(\frac{1}{\omega}\right) = 20\left(-\log(\omega)\right) = -20\log(\omega)$





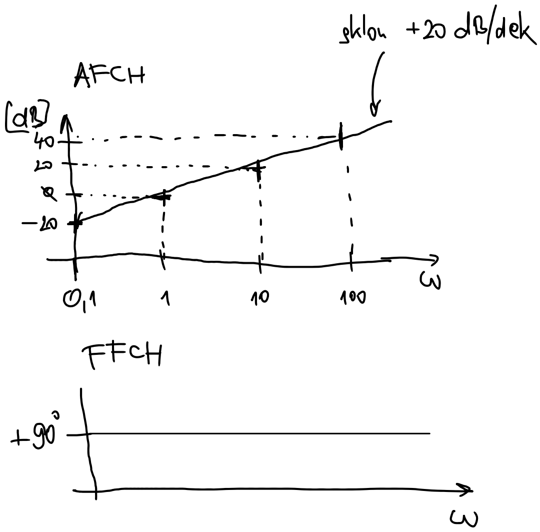
$\phi = -\,\mathrm{atan}\left(\frac{\frac{1}{\omega}}{0}\right) = -\mathrm{atan}(\infty) = -90°$



sklon +20 dB/dek

Bodeho diagram pre $G(s) = s$ $\quad$ AFCH

Bodeho diagram pre $G(s) = s$



T má rozmer [čas]

$\frac{1}{T}$ je frekvencia ...

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{Tj\omega+1}$$

$$\frac{1}{1+jT\omega} \cdot \frac{1-jT\omega}{1-jT\omega} = \frac{1-jT\omega}{1+T^2\omega^2} = \frac{1}{1+T^2\omega^2} + j\frac{T\omega}{1+T^2\omega^2}$$

$$|G(j\omega)| = \sqrt{\left(\frac{1}{1+T^2\omega^2}\right)^2 + \left(\frac{T\omega}{1+T^2\omega^2}\right)^2}$$

$$= \sqrt{\frac{1+T^2\omega^2}{(1+T^2\omega^2)^2}}$$

$$= \sqrt{\frac{1}{1+T^2\omega^2}} = \frac{1}{\sqrt{1+T^2\omega^2}}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+T^2\omega^2}}$$

$$20\log\left(\frac{1}{\sqrt{1+T^2\omega^2}}\right) = -20\log\left(\sqrt{1+T^2\omega^2}\right)$$

$$\angle G(j\omega) = -\operatorname{atan}\left(\frac{\frac{T\omega}{(1+T^2\omega^2)}}{\frac{1}{(1+T^2\omega^2)}}\right) = -\operatorname{atan}(T\omega)$$

$$\Downarrow$$

$\omega \ll \frac{1}{T}$

$$|G(j\omega)| = -20\log(1) = 0\ [dB]$$

$\omega \gg \frac{1}{T}$

$$|G(j\omega)| = -20\log\left(\sqrt{1+T^2\omega^2}\right) \approx -20\log(T\omega)\ [dB]$$

$$1 \ll T^2\omega^2$$

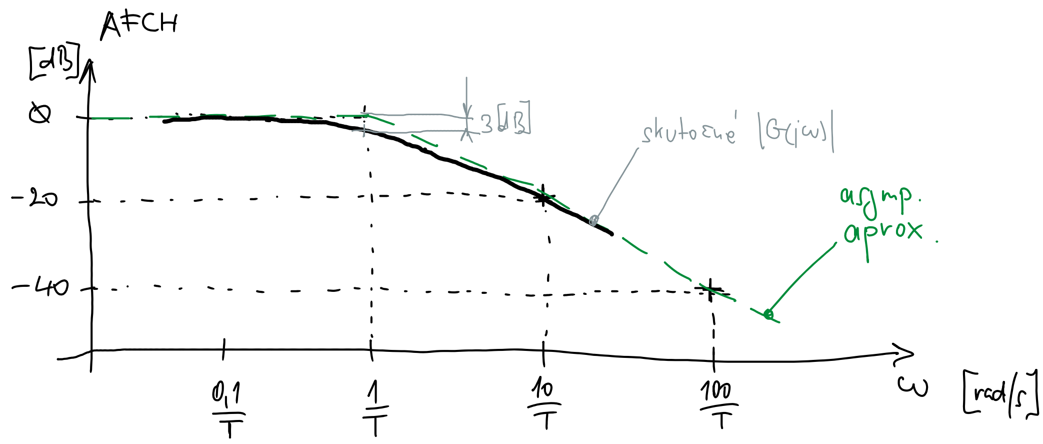
$$\Downarrow$$

$$\sqrt{T^2\omega^2}$$

ω	aprox. $\lvert G(j\omega)\rvert$ [dB]
$\frac{1}{T}$	0
$\frac{10}{T}$	-20
$\frac{100}{T}$	-40

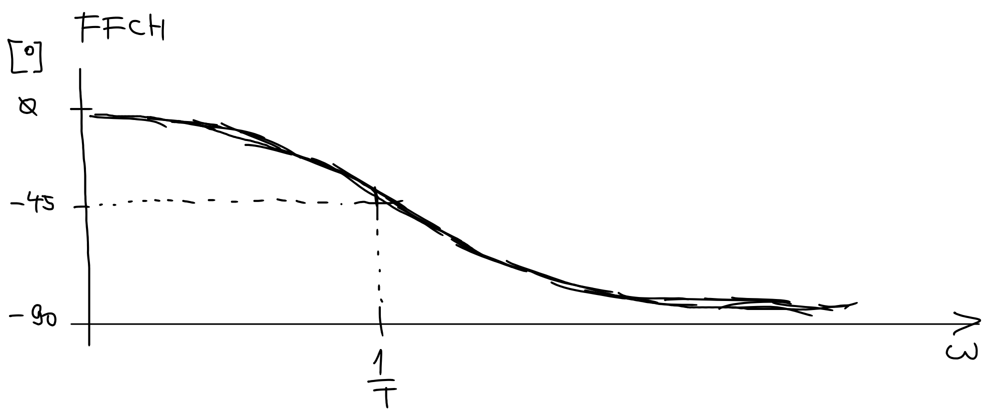
$\omega = \frac{1}{T}$

$$|G(j\omega)| = -20\log\left(\sqrt{1+1}\right) = -20\log\left(\sqrt{2}\right) \approx -20 \cdot 0{,}15 = -3\ [dB]$$



$$\phi = -\,\mathrm{atan}(T\omega)$$

ω	φ [°]
Ø	Ø
$\frac{1}{T}$	−45
∞	−90



$$G(s) = (Ts+1)$$ ← obrátený výraz oproti $\frac{1}{(Ts+1)}$

Výhoda Bodeho diagramu...
(použitie $20\log(|G(j\omega)|)$

platí totiž:

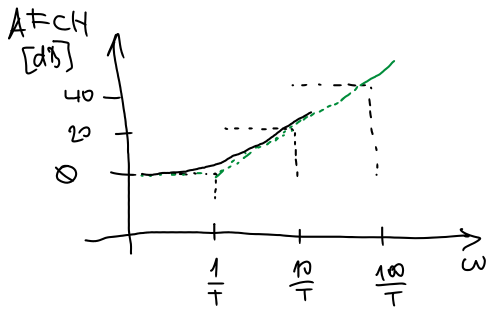
$$+\,20\log\left(|Tj\omega+1|\right) = -\,20\log\left(\left|\frac{1}{Tj\omega+1}\right|\right)$$

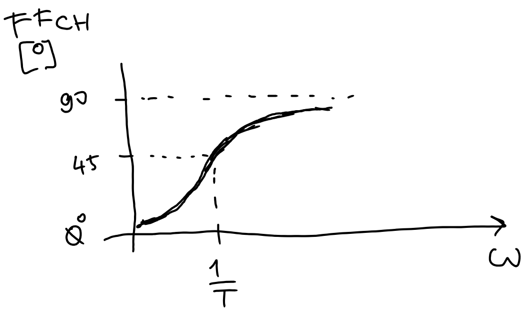
len opačné znamienko...

$$+\,\angle Tj\omega+1 = \mathrm{atan}(T\omega) = -\,\angle\frac{1}{Tj\omega+1}$$

$$(+)\ \angle\, Tj\omega+1 = \operatorname{atan}(T\omega) = (-)\ \angle\, \frac{1}{Tj\omega+1}$$

Takže:



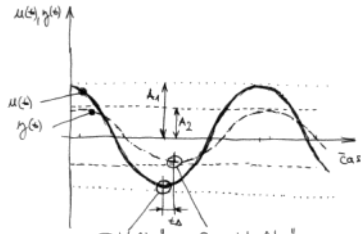


MRS 08 (Beta verzia)

strana 1

$u(t) = A_1 \cos(\omega t)$

amplitúda A_1 — frekvencia ω



ustálený stav
„ustálená situácia"

$y(t) = ?$

||| frekvencia rovnaká ako $u(t)$

– iná amplitúda A_2
– tzv. fázový posun

„Istá fáza" signálu $u(t)$

Rovnaká „fáza" ako, ale v inom čase

...fázové posunutie

fázové posunutie
ako čas: t_Δ
ako uhol: $\omega \cdot t_\Delta$
označme $\varphi = \omega t_\Delta$

$y(t) = A_2 \cos(\omega t + \varphi)$

Amplitúdové zosilnenie A

Nech $A_1 = 1$ [jednotka]

$A = \frac{A_2}{A_1}$

Závislé od ω...

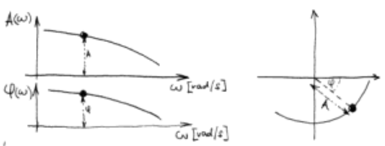
$A(\omega)$
$\varphi(\omega)$

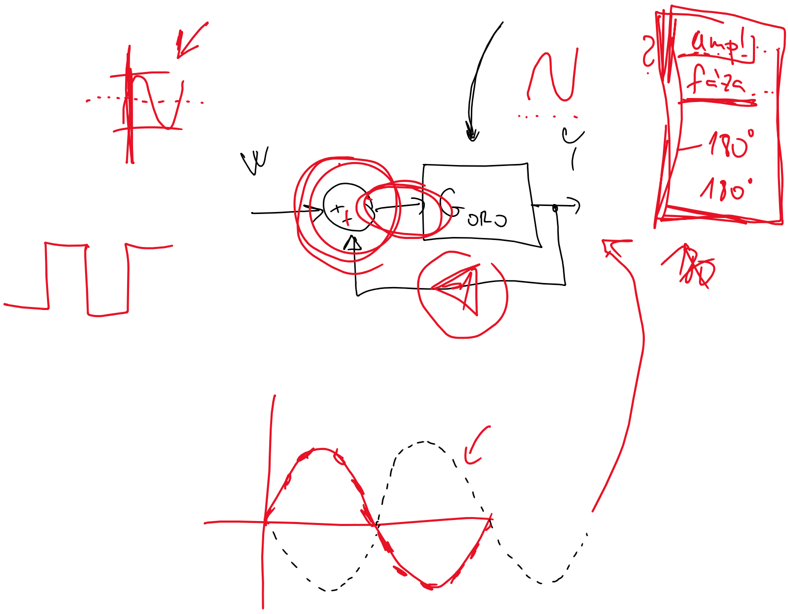
Amplitúdová frekvenčná charakteristika
APCH

~~Frekvenčná~~ Fázová frekvenčná charakteristika
FFCH

„xy" graf:

polárne súradnice:

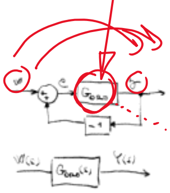




MRS 08 (Beta verzia)

strana 5

URO:

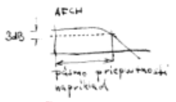


ORO:

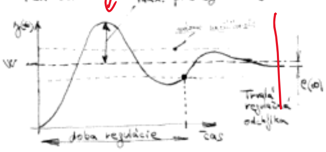
FCH ORO a návrh URO

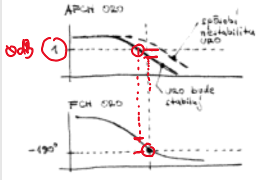
Na základe FCH ORO je možné zaoberať sa nasledovnými vlastnosťami URO

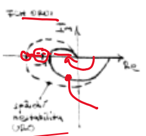
- stabilita URO → Nyquistove kritérium
- Trvalá regulačná odchýlka URO $e(\infty)$ — $e(\infty)$ sa zmenšuje ak $A(\omega)$ sa zvyšuje pri nízkych ω — amplitúdové zosilnenie ORO
- Preregulovanie URO → je menšie ak bezpečnosť (rezerva) vo fáze je väčšia
- Doba regulácie URO → je kratšia ak pásmo priepustnosti ORO je väčšie



PCH URO



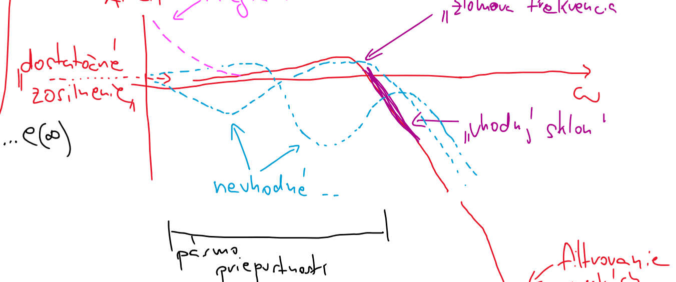




Nyquistove kritérium stability (bez dôkazu...)

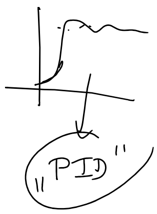
URO je stabilný ak APCH ORO má hodnotu menej ako 1 (0 dB) pri frekvencii kde FFCH ORO má hodnotu 180°

Požadovaná FCH ORO ?

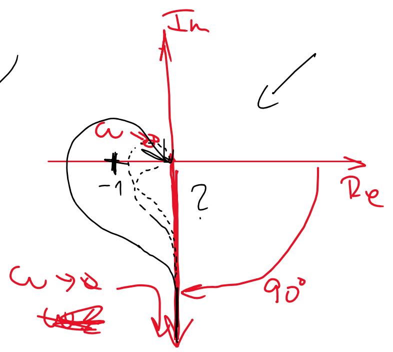


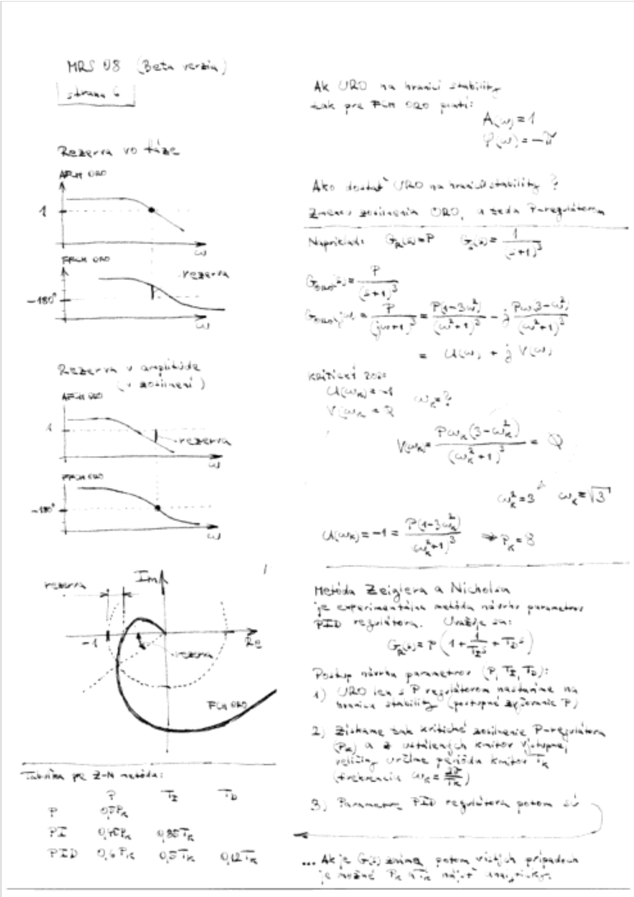
...$e(\infty)$

Filtrovanie vysokých frekv. (šum...)



„PID"




MRS 08 (Beta verzia)

strana 6

Rezerva vo fáze

Rezerva v amplitúde (v zosilnení)

Ak URO na hranici stability tak pre FCH ORO platí:

$A(\omega) = 1$
$\varphi(\omega) = -\pi$

Ako dostať URO na hranicu stability?

Zmenou zosilnenia ORO, a teda P regulátorom

Napríklad: $G_R(s) = P$ $\quad G_S(s) = \frac{1}{(s+1)^3}$

$G_{ORO}(s) = \frac{P}{(s+1)^3}$

$G_{ORO}(j\omega) = \frac{P}{(j\omega+1)^3} = \frac{P(1-3\omega^2)}{(\omega^2+1)^3} - j\frac{P\omega(3-\omega^2)}{(\omega^2+1)^3}$

$= U(\omega) + jV(\omega)$

kritický bod:
$U(\omega_K) = -1$ $\quad \omega_K = ?$
$V(\omega_K) = 0$

$V(\omega_K) = \frac{P\omega_K(3-\omega_K^2)}{(\omega_K^2+1)^3} = 0$

$\omega_K^2 = 3$ $\quad \omega_K = \sqrt{3}$

$U(\omega_K) = -1 = \frac{P(1-3\omega_K^2)}{(\omega_K^2+1)^3} \Rightarrow P_K = 8$

Metóda Zeiglera a Nicholsa
je experimentálna metóda návrhu parametrov PID regulátora. Uvažuje sa:

$G_R(s) = P\left(1 + \frac{1}{T_I s} + T_D s\right)$

Postup návrhu parametrov (P, T_I, T_D):
1) URO len s P regulátorom nastavíme na hranicu stability (postupné zvyšovanie P)
2) Získame tak kritické zosilnenie P-regulátora (P_K) a z ustálených kmitov výstupnej veličiny určíme periódu kmitov T_K (frekvencia $\omega_K = \frac{2\pi}{T_K}$)
3) Parametre PID regulátora potom sú

Tabuľka pre Z-N metódu:

	P	T_I	T_D
P	$0{,}5P_K$		
PI	$0{,}45P_K$	$0{,}85T_K$	
PID	$0{,}6P_K$	$0{,}5T_K$	$0{,}12T_K$

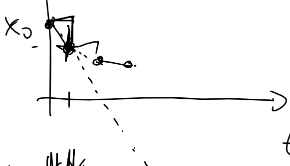
... Ak je G(s) známa, potom v istých prípadoch je možné P_K a T_K nájsť analyticky.

num. sim

~~ode~~ solver(fcn)

$\dot{x} = ax + u$ $\quad x(t)$

$a<0$ $\quad x(0)=x_0$



inf: dotx

dotx=fcn(.....)

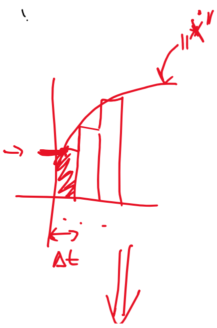
x(k)

$x(k) = x(k-1) + \text{dotx}\cdot\Delta t$

cyklus .... k

dotx = ...? $\quad u_k$??? $\quad u(k-1)$

$x(k) = x(k-1) + \text{dotx}\cdot\Delta t$



$$x(k) = x(k-1) + dotx \cdot \Delta t$$

Δt

$\Downarrow$

x

$$u(k) = ?$$

PID

$$e(k) = w(k) - x(k)$$

$$inte(k) = inte(k-1) + e(k)\,\Delta t$$

$$dere(k) = \big(e(k-1) - e(k)\big)/\Delta t$$