Príklady

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Pojmy (a dojmy)

$$\dot{y}(t) + a y(t) = 0 \qquad \boxed{y(0) = y_0}$$

CHR: $s + a = 0$

korene: $s_1 = -a$

f.r. $y_{fr1} = e^{s_1 t} = e^{-at}$

vš. r. $y(t) = C_1 e^{-at}$

„konkrétne c"

$$y(0) = C_1 = y_0$$

$$\underline{\underline{y(t) = y_0 e^{-at}}};$$

$$\ddot{y} + 6\dot{y} + 5y = 0 \qquad y(0) = 7$$
$$\dot{y}(0) = 0$$

CHR: $s^2 + 6s + 5 = 0$

korene: $s_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$s_1 = -1$

$s_2 = -5$

$s_2 = -5$

f.r. $y_{f1} = e^{-t}$ $\quad y_{f2} = e^{-5t}$

v.r. $y(t) = c_1 e^{-t} + c_2 e^{-5t}$

$y(0) = \underline{c_1 + c_2 = 7}$

$\dot{y}(t) = c_1 e^{-t}(-1) + c_2 e^{-5t}(-5)$

$\dot{y}(0) = \underline{-c_1 - 5c_2 = 0}$

$c_1 = 7 - c_2$

$-7 + c_2 - 5c_2 = 0$

$-4c_2 = 7$

$c_2 = -\frac{7}{4}$

$c_1 = 7 + \frac{7}{4} = \frac{28}{4} + \frac{7}{4} = \frac{35}{4}$

LT !!!

$\dot{y}(t) + a\,y(t) = 0 \qquad y(0) = y_0$

Lap... t.

$\left(sY(s) - y(0)\right) + aY(s) = 0$

$Y(s) = ?$

$Y(s) = \frac{y(0)}{s+a} = y_0 \frac{1}{s+a}$

$y(t) \longleftarrow Y(s)$

$\dot{y}(t) \longrightarrow \boxed{s}Y(s) - \underline{y(0)}$

$e^{-t} \longleftrightarrow \frac{1}{s+1}$

$$y(t) = y_0 e^{-at}$$

LT !!!!

$$\ddot{y} + (a+b)\dot{y} + ab\,y = u \qquad y(0) = y_0 \qquad \dot{y}(0) = z_0$$

$\boxed{u = \delta(t)}$? dirack ...

$a, b \in \mathbb{R}$

y_0, z_0

$\delta(t) \longrightarrow 1$

$U(s) = 1$

$$y \to Y(s)$$

$$\dot{y} \to sY(s) - y(0)$$

$$\ddot{y} \to s\,(sY(s) - y(0)) - \dot{y}(0)$$

$$s\big(sY(s) - \underbrace{y(0)}_{y_0}\big) - \underbrace{\dot{y}(0)}_{z_0} + (a+b)\big(sY(s) - y(0)\big) + ab\,Y(s) = 1$$

$Y(s) = ?$

$$\underbrace{s^2Y(s) + (a+b)sY(s) + abY(s)} - s y_0 - z_0 - (a+b)y_0 = 1$$

$$Y(s)\left(s^2 + (a+b)s + ab\right) = 1 + s y_0 + z_0 + (a+b)y_0$$

$$Y(s) = \frac{1 + s y_0 + z_0 + (a+b)y_0}{\underbrace{s^2 + (a+b)s + ab}_{(s + \text{koreň}_1)(s + \text{koreň}_2)}} = \frac{A}{(s+a)} + \frac{B}{(s+b)}$$

$$(s+a)(s+b)$$

$$1 + s y_0 + z_0 + (a+b)y_0 = A(s+b) + B(s+a)$$

$$s = -a \qquad 1 - \cancel{a y_0} + z_0 + \cancel{a y_0} + b y_0 = A(-a+b)$$

$$A = \frac{1 + z_0 + b y_0}{-a+b}$$

$s=-b \qquad 1-b\cancel{y_0}+z_0+ay_0+\cancel{by_0} = B(-b+a)$

$$B=\frac{1+z_0+ay_0}{-b+a}$$

$$Y(s)=\frac{A}{s+a}+\frac{B}{s+b}$$

$\Downarrow$ inverzná LT (pomocou tabuľky ...)

$$y(t)=Ae^{-at}+Be^{-bt}$$

LT !!!!

$$\ddot{y}+4\dot{y}+3y=u \qquad y(0)=3 \qquad \dot{y}(0)=-2 \qquad \boxed{u=1} \qquad U(s)=\frac{1}{s}$$

$\Big\downarrow$ LT

$$s\left(sY(s)-3\right)+2+4\left(sY(s)-3\right)+3Y(s)=\frac{1}{s}$$

$$s^2Y(s)-3s+2+4sY(s)-12+3Y(s)=\frac{1}{s}$$

$$Y(s)\left(s^2+4s+3\right)\;-3s+2-12=\frac{1}{s}$$

$-3s-10$

$Y(s)=?$

$$Y(s)=\frac{3s+10}{(s^2+4s+3)}+\frac{1}{s}\,\frac{1}{(s^2+4s+3)}$$

$$Y(s) = \frac{3s+10}{\underbrace{(s^2+4s+3)}_{(s+1)(s+3)}} + \frac{1}{s\,\underbrace{(s^2+4s+3)}_{(s+1)(s+3)}}$$

$$Y(s) = \frac{A}{(s+1)} + \frac{B}{(s+3)} + \frac{C}{s} + \frac{D}{(s+1)} + \frac{E}{(s+3)}$$

$$3s+10 = A(s+3)+B(s+1)$$

$s=-3 \qquad 1 = B\cdot(-2) \qquad B = -\frac{1}{2}$

$s=-1 \qquad 7 = A\cdot(2) \qquad A = \frac{7}{2}$

inverzní LT

$$y(t) = \frac{7}{2}e^{-t} - \frac{1}{2}e^{-3t} + \quad \dots$$

$$\ddot{y} + a_1\dot{y} + a_0 y = b_0 u \longrightarrow \ddot{y} = \dots$$

$$\boxed{\begin{aligned}\dot{x}_1 &= ?\\ \dot{x}_2 &= ?\end{aligned}}$$

$$\ddot{y} = -a_1\underbrace{\dot{y}}_{x_2} - a_0\underbrace{y}_{x_1} + b_0 u$$

zvolíme $x_1 = y$

$\dot{x}_1 = \dot{y}$

zvolíme $x_2 = \dot{y}$

zvolíme $x_2 = \dot{y}$

$\dot{x}_2 = \ddot{y} = \ldots = -a_1 x_2 - a_0 x_1 + b_0 u$

Výsledok:

1. rov... $\dot{x}_1 = x_2$

2. rov... $\dot{x}_2 = -a_1 x_2 - a_0 x_1 + b_0 u$

ďalšie úloha prepísať na:

$$\dot{x} = Ax + bu$$

$$y = c^T x$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$