# GUROBI OPTIMIZER QUICK START GUIDE



Version 9.0, Copyright © 2019, Gurobi Optimization, LLC

# Contents

| 1 | Introduction         |   |    |  |  |  |  |  |  |  |  |
|---|----------------------|---|----|--|--|--|--|--|--|--|--|
| 2 | Obt                  | taining a Gurobi License  | 3  |  |  |  |  |  |  |  |  |
|   | 2.1                  | Creating a new academic license                                 | 3  |  |  |  |  |  |  |  |  |
| 3 | Sof                  | tware Installation Guide  | 4  |  |  |  |  |  |  |  |  |
|   | 3.1                  | SELinux   | 5  |  |  |  |  |  |  |  |  |
| 4 | Ret                  | trieving and Setting Up a Gurobi License                        | 7  |  |  |  |  |  |  |  |  |
|   | 4.1                  | Retrieving a Free Academic license                              | 8  |  |  |  |  |  |  |  |  |
|   |                      | 4.1.1 Academic validation                                       | 9  |  |  |  |  |  |  |  |  |
|   | 4.2                  | Retrieving a Named-User or Single-Machine or Single-Use license | 10 |  |  |  |  |  |  |  |  |
|   | 4.3                  | Setting up and using a Floating license                         | 11 |  |  |  |  |  |  |  |  |
|   |                      | 4.3.1 Retrieving a Floating license                             | 12 |  |  |  |  |  |  |  |  |
|   |                      | 4.3.2 Starting a token server                                   | 13 |  |  |  |  |  |  |  |  |
|   |                      | 4.3.3 Upgrading a token server                                  | 14 |  |  |  |  |  |  |  |  |
|   |                      | 4.3.4 Creating a token server client license                    | 14 |  |  |  |  |  |  |  |  |
|   | 4.4                  | Setting up and using a Compute Server license                   | 16 |  |  |  |  |  |  |  |  |
|   |                      | 4.4.1 Retrieving a Compute Server license                       | 16 |  |  |  |  |  |  |  |  |
|   |                      | 4.4.2 Creating a Compute Server client license                  | 17 |  |  |  |  |  |  |  |  |
|   | 4.5                  | Starting Gurobi Remote Services                                 | 18 |  |  |  |  |  |  |  |  |
|   | 4.6                  | Using an Instant Cloud license                                  | 19 |  |  |  |  |  |  |  |  |
|   | Testing your license | 20  |    |  |  |  |  |  |  |  |  |
|   | 4.8                  | Setting environment variables                                   | 21 |  |  |  |  |  |  |  |  |
| 5 | Sol                  | ving a Simple Model - The Gurobi Command Line                   | 22 |  |  |  |  |  |  |  |  |
| 6 | Inte                 | eractive Shell  | 27 |  |  |  |  |  |  |  |  |
| 7 | Att                  | tributes  | 40 |  |  |  |  |  |  |  |  |

| 8         | C Interface                                 | 41  |
|-----------|---|-----|
| 9         | C++ Interface                               | 48  |
| 10        | Java Interface                              | 54  |
| 11        | .NET Interface (C#)                         | 60  |
| 12        | Python Interface                            | 66  |
|           | 12.1 Simple Python Example                  | 67  |
|           | 12.2 Python Matrix Example                  | 71  |
|           | 12.3 Python Dictionary Example              | 75  |
|           | 12.4 Building and running the examples      | 87  |
| 13        | MATLAB Interface                            | 88  |
| 14        | R Interface                                 | 92  |
| <b>15</b> | Recommended Reading                         | 96  |
| 16        | Installing the Anaconda Python distribution | 97  |
|           | 16.1 Using the Spyder IDE                   | 99  |
|           | 16.2 Using Jupyter                          | 101 |
| <b>17</b> | File Overview                               | 103 |

ii ii

# Introduction

Welcome to the Gurobi<sup>TM</sup> Optimizer Quick Start Guide for Linux users! This document provides a basic introduction to the Gurobi Optimizer, including:

- Information on Obtaining a Gurobi License.
- A Software Installation Guide, which includes information on Retrieving and Setting Up your License.
- An example of how to create a simple optimization model and solve it with the Gurobi Command Line, and
- A discussion of the Gurobi Interactive Shell.

We suggest that all users read these first five sections.

Once you have done this, you will probably want to choose a programming environment from which to use Gurobi. If you don't have a strong preference, we recommend that you use our Python® interface, which provides a number of benefits. First, Python is a very nice programming language that can be used for anything from experimentation to prototyping to deployment. Beyond this, though, our Python interface includes a set of higher-level modeling constructs that make it much easier to build optimization models. We also include instructions for installing the Anaconda Python distribution, which includes both a graphical development environment (Spyder) and a notebook-style interface (Jupyter).

If you already have a preferred programming language, you can select from among our available interfaces:

- C interface,
- C++ interface,
- Java® interface,
- Microsoft® .NET interface,
- Python interface,
- MATLAB® interface, or
- R interface.

At the end of the Quick Start Guide, you'll find a File Overview that lists the files included in the Gurobi distribution.

#### **Additional resources**

Once you are done with the Quick Start Guide, the next step is to explore these additional resources:

- If you are familiar with mathematical modeling and are ready to use Gurobi from one of our programming language APIs, consult the Gurobi Reference Manual.
- If you would like to see examples of ways to use Gurobi, consult the Gurobi Example Tour.
- If you are a Gurobi Compute Server user, consult the Gurobi Remote Services Reference Manual.
- If you would like to learn more about mathematical programming or modeling, we've collected a set of references in our recommended reading section.

# **Getting help**

If you have a question that is not answered in this document, please visit the Gurobi support site at <a href="https://support.gurobi.com">https://support.gurobi.com</a>. There, you can read knowledge base articles and join the community discussion forum. Also, if you have a current maintenance contract, you can use the Gurobi support site to submit a request to the Gurobi support team.

Ready to get started? Your first step is to Obtain a License.

# **Obtaining a Gurobi License**

You will need a license in order to install and use the Gurobi Optimizer. There are several ways to obtain one, depending on your situation:

- If you would like a free evaluation license, please email us at sales@gurobi.com to request one.
- If you are an academic user, you can obtain a free academic license on our website.
- If you have purchased a license from us, that license should be visible through the Current tab of your *Licenses* page on our website (you will need to login to your account to see this page).
- If you plan to use Gurobi as a client of a Gurobi Compute Server, you will need to create a compute server client license.
- If you plan to use Gurobi as a client of a Gurobi token server, you will need to create a token server client license.

Once you have a license, your next step is to install the software.

# 2.1 Creating a new academic license

If you are an academic user at a degree-granting institution and wish to use an academic Gurobi license, you can create one yourself. To do so, visit the Free Academic License page on our website. You will need to read and agree to the End User License Agreement and the Conditions for academic use. Once you have done so, click on *Request License*. Your new license will be visible immediately on the Current page. You can create as many academic licenses as you like.

Your next step is to install the software.

# **Software Installation Guide**

Before using the Gurobi Optimizer, you'll need to install the software on your computer. If you haven't already done so, please go to our download page, find your platform (we'll assume Linux in this document), and choose the corresponding file to download.



# Gurobi Optimizer - Get the Software

#### Get the software

Gurobi Optimizer is the Gurobi optimization libraries. In addition to the software, the corresponding README file contains installation instructions. Here is the list of bug fixes for each release.

| Current version |                                  | 64-bit Windows                   | 32-bit Windows                   | 64-bit Linux                     | 64-bit macOS                     | 64-bit AIX                       |  |  |  |  |
|-----------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|--|--|--|--|
| 8.1.1           | README                           | Gurobi-8.1.1-win64.msi           |                                  | gurobi8.1.1_linux64.tar.gz       | gurobi8.1.1_mac64.pkg            | gurobi8.1.1_power64.tar.gz       |  |  |  |  |
| md5<br>Checksum | 1/dfc21f0ed64daaa4bdf/634eab/05b |                                  |                                  | 05cbb96072e393bd4ebb1d8b9526ce01 | d05a73c0df6622851b4371dc1d292579 | 3d1a756695d52065eeefc15516d9aac6 |  |  |  |  |
| Old versions    |                                  |                                  |                                  |                                  |                                  |                                  |  |  |  |  |
| 8.0.1           | README                           | Gurobi-8.0.1-win64.msi           |                                  | gurobi8.0.1_linux64.tar.gz       | gurobi8.0.1_mac64.pkg            | gurobi8.0.1_power64.tar.gz       |  |  |  |  |
| md5<br>Checksum |                                  | d9363f13daa63b79c0cdaa37ad92e8b6 |                                  | cfc595ddf9482734bdc0268749093cc4 | a02d04ef884e64e7091ef7a7439cfe68 | 877f94a02e602346ee767b9894df4030 |  |  |  |  |
| 7.5.2           | README                           | Gurobi-7.5.2-win64.msi           | Gurobi-7.5.2-win32.msi           | gurobi7.5.2_linux64.tar.gz       | gurobi7.5.2_mac64.pkg            | gurobi7.5.2_power64.tar.gz       |  |  |  |  |
| md5<br>Checksum |                                  | 8216e812ff183a966210d9d8b3059626 | 9d3e10b408d4a83e5ca93359d7b26b49 | 01f6dbb8d165838cca1664a1a14e4a85 | e42651aa319ee08c56f52c8532b7f458 | f99ef512f0a8f97d3566b2c3dd136958 |  |  |  |  |
| 7.0.2           | README                           | Gurobi-7.0.2-win64.msi           | Gurobi-7.0.2-win32.msi           | gurobi7.0.2_linux64.tar.gz       | gurobi7.0.2_mac64.pkg            | gurobi7.0.2_power64.tar.gz       |  |  |  |  |
| md5<br>Checksum |                                  | 0924e658882589fa2508e04381f50973 | 30e0b6f8b1fef6b018eee6b40c494eab | 447a89f1bab64db4fc9ecc78f7b2f546 | 91a721ef5d6c41633dd01b5f8e97c8bc | f96300af4377320528e9cceeb2e2e4fd |  |  |  |  |

Make a note of the name and location of the downloaded file.

Your next step is to choose a destination directory. We recommend /opt for a shared installation, but other directories will work as well. Copy the Gurobi distribution to the destination directory and extract the contents. Extraction is done with the following command:

## tar xvfz gurobi9.0.0\_linux64.tar.gz

This command will create a sub-directory /opt/gurobi900/linux64 that contains the complete Gurobi distribution (assuming you chose /opt). Your <installdir> (which we'll refer to throughout this document) will be /opt/gurobi900/linux64.

The Gurobi Optimizer makes use of several executable files. In order to allow these files to be found when needed, you will have to modify a few environment variables:

- GUROBI\_HOME should point to your <installdir>.
- PATH should be extended to include <installdir>/bin.
- LD LIBRARY PATH should be extended to include <installdir>/lib.

Users of the bash shell should add the following lines to their .bashrc files:

```
export GUROBI_HOME="/opt/gurobi900/linux64"
export PATH="${PATH}:${GUROBI_HOME}/bin"
export LD_LIBRARY_PATH="${LD_LIBRARY_PATH}:${GUROBI_HOME}/lib"

Users of the csh shell should add the following lines to their .cshrc files:
    setenv GUROBI_HOME /opt/gurobi900/linux64
    setenv PATH ${PATH}:${GUROBI_HOME}/bin
    setenv LD_LIBRARY_PATH ${LD_LIBRARY_PATH}:${GUROBI_HOME}/lib

If LD_LIBRARY_PATH is not already set, use the following instead:
    export LD_LIBRARY_PATH="${GUROBI_HOME}/lib"

Or
    setenv LD_LIBRARY_PATH ${GUROBI_HOME}/lib
```

These paths should be adjusted to reflect your chosen <installdir>.

You'll need to close your current terminal window and open a new one after you have made these changes in order to pick up the new settings.

In some Linux distributions, applications launched from the Linux desktop won't read .bashrc (or .cshrc). You may need to set the Gurobi environment variables in .bash\_profile or .profile instead. Unfortunately, the details of where to set these variables vary widely among different Linux distributions. We suggest that you consult the documentation for your distribution if you run into trouble.

You are now ready to proceed to the section on Retrieving Your Gurobi License.

If you would like an overview of the files included in the Gurobi distribution, you can also view the File Overview section.

If you are running Gurobi on a system with SELinux enforced, you should read this section.

# 3.1 SELinux

When you run a program that uses Gurobi on some SELinux systems, you may run into the following error message:

error while loading shared libraries: libgurobi90.so: cannot restore segment prot after reloc: Permission denied The simplest workaround is to set alloc\_execmed on your system (as root):

> setsebool -P allow\_execmod on

The on setting is the default on most SELinux systems, but it may be off in some cases. Note that this is a system-wide setting, so you should consult your system administrator before making this change.

A more local workaround is to modify the security context for the Gurobi shared library:

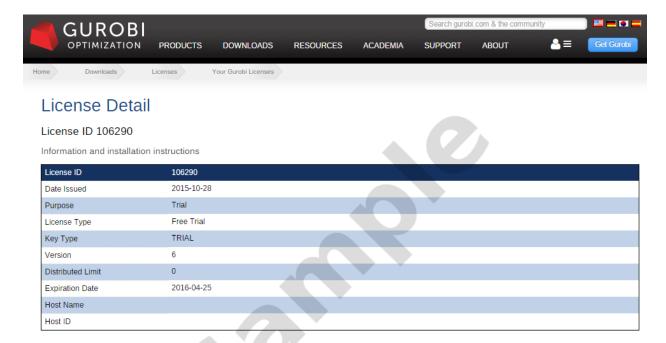
> chcon -t textrel\_shlib\_t libgurobi90.so

You can also use the semanage command to set SELinux policy for this library.

Please consult the SELinux documentation for more information on these commands.

# Retrieving and Setting Up a Gurobi License

Once your license is visible on the Current Page of the website, click on the *License ID* to view the License Detail page:



To install this license on a computer where Gurobi Optimizer is installed, copy and paste the following command to the Start/Run menu (Windows only) or a command/terminal prompt (any system):

grbgetkey 9f712a83-32db-0e10-8285-5630dfa56756

The <code>grbgetkey</code> command requires an active internet connection. If you get no response or an error message such as "Unable to contact key server", please click here for additional instructions.

Your next step is to install this Gurobi license on your machine. You do this by obtaining a license key file.

We strongly recommend that you place your client gurobi.lic file in a default location for your platform (either your home directory or /opt/gurobi). Setting up a non-default location is errorprone and a frequent source of trouble.

Please consult the License Type field on the License Detail page to identify your license type, and click on the appropriate link below to proceed:

- Free Academic
- Named User

- Single-Machine
- Single-Use
- Floating
- Compute Server
- Instant Cloud

If your license includes the Distributed Add-On and you plan to use any of the Gurobi distributed algorithms, you'll also need to set up Gurobi Remote Services on your distributed worker machines.

# 4.1 Retrieving a Free Academic license

To obtain a Gurobi license key you'll need to run the grbgetkey command on your machine to retrieve your Gurobi license key. Note that the machine must be connected to the Internet in order to run this command. An Internet connection is not required after you have obtained your license key.

The exact grbgetkey command to run for a specific license is indicated at the bottom of the License Detail page (e.g., grbgetkey 253e22f3-...). We recommend that you use copy-paste to copy the entire grbgetkey command from our website and paste it into a shell window.

The grbgetkey program passes identifying information about your machine back to our website, and the website responds with your license key. Once this exchange has occurred, grbgetkey will ask for the name of the directory in which to store your license key file (gurobi.lic). You should see a prompt that looks like this:

You can store the license key file anywhere, but we strongly recommend that you accept the default location (either your home directory or /opt/gurobi) by hitting *Enter*. Setting up a non-default location is error-prone and a frequent source of trouble.

If you receive an error message at this stage, it typically means that we were unable to validate your academic domain. Please consult the Academic Validation section for more information.

#### Using a non-default license file location

When you run the Linux version of the Gurobi Optimizer, it will look for the gurobi.lic key file in three different default locations. It will always look in your home directory. In addition, Gurobi Optimizer 9.0.0 will also look in /opt/gurobi and /opt/gurobi900. If you would like to use a non-default license key file location, you can do so by setting an environment variable GRB\_LICENSE\_FILE to point to the license key file. See Setting environment variables for details on how to do this.

Important note: the environment variable should point to the license key file itself, not to the directory that contains the file.

#### Next steps

If your license includes the Distributed Add-On and you plan to use any of the Gurobi distributed algorithms, you'll also need to set up Gurobi Remote Services on your distributed worker machines.

Once you have followed the steps above and have obtained a license key file, your next step is to test your license.

#### 4.1.1 Academic validation

If you are using a free academic license, grbgetkey will perform an academic validation step before retrieving your license key. This step checks your domain name against our list of known academic domains. This section will help you resolve validation errors.

#### Not a recognized academic domain

If grbgetkey produces a message that looks like this:

```
ERROR 303: hostname mymachine.mydomain (234.28.234.144) not recognized as belonging to an academic domain
```

it means your domain isn't on our academic domain list. Please make sure you are connected to your university network. If you are validating a home machine and the university provides a VPN, please connect to it before retrieving your license. If the reported host name is a valid university address, please send the specific error message you received to key@gurobi.com and we'll add your domain.

If you are having trouble validating your license through a VPN, note that some VPNs are configured to use *split tunneling*, where traffic to public internet sites is routed through your ISP. You should ask your network administrator whether the VPN can be configured to route traffic to gurobi.com through the private network.

Some machines connect to the internet through a proxy server. You can use the HTTPS\_PROXY or HTTP\_PROXY environment variables to enable grbgetkey to work with your proxy server. Type grbgetkey --help for details.

#### No reverse DNS information

If grbgetkey produces a message that only references a numerical IP address, like this:

```
ERROR 303: hostname 234.28.234.12 (234.28.234.100) not recognized as belonging to an academic domain
```

it means your machine has no reverse DNS information. This usually happens when you are connecting to the Internet through a DHCP server that does NAT (network address translation) or PAT (port address translation), but does not provide DNS information for its clients. The simplest way to resolve this issue is to ask your network administrator to add a DNS entry (a *PTR record*) for the DHCP device itself.

There is unfortunately no way for us to validate your academic license without reverse DNS information. You can visit this site to check DNS information for your IP address and to obtain more information about reverse DNS.

# 4.2 Retrieving a Named-User or Single-Machine or Single-Use license

To obtain a Gurobi license key you'll need to run the grbgetkey command on your machine to retrieve your Gurobi license key. Note that the machine must be connected to the Internet in order to run this command. An Internet connection is not required after you have obtained your license key.

If your computer isn't connected to the Internet or if your network security system does not allow the command below to function, we also offer a manual license key process. You'll find manual instructions at the bottom of the License Detail page (by following the link labeled *click here for additional instructions*).

The exact grbgetkey command to run for a specific license is indicated at the bottom of the License Detail page (e.g., grbgetkey 253e22f3-...). We recommend that you use copy-paste to copy the entire grbgetkey command from our website and paste it into a shell window.

The grbgetkey program passes identifying information about your machine back to our website, and the website responds with your license key. Once this exchange has occurred, grbgetkey will ask for the name of the directory in which to store your license key file (gurobi.lic). You should see a prompt that looks like this:

```
Gurobi license key client (version 9.0.0)
Copyright (c) 2019, Gurobi Optimization, LLC

-----
Contacting Gurobi key server...
```

You can store the license key file anywhere, but we strongly recommend that you accept the default location (either your home directory or /opt/gurobi) by hitting *Enter*. Setting up a non-default location is error-prone and a frequent source of trouble.

# Using a non-default license file location

When you run the Linux version of the Gurobi Optimizer, it will look for the gurobi.lic key file in three different default locations. It will always look in your home directory. In addition, Gurobi Optimizer 9.0.0 will also look in /opt/gurobi and /opt/gurobi900. If you would like to use a non-default license key file location, you can do so by setting an environment variable GRB\_LICENSE\_FILE to point to the license key file. See Setting environment variables for details on how to do this.

Important note: the environment variable should point to the license key file itself, not to the directory that contains the file.

#### Next steps

If your license includes the Distributed Add-On and you plan to use any of the Gurobi distributed algorithms, you'll also need to set up Gurobi Remote Services on your distributed worker machines.

Once you have followed the steps above and have obtained a license key file, your next step is to test your license.

# 4.3 Setting up and using a Floating license

When using a floating license, a program that calls the Gurobi Optimizer must obtain a token from a Gurobi token server before it can solve an optimization model. There are a few steps involved in setting up such licenses. The first is to retrieve your license key. The key should be installed on the machine that will act as your token server. Once you have your key, you will need to start the Gurobi token server. The token server is a process that runs in the background, handing out available tokens to programs as they request them. Finally, each client for the token server will need to create a token server client license to allow client programs to find the token server.

Note that if you are setting up a machine as a client of an existing token server, you just need to create a token server client license.

## 4.3.1 Retrieving a Floating license

If you are using a floating license, you will need to choose a machine to act as your Gurobi token server. This token server doles out tokens to client machines. A client will request a token from the token server when it creates a Gurobi environment, and it will return the token when it destroys that environment. The client machine can be any machine that can reach the token server over your network (including the token server itself). The client can run any supported operating system. Thus, for example, a Linux client can request tokens from a Windows token server.

Once you've chosen a machine to act as your token server, you'll need to run the grbgetkey command on your machine to retrieve your Gurobi license key. Note that the machine must be connected to the Internet in order to run this command. An Internet connection is not required after you have obtained your license key.

If your computer isn't connected to the Internet or if your network security system does not allow the command below to function, we also offer a manual license key process. You'll find manual instructions at the bottom of the License Detail page (by following the link labeled *click here for additional instructions*).

The exact grbgetkey command to run for a specific license is indicated at the bottom of the License Detail page (e.g., grbgetkey 253e22f3-...). We recommend that you use copy-paste to copy the entire grbgetkey command from our website and paste it into a shell window.

The grbgetkey program passes identifying information about your machine back to our website, and the website responds with your license key. Once this exchange has occurred, grbgetkey will ask for the name of the directory in which to store your license key file (gurobi.lic). You should see a prompt that looks like this:

You can store the license key file anywhere, but we strongly recommend that you accept the default location (either your home directory or /opt/gurobi) by hitting *Enter*. Setting up a non-default location is error-prone and a frequent source of trouble.

#### Setting a password for your token server

If you want to require clients of your token server to specify a password in order to check out a token, you'll need to add one line to your gurobi.lic file:

#### PASSWORD=abcd1234

You should of course choose your own password. Clients will need to place this same line in their client license files.

When adding this line to your gurobi.lic file, please be careful not to modify anything else in the file.

## Using a non-default license file location

When you run the Linux version of the Gurobi Optimizer, it will look for the gurobi.lic key file in three different default locations. It will always look in your home directory. In addition, Gurobi Optimizer 9.0.0 will also look in /opt/gurobi and /opt/gurobi900. If you would like to use a non-default license key file location, you can do so by setting an environment variable GRB\_LICENSE\_FILE to point to the license key file. See Setting environment variables for details on how to do this.

Important note: the environment variable should point to the license key file itself, not to the directory that contains the file.

Once you have followed the steps above and have obtained a license key file, your next step is to start the token server.

#### 4.3.2 Starting a token server

Important note: most Gurobi licenses do not use the token server. You should only follow these instructions if you are setting up a floating license. If you are not sure whether you need to start a token server, you can examine the contents of your gurobi.lic file. If it contains the line TYPE=TOKEN, and does not contain the line MACHINELIMIT=0, then you need a token server.

To start the Linux token server daemon, run the program <code>grb\_ts</code> (with no arguments) on your token server machine. You only need to do this once -- the token server will keep running until you stop it (or until the machine is shut down). Be sure that the license key file has been installed before running this program. Note that the token server runs as a user process, so you do not need root privileges to start it.

If you would like the token server to restart automatically when the machine is rebooted, there are a number of options for doing so (including /etc/rc.local and upstart). You should talk to your system administrator.

To stop a running token server, you can issue the grb\_ts -s command. You can also use the ps command to find the relevant process ID, and the kill command to terminate that process.

Output from the token server goes to the system log (/var/log/syslog). You should see a message similar to the following when you start the server:

Mar 5 12:37:21 mymachine grb[7917]: Gurobi Token Server started: Sat Mar 5 12:37:21 2019

By default, the token server only produces logging output when it starts. To obtain more detailed logging information, start the token server with the -v switch. This will produce a log message each time a token is checked in or out.

The token server forks off a separate process that runs in the background. Use the -n switch to run it in the foreground instead.

#### **Firewalls**

If you run into trouble accessing the token server, check to see if the server is running firewall software that is blocking access to some ports. The Gurobi token server uses port number 41954 by default, so you'll need to open access to that port on the server. Please consult the documentation for your firewall software to determine how to do this. If there's a conflict on the default port, you can choose a different one by adding a PORT line to both the server and the client license key files:

#### PORT=46325

You can choose any available port number.

## Next steps

Clients of the token server also need simple license files. Your next step is to set up a client license.

Once your token server is running and you've set up a client license, you can move on to testing the license.

Once you've set up a client license, you can test the state of the token server at any time, as well as get a list of the clients that are currently using tokens, by typing gurobi\_cl --tokens.

## 4.3.3 Upgrading a token server

To upgrade your token server from an earlier version of the Gurobi Optimizer, you will need to perform the following steps (on the machine running the token server):

- 1. Stop the old token server.
- 2. Install the new version of the Gurobi Optimizer.
- 3. Upgrade your license file (or modify GRB\_LICENSE\_FILE to point to the new license file).
- 4. Start the new token server.

# 4.3.4 Creating a token server client license

The purpose of a token server client license is quite simple: it tells the client where to find the Gurobi token server. You can create this file yourself (using a text editor like vim, for example). The client gurobi.lic file typically contains a single line of text:

TOKENSERVER=mymachine.mydomain.com

or:

TOKENSERVER=192.168.1.100

You should of course substitute the name or IP address of your token server in the example above. If your token server was configured to use a non-default port, you'll also need a line that provides the port number:

PORT=46325

The client license file may also include four optional lines. A QUEUETIMEOUT line allows you to set a limit (in seconds) on how long a new Compute Server job will wait in queue before it gives up (and reports a JOB\_REJECTED error). Any negative value will allow a job to sit in the Compute Server queue indefinitely.

An IDLETIMEOUT line allows you to set a limit on how long a Compute Server job can sit idle before the server kills the job (in seconds). A job is considered idle if the server is not currently performing an optimization and the client has not issued any additional commands. The default value will allow a job to sit idle indefinitely in all but a few circumstances. The first exception is the Gurobi Instant Cloud, where the default setting will automatically impose a 30 minute idle time limit (1800 seconds). If you are using an Instant Cloud pool, the actual value will be the maximum between this parameter value and the idle timeout defined by the pool. The second exception is any program that uses the Gurobi Python interface (including the Gurobi Interactive Shell). Such programs will also get a 30 minute idle time limit by default.

A SERVERTIMEOUT line allows you to specify the timeout (in seconds) in case the token server is unavailable. The default value is 30 seconds. If the client program is unable to contact the server for more than the specified amount of time, the client will quit with a network error.

A PASSWORD line allows you to connect to a password-protected token server (you'll need to get the password from the owner of the token server).

A more complex client token file might look like this:

TOKENSERVER=192.168.1.100 IDLETIMEOUT=60 QUEUETIMEOUT=120 SERVERTIMEOUT=10 PASSWORD=abcd1234

We strongly recommend that you place your client gurobi.lic file in a default location for your platform (either your home directory or /opt/gurobi). Setting up a non-default location is errorprone and a frequent source of trouble. (If you still want to use a non-default location, please refer to the instructions that appeared earlier in this section).

If your client and the token server are both running on the same machine, they can share a single gurobi.lic file. You just need to add the following line to the gurobi.lic file you obtained from our website:

TOKENSERVER=localhost

The token server will ignore this line, and the client will ignore everything but this line. Your other option when both client and server are running on the same machine is to create a separate gurobi.lic file for the client, and to set the GRB\_LICENSE\_FILE environment variable to point to this file (following the earlier instructions for using a non-default license location).

Once your client license is in place, you can test the license. If you are unable to connect to the server, you'll need to make sure the server is installed and running. Please consult the instructions for starting a token server for more information.

# 4.4 Setting up and using a Compute Server license

When using a Compute Server license, programs that call the Gurobi Optimizer can offload Gurobi computations onto one or more machines. There are a few steps involved in setting up such licenses. The first is to retrieve your license key. The key should be installed on the machine that will act as a Compute Server. Once you have your key, you will need to start Gurobi Remote Services. Finally, client machines will need a Compute Server client license in order to find the Compute Server(s).

Note that if you are setting up a machine as a client of an existing Compute Server, you just need to create a Compute Server client license.

# 4.4.1 Retrieving a Compute Server license

If you have purchased one or more Gurobi Compute Server licenses, you'll need to perform a few setup steps in order to start your Compute Servers. Once started, client machines will be able to offload the work of solving an optimization model onto these servers. The clients and the Compute Servers can run any mix of supported operating systems. Thus, for example, multiple Linux machines could submit jobs to a pair of Compute Servers, one running Windows and the other running Linux. Any machine that can reach the Compute Server(s) over your network can be a client (including the Compute Servers themselves).

Once you've chosen a machine to act as a Compute Server (or a node in a Compute Server cluster), you'll need to run the grbgetkey command on your machine to retrieve your Gurobi license key. Note that the machine must be connected to the Internet in order to run this command. An Internet connection is not required after you have obtained your license key.

If your computer isn't connected to the Internet or if your network security system does not allow the command below to function, we also offer a manual license key process. You'll find manual instructions at the bottom of the License Detail page (by following the link labeled *click here for additional instructions*).

The exact grbgetkey command to run for a specific license is indicated at the bottom of the License Detail page (e.g., grbgetkey 253e22f3-...). We recommend that you use copy-paste to copy the entire grbgetkey command from our website and paste it into a shell window.

The grbgetkey program passes identifying information about your machine back to our website, and the website responds with your license key. Once this exchange has occurred, grbgetkey will ask for the name of the directory in which to store your license key file (gurobi.lic). You should

You can store the license key file anywhere, but we strongly recommend that you accept the default location (either your home directory or /opt/gurobi) by hitting *Enter*. Setting up a non-default location is error-prone and a frequent source of trouble.

#### Using a non-default license file location

When you run the Linux version of the Gurobi Optimizer, it will look for the gurobi.lic key file in three different default locations. It will always look in your home directory. In addition, Gurobi Optimizer 9.0.0 will also look in /opt/gurobi and /opt/gurobi900. If you would like to use a non-default license key file location, you can do so by setting an environment variable GRB\_LICENSE\_FILE to point to the license key file. See Setting environment variables for details on how to do this.

Important note: the environment variable should point to the license key file itself, not to the directory that contains the file.

Once you have followed the steps above and have obtained a license key file, your next step is to start Gurobi Remote Services.

## 4.4.2 Creating a Compute Server client license

If you are a Compute Server user, we recommend that you read the Gurobi Remote Services Reference Manual for information about configuring and using Remote Services. We'll provide a few relevant details here, but this other document provides a much broader overview.

You have two options for indicating that a Gurobi program will act as a client of a Compute Server. If you are writing a program that calls the Gurobi C, C++, Java, .NET, or Python APIs, these APIs provide routines that allow you to specify the name of a Compute Server node (by creating an

empty environment and then setting parameters related to Compute Server on that environment). If you use these routines, Gurobi licenses aren't required on the client.

Alternately, you can set up a gurobi.lic file that points to the Compute Server. This option allows you to use a Compute Server with nearly any program that calls Gurobi, without the need to modify the calling program. You can create your client gurobi.lic with a text editor like vim. The file should contain a line that looks like this:

COMPUTESERVER=server.mydomain.com:61000

or like this:

COMPUTESERVER=192.168.1.100:61000

This line should provide the name or IP address of any machine in your Compute Server cluster, optionally followed by the chosen port number on that server (which was chosen when you set up the Compute Server on that machine). If your Compute Server uses a password, you should also include a line that gives the password:

PASSWORD=cspwd

Please consult the *Using Remote Services* section of the Gurobi Remote Services Reference Manual for more information.

Note that if your client and server are both running on the same machine, you'll need to create a separate gurobi.lic file for the client, and set the GRB\_LICENSE\_FILE environment variable to point to this file (following the earlier instructions for using a non-default license location).

Once your client license is in place, you can test the license. If you are unable to connect to the server, you'll need to make sure the server is installed and running. Please consult the *Cluster Setup* and *Administration* section of the Gurobi Remote Services Reference Manual for more information.

# 4.5 Starting Gurobi Remote Services

Important note: you only need to start Gurobi Remote Services if you are setting up a Compute Server or a distributed worker (for use in distributed algorithms). If you are not sure whether you need to start Gurobi Remote Services, you can examine the contents of your gurobi.lic file. If it contains the line CSENABLED=1, then you need Gurobi Remote Services. If it contains a line that begins with DISTRIBUTED=, and if you plan to run distributed algorithms, then you also need Gurobi Remote Services.

Gurobi Remote Services is a Linux service that allows one or more machines to perform Gurobi computations on behalf of other client machines. The set of services a server can provide will depend on your license. If you are setting up a machine as a distributed worker, no license is required. In this case, the only service provided by the server is to act as a worker in a distributed algorithm. If you have a Compute Server license, then servers running Gurobi Remote Services can provide a variety of services, including offloading computation from a set of clients, balancing computational load among the servers, and providing failover capabilities, in addition to acting as a distributed worker. In this case, be sure that your license key file has been installed before starting Gurobi Remote Services.

You'll find instructions for installing, starting, and stopping Gurobi Remote Services in the *Cluster Setup and Administration* section of the Gurobi Remote Services Reference Manual.

Note that Gurobi Remote Services is distributed as a separate installer (named gurobi\_server8.0.0\_linux64.tar.gzon the Linuxplatform). You'll need to download that file separately from our download page.

#### Next steps

Once you've set up Gurobi Remote Services, you should test the state of the server. Type this command on your server (assuming you have configured your Compute Server to use port 61000):

```
gurobi_cl --server=localhost:61000
```

If the output includes the following line:

```
Capacity available on 'localhost:61000' - connecting...
```

then Remote Services is ready for use.

Client programs will need to know how to reach your server. If you are using Gurobi Compute Server, this is typically done with a client license file. You should set that up now.

# 4.6 Using an Instant Cloud license

You have two options for indicating that a Gurobi program will act as a client of a Gurobi Instant Cloud instance. If you are writing a program that calls the Gurobi C, C++, Java, .NET, or Python APIs, these APIs provide routines that allow you to launch cloud instances (by creating an empty environment and then setting parameters related to the cloud on that environment). If you use these routines, Gurobi licenses aren't required on the client.

Alternately, you can install a gurobi.lic file that points to your Gurobi Instant Cloud. This option allows you to use Instant Cloud with nearly any program that calls Gurobi, without the need to modify the calling program. You can download a client gurobi.lic file from your account on the Instant Cloud website. You should place this file in a default location (either your home directory or /opt/gurobi), or set the GRB\_LICENSE\_FILE environment variable to point to it.

This file will contain two lines that look like this:

```
CLOUDACCESSID=312e9gef-e0bc-4114-b6fb-26ed7klaeff9
CLOUDKEY=ae32L0H321dgaL
```

If you are using a non-default pool, the file should also indicate the name of that pool:

```
CLOUDPOOL=myPool
```

These lines allow your client program to launch and use cloud instances from your account. You should keep this information private, since anyone with access to it can also use your cloud instances.

Please consult the *Gurobi Instant Cloud* section of the Gurobi Remote Services Reference Manual for more information.

# 4.7 Testing your license

Once you have obtained a license key for your machine, you are ready to test your license using the Gurobi Interactive Shell. To do this, type gurobi.sh in a *Terminal* window. The shell should produce the following output:

```
Using license file /opt/gurobi/gurobi.lic
Set parameter LogFile to value gurobi.log
Gurobi Interactive Shell, Version 9.0.0
Copyright (c) 2019, Gurobi Optimization, LLC
Type "help()" for help
```

gurobi>

If you are running as a client of a Gurobi Compute Server, the message above will be preceded by a message like this:

```
Capacity available on 'myserver' - connecting...
Established HTTP unencrypted connection
```

Congratulations, your license is functioning correctly! You are now ready to use the Gurobi Optimizer. The next section will show you how to solve a simple optimization model.

#### Possible errors

If the Gurobi shell didn't produce the desired output, there's a problem with your license. We'll list a few common errors here.

The following message...

```
ERROR: No Gurobi license found (user smith, host mymachine, hostid 9d3128ce)
```

indicates that your gurobi.lic file couldn't be found.

Did you use a non-default license file location? When you run the Linux version of the Gurobi Optimizer, it will look for the gurobi.lic key file in three different default locations. It will always look in your home directory. In addition, Gurobi Optimizer 9.0.0 will also look in /opt/gurobi and /opt/gurobi900. If you used a non-default license key file location, you should set environment variable GRB\_LICENSE\_FILE to point to the license key file. See Setting environment variables for details on how to do this.

Important note: the environment variable should point to the license key file itself, not to the directory that contains the file.

The following message:

```
ERROR: HostID mismatch (licensed to 9d3128ce, hostid is 7de025e9)
```

indicates that your gurobi.lic isn't valid for this machine. You should make sure that you are using the right gurobi.lic file.

If you are running as a client of a Gurobi token server and receive this message:

ERROR: Failed to connect to token server 'myserver' (port 41954)

the token server isn't currently running. If you receive this message:

ERROR: No TOKENSERVER specified for TOKEN license

your license file is missing the TOKENSERVER= line that provides the name of your token server. Please consult the section on setting up a token server.

If you are running as a client of a Gurobi Compute Server and receive this message:

ERROR: No server available

the Compute Server isn't currently running. Please consult the section on setting up a Compute Server.

If, after following the instructions, you still experience problems while setting up or testing your license, you can contact us at key@gurobi.com. Please include the exact text of the error message you are receiving.

# 4.8 Setting environment variables

Gurobi uses environment variables for multiple configuration purposes. For example, to specify where to look for a license file, you must set the variable GRB\_LICENSE\_FILE.

You can set an environment variable by adding a line to your .bashrc file:

export GRB\_LICENSE\_FILE=/usr/home/jones/gurobi.lic

For csh shell users, you should add the following to your .cshrc file:

setenv GRB\_LICENSE\_FILE /usr/home/jones/gurobi.lic

You should of course set the variable to point to the actual location of your license key file.

# Solving a Simple Model - The Gurobi Command Line

Now that the Gurobi Optimizer is installed and the license key has been tested, you're ready to solve a simple math programming model.

This section includes instructions on how to create a simple math programming model and how to use the Gurobi command-line interface to compute an optimal solution. If you are already familiar with mathematical modeling and LP-format files, feel free to skip to the end of this section.

Note that this section gives only a brief glimpse into the capabilities of the Gurobi command-line interface. This interface plays a number of different roles. In addition to providing a simple interface for solving a model, it can also be used to launch a model on a Compute Server or on a cloud system, and it can check on the status of a token server. If you'd like additional information, consult the Command-Line Tool section of our Reference Manual.

# The problem statement - producing coins

We'll begin by stating the problem to be solved.

Imagine that it is the end of the calendar year at the United States Mint. The Mint keeps an inventory of the various minerals used to produce the coins that are put into circulation, and it wants to use up the minerals on hand before retooling for next year's coins.

The Mint produces several different types of coins, each with a different composition. The table below shows the make-up of each coin type (as reported in the US Mint coin specifications).

|                | Penny | Nickel | Dime | Quarter | Dollar |
|----------------|-------|--------|------|---------|--------|
| Copper (Cu)    | 0.06g | 3.8g   | 2.1g | 5.2g    | 7.2g   |
| Nickel (Ni)    |       | 1.2g   | 0.2g | 0.5g    | 0.2g   |
| Zinc (Zi)      | 2.4g  |        |      |         | 0.5g   |
| Manganese (Mn) |       |        |      |         | 0.3g   |

Suppose the Mint wants to use the available materials to produce coins with the maximum total dollar value. Which coins should they produce?

# The optimization model

In order to formulate this as an optimization problem, we'll need to do three things.

• First, we'll need to define the decision variables. The goal of the optimization is to choose values for these variables.

- Second, we'll define a linear objective function. This is the function we'd like to minimize (or maximize).
- Third, we'll define the linear constraints.

The Gurobi Optimizer will consider all assignments of values to decision variables that satisfy the specified linear constraints, and return one that optimizes the stated objective function.

The variables in this problem are quite straightforward. The solver will need to decide how many of each coin to produce. It is convenient to give the decision variables meaningful names. In this case, we'll call the variables Pennies, Nickels, Dimes, Quarters, and Dollars. We'll also introduce variables that capture the quantities of the various minerals actually used by the solution. We'll call them Cu, Ni, Zi, and Mn.

Recall that the objective of our optimization problem is to maximize the total dollar value of the coins produced. Each penny produced is worth 0.01 dollars, each nickel is worth 0.05 dollars, etc. This gives the following linear objective:

```
maximize: 0.01 Pennies + 0.05 Nickels + 0.1 Dimes + 0.25 Quarters + 1 Dollars
```

The constraints of this model come from the fact that producing a coin consumes certain quantities of the available minerals, and the supplies of those minerals are limited. We'll capture these relationships in two parts. First, we'll create an equation for each mineral that captures the amount of that mineral that is consumed. For copper, that equation would be:

```
Cu = 0.06 Pennies + 3.8 Nickels + 2.1 Dimes + 5.2 Quarters + 7.2 Dollars
```

The coefficients for this equation come from the earlier coin specification table: one penny uses 0.06g of copper, one nickel uses 3.8g, etc.

The model must also capture the available quantities of each mineral. If we have 1000 grams of copper available, then the constraint would be:

```
Cu <= 1000
```

For our example, we'll assume we have 1000 grams of copper and 50 grams of the other minerals.

There is actually one other set of constraints that must be captured in order for our model to accurately reflect the physical realities of our problem. While a dime is worth 10 cents, half of a dime isn't worth 5 cents. The variables that capture the number of each coin produced must take integer values.

# The model file

The Gurobi Optimizer provides a variety of options for expressing an optimization model. Typically, you would build the model using an interface to a programming languages (C, C++, C#, Java, etc.) or using a higher-level application environment (a notebook, spreadsheet, a modeling system, MATLAB, R, etc.). However, to keep our example as simple as possible, we're going to read the model from an LP format file. The LP format was designed to be human readable, and as such it is well suited for our needs.

The LP format is mostly self-explanatory. Here is our model:

```
Maximize
  .01 Pennies + .05 Nickels + .1 Dimes + .25 Quarters + 1 Dollars
Subject To
 Copper: .06 Pennies + 3.8 Nickels + 2.1 Dimes + 5.2 Quarters + 7.2 Dollars -
     Cu = 0
 Nickel: 1.2 Nickels + .2 Dimes + .5 Quarters + .2 Dollars -
    Ni = 0
 Zinc: 2.4 Pennies + .5 Dollars - Zi = 0
 Manganese: .3 Dollars - Mn = 0
Bounds
  Cu <= 1000
 Ni <= 50
 Zi <= 50
 Mn \le 50
Integers
 Pennies Nickels Dimes Quarters Dollars
```

You'll find this model in file coins.lp in the <installdir>/examples/data directory of your Gurobi distribution. Specifically, assuming you've installed Gurobi 9.0.0 in the recommended location, you'll find the file in /opt/gurobi900/linux64/examples/data/coins.lp.

To modify this file, open it in a text editor (like vim).

## **Editing the LP file**

Before you consider making any modifications to this file or creating your own, we should point out a few rules about LP format files.

#### 1. Ordering of the sections

Our example contains an objective section (Maximize...), a constraint section (Subject To...), a variable bound section (Bounds...), and an integrality section (Integers...). The sections must come in that order. The complete list of section types and the associated ordering rules can be found in the file format section of the Gurobi Reference Manual.

#### 2. Separating tokens

Tokens must be separated by either a space or a newline. Thus, for example, the term:

```
+ .1 Dimes
```

must include a space or newline between + and .1, and another between .1 and Dimes.

## 3. Arranging variables

Variables must always appear on the left-hand side of a constraint. The right-hand side is always a constant. Thus, our constraint:

```
Cu = .06 Pennies + 3.8 Nickels + 2.1 Dimes + 5.2 Quarters + 7.2 Dollars
...becomes...

.06 Pennies + 3.8 Nickels + 2.1 Dimes + 5.2 Quarters + 7.2 Dollars - Cu = 0
```

#### 4. Variable default bounds

Unless stated otherwise, a variable has a zero lower bound and an infinite upper bound. Thus,  $Cu \le 1000$  really means  $0 \le Cu \le 1000$ . Similarly, any variable not mentioned in the Bounds section may take any non-negative value.

Full details on the LP file format are provided in the file format section of the Gurobi Reference Manual.

# Solving the model using the Gurobi command-line interface

The final step in solving our optimization problem is to pass the model to the Gurobi Optimizer. We'll use the Gurobi command-line interface, as it is typically the simplest of our interfaces to use when solving a model stored in a file.

To use the command-line interface, you'll first need to bring up a window that allows you to run command-line programs. On a Linux system, you can use a *Terminal* window. (Note that the Gurobi Interactive Shell, which was used earlier to test your license, does *not* directly accept command-line program input).

The name of the Gurobi command-line tool is gurobi\_cl. To invoke it, type gurobi\_cl, followed by the name of the model file. For example, if our model is stored in the file

/opt/gurobi900/linux64/examples/data/coins.lp, you would type the following command into
your command-line window...

```
>gurobi_cl /opt/gurobi900/linux64/examples/data/coins.lp
```

This command should produce the following output...

```
Using license file /opt/gurobi/gurobi.lic
Set parameter LogFile to value gurobi.log
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Copyright (c) 2019, Gurobi Optimization, LLC
Read LP format model from file /opt/gurobi900/linux64/examples/data/coins.lp
Reading time = 0.00 seconds
: 4 rows, 9 columns, 16 nonzeros
Optimize a model with 4 rows, 9 columns and 16 nonzeros
Model fingerprint: 0xa0c5449c
Variable types: 4 continuous, 5 integer (0 binary)
Coefficient statistics:
 Matrix range
                   [6e-02, 7e+00]
 Objective range [1e-02, 1e+00]
 Bounds range
                   [5e+01, 1e+03]
                   [0e+00, 0e+00]
 RHS range
Found heuristic solution: objective -0.0000000
Presolve removed 1 rows and 5 columns
Presolve time: 0.00s
Presolved: 3 rows, 4 columns, 9 nonzeros
Variable types: 0 continuous, 4 integer (0 binary)
Root relaxation: objective 1.134615e+02, 2 iterations, 0.00 seconds
   Nodes
                  Current Node
                                        Objective Bounds
                                                                    Work
```

```
Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
    0
          0 113.46154
                               1 -0.00000 113.46154
                                                                       0s
                                113.4500000 113.46154
    0
          0
Н
                                                        0.01%
                                                                       0s
                          0
                               1 113.45000 113.46154
            113.46154
                                                                       0s
Explored 1 nodes (2 simplex iterations) in 0.00 seconds
Thread count was 8 (of 8 available processors)
Solution count 2: 113.45 -0
Optimal solution found (tolerance 1.00e-04)
Best objective 1.134500000000e+02, best bound 1.134500000000e+02, gap 0.0000%
```

Details on the format of the Gurobi log file can be found in the Gurobi Reference Manual. For now, you can simply note that the optimal objective value is 113.45. Recall that the objective is denoted in dollars. We can therefore conclude that by a proper choice of production plan, the Mint can produce \$113.45 worth of coins using the available minerals. Moreover, because this value is optimal, we know that it is not possible to produce coins with value greater than \$113.45!

It would clearly be useful to know the exact number of each coin produced by this optimal plan. The <code>gurobi\_cl</code> command allows you to set Gurobi parameters through command-line arguments. One particularly useful parameter for the purposes of this example is <code>ResultFile</code>, which instructs the Gurobi Optimizer to write a file once optimization is complete. The type of the file is encoded in the suffix. To request a <code>.sol</code> file:

```
> gurobi_cl ResultFile=coins.sol coins.lp
```

The command will produce a file that contains solution values for the variables in the model:

```
# Objective value = 113.45
Pennies 0
Nickels 0
Dimes 2
Quarters 53
Dollars 100
Cu 999.8
Ni 46.9
Zi 50
Mn 30
```

In the optimal solution, we'll produce 100 dollar coins, 53 quarters, and 2 dimes.

If we wanted to explore the parameters of the model (for example, to consider how the optimal solution changes with different quantities of available minerals), we could use a text editor to modify the input file. However, it is typically better to do such tests within a more powerful system. We'll now describe the Gurobi Interactive Shell, which provides an environment for creating, modifying, and experimenting with optimization models.

# Interactive Shell

The Gurobi interactive shell allows you to perform hands-on interaction and experimentation with optimization models. You can read models from files, perform complete or partial optimization runs on them, change parameters, modify the models, reoptimize, and so on. The Gurobi shell is actually a set of extensions to the Python shell. Python is a rich and flexible programming language, and any capabilities that are available from Python are also available from the Gurobi shell. We'll touch on these capabilities here, but we encourage you to explore the help system and experiment with the interface in order to gain a better understanding of what is possible.

One big advantage of working within Python is that the Python language is popular and well supported. One aspect of this support is the breadth of powerful Python Integrated Development Environments (IDEs) that are available, most of which can be downloaded for free from the internet. This document includes instructions for setting up Gurobi for use within the Anaconda distribution. Anaconda includes a powerful IDE (Spyder), as well as a notebook-style interface (Jupyter).

Before diving into the details of the Gurobi interactive shell, we should remind you that Gurobi also provides a lightweight command line interface. If you just need to do a quick test on a model stored in a file, you will probably find that that interface is better suited to simple tasks than the interactive shell.

**Important note for AIX users:** due to limited Python support on AIX, our AIX port does not include the Interactive Shell or the Python interface. You can use the command line, or the C, C++, or Java interfaces.

We will now work through a few simple examples of how the Gurobi shell might be used, in order to give you a quick introduction to its capabilities. More thorough documentation on this and other interfaces is available in the Gurobi Reference Manual.

# Reading and optimizing a model

There are several ways to access the Gurobi Interactive Shell.

From Linux, you can simply type gurobi.sh from the command prompt. If you've installed a Python IDE, the shell will also be available from that environment.

Once the optimizer has started, you are ready to load and optimize a model. We'll consider model coins.lp from <installdir>/examples/data...

```
> gurobi.sh
```

```
Using license file /opt/gurobi/gurobi.lic
Set parameter LogFile to value gurobi.log
```

Gurobi Interactive Shell, Version 9.0.0

```
Copyright (c) 2019, Gurobi Optimization, LLC
Type "help()" for help
gurobi> m = read('/opt/gurobi900/linux64/examples/data/coins.lp')
Read LP format model from file /opt/gurobi900/linux64/examples/data/coins.lp
Reading time = 0.01 seconds
: 4 rows, 9 columns, 16 nonzeros
gurobi> m.optimize()
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Optimize a model with 4 rows, 9 columns and 16 nonzeros
Model fingerprint: 0xa0c5449c
Variable types: 4 continuous, 5 integer (0 binary)
Coefficient statistics:
 Matrix range
                   [6e-02, 7e+00]
 Objective range [1e-02, 1e+00]
                   [5e+01, 1e+03]
 Bounds range
 RHS range
                   [0e+00, 0e+00]
Found heuristic solution: objective -0.0000000
Presolve removed 1 rows and 5 columns
Presolve time: 0.00s
Presolved: 3 rows, 4 columns, 9 nonzeros
Variable types: 0 continuous, 4 integer (0 binary)
Root relaxation: objective 1.134615e+02, 2 iterations, 0.00 seconds
   Nodes
                 Current Node
                                  1
                                       Objective Bounds
                                                             1
                                                                    Work
Expl Unexpl | Obj Depth IntInf | Incumbent
                                                 BestBd
                                                         Gap | It/Node Time
                          Ω
    0
          0 113.46154
                             1 -0.00000 113.46154
                                                                        0s
    0
Н
          0
                                113.4500000 113.46154 0.01%
                                                                        0s
           0 113.46154
                          0
                              1 113.45000 113.46154 0.01%
                                                                        0s
Explored 1 nodes (2 simplex iterations) in 0.01 seconds
Thread count was 8 (of 8 available processors)
Solution count 2: 113.45 -0
Optimal solution found (tolerance 1.00e-04)
Best objective 1.134500000000e+02, best bound 1.134500000000e+02, gap 0.0000%
```

The read() command reads a model from a file and returns a Model object. In the example, that object is placed into variable m. There is no need to declare variables in the Python language; you simply assign a value to a variable.

Note that read() accepts wildcard characters, so you could also have said:

```
gurobi> m = read('/opt/gurobi900/linux64/*/*/coin*')
```

Note also that Gurobi commands that read or write files will also function correctly with compressed files. If gzip, bzip2, or 7zip are installed on your machine and available in your default path, then you simply need to add the appropriate suffix (.gz, .bz2, .zip, or .7z) to the file name to read or write compressed versions.

The next statement in the example, m.optimize(), invokes the optimize method on the Model object (you can obtain a list of all methods on Model objects by typing help(Model) or help(m)). The Gurobi optimization engine finds an optimal solution with objective 113.45.

# Inspecting the solution

Once a model has been solved, you can inspect the values of the model variables in the optimal solution with the *printAttr()* method on the Model object:

| <pre>gurobi&gt; m.printAttr</pre> | ('X') |
|-----------------------------------|-------|
| Variable                          | Х     |
|                                   |       |
| Dimes                             | 2     |
| Quarters                          | 53    |
| Dollars                           | 100   |
| Cu                                | 999.8 |
| Ni                                | 46.9  |
| Zi                                | 50    |
| Mn                                | 30    |
|                                   |       |

This routine prints all non-zero values of the specified attribute X. Attributes play a major role in the Gurobi optimizer. We'll discuss them in more detail shortly.

You can also inspect the results of the optimization at a finer grain by retrieving a list of all the variables in the model using the getVars() method on the Model object (m.getVars() in our example):

```
gurobi> v = m.getVars()
gurobi> print(len(v))
9
```

The first command assigns the list of all Var objects in model m to variable v. The Python len() command gives the length of the array (our example model coins has 9 variables). You can then query various attributes of the individual variables in the list. For example, to obtain the variable name and solution value for the first variable in list v, you would issue the following command:

```
gurobi> print(v[0].varName, v[0].x)
Pennies 0.0
```

You can type help(Var) or help(v[0]) to get a list of all methods on a Var object. You can type help(GRB.Attr) to get a list of all attributes.

# Simple model modification

We will now demonstrate a simple model modification. Imagine that you only want to consider solutions where you make at least 10 pennies (i.e., where the Pennies variable has a lower bound of 10.0). This is done by setting the 1b attribute on the appropriate variable (the first variable in the list v in our example)...

```
gurobi> v = m.getVars()
gurobi> v[0].lb = 10
```

The Gurobi optimizer keeps track of the state of the model, so it knows that the currently loaded solution is not necessarily optimal for the modified model. When you invoke the *optimize()* method again, it performs a new optimization on the modified model...

gurobi> m.optimize()

```
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
```

```
Optimize a model with 4 rows, 9 columns and 16 nonzeros
Variable types: 4 continuous, 5 integer (0 binary)
Coefficient statistics:
                  [6e-02, 7e+00]
 Matrix range
 Objective range [1e-02, 1e+00]
 Bounds range
                  [1e+01, 1e+03]
 RHS range
                  [0e+00, 0e+00]
Presolve removed 2 rows and 5 columns
Presolve time: 0.00s
Presolved: 2 rows, 4 columns, 5 nonzeros
MIP start from previous solve did not produce a new incumbent solution
Variable types: 0 continuous, 4 integer (0 binary)
Found heuristic solution: objective 25.9500000
Root relaxation: objective 7.190000e+01, 2 iterations, 0.00 seconds
                 Current Node
                                 Objective Bounds
Expl Unexpl | Obj Depth IntInf | Incumbent
                                                BestBd
                                                         Gap | It/Node Time
             71.90000
                               1 25.95000
                                              71.90000
                                                                       0s
    0
                                 71.8500000
                                              71.90000 0.07%
                                                                       0s
Η
          0
н
     0
                                 71.9000000 71.90000 0.00%
                                                                       0s
          0
Explored 0 nodes (2 simplex iterations) in 0.01 seconds
Thread count was 8 (of 8 available processors)
Solution count 3: 71.9 71.85 25.95
Optimal solution found (tolerance 1.00e-04)
Best objective 7.190000000000e+01, best bound 7.19000000000e+01, gap 0.0000%
```

The result shows that, if you force the solution to include at least 10 pennies, the maximum possible objective value for the model decreases from 113.45 to 71.9. A simple check confirms that the new lower bound is respected:

```
gurobi> print(v[0].x)
10.0
```

# Simple experimentation with a more difficult model

Let us now consider a more difficult model, glass4.mps. Again, we read the model and begin the optimization:

```
gurobi> m = read('/opt/gurobi900/linux64/examples/data/glass4')
Read MPS format model from file /opt/gurobi900/linux64/examples/data/glass4.mps
Reading time = 0.00 seconds
glass4: 396 Rows, 322 Columns, 1815 NonZeros
gurobi> m.optimize()
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Optimize a model with 396 rows, 322 columns and 1815 nonzeros
Model fingerprint: 0x541d0ad3
Variable types: 20 continuous, 302 integer (0 binary)
```

#### Coefficient statistics:

Matrix range [1e+00, 8e+06]
Objective range [1e+00, 1e+06]
Bounds range [1e+00, 8e+02]
RHS range [1e+00, 8e+06]
Presolve removed 4 rows and 5 columns

Presolve time: 0.00s

Presolved: 392 rows, 317 columns, 1815 nonzeros

Variable types: 19 continuous, 298 integer (298 binary)

Found heuristic solution: objective 3.133356e+09

Root relaxation: objective 8.000024e+08, 72 iterations, 0.00 seconds

| Nodes       |      |        | Current Node |       |      |    | 1  | Objective Bounds |      |        |       | Work     |         |      |
|-------------|------|--------|--------------|-------|------|----|----|------------------|------|--------|-------|----------|---------|------|
| Expl Unexpl |      | Jnexpl | l Obj        | Depth | IntI | nf | 1  | Incumbent        | t 1  | BestBd | Gap   | 1        | It/Node | Time |
|             |      |        |              |       |      |    |    |                  |      |        |       |          |         |      |
|             | 0    | 0      | 8.0000e      | +08   | 0    | 72 | 3. | 1334e+09         | 8.00 | 00e+08 | 74.5% | <b>,</b> | -       | 0s   |
| Η           | 0    | 0      |              |       |      | 2. | 40 | 0019e+09         | 8.00 | 00e+08 | 66.7% | <b>,</b> | -       | 0s   |
| Η           | 0    | 0      |              |       |      | 2. | 22 | 20019e+09        | 8.00 | 00e+08 | 64.0% | <b>,</b> | -       | 0s   |
|             | 0    | 0      | 8.0000e      | +08   | 0    | 72 | 2. | 2200e+09         | 8.00 | 00e+08 | 64.0% | <b>,</b> | -       | 0s   |
| Η           | 0    | 0      |              |       |      | 2. | 20 | 0019e+09         | 8.00 | 00e+08 | 63.6% | <b>,</b> | -       | 0s   |
|             | 0    | 0      | 8.0000e      | +08   | 0    | 81 | 2. | 2000e+09         | 8.00 | 00e+08 | 63.6% | <b>,</b> | -       | 0s   |
|             | 0    | 0      | 8.0000e      | +08   | 0    | 77 | 2. | 2000e+09         | 8.00 | 00e+08 | 63.6% | <b>,</b> | -       | 0s   |
|             | 0    | 2      | 8.0000e      | +08   | 0    | 77 | 2. | 2000e+09         | 8.00 | 00e+08 | 63.6% | <b>,</b> | -       | 0s   |
| Η           | 307  | 609    |              |       |      | 2. | 06 | 6686e+09         | 8.00 | 00e+08 | 61.3% | <b>,</b> | 5.8     | 0s   |
| Н :         | 1126 | 885    |              |       |      | 1. | 95 | 0016e+09         | 8.00 | 00e+08 | 59.0% | <b>,</b> | 6.0     | 0s   |
| Н :         | 1317 | 983    |              |       |      | 1. | 90 | 00015e+09        | 8.00 | 00e+08 | 57.9% | <b>,</b> | 5.6     | 0s   |
| Н :         | 1817 | 1173   |              |       |      | 1. | 90 | 00015e+09        | 8.00 | 00e+08 | 57.9% | <b>,</b> | 5.0     | 0s   |
| Н :         | 2656 | 1796   |              |       |      | 1. | 90 | 00015e+09        | 8.00 | 00e+08 | 57.9% | <b>,</b> | 4.8     | 0s   |
| Н 8         | 8305 | 6287   |              |       |      | 1. | 90 | 00015e+09        | 8.00 | 00e+08 | 57.9% | <b>,</b> | 3.4     | 0s   |
| *10         | 0878 | 6870   |              | :     | 99   | 1. | 80 | 8351e+09         | 8.00 | 00e+08 | 55.8% | <b>,</b> | 3.3     | 0s   |
| *12         | 2677 | 7866   |              |       | 62   | 1. | 80 | 0016e+09         | 8.00 | 00e+08 | 55.6% | <b>,</b> | 3.3     | 1s   |
| *1          | 7157 | 10811  |              | 1     | 18   | 1. | 80 | 00015e+09        | 8.00 | 00e+08 | 55.6% | <b>'</b> | 3.2     | 1s   |
| H19         | 9145 | 11166  |              |       |      | 1. | 75 | 0016e+09         | 8.00 | 00e+08 | 54.3% | <b>'</b> | 3.2     | 1s   |
| H24         | 4736 | 14317  |              |       |      | 1. | 70 | 00015e+09        | 8.00 | 00e+08 | 52.9% | <b>'</b> | 3.2     | 1s   |
| H24         | 4874 | 14315  |              |       |      | 1. | 70 | 00015e+09        | 8.00 | 00e+08 | 52.9% | <b>'</b> | 3.2     | 1s   |
| Н32         | 2097 | 17197  |              |       |      | 1. | 63 | 3347e+09         | 8.06 | 65e+08 | 50.6% | <b>'</b> | 3.2     | 2s   |
| Н32         | 2123 | 16354  |              |       |      | 1. | 60 | 0013e+09         | 8.18 | 73e+08 | 48.8% | <b>'</b> | 3.2     | 3s   |
| 32          | 2158 | 16378  | 1.6000e      | +09 1 | 18 1 | 10 | 1. | 6000e+09         | 8.45 | 64e+08 | 47.1% | <b>'</b> | 3.2     | 5s   |
| Н32         | 2215 | 15596  |              |       |      | 1. | 53 | 3346e+09         | 8.60 | 63e+08 | 43.9% | <b>'</b> | 3.3     | 6s   |
| Н32         | 2284 | 14860  |              |       |      | 1. | 50 | 0013e+09         | 8.81 | 36e+08 | 41.2% | <b>'</b> | 3.4     | 9s   |
| 32          | 2294 | 14867  | 1.2500e      | +09   | 46   | 93 | 1. | 5000e+09         | 8.81 | 36e+08 | 41.2% | <b>'</b> | 3.4     | 10s  |
| 32          | 2446 | 14975  | 1.2500e      | +09   | 52   | 95 | 1. | 5000e+09         | 9.00 | 01e+08 | 40.0% | <b>'</b> | 3.5     | 15s  |

# Interrupt request received

## Cutting planes:

Gomory: 8

Implied bound: 13

Projected implied bound: 1

MIR: 19 Flow cover: 17 Zero half: 1 RLT: 4

Relax-and-lift: 17

```
Explored 57196 nodes (301282 simplex iterations) in 19.00 seconds
Thread count was 8 (of 8 available processors)

Solution count 10: 1.50001e+09 1.53335e+09 1.60001e+09 ... 1.80835e+09

Solve interrupted
Best objective 1.500012666667e+09, best bound 1.000006945369e+09, gap 33.3334%
```

It quickly becomes apparent that this model is quite a bit more difficult than the earlier coins model. The optimal solution is actually 1,200,000,000, but finding that solution takes a while. After letting the model run for 10 seconds, we interrupt the run (by hitting CTRL-C, which produces the *Interrupt request received* message) and consider our options. Typing m.optimize() would resume the run from the point at which it was interrupted.

# **Changing parameters**

Rather than continuing optimization on a difficult model like glass4, it is sometimes useful to try different parameter settings. When the lower bound moves slowly, as it does on this model, one potentially useful parameter is MIPFocus, which adjusts the high-level MIP solution strategy. Let us now set this parameter to value 1, which changes the focus of the MIP search to finding good feasible solutions. There are two ways to change the parameter value. You can either use method m.setParam():

```
gurobi> m.setParam('MIPFocus', 1)
Changed value of parameter MIPFocus to 1
  Prev: 0 Min: 0 Max: 3 Default: 0
...or you can use the m.Params class...
gurobi> m.Params.MIPFocus = 1
Changed value of parameter MIPFocus to 1
  Prev: 0 Min: 0 Max: 3 Default: 0
Once the parameter has been changed, we call m.reset() to reset the optimization on our model
and then m.optimize() to start a new optimization run:
gurobi> m.reset()
Discarded solution information
gurobi> m.optimize()
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Optimize a model with 396 rows, 322 columns and 1815 nonzeros
Model fingerprint: 0x541d0ad3
Variable types: 20 continuous, 302 integer (0 binary)
Coefficient statistics:
                   [1e+00, 8e+06]
 Matrix range
 Objective range [1e+00, 1e+06]
 Bounds range
                  [1e+00, 8e+02]
                  [1e+00, 8e+06]
 RHS range
Presolve removed 4 rows and 5 columns
Presolve time: 0.00s
Presolved: 392 rows, 317 columns, 1815 nonzeros
Variable types: 19 continuous, 298 integer (298 binary)
```

Found heuristic solution: objective 3.133356e+09

Root relaxation: objective 8.000024e+08, 72 iterations, 0.00 seconds

| * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 1   * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 1   H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 1   4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 1   H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 1   H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9   H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8 1  | Nodes |      |        | I  |        | rrent |     |     |      | _         |     | ve Bounds |     | 1    | Wor     |      |
|--|-------|------|--------|----|--------|-------|-----|-----|------|-----------|-----|-----------|-----|------|---------|------|
| H 0 0 0 2.400019e+09 8.0000e+08 66.7% - H 0 0 0 2.220019e+09 8.0000e+08 64.0% - 0 0 8.0000e+08 0 72 2.2200e+09 8.0000e+08 64.0% - 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 77 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 80 2.133351e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.5500e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 61.00 -  | ]     | Expl | Unexpl | I  | Obj    | Depth | Int | Inf | 1    | Incumbent | t   | BestBd    | Ga  | ap   | It/Node | Time |
| H 0 0 0 2.400019e+09 8.0000e+08 66.7% - H 0 0 0 2.220019e+09 8.0000e+08 64.0% - 0 0 8.0000e+08 0 72 2.2200e+09 8.0000e+08 64.0% - 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 77 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 80 2.133351e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.5500e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 65 2.050017e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+09 8.0000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 66 2.05000e+08 61.0% - 0 1 2 8.0000e+08 0 61.00 -  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 72  | 3    | .1334e+09 | 8.0 | 0000e+08  | 74. | .5%  | _       | 0s   |
| 0         0         8.0000e+08         0         72         2.2200e+09         8.0000e+08         64.0%         −           H         0         0         8.0000e+08         63.1%         −           0         0         8.0000e+08         0         72         2.1667e+09         8.0000e+08         63.1%         −           0         0         8.0000e+08         0         77         2.1667e+09         8.0000e+08         63.1%         −           0         0         8.0000e+08         0         77         2.1667e+09         8.0000e+08         63.1%         −           0         0         8.0000e+08         0         80         2.1334e+09         8.0000e+08         62.5%         −           0         0         8.0000e+08         0         83         2.1334e+09         8.0000e+08         62.5%         −           0         0         8.0000e+08         0         83         2.1334e+09         8.0000e+08         62.5%         −           0         0         8.0000e+08         0         83         2.1334e+09         8.0000e+08         62.5%         −           0         0         8.0000e+08         0         82.1334e+09 <td>Н</td> <td>0</td> <td>0</td> <td></td> <td></td> <td></td> <td></td> <td>2.</td> <td>. 40</td> <td>00019e+09</td> <td>8.0</td> <td>0000e+08</td> <td>66</td> <td>.7%</td> <td>-</td> <td>0s</td>   | Н     | 0    | 0      |    |        |       |     | 2.  | . 40 | 00019e+09 | 8.0 | 0000e+08  | 66  | .7%  | -       | 0s   |
| H 0 0 0 2.166685e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 77 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 77 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 80 2.13334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 78 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 1 8.0000e+08 0 62.5% - 0 0 8 | Н     | 0    | 0      |    |        |       |     | 2.  | . 2: | 20019e+09 | 8.0 | 0000e+08  | 64  | .0%  | -       | 0s   |
| 0 0 8.0000e+08 0 72 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 77 2.1667e+09 8.0000e+08 63.1% - 0 0 8.0000e+08 0 80 2.133351e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2 2.050017e+09 8.0000e+08 61.0% - H 6 8 2.050017e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% - H 199 203 1.644459e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5  * 4233 2185 66 1.600017e+09 8.0001e+08 51.0% 20.5  * 4233 2082 67 1.550017e+09 8.0001e+08 44.7% 21.5 14.4  H 4008 2026 1.5000e+09 8.0001e+08 46.7% 21.6 14.4  H 4809 2136 1.450016e+09 8.0001e+08 46.7% 22.6  H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4  H 4908 2043 1.400013e+09 8.0001e+08 44.8% 23.4  H 4908 2043 1.400013e+09 8.0001e+08 40.7% 24.8  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 72  | 2    | .2200e+09 | 8.0 | 0000e+08  | 64. | .0%  | -       | 0s   |
| 0         0         8.0000e+08         0         77 2.1667e+09 8.0000e+08         63.1%         −           H         0         0         2.133351e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         80 2.1334e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         80 2.1334e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         83 2.1334e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         78 2.1334e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         83 2.1334e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         83 2.1334e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         83 2.1334e+09 8.0000e+08         62.5%         −           0         0         8.0000e+08         0         66 2.1334e+09 8.0000e+08         62.5%         −           H         0         0         2.050017e+09 8.0000e+08         61.0%         −           H  | Н     | 0    | 0      |    |        |       |     | 2.  | . 16 | 66685e+09 | 8.0 | 0000e+08  | 63  | . 1% | -       | 0s   |
| H 0 0 0 2.133351e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 78 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 61.0% - H 0 0 2.050017e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 61.0% 74.0 H 199 203 1.644459e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 1423 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 14457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 21.6 64 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 64 H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 44.8% 23.9  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 72  | 2    | .1667e+09 | 8.0 | 0000e+08  | 63  | . 1% | -       | 0s   |
| 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 78 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 * 44308 2026 1.550016e+09 8.0001e+08 48.4% 21.5 * 44308 2026 1.550016e+09 8.0001e+08 44.8% 23.4 * 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 44.8% 23.4 * H 4908 2043 1.400013e+09 8.0001e+08 44.8% 23.9 * H 5098 2027 1.350013e+09 8.0001e+08 44.8% 23.9   |       | 0    | 0      | 8. | .0000e | +08   | 0   | 77  | 2    | .1667e+09 | 8.0 | 0000e+08  | 63  | . 1% | -       | 0s   |
| 0 0 8.0000e+08 0 80 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 78 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% 74.0 H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 51.0% 20.5 * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 40.7% 24.8   | Н     | 0    | 0      |    |        |       |     | 2.  | . 13 | 33351e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 78 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 68 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 * 4238 2082 67 1.550017e+09 8.0001e+08 50.0% 21.5 * 4423 2082 67 1.550017e+09 8.0001e+08 46.7% 21.6 * 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 * 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 * 4459 2136 1.450016e+09 8.0001e+08 44.8% 23.4 * H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 * H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 80  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| 0 0 8.0000e+08 0 78 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 51.0% 20.5 * 4238 2082 67 1.550017e+09 8.0001e+08 50.0% 21.5 * 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 21.6 * 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 * 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 * 4459 2136 1.450016e+09 8.0001e+08 44.8% 23.4 * H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 * H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 80  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - H 0 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% - H 6 8 2.000016e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 15.5 3138 2082 67 1.550017e+09 8.0001e+08 50.0% 21.5 144308 2026 1.500016e+09 8.0001e+08 48.4% 21.5 144409 2136 1.450016e+09 8.0001e+08 44.8% 23.4 14908 2043 1.400013e+09 8.0001e+08 44.8% 23.4 14908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 14.5008 2027 1.350013e+09 8.0001e+08 42.9% 23.9 14.50016e+09 8.0001e+08 40.7% 24.8 14.50016e+09 8.0001e+08 4 |       | 0    | 0      | 8. | .0000e | +08   | 0   | 83  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| 0 0 8.0000e+08 0 83 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - H 0 0 0 28.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 51.0% 20.5 * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 H 4308 2026 1.50001e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 42.9% 23.9  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 78  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| 0 0 8.0000e+08 0 88 2.1334e+09 8.0000e+08 62.5% - 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - H 0 0 2.050017e+09 8.0000e+08 61.0% - 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 51.0% 20.5 * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 42.9% 23.9  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 83  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| 0 0 8.0000e+08 0 66 2.1334e+09 8.0000e+08 62.5% - H 0 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - O 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% 74.0 H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8   |       | 0    | 0      | 8. | .0000e | +08   | 0   | 83  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| H 0 0 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 88  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| 0 2 8.0000e+08 0 65 2.0500e+09 8.0000e+08 61.0% - H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8  |       | 0    | 0      | 8. | .0000e | +08   | 0   | 66  | 2    | .1334e+09 | 8.0 | 0000e+08  | 62  | . 5% | -       | 0s   |
| H 1 4 2.050017e+09 8.0000e+08 61.0% 74.0 H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 18 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 18 4308 2026 1.500016e+09 8.0001e+08 46.7% 22.6 1.450016e+09 8.0001e+08 46.7% 22.6 1.450016e+09 8.0001e+08 44.8% 23.4 18 4908 2043 1.450016e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8 18   | Η     | 0    | 0      |    |        |       |     | 2.  | . 0! | 50017e+09 | 8.0 | 0000e+08  | 61  | .0%  | -       | 0s   |
| H 6 8 2.000016e+09 8.0000e+08 60.0% 41.8 H 130 128 1.700015e+09 8.0000e+08 52.9% 12.7 H 199 203 1.644459e+09 8.0000e+08 51.4% 10.8 H 213 213 1.644459e+09 8.0000e+08 51.4% 10.8 H 244 269 1.633347e+09 8.0001e+08 51.0% 11.0 1428 1027 1.5333e+09 40 44 1.6333e+09 8.0001e+08 51.0% 15.5 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 H 4308 2026 1.50001e+09 8.0001e+08 46.7% 21.6 H 4809 2136 1.450016e+09 8.0001e+08 46.7% 22.6 H 4808 2043 1.400013e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8  |       | 0    | 2      | 8. | .0000e | +08   | 0   | 65  | 2    | .0500e+09 | 8.0 | 0000e+08  | 61. | .0%  | -       | 0s   |
| H 130 128  | Η     | 1    | 4      |    |        |       |     | 2.  | . 0! | 50017e+09 | 8.0 | 0000e+08  |     |      | 74.0    | 0s   |
| H 199 203  | Η     | 6    | 8      |    |        |       |     | 2.  | . 00 | 00016e+09 | 8.0 | 0000e+08  | 60  | .0%  | 41.8    | 0s   |
| H 213 213  | Η     |      |        |    |        |       |     | 1.  | . 70 | 00015e+09 | 8.0 | 0000e+08  |     |      | 12.7    | 0s   |
| H 244 269  | Η     |      |        |    |        |       |     |     |      |           |     |           |     |      |         | 0s   |
| 1428       1027       1.5333e+09       40       44       1.6333e+09       8.0001e+08       51.0%       15.5         3138       1602       1.3750e+09       58       22       1.6333e+09       8.0001e+08       51.0%       20.5       1         * 4233       2185       66       1.600017e+09       8.0001e+08       50.0%       21.5       1         * 4238       2082       67       1.550017e+09       8.0001e+08       48.4%       21.5       1         H 4308       2026       1.500016e+09       8.0001e+08       46.7%       21.6       1         4457       2226       1.1000e+09       36       65       1.5000e+09       8.0001e+08       46.7%       22.6       1         H 4809       2136       1.450016e+09       8.0001e+08       44.8%       23.4       1         H 4908       2043       1.400013e+09       8.0001e+08       42.9%       23.9       1         H 5098       2027       1.350013e+09       8.0001e+08       40.7%       24.8       1  | Η     | 213  |        |    |        |       |     | 1.  | . 64 | 44459e+09 | 8.0 | 0000e+08  |     |      | 10.8    | 1s   |
| 3138 1602 1.3750e+09 58 22 1.6333e+09 8.0001e+08 51.0% 20.5 1   * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 1   * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 1   H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 1   4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 1   H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 1   H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9   H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8 1  | Η     |      |        |    |        |       |     |     |      |           |     |           |     |      |         | 1s   |
| * 4233 2185 66 1.600017e+09 8.0001e+08 50.0% 21.5 1   * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 1   H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 1   4457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 1   H 4809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 1   H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9   H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8 1  |       |      |        |    |        |       |     |     |      |           |     |           |     |      |         | 5s   |
| * 4238 2082 67 1.550017e+09 8.0001e+08 48.4% 21.5 1  |       |      |        | 1. | .3750e |       |     | 22  | 1    | .6333e+09 | 8.0 | 0001e+08  |     |      |         | 10s  |
| H 4308 2026 1.500016e+09 8.0001e+08 46.7% 21.6 14457 2226 1.1000e+09 36 65 1.5000e+09 8.0001e+08 46.7% 22.6 144809 2136 1.450016e+09 8.0001e+08 44.8% 23.4 14908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 145098 2027 1.350013e+09 8.0001e+08 40.7% 24.8 15   |       |      |        |    |        |       | 66  |     |      |           |     |           |     |      |         | 12s  |
| 4457       2226       1.1000e+09       36       65       1.5000e+09       8.0001e+08       46.7%       22.6       1         H       4809       2136       1.450016e+09       8.0001e+08       44.8%       23.4       1         H       4908       2043       1.400013e+09       8.0001e+08       42.9%       23.9       1         H       5098       2027       1.350013e+09       8.0001e+08       40.7%       24.8       1   |       |      |        |    |        |       | 67  |     |      |           |     |           |     |      |         | 12s  |
| H 4809 2136  | Η     | 4308 |        |    |        |       |     |     |      |           |     |           |     |      |         | 14s  |
| H 4908 2043 1.400013e+09 8.0001e+08 42.9% 23.9 H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8  |       |      |        | 1. | .1000e | +09   | 36  |     |      |           |     |           |     |      |         | 15s  |
| H 5098 2027 1.350013e+09 8.0001e+08 40.7% 24.8   |       |      |        |    |        |       |     | 1.  | . 4! | 50016e+09 | 8.0 | 0001e+08  |     |      |         | 16s  |
|  |       |      |        |    |        |       |     |     |      |           |     |           |     |      |         | 17s  |
| II E000 47E0 4 000049-100 0 000 0F 7 4   |       |      |        |    |        |       |     |     |      |           |     |           |     |      |         | 18s  |
| H 5282 1752 1.200013e+09 8.0001e+08 33.3% 25.7   | Н     | 5282 | 1752   |    |        |       |     | 1.  | . 20 | 00013e+09 | 8.0 | 0001e+08  | 33. | . 3% | 25.7    | 18s  |

Interrupt request received

# Cutting planes:

Gomory: 37 Cover: 9

Implied bound: 41

MIR: 51

Flow cover: 266

RLT: 107

Relax-and-lift: 99

Explored 5332 nodes (140122 simplex iterations) in 19.00 seconds Thread count was 8 (of 8 available processors)

```
Solution count 10: 1.20001e+09 1.35001e+09 1.40001e+09 ... 1.64446e+09

Solve interrupted

Best objective 1.200012600000e+09, best bound 8.000066838804e+08, gap 33.3335%
```

Results are consistent with our expectations. We find a better solution sooner by shifting the focus towards finding feasible solutions (objective value 1.2e9 versus 1.5e9).

The setParam() method is designed to be quite flexible and forgiving. It accepts wildcards as arguments, and it ignores character case. Thus, the following commands are all equivalent:

```
gurobi> m.setParam('NODELIMIT', 100)
gurobi> m.setParam('NodeLimit', 100)
gurobi> m.setParam('Node*', 100)
gurobi> m.setParam('N???Limit, 100)
```

You can use wildcards to get a list of matching parameters:

```
gurobi> m.setParam('*Cuts', 2)
Matching parameters: ['Cuts', 'CliqueCuts', 'CoverCuts', 'FlowCoverCuts',
'FlowPathCuts', 'GUBCoverCuts', 'ImpliedCuts', 'MIPSepCuts', 'MIRCuts', 'ModKCuts',
'NetworkCuts', 'SubMIPCuts', 'ZeroHalfCuts']
```

Note that Model.Params is a bit less forgiving than setParam(). In particular, wildcards are not allowed with this approach. You don't have to worry about capitalization of parameter names in either approach, though, so m.Params.Heuristics and m.Params.heuristics are equivalent.

The full set of available parameters can be browsed using the paramHelp() command. You can obtain further information on a specific parameter (e.g., MIPGap) by typing paramHelp('MIPGap').

# Parameter tuning tool

When confronted with the task of choosing parameter values that might lead to better performance on a model, the long list of Gurobi parameters may seem intimidating. To simplify the process, we include a simple automated parameter tuning tool. From the interactive shell, the command is tune:

```
gurobi> m = read('misc07')
gurobi> m.tune()
```

The tool tries a number of different parameter settings, and eventually outputs the best ones that it finds. For example:

```
Tested 29 parameter sets in 99.33s

Baseline parameter set: mean runtime 1.48s

Improved parameter set 1 (mean runtime 1.13s):

FlowCoverCuts 1
```

```
Aggregate 0
```

Improved parameter set 2 (mean runtime 1.22s):

MIPFocus 1

In this case, it found that setting the MIPFocus parameter to 1 for model misc07 reduced the runtime from 1.48 to 1.22.

Note that tuning is meant to give general suggestions for parameters that might help performance. You should make sure that the results it gives on one model are helpful on the full range of models you plan to solve. You may sometimes find that performance problems can't be fixed with parameter changes alone, particularly if your model has severe numerical issues.

Tuning is also available as a standalone program. From a command prompt, you can type:

> grbtune /opt/gurobi900/linux64/examples/data/p0033

Please consult the Automated Tuning Tool section of the Gurobi Reference Manual for more information.

#### Using a gurobi.env file

When you want to change the values of Gurobi parameters, you actually have several options available for doing so. We've already discussed parameter changes through the command-line tool (e.g., gurobi\_cl Threads=1 coins.lp), and through interactive shell commands (e.g., m.setParam('Threads', 1)). Each of our language APIs also provides methods for setting parameters. The other option we'd like to mention now is the gurobi.env file.

Whenever the Gurobi library starts, it will look for file gurobi.env in the current working directory, and will apply any parameter changes contained therein. This is true whether the Gurobi library is invoked from the command-line, from the interactive shell, or from any of the Gurobi APIs. Parameter settings are stored one per line in this file, with the parameter name first, followed by at least one space, followed by the desired value. Lines beginning with the # sign are comments and are ignored. To give an example, the following (Linux) commands:

```
> echo "Threads 1" > gurobi.env
> gurobi_cl coins.lp
Using license file /opt/gurobi/gurobi.lic
Using gurobi.env file
Set parameter LogFile to value gurobi.log
Set parameter Threads to value 1

Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Copyright (c) 2019, Gurobi Optimization, LLC

Read LP format model from file coins.lp
Reading time = 0.00 seconds
: 4 rows, 9 columns, 16 nonzeros
Optimize a model with 4 rows, 9 columns and 16 nonzeros
Model fingerprint: 0xa0c5449c
Variable types: 4 continuous, 5 integer (0 binary)
```

```
Coefficient statistics:
 Matrix range
                  [6e-02, 7e+00]
 Objective range [1e-02, 1e+00]
 Bounds range [5e+01, 1e+03]
 RHS range
                  [0e+00, 0e+00]
Found heuristic solution: objective -0.0000000
Presolve removed 1 rows and 5 columns
Presolve time: 0.00s
Presolved: 3 rows, 4 columns, 9 nonzeros
Variable types: 0 continuous, 4 integer (0 binary)
Root relaxation: objective 1.134615e+02, 2 iterations, 0.00 seconds
                 Current Node
                               Objective Bounds
Expl Unexpl | Obj Depth IntInf | Incumbent
                                               BestBd Gap | It/Node Time
          0 113.46154 0 1 -0.00000 113.46154
    0
                              113.4500000 113.46154 0.01%
Η
    0
          Ω
                                                                     0s
          0 113.46154
                         0
                            1 113.45000 113.46154 0.01%
Explored 1 nodes (2 simplex iterations) in 0.00 seconds
Thread count was 1 (of 8 available processors)
Solution count 2: 113.45 -0
Optimal solution found (tolerance 1.00e-04)
Best objective 1.134500000000e+02, best bound 1.134500000000e+02, gap 0.0000%
```

would read the new value of the Threads parameter from file gurobi.env and then optimize model coins.lp using one thread. Note that if the same parameter is changed in both *gurobi.env* and in your program (or through the Gurobi command-line), the value from gurobi.env will be overridden.

The distribution includes a sample gurobi.env file (in the bin directory). The sample includes every parameter, with the default value for each, but with all settings commented out.

### Working with multiple models

The Gurobi shell allows you to work with multiple models simultaneously. For example...

```
gurobi> a = read('/opt/gurobi900/linux64/examples/data/p0033')
Read MPS format model from file /opt/gurobi900/linux64/examples/data/p0033.mps
Reading time = 0.00 seconds
P0033: 16 Rows, 33 Columns, 98 NonZeros.
gurobi> b = read('/opt/gurobi900/linux64/examples/data/stein9')
Read MPS format model from file /opt/gurobi900/linux64/examples/data/stein9.mps
Reading time = 0.00 seconds
STEIN9: 13 Rows, 9 Columns, 45 NonZeros.
The models() command gives a list of all active models.
gurobi> models()
```

```
Currently loaded models:
a : <gurobi.Model MIP instance P0033: 16 constrs, 33 vars, Parameter changes: LogFile=gurobi.log>
b : <gurobi.Model MIP instance STEIN9: 13 constrs, 9 vars, Parameter changes: LogFile=gurobi.log>
```

Note that parameters can be set for a particular model with the Model.setParam() method or the Model.Params class, or they can be changed for all models in the Gurobi shell by using the global setParam() method.

#### Help

The interactive shell includes an extensive help facility. To access it, simply type help() at the prompt. As previously mentioned, help is available for all of the important objects in the interface. For example, as explained in the help facility, you can type help(Model), help(Var), or help(Constr). You can also obtain detailed help on any of the available methods on these objects. For example, help(Model.setParam) gives help on setting model parameters. You can also use a variable, or a method on a variable, to ask for help. For example, if variable m contains a Model object, then help(m) is equivalent to help(Model.setParam) is equivalent to help(Model.setParam).

#### Interface customization

The Gurobi interactive shell lives within a full-featured scripting language. This allows you to perform a wide range of customizations to suit your particular needs. Creating custom functions requires some knowledge of the Python language, but you can achieve a lot by using a very limited set of language features.

Let us consider a simple example. Imagine that you store your models in a certain directory on your disk. Rather than having to type the full path whenever you read a model, you can create your own custom read method:

```
gurobi> def myread(filename):
..... return read('/home/john/models/'+filename)
```

Note that the indentation of the second line is required.

Defining this function allows you to do the following:

```
gurobi> m = myread('stein9')
Read MPS format model from file /home/john/models/stein9.mps
```

If you don't want to type this function in each time you start the Gurobi shell, you can store it in a file. The file would look like the following:

```
from gurobipy import *

def myread(filename):
   return read('/home/john/models/'+filename)
```

The from gurobipy import \* line is required in order to allow you to use the read method from the Gurobi shell in your custom function. The name of your customization file must end with a .py suffix. If the file is named custom.py, you would then type the following to import this function:

```
gurobi> from custom import *
```

One file can contain as many custom functions as you'd like (see custom.py in <installdir>/examples/python for an example). If you wish to make site-wide customizations, you can also customize the gurobi.py file that is included in <installdir>/lib.

#### **Customization through callbacks**

Another type of customization we'd like to touch on briefly can be achieved through Gurobi callbacks. Callbacks allow you to track the progress of the optimization process. For the sake of our example, let's say you want the MIP optimizer to run for 10 seconds before quitting, but you don't want it to terminate before it finds a feasible solution. The following callback method would implement this condition:

```
from gurobipy import *

def mycallback(model, where):
   if where == GRB.Callback.MIP:
      time = model.cbGet(GRB.Callback.RUNTIME)
      best = model.cbGet(GRB.Callback.MIP_OBJBST)
   if time > 10 and best < GRB.INFINITY:
      model.terminate()</pre>
```

Once you import this function (from custom import \*), you can then say m.optimize(mycallback) to obtain the desired termination behavior. Alternatively, you could define your own custom optimize method that always invokes the callback:

```
def myopt(model):
  model.optimize(mycallback)
```

This would allow you to say myopt(m).

You can pass arbitrary data into your callback through the model object. For example, if you set m.\_mydata = 1 before calling optimize(), you can query m.\_mydata inside your callback function. Note that the names of user data fields must begin with an underscore.

This callback example is included in <installdir>/examples/python/custom.py. Type from custom import \* to import the callback and the myopt() function.

You can type help(GRB.Callback) for more information on callbacks. You can also refer to the Callback class documentation in the Gurobi Reference Manual.

# The Gurobi Python Interface for Python Users

While the Gurobi installation includes everything you need to use Gurobi from within Python, we understand that some users would prefer to use Gurobi from within their own Python environment.

Doing so requires you to install the gurobipy module. The steps for doing this depend on your platform. On Linux, you will need to open a terminal window, change your current directory to the Gurobi <installdir> (the directory that contains the file setup.py), and issue the following command:

#### python setup.py install

Unless you are using your own private Python installation, you will need to run this command as super-user. Once gurobipy is successfully installed, you can type import gurobipy or from gurobipy import \* from your Python shell and access all of the Gurobi classes and methods.

Note that Gurobi distributes an Anaconda package, so you don't need to perform these steps if you are using Anaconda Python. Please refer to the instructions for setting up Gurobi for use within the Anaconda distribution for details.

Note that for this installation to succeed, your Python environment must be compatible with the Gurobi Python module. You should only install Gurobi libraries into a 64-bit Python shell. In addition, your Python version must be compatible. With this release, gurobipy can be used with Python 2.7, 3.6, or 3.7 on Linux.

As mentioned in the previous section, most of the information associated with a Gurobi model is stored in a set of *attributes*. Some attributes are associated with the variables of the model, some with the constraints of the model, and some with the model itself. After you optimize a model, for example, the solution is stored in the X variable attribute. Attributes that are computed by the Gurobi optimizer (such as the solution attribute) cannot be modified directly by the user, while those that represent input data (such as the LB attribute which stores variable lower bounds) can.

Each of the Gurobi language interfaces contains routines for querying or modifying attribute values. To retrieve or modify the value of a particular attribute, you simply pass the name of the attribute to the appropriate query or modification routine. In the C interface, for example, you'd make the following call to query the current solution value on variable 1:

```
double x1;
error = GRBgetdblattrelement(model, GRB_DBL_ATTR_X, 1, &x1);
```

This routine returns a single element from an array-valued attribute containing double-precision data. Routines are provided to query and modify scalar-valued and array-valued attributes of type int, double, char, or char \*.

In the object oriented interfaces, you query or modify attribute values through the appropriate objects. For example, if variable v is a Gurobi variable object (a GRBVar), then the following calls would be used to modify the lower bound on v:

```
C++: v.set(GRB_DoubleAttr_LB, 0.0)
Java: v.set(GRB.DoubleAttr.LB, 0.0)
C#: v.Set(GRB.DoubleAttr.LB, 0.0) or v.LB = 0.0
Python: v.lb = 0.0
```

The exact syntax for querying or modifying an attribute varies slightly from one language to another, but the basic approach remains consistent: you call the appropriate query or modification method using the name of the desired attribute as an argument.

The full list of Gurobi attributes can be found in the *Attributes* section of the Gurobi Reference Manual.

This section will work through a simple C example in order to illustrate the use of the Gurobi C interface. The example builds a simple Mixed Integer Programming model, optimizes it, and outputs the optimal objective value. This section assumes that you are already familiar with the C programming language. If not, a variety of books are available for learning the language (for example, *The C Programming Language*, by Kernighan and Ritchie).

Our example optimizes the following model:

#### Example mip1\_c.c

This is the complete source code for our example (also available as <installdir>/examples/c/mip1\_c.c)...

```
/* Copyright 2019, Gurobi Optimization, LLC */
/* This example formulates and solves the following simple MIP model:
    maximize
                x +
                      y + 2z
    subject to x + 2 y + 3 z \le 4
                x + y
                x, y, z binary
*/
#include <stdlib.h>
#include <stdio.h>
#include "gurobi_c.h"
int
main(int
         argc,
    char *argv[])
{
  GRBenv *env = NULL;
  GRBmodel *model = NULL;
  int
         error = 0;
  double sol[3];
          ind[3];
  int
  double
           val[3];
  double
           obj[3];
  char
           vtype[3];
  int
           optimstatus;
```

```
double
          objval;
/* Create environment */
error = GRBemptyenv(&env);
if (error) goto QUIT;
error = GRBsetstrparam(env, "LogFile", "mip1.log");
if (error) goto QUIT;
error = GRBstartenv(env);
if (error) goto QUIT;
/* Create an empty model */
error = GRBnewmodel(env, &model, "mip1", 0, NULL, NULL, NULL, NULL, NULL);
if (error) goto QUIT;
/* Add variables */
obj[0] = 1; obj[1] = 1; obj[2] = 2;
vtype[0] = GRB_BINARY; vtype[1] = GRB_BINARY; vtype[2] = GRB_BINARY;
error = GRBaddvars(model, 3, 0, NULL, NULL, NULL, obj, NULL, NULL, vtype,
                   NULL);
if (error) goto QUIT;
/* Change objective sense to maximization */
error = GRBsetintattr(model, GRB_INT_ATTR_MODELSENSE, GRB_MAXIMIZE);
if (error) goto QUIT;
/* First constraint: x + 2 y + 3 z \le 4 */
ind[0] = 0; ind[1] = 1; ind[2] = 2;
val[0] = 1; val[1] = 2; val[2] = 3;
error = GRBaddconstr(model, 3, ind, val, GRB_LESS_EQUAL, 4.0, "c0");
if (error) goto QUIT;
/* Second constraint: x + y \ge 1 */
ind[0] = 0; ind[1] = 1;
val[0] = 1; val[1] = 1;
error = GRBaddconstr(model, 2, ind, val, GRB_GREATER_EQUAL, 1.0, "c1");
if (error) goto QUIT;
/* Optimize model */
error = GRBoptimize(model);
if (error) goto QUIT;
/* Write model to 'mip1.lp' */
error = GRBwrite(model, "mip1.lp");
if (error) goto QUIT;
/* Capture solution information */
error = GRBgetintattr(model, GRB_INT_ATTR_STATUS, &optimstatus);
if (error) goto QUIT;
error = GRBgetdblattr(model, GRB_DBL_ATTR_OBJVAL, &objval);
if (error) goto QUIT;
```

```
error = GRBgetdblattrarray(model, GRB_DBL_ATTR_X, 0, 3, sol);
  if (error) goto QUIT;
  printf("\nOptimization complete\n");
  if (optimstatus == GRB_OPTIMAL) {
   printf("Optimal objective: %.4e\n", objval);
   printf(" x=\%.0f, y=\%.0f, z=\%.0f n", sol[0], sol[1], sol[2]);
 } else if (optimstatus == GRB_INF_OR_UNBD) {
   printf("Model is infeasible or unbounded\n");
  } else {
   printf("Optimization was stopped early\n");
QUIT:
  /* Error reporting */
  if (error) {
   printf("ERROR: %s\n", GRBgeterrormsg(env));
    exit(1);
  /* Free model */
  GRBfreemodel(model);
  /* Free environment */
  GRBfreeenv(env);
 return 0;
```

### **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

The example begins by including a few include files. Gurobi C applications should always start by including gurobi\_c.h, along with the standard C include files (stdlib.h and stdio.h).

### Creating the environment

After declaring the necessary program variables, the example continues by creating an environment, by first requesting an empty environment, then setting some options -- as log file name -- and then starting the environment.

```
/* Create environment */
error = GRBemptyenv(&env);
if (error) goto QUIT;

error = GRBsetstrparam(env, "LogFile", "mip1.log");
if (error) goto QUIT;
```

```
error = GRBstartenv(env);
if (error) goto QUIT;
```

Later requests to create optimization models will always require an active environment, so environment creation should always be the first step when using the Gurobi optimizer.

Note that environment creation may fail, so you should check the return value of the call.

### Creating the model

Once an environment has been created, the next step is to create a model. A Gurobi model holds a single optimization problem. It consists of a set of variables, a set of constraints, and the associated attributes (variable bounds, objective coefficients, variable integrality types, constraint senses, constraint right-hand side values, etc.). The first step towards building a model that contains all of this information is to create an empty model object:

```
/* Create an empty model */
error = GRBnewmodel(env, &model, "mip1", 0, NULL, NULL, NULL, NULL, NULL);
if (error) goto QUIT;
```

The first argument to GRBnewmodel() is the previously created environment. The second is a pointer to the location where the pointer to the new model should be stored. The third is the name of the model. The fourth is the number of variables to initially add to the model. Since we're creating an empty model, the number of initial variables is 0. The remaining arguments would describe the initial variables (lower bounds, upper bounds, variable types, etc.), had they been present.

# Adding variables to the model

Once we create a Gurobi model, we can start adding variables and constraints to it. In our example, we'll begin by adding variables:

The first argument to GRBaddvars() is the model to which the variables are being added. The second is the number of added variables (3 in our example).

Arguments three through six describe the constraint matrix coefficients associated with the new variables. The third argument gives the number of non-zero constraint matrix entries associated with the new variables, and the next three arguments give details on these non-zeros. In our example, we'll be adding these non-zeros when we add the constraints. Thus, the non-zero count here is zero, and the following three arguments are all NULL.

The seventh argument to GRBaddvars() is the linear objective coefficient for each new variable. Since our example aims to maximize the objective, and by default Gurobi will minimize the objective, we'll need to change the objective sense. This is done in the next statement. Note we could have multiplied the objective coefficients by -1 instead (since maximizing c'x is equivalent to minimizing -c'x).

The next two arguments specify the lower and upper bounds of the variables, respectively. The NULL values indicate that these variables should take their default values (0.0 and 1.0 for binary variables).

The tenth argument specifies the types of the variables. In this example, the variables are all binary (GRB\_BINARY).

The final argument gives the names of the variables. In this case, we allow the variable names to take their default values (x0, x1, and x2).

#### Changing the objective sense

As we just noted, the default sense for the objective function is minimization. Since our example aims to maximize the objective, we need to modify the ModelSense attribute:

```
/* Change objective sense to maximization */
error = GRBsetintattr(model, GRB_INT_ATTR_MODELSENSE, GRB_MAXIMIZE);
if (error) goto QUIT;
```

### Adding constraints to the model

Once the new variables are integrated into the model, the next step is to add our two linear constraints. These constraints are added through the GRBaddconstr() routine. To add a constraint, you must specify several pieces of information, including the non-zero values associated with the constraint, the constraint sense, the right-hand side value, and the constraint name. These are all specified as arguments to GRBaddconstr():

```
/* First constraint: x + 2 y + 3 z <= 4 */
ind[0] = 0; ind[1] = 1; ind[2] = 2;
val[0] = 1; val[1] = 2; val[2] = 3;

error = GRBaddconstr(model, 3, ind, val, GRB_LESS_EQUAL, 4.0, "c0");
if (error) goto QUIT;</pre>
```

The first argument of GRBaddconstr() is the model to which the constraint is being added. The second is the total number of non-zero coefficients associated with the new constraint. The next two arguments describe the non-zeros in the new constraint. Constraint coefficients are specified using a list of index-value pairs, one for each non-zero value. In our example, the first constraint to be added is  $x + 2y + 3z \le 4$ . We have chosen to make x the first variable in our constraint matrix, y the second, and z the third (note that this choice is arbitrary). Given our variable ordering choice,

the index-value pairs that are required for our first constraint are (0, 1.0), (1, 2.0), and (2, 3.0). These pairs are placed in the ind and val arrays.

The fifth argument to GRBaddconstr() provides the sense of the new constraint. Possible values are  $GRB\_LESS\_EQUAL$ ,  $GRB\_GREATER\_EQUAL$ , or  $GRB\_EQUAL$ . The sixth argument gives the right-hand side value. The final argument gives the name of the constraint (we allow the constraint to take its default name here by specifying NULL for the argument). The second constraint is added in a similar fashion.

Note that routine GRBaddconstrs() would allow you to add both constraints in a single call. The arguments for this routine are much more complex, though, without providing any significant advantages, so we recommend that you add one constraint at a time.

#### Optimizing the model

Now that the model has been built, the next step is to optimize it:

```
/* Optimize model */
error = GRBoptimize(model);
if (error) goto QUIT;
```

This routine performs the optimization and populates several internal model attributes, including the status of the optimization, the solution, etc. Once the function returns, we can query the values of these attributes. In particular, we can query the status of the optimization process by retrieving the value of the Status attribute...

```
/* Capture solution information */
error = GRBgetintattr(model, GRB_INT_ATTR_STATUS, &optimstatus);
if (error) goto QUIT;
```

The optimization status has many possible values. An optimal solution to the model may have been found, or the model may have been determined to be infeasible or unbounded, or the solution process may have been interrupted. A list of possible statuses can be found in the Gurobi Reference Manual. For our example, we know that the model is feasible, and we haven't modified any parameters that might cause the optimization to stop early (e.g., a time limit), so the status will be GRB\_OPTIMAL.

Another important model attribute is the value of the objective function for the computed solution. This is accessed through this call:

```
error = GRBgetdblattr(model, GRB_DBL_ATTR_OBJVAL, &objval);
if (error) goto QUIT;
```

Note that this call would return a non-zero error result if no solution was found for this model.

Once we know that the model was solved, we can extract the X attribute of the model, which contains the value for each variable in the computed solution:

```
error = GRBgetdblattrarray(model, GRB_DBL_ATTR_X, 0, 3, sol);
if (error) goto QUIT;
```

This routine retrieves the values of an array-valued attribute. The third and fourth arguments indicate the index of the first array element to be retrieved, and the number of elements to retrieve, respectively. In this example we retrieve entries 0 through 2 (i.e., all three of them)

### **Error reporting**

We would like to point out one additional aspect of the example. Almost all of the Gurobi methods return an error code. The code will typically be zero, indicating that no error was encountered, but it is important to check the value of the code in case an error arises.

While you may want to print a specialized error code at each point where an error may occur, the Gurobi interface provides a more flexible facility for reporting errors. The GRBgeterrormsg() routine returns a textual description of the most recent error associated with an environment:

```
/* Error reporting */
if (error) {
  printf("ERROR: %s\n", GRBgeterrormsg(env));
  exit(1);
}
```

Once the error reporting is done, the only remaining task in our example is to release the resources associated with our optimization task. In this case, we populated one model and created one environment. We call <code>GRBfreemodel(model)</code> to free the model, and <code>GRBfreeenv(env)</code> to free the environment (in that order).

### Building and running the example

To build and run the example, please refer to the files in <installdir>/examples/build. For Linux platforms, the <installdir>/examples/build directory contains an example Makefile. Typing make mip1\_c will build and run this example.

The C example directory <installdir>/examples/c contains a number of examples. We encourage you to browse and modify them in order to become more familiar with the Gurobi C interface. We also encourage you to read the Gurobi Example Tour for more information.

This section will work through a simple C++ example in order to illustrate the use of the Gurobi C++ interface. The example builds a model, optimizes it, and outputs the optimal objective value. This section assumes that you are already familiar with the C++ programming language. If not, a variety of books are available for learning the language (for example, The C++ Programming Language, by Stroustrup).

Our example optimizes the following model:

Note that this is the same model that was modeled and optimized in the C Interface section.

### Example mip1\_c++.cpp

This is the complete source code for our example (also available in <installdir>/examples/c++/mip1\_c++.cpp)...

```
/* Copyright 2019, Gurobi Optimization, LLC */
/* This example formulates and solves the following simple MIP model:
    maximize x + y + 2z
    subject to x + 2 y + 3 z \le 4
                x + y
                x, y, z binary
*/
#include "gurobi_c++.h"
using namespace std;
int
main(int argc,
    char *argv[])
  try {
   // Create an environment
   GRBEnv env = GRBEnv(true);
    env.set("LogFile", "mip1.log");
    env.start();
```

```
// Create an empty model
  GRBModel model = GRBModel(env);
  // Create variables
  GRBVar x = model.addVar(0.0, 1.0, 0.0, GRB_BINARY, "x");
  GRBVar y = model.addVar(0.0, 1.0, 0.0, GRB_BINARY, "y");
  GRBVar z = model.addVar(0.0, 1.0, 0.0, GRB_BINARY, "z");
  // Set objective: maximize x + y + 2z
  model.setObjective(x + y + 2 * z, GRB_MAXIMIZE);
  // Add constraint: x + 2 y + 3 z \le 4
  model.addConstr(x + 2 * y + 3 * z \le 4, "c0");
  // Add constraint: x + y >= 1
  model.addConstr(x + y >= 1, "c1");
  // Optimize model
  model.optimize();
  cout << x.get(GRB_StringAttr_VarName) << " "</pre>
       << x.get(GRB_DoubleAttr_X) << endl;
  cout << y.get(GRB_StringAttr_VarName) << " "</pre>
       << y.get(GRB_DoubleAttr_X) << endl;</pre>
  cout << z.get(GRB_StringAttr_VarName) << " "</pre>
       << z.get(GRB_DoubleAttr_X) << endl;</pre>
  cout << "Obj: " << model.get(GRB_DoubleAttr_ObjVal) << endl;</pre>
} catch(GRBException e) {
  cout << "Error code = " << e.getErrorCode() << endl;</pre>
  cout << e.getMessage() << endl;</pre>
} catch(...) {
  cout << "Exception during optimization" << endl;</pre>
return 0;
```

### **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

The example begins by including file gurobi\_c++.h. Gurobi C++ applications should always start by including this file.

# Creating the environment

The first executable statement in our example obtains a Gurobi environment (using the GRBEnv() constructor):

```
// Create an environment
GRBEnv env = GRBEnv(true);
env.set("LogFile", "mip1.log");
env.start();
```

In this call we requested an *empty* environment, choose a log file, and started the environment.

Later calls to create an optimization model will always require an environment, so environment creation is typically the first step in a Gurobi application.

### Creating the model

Once an environment has been created, the next step is to create a model. A Gurobi model holds a single optimization problem. It consists of a set of variables, a set of constraints, and the associated attributes (variable bounds, objective coefficients, variable integrality types, constraint senses, constraint right-hand side values, etc.). The first step towards building a model that contains all of this information is to create an empty model object:

```
// Create an empty model
GRBModel model = GRBModel(env);
```

The constructor takes the previously created environment as its argument.

### Adding variables to the model

The next step in our example is to add variables to the model.

```
// Create variables
GRBVar x = model.addVar(0.0, 1.0, 0.0, GRB_BINARY, "x");
GRBVar y = model.addVar(0.0, 1.0, 0.0, GRB_BINARY, "y");
```

Variables are added through the addVar() method on the model object (or addVars() if you wish to add more than one at a time). A variable is always associated with a particular model.

The first and second arguments to the addVar() call are the variable lower and upper bounds, respectively. The third argument is the linear objective coefficient (zero here - we'll set the objective later). The fourth argument is the variable type. Our variables are all binary in this example. The final argument is the name of the variable.

The addVar() method has been overloaded to accept several different argument lists. Please refer to the Gurobi Reference Manual for further details.

### Setting the objective

The next step in the example is to set the optimization objective:

```
// Set objective: maximize x + y + 2 z
model.setObjective(x + y + 2 * z, GRB_MAXIMIZE);
```

The objective is built here using overloaded operators. The C++ API overloads the arithmetic operators to allow you to build linear and quadratic expressions involving Gurobi variables.

The second argument indicates that the sense is maximization.

Note that while this simple example builds the objective in a single statement using an explicit list of terms, more complex programs will typically build it incrementally. For example:

```
GRBLinExpr obj = 0.0;
obj += x;
obj += y;
obj += 2*z;
model.setObjective(obj, GRB_MAXIMIZE);
```

### Adding constraints to the model

The next step in the example is to add the linear constraints. The first constraint is added here:

```
// Add constraint: x + 2 y + 3 z <= 4
model.addConstr(x + 2 * y + 3 * z <= 4, "c0");
```

As with variables, constraints are always associated with a specific model. They are created using the addConstr() or addConstrs() methods on the model object.

We again use overloaded arithmetic operators to build the linear expression. The comparison operators are also overloaded to make it easier to build constraints.

The second argument to addConstr gives the (optional) constraint name.

Again, this simple example builds the linear expression for the constraint in a single statement using an explicit list of terms. More complex programs will typically build the expression incrementally.

The second constraint in our model is added with this similar call:

```
// Add constraint: x + y >= 1
model.addConstr(x + y >= 1, "c1");
```

### Optimizing the model

Now that the model has been built, the next step is to optimize it:

```
// Optimize model
model.optimize();
```

This routine performs the optimization and populates several internal model attributes (including the status of the optimization, the solution, etc.).

#### Reporting results - attributes

Once the optimization is complete, we can query the values of the attributes. In particular, we can query the VarName and X attributes to obtain the name and solution value of each variable:

We can also query the ObjVal attribute on the model to obtain the objective value for the current solution:

```
cout << "Obj: " << model.get(GRB_DoubleAttr_ObjVal) << endl;</pre>
```

The names and types of all model, variable, and constraint attributes can be found in the Attributes section of the Gurobi Reference Manual.

#### **Error handling**

Errors in the Gurobi C++ interface are handled through the C++ exception mechanism. In the example, all Gurobi statements are enclosed inside a try block, and any associated errors would be caught by the catch block:

```
} catch(GRBException e) {
  cout << "Error code = " << e.getErrorCode() << endl;
  cout << e.getMessage() << endl;
} catch(...) {
  cout << "Exception during optimization" << endl;
}</pre>
```

# Building and running the example

To build and run the example, we refer the user to the files in <installdir>/examples/build. For Linux platforms, the <installdir>/examples/build directory contains an example Makefile. Typing make mip1\_c++ will build and run this example.

If you want to create your own project or makefile to build a C++ program that calls Gurobi, the details will depend on your platform and development environment, but we'd like to point out a few common pitfalls:

- A C++ program that uses Gurobi must link in both the Gurobi C++ library libgurobi\_c++.a and the Gurobi C library libgurobi90.so.
- The Gurobi Linux distribution includes multiple C++ libraries. You may need to choose an alternate version (e.g., libgurobi\_c++5.2.a), depending on the version of g++ you are using. You should consult the supported platform list for additional information.

The C++ example directory  $\mbox{\sc installdir}/\mbox{\sc examples/c++}$  contains a number of examples. We encourage you to browse and modify them in order to become more familiar with the Gurobi C++ interface. We also encourage you to read the Gurobi Example Tour for more information.

This section will work through a simple Java example in order to illustrate the use of the Gurobi Java interface. The example builds a model, optimizes it, and outputs the optimal objective value. This section assumes that you are already familiar with the Java programming language. If not, a variety of books and websites are available for learning the language (for example, the online Java tutorials).

If you are using an integrated development environment like Eclipse®, we recommend that you import the file <installdir>/lib/gurobi-javadoc.jar, which contains Javadoc documentation for the Gurobi Java interface.

Our example optimizes the following model:

Note that this is the same model that was modeled and optimized in the C Interface section.

### Example Mip1.java

This is the complete source code for our example (also available in <installdir>/examples/java/Mip1.java)...

```
// Create empty model
    GRBModel model = new GRBModel(env);
    // Create variables
    GRBVar x = model.addVar(0.0, 1.0, 0.0, GRB.BINARY, "x");
    GRBVar y = model.addVar(0.0, 1.0, 0.0, GRB.BINARY, "y");
    GRBVar z = model.addVar(0.0, 1.0, 0.0, GRB.BINARY, "z");
    // Set objective: maximize x + y + 2z
    GRBLinExpr expr = new GRBLinExpr();
    expr.addTerm(1.0, x); expr.addTerm(1.0, y); expr.addTerm(2.0, z);
    model.setObjective(expr, GRB.MAXIMIZE);
    // Add constraint: x + 2 y + 3 z \le 4
    expr = new GRBLinExpr();
    expr.addTerm(1.0, x); expr.addTerm(2.0, y); expr.addTerm(3.0, z);
    model.addConstr(expr, GRB.LESS_EQUAL, 4.0, "c0");
    // Add constraint: x + y >= 1
    expr = new GRBLinExpr();
    expr.addTerm(1.0, x); expr.addTerm(1.0, y);
    model.addConstr(expr, GRB.GREATER_EQUAL, 1.0, "c1");
    // Optimize model
    model.optimize();
    System.out.println(x.get(GRB.StringAttr.VarName)
                       + " " +x.get(GRB.DoubleAttr.X));
    {\tt System.out.println(y.get(GRB.StringAttr.VarName)}
                       + " " +y.get(GRB.DoubleAttr.X));
    System.out.println(z.get(GRB.StringAttr.VarName)
                       + " " +z.get(GRB.DoubleAttr.X));
    System.out.println("Obj: " + model.get(GRB.DoubleAttr.ObjVal));
    // Dispose of model and environment
    model.dispose();
    env.dispose();
  } catch (GRBException e) {
    System.out.println("Error code: " + e.getErrorCode() + ". " +
                       e.getMessage());
  }
}
```

### **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

The example begins by importing the Gurobi classes (import gurobi.\*). Gurobi Java applications

should always start with this line.

#### Creating the environment

The first executable statement in our example obtains a Gurobi environment (using the GRBEnv() constructor):

```
// Create empty environment, set options, and start
GRBEnv env = new GRBEnv(true);
env.set("logFile", "mip1.log");
env.start();
```

In this call we requested an *empty* environment, choose a log file, and started the environment.

Later calls to create an optimization model will always require an environment, so environment creation is typically the first step in a Gurobi application. The constructor argument specifies the name of the log file.

#### Creating the model

Once an environment has been created, the next step is to create a model. A Gurobi model holds a single optimization problem. It consists of a set of variables, a set of constraints, and the associated attributes (variable bounds, objective coefficients, variable integrality types, constraint senses, constraint right-hand side values, etc.). The first step towards building a model that contains all of this information is to create an empty model object:

```
// Create empty model
GRBModel model = new GRBModel(env);
```

The constructor takes the previously created environment as its argument.

# Adding variables to the model

The next step in our example is to add variables to the model.

```
// Create variables
GRBVar x = model.addVar(0.0, 1.0, 0.0, GRB.BINARY, "x");
GRBVar y = model.addVar(0.0, 1.0, 0.0, GRB.BINARY, "y");
```

Variables are added through the addVar() method on a model object (or addVars() if you wish to add more than one at a time). A variable is always associated with a particular model.

The first and second arguments to the addVar() call are the variable lower and upper bounds, respectively. The third argument is the linear objective coefficient (zero here - we'll set the objective later). The fourth argument is the variable type. Our variables are all binary in this example. The final argument is the name of the variable.

The addVar() method has been overloaded to accept several different argument lists. Please refer to the Gurobi Reference Manual for further details.

### Setting the objective

The next step in the example is to set the optimization objective:

```
// Set objective: maximize x + y + 2 z
GRBLinExpr expr = new GRBLinExpr();
expr.addTerm(1.0, x); expr.addTerm(1.0, y); expr.addTerm(2.0, z);
model.setObjective(expr, GRB.MAXIMIZE);
```

The objective must be a linear or quadratic function of the variables in the model. In our example, we build our objective by first constructing an empty linear expression and adding three terms to it.

The second argument to setObjective indicates that the optimization sense is maximization.

#### Adding constraints to the model

The next step in the example is to add the linear constraints. The first constraint is added here:

```
// Add constraint: x + 2 y + 3 z <= 4
expr = new GRBLinExpr();
expr.addTerm(1.0, x); expr.addTerm(2.0, y); expr.addTerm(3.0, z);
model.addConstr(expr, GRB.LESS_EQUAL, 4.0, "c0");</pre>
```

As with variables, constraints are always associated with a specific model. They are created using the addConstr() or addConstrs() methods on the model object.

The first argument to addConstr() is the left-hand side of the constraint. We built the left-hand side by first creating an empty linear expression object, and then adding three terms to it. The second argument is the constraint sense (GRB\_LESS\_EQUAL, GRB\_GREATER\_EQUAL, or GRB\_EQUAL). The third argument is the right-hand side (a constant in our example). The final argument is the constraint name. Several signatures are available for addConstr(). Please consult the Gurobi Reference Manual for details.

The second constraint is created in a similar manner:

```
// Add constraint: x + y >= 1
expr = new GRBLinExpr();
expr.addTerm(1.0, x); expr.addTerm(1.0, y);
model.addConstr(expr, GRB.GREATER_EQUAL, 1.0, "c1");
```

# Optimizing the model

Now that the model has been built, the next step is to optimize it:

```
// Optimize model
model.optimize();
```

This routine performs the optimization and populates several internal model attributes (including the status of the optimization, the solution, etc.).

#### Reporting results - attributes

Once the optimization is complete, we can query the values of the attributes. In particular, we can query the VarName and X attributes to obtain the name and solution value for each variable:

We can also query the ObjVal attribute on the model to obtain the objective value for the current solution:

```
System.out.println("Obj: " + model.get(GRB.DoubleAttr.ObjVal));
```

The names and types of all model, variable, and constraint attributes can be found in the Attributes section of the Gurobi Reference Manual.

#### Cleaning up

The example concludes with dispose calls:

```
// Dispose of model and environment
model.dispose();
```

These reclaim the resources associated with the model and environment. Garbage collection would reclaim these eventually, but if your program doesn't exit immediately after performing the optimization, it is best to reclaim them explicitly.

Note that all models associated with an environment must be disposed before the environment itself is disposed.

### **Error handling**

Errors in the Gurobi Java interface are handled through the Java exception mechanism. In the example, all Gurobi statements are enclosed inside a try block, and any associated errors would be caught by the catch block:

# Building and running the example

To build and run the example, please refer to the files in <installdir>/examples/build. For Linux platforms, the <installdir>/examples/build directory contains an example Makefile. Typing make Mip1 will build and run this example.

The Java example directory <installdir>/examples/java contains a number of examples. We encourage you to browse and modify them in order to become more familiar with the Gurobi Java interface. We also encourage you to read the Gurobi Example Tour for more information.

This section assumes that you are already familiar with the C# programming language. If not, a variety of books and websites are available for learning the language (for example, the Microsoft online C# documentation).

While C# programs have historically only run on Windows machines, Microsoft has recently released an open-source, cross-platform .NET implementation (.NET Core) that allows you to run C# programs on Linux and Mac OS as well. You will find the Gurobi library that supports .NET Core 2 in <installdir>/lib/gurobi90.netstandard20.dll. For more information on how to obtain and use .NET Core, please refer to the .NET Tutorial.

We will work through a simple C# example in order to illustrate the use of the Gurobi .NET interface. The example builds a model, optimizes it, and outputs the optimal objective value.

Our example optimizes the following model:

Note that this is the same model that was modeled and optimized in the C Interface section.

### Example mip1\_cs.cs

This is the complete source code for our example (also available in <installdir>/examples/c#/mip1\_cs.cs)...

```
try {
    // Create an empty environment, set options and start
            env = new GRBEnv(true);
    env.Set("LogFile", "mip1.log");
    env.Start();
    // Create empty model
    GRBModel model = new GRBModel(env);
    // Create variables
    GRBVar x = model.AddVar(0.0, 1.0, 0.0, GRB.BINARY, "x");
    GRBVar y = model.AddVar(0.0, 1.0, 0.0, GRB.BINARY, "y");
    GRBVar z = model.AddVar(0.0, 1.0, 0.0, GRB.BINARY, "z");
    // Set objective: maximize x + y + 2 z
    model.SetObjective(x + y + 2 * z, GRB.MAXIMIZE);
    // Add constraint: x + 2 y + 3 z \le 4
    model.AddConstr(x + 2 * y + 3 * z \le 4.0, "c0");
    // Add constraint: x + y >= 1
    model.AddConstr(x + y >= 1.0, "c1");
    // Optimize model
    model.Optimize();
    Console.WriteLine(x.VarName + " " + x.X);
    Console.WriteLine(y.VarName + " " + y.X);
    Console.WriteLine(z.VarName + " " + z.X);
    Console.WriteLine("Obj: " + model.ObjVal);
    // Dispose of model and env
    model.Dispose();
    env.Dispose();
  } catch (GRBException e) {
    Console.WriteLine("Error code: " + e.ErrorCode + ". " + e.Message);
}
```

### **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

The example begins by importing the Gurobi namespace (using Gurobi). Gurobi .NET applications should always start with this line.

#### Creating the environment

The first executable statement in our example obtains a Gurobi environment (using the GRBEnv() constructor):

```
// Create an empty environment, set options and start
GRBEnv env = new GRBEnv(true);
env.Set("LogFile", "mip1.log");
env.Start();
```

In this case, we create an empty environment, select a log file to use, and start the environment for latter use.

Later calls to create an optimization model will always require an active environment, so environment creation is typically the first step in a Gurobi application. The constructor argument specifies the name of the log file.

### Creating the model

Once an environment has been created, the next step is to create a model. A Gurobi model holds a single optimization problem. It consists of a set of variables, a set of constraints, and the associated attributes (variable bounds, objective coefficients, variable integrality types, constraint senses, constraint right-hand side values, etc.). The first step towards building a model that contains all of this information is to create an empty model object:

```
// Create empty model
GRBModel model = new GRBModel(env);
```

The constructor takes the previously created environment as its argument.

# Adding variables to the model

The next step in our example is to add variables to the model.

```
// Create variables
GRBVar x = model.AddVar(0.0, 1.0, 0.0, GRB.BINARY, "x");
GRBVar y = model.AddVar(0.0, 1.0, 0.0, GRB.BINARY, "y");
```

Variables are added through the AddVar() method on a model object (or AddVars if you wish to add more than one at a time). A variable is always associated with a particular model.

The first and second arguments to the AddVar() call are the variable lower and upper bounds, respectively. The third argument is the linear objective coefficient (zero here - we'll set the objective later). The fourth argument is the variable type. Our variables are all binary in this example. The final argument is the name of the variable.

The AddVar() method has been overloaded to accept several different argument lists. Please refer to the Gurobi Reference Manual for further details.

### Setting the objective

The next step in the example is to set the optimization objective:

```
// Set objective: maximize x + y + 2 z
model.SetObjective(x + y + 2 * z, GRB.MAXIMIZE);
```

The objective is built here using overloaded operators. The C# API overloads the arithmetic operators to allow you to build linear and quadratic expressions involving Gurobi variables.

The second argument indicates that the sense is maximization.

Note that while this simple example builds the objective in a single statement using an explicit list of terms, more complex programs will typically build it incrementally. For example:

```
GRBLinExpr obj = 0.0;
obj.AddTerm(1.0, x);
obj.AddTerm(1.0, y);
obj.AddTerm(2.0, z);
model.SetObjective(obj, GRB.MAXIMIZE);
```

### Adding constraints to the model

The next step in the example is to add the linear constraints:

```
// Add constraint: x + 2 y + 3 z <= 4
model.AddConstr(x + 2 * y + 3 * z <= 4.0, "c0");
```

As with variables, constraints are always associated with a specific model. They are created using the AddConstr() or AddConstrs() methods on the model object.

We again use overloaded arithmetic operators to build linear expressions. The comparison operators are also overloaded to make it easier to build constraints.

The second argument to AddConstr gives the constraint name.

The Gurobi .NET interface also allows you to add constraints by building linear expressions in a term-by-term fashion:

```
GRBLinExpr expr = 0.0;
expr.AddTerm(1.0, x);
expr.AddTerm(2.0, y);
expr.AddTerm(3.0, z);
model.AddConstr(expr, GRB.LESS EQUAL, 4.0, "c0");
```

This particular AddConstr() signature takes a linear expression that captures the left-hand side of the constraint as its first argument, the sense of the constraint as its second argument, and a linear expression that captures the right-hand side of the constraint as its third argument. The constraint name is given as the fourth argument.

### Optimizing the model

Now that the model has been built, the next step is to optimize it:

```
// Optimize model
model.Optimize();
```

This routine performs the optimization and populates several internal model attributes (including the status of the optimization, the solution, etc.).

#### Reporting results - attributes

Once the optimization is complete, we can query the values of the attributes (which are implemented as .NET *properties*). In particular, we can query the VarName and X attributes to obtain the name and solution value for each variable:

```
Console.WriteLine(x.VarName + " " + x.X);
Console.WriteLine(y.VarName + " " + y.X);
```

We can also query the ObjVal attribute on the model to obtain the objective value for the current solution:

```
Console.WriteLine("Obj: " + model.ObjVal);
```

The names and types of all model, variable, and constraint attributes can be found in the *Attributes* section of the Gurobi Reference Manual.

# Cleaning up

The example concludes with Dispose calls:

```
// Dispose of model and env
model.Dispose();
```

These reclaim the resources associated with the model and environment. Garbage collection would reclaim these eventually, but if your program doesn't exit immediately after performing the optimization, it is best to reclaim them explicitly.

Note that all models associated with an environment must be disposed before the environment itself is disposed.

### **Error handling**

Errors in the Gurobi .NET interface are handled through the .NET exception mechanism. In the example, all Gurobi statements are enclosed inside a try block, and any associated errors would be caught by the catch block:

```
} catch (GRBException e) {
  Console.WriteLine("Error code: " + e.ErrorCode + ". " + e.Message);
}
```

### **Building and running the example**

The C# and Visual Basic example directories (<installdir>/examples/c# and <installdir>/examples/vb) contain a number of examples. We encourage you to browse and modify them in order to become more familiar with the Gurobi .NET interface. We also encourage you to read the Gurobi Example Tour for more information.

To run the C# examples on Linux, you can use the Visual Studio project file, dotnetcore2.csproj (in <installdir>/examples/build/DOTNETCore2), or you can use our standard example Makefile (in <installdir>/examples/build). To run the full set of examples using the Makefile, just execute the command make run\_dotnet. You can also run an individual example with make run\_mip1\_cs (e.g.).

# **Python Interface**

The Gurobi Python interface can be used in a number of ways. It is the basis of our Interactive Shell, where it is typically used to work with existing models. It can also be used to write standalone programs that create and solve models, in much the same way that you would use our other language interfaces. The Gurobi distribution includes a Python interpreter and a basic set of Python modules. If you'd like additional capabilities, you can also install the Anaconda Python distribution, which includes both a graphical development environment (Spyder) and a notebook-style interface (Jupyter).

When comparing our Python interface against our other language interfaces, you will find that our Python interface provides a lot more options for building a model. You can work with individual variables and constraints, like you do in other object-oriented interfaces. You can work with matrices, like you do in our matrix-oriented interfaces. Our Python interface also includes a few higher-level constructs that allow you to build models using a more mathematical syntax, similar to the way you might work with a traditional modeling language.

This section will work through three Python modeling examples. The first will present a Python program that is similar to the C, C++, Java, and C# programs presented in previous sections. The second shows how to build models using matrices. The third demonstrates some of the higher-level modeling capabilities of our Python interface.

This section assumes that you are already familiar with the Python programming language, and that you have read the preceding section on the Gurobi Interactive Shell. If you would like to learn more about the Python language, we recommend that you visit the Python online tutorial.

As we noted, the Gurobi distribution includes all the tools you will need to run Python programs. However, if you would prefer to use your own Python installation, we also provide tools for installing the gurobipy module into your Python environment. You should refer to the instructions for building and running the examples for further details.

One big advantage of working within Python is that the Python language is popular and well supported. One aspect of this support is the breadth of powerful Python Integrated Development Environments (IDEs) that are available, most of which can be downloaded for free from the internet. This document includes instructions for setting up Gurobi for use within the Anaconda distribution. Anaconda greatly simplifies the task of installing Python packages, and it includes both a graphical development environment (Spyder) and a notebook-style interface (Jupyter). If you plan to do significant Python development, we recommend that you install Anaconda now. You will also find pointers to other useful Python tools there.

**Important note for AIX users:** due to limited Python support on AIX, our AIX port does not include the Interactive Shell or the Python interface. You can use the C, C++, or Java interfaces.

The Python example directory contains a number of examples. We encourage you to browse and modify them in order to become more familiar with the Gurobi Python interface. We also encourage

you to read the Gurobi Example Tour for more information.

### 12.1 Simple Python Example

This section will work through a simple Python example in order to illustrate the use of the Gurobi Python interface. The example builds a model, optimizes it, and outputs the optimal objective value.

Our example optimizes the following model:

Note that this is the same model that was modeled and optimized in the C Interface section.

#### Example mip1.py

This is the complete source code for our example (also available in <installdir>/examples/python/mip1.py)...

```
#!/usr/bin/env python3.7
# Copyright 2019, Gurobi Optimization, LLC
# This example formulates and solves the following simple MIP model:
 maximize
              y + 2 z
       x +
  subject to
        x + 2 y + 3 z <= 4
        x + y >= 1
        x, y, z binary
import gurobipy as gp
from gurobipy import GRB
try:
    # Create a new model
    m = gp.Model("mip1")
    # Create variables
    x = m.addVar(vtype=GRB.BINARY, name="x")
    y = m.addVar(vtype=GRB.BINARY, name="y")
    z = m.addVar(vtype=GRB.BINARY, name="z")
    # Set objective
    m.setObjective(x + y + 2 * z, GRB.MAXIMIZE)
    # Add constraint: x + 2 y + 3 z \le 4
```

```
m.addConstr(x + 2 * y + 3 * z <= 4, "c0")

# Add constraint: x + y >= 1
m.addConstr(x + y >= 1, "c1")

# Optimize model
m.optimize()

for v in m.getVars():
    print('%s %g' % (v.varName, v.x))

print('Obj: %g' % m.objVal)

except gp.GurobiError as e:
    print('Error code ' + str(e.errno) + ': ' + str(e))

except AttributeError:
    print('Encountered an attribute error')
```

#### **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

The example begins by importing Gurobi functions and classes:

```
import gurobipy as gp
```

The first line makes all Gurobi functions and classes available through a gp. prefix (e.g., gp.Model()). The second makes everything in class GRB available without a prefix (e.g., GRB.OPTIMAL). You can also start a program with from gurobipy import \* to remove the need for the gp. prefix, but if your program uses multiple Python modules it is usually preferred to access each through its own prefix.

In order for these commands to succeed, the Python application needs to know how to find the Gurobi functions and classes. Recall that you have three options here. The first is to use the Python files that are included with our distribution. You would run this example by typing gurobi.sh mip1.py. The second is to install the Anaconda Python distribution. The third is to install the Gurobi functions and classes into your own Python installation.

#### Creating the model

The first step in our example is to create a model. A Gurobi model holds a single optimization problem. It consists of a set of variables, a set of constraints, and the associated attributes (variable bounds, objective coefficients, variable integrality types, constraint senses, constraint right-hand side values, etc.). We start this example with an empty model object:

```
# Create a new model
m = gp.Model("mip1")
```

This function takes the desired model name as its argument.

#### Adding variables to the model

The next step in our example is to add variables to the model.

```
# Create variables
x = m.addVar(vtype=GRB.BINARY, name="x")
```

Variables are added through the addVar() method on a model object (or addVars() if you wish to add more than one at a time). A variable is always associated with a particular model.

Python allows you to pass arguments by position or by name. We've passed them by name here. Each variable gets a type (binary), and a name. We use the default values for the other arguments. Please refer to the online help (help(Model.addVar) in the Gurobi Shell) for further details on addVar().

#### Setting the objective

The next step in the example is to set the optimization objective:

```
# Set objective
m.setObjective(x + y + 2 * z, GRB.MAXIMIZE)
```

The objective is built here using overloaded operators. The Python API overloads the arithmetic operators to allow you to build linear and quadratic expressions involving Gurobi variables.

The second argument indicates that the sense is maximization.

Note that while this simple example builds the objective in a single statement using an explicit list of terms, more complex programs will typically build it incrementally. For example:

```
obj = LinExpr();
obj += x;
obj += y;
obj += 2*z;
model.setObjective(obj, GRB.MAXIMIZE);
```

#### Adding constraints to the model

The next step in the example is to add the linear constraints. The first constraint is added here:

```
# Add constraint: x + 2 y + 3 z <= 4
m.addConstr(x + 2 * y + 3 * z <= 4, "c0")
```

As with variables, constraints are always associated with a specific model. They are created using the addConstr() method on the model object.

We again use overloaded arithmetic operators to build linear expressions. The comparison operators are also overloaded to make it easier to build constraints.

The second argument to addConstr gives the (optional) constraint name.

Again, this simple example builds the linear expression for the constraint in a single statement using an explicit list of terms. More complex programs will typically build the expression incrementally.

The second constraint is created in a similar manner:

```
# Add constraint: x + y >= 1
m.addConstr(x + y >= 1, "c1")
```

### Optimizing the model

Now that the model has been built, the next step is to optimize it:

```
# Optimize model
m.optimize()
```

This routine performs the optimization and populates several internal model attributes (including the status of the optimization, the solution, etc.).

### Reporting results - attributes

Once the optimization is complete, we can query the values of the attributes. In particular, we can query the varName and x variable attributes to obtain the name and solution value for each variable:

```
for v in m.getVars():
    print('%s %g' % (v.varName, v.x))
```

We can also query the objVal attribute on the model to obtain the objective value for the current solution:

```
print('Obj: %g' % m.objVal)
```

The names and types of all model, variable, and constraint attributes can be found in the online Python documentation. Type help(GRB.Attr) in the Gurobi Shell for details.

#### **Error handling**

Errors in the Gurobi Python interface are handled through the Python exception mechanism. In the example, all Gurobi statements are enclosed inside a try block, and any associated errors would be caught by the except block:

```
except gp.GurobiError as e:
    print('Error code ' + str(e.errno) + ': ' + str(e))

except AttributeError:
    print('Encountered an attribute error')
```

### Running the example

When you run the example gurobi.sh mip1.py, you should see the following output:

```
Using license file /opt/gurobi/gurobi.lic
Set parameter LogFile to value gurobi.log
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Optimize a model with 2 rows, 3 columns and 5 nonzeros
Model fingerprint: 0xb2adf8c4
Variable types: 0 continuous, 3 integer (3 binary)
Coefficient statistics:
 Matrix range
                  [1e+00, 3e+00]
 Objective range [1e+00, 2e+00]
                  [1e+00, 1e+00]
 Bounds range
 RHS range
                  [1e+00, 4e+00]
Found heuristic solution: objective 2.0000000
Presolve removed 2 rows and 3 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
Explored O nodes (O simplex iterations) in 0.00 seconds
Thread count was 1 (of 4 available processors)
Solution count 2: 3
Optimal solution found (tolerance 1.00e-04)
Best objective 3.000000000000e+00, best bound 3.00000000000e+00, gap 0.0000%
у 0
z 1
Obj: 3
```

# 12.2 Python Matrix Example

You can also build optimization models in Python using dense and sparse matrices, much like you would in our MATLAB and R interfaces. We'll show an example that works through the same simple model from the first part of this section:

### Example matrix1.py

This is the complete source code for our example (also available in <installdir>/examples/python/matrix1.py)...

```
#!/usr/bin/env python3.7

# Copyright 2019, Gurobi Optimization, LLC

# This example formulates and solves the following simple MIP model
# using the matrix API:
```

```
maximize
   x + y + 2z
#
 subject to
#
       x + 2 y + 3 z <= 4
#
        x + y >= 1
       x, y, z binary
import numpy as np
import scipy.sparse as sp
import gurobipy as gp
from gurobipy import GRB
try:
    # Create a new model
    m = gp.Model("matrix1")
    # Create variables
    x = m.addMVar(shape=3, vtype=GRB.BINARY, name="x")
    # Set objective
    obj = np.array([1.0, 1.0, 2.0])
    m.setObjective(obj @ x, GRB.MAXIMIZE)
    # Build (sparse) constraint matrix
    data = np.array([1.0, 2.0, 3.0, -1.0, -1.0])
    row = np.array([0, 0, 0, 1, 1])
    col = np.array([0, 1, 2, 0, 1])
    A = sp.csr_matrix((data, (row, col)), shape=(2, 3))
    # Build rhs vector
    rhs = np.array([4.0, -1.0])
    # Add constraints
    m.addConstr(A @ x <= rhs, name="c")</pre>
    # Optimize model
    m.optimize()
    print(x.X)
    print('Obj: %g' % m.objVal)
except gp.GurobiError as e:
    print('Error code ' + str(e.errno) + ": " + str(e))
except AttributeError:
    print('Encountered an attribute error')
```

You will need to have both the NumPy package and the SciPy sparse matrix package in your Python environment to run this example. The easiest way to obtain a suitable Python environment is to install the Anaconda Python distribution.

### **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

The example begins by importing the Gurobi functions and classes, as well as the NumPy and SciPy packages:

```
import numpy as np
```

Gurobi Python applications should always start with the first line. If you want to use the matrix interface, the next two lines are needed as well.

### Creating the model

The first step in our example is to create a model. We start with an empty model object:

```
# Create a new model
m = gp.Model("matrix1")
```

This function takes the desired model name as its argument.

### Adding variables to the model

The next step adds a matrix variable to the model:

```
# Create variables
x = m.addMVar(shape=3, vtype=GRB.BINARY, name="x")
```

A matrix variable is added through the addMVar() method on a model object. In this case the matrix variable consists of a 1-D array of 3 binary variables. Variables are always associated with a particular model.

#### Setting the objective

The next step is to set the optimization objective:

```
obj = np.array([1.0, 1.0, 2.0])
m.setObjective(obj @ x, GRB.MAXIMIZE)
```

The objective is built here by computing a dot product between a constant vector and our matrix variable using the overloaded @ operator. Note that the constant vector must have the same length as our matrix variable.

The second argument indicates that the sense is maximization.

#### Adding constraints to the model

The next step in the example is to add our two linear constraints. This is done by building a sparse matrix that captures the constraint matrix:

```
# Build (sparse) constraint matrix
data = np.array([1.0, 2.0, 3.0, -1.0, -1.0])
row = np.array([0, 0, 0, 1, 1])
col = np.array([0, 1, 2, 0, 1])
```

The matrix has two rows, one for each constraint, and three columns, one for each variable in our matrix variable. Note that we multiply the greater-than constraint by -1 to transform it to a less-than constraint.

We also capture the right-hand side in a NumPy array:

```
# Build rhs vector
```

We then use the overloaded @ operator to build a linear matrix expression, and then use the overloaded less-than-or-equal operator to add two constraints (one for each row in the matrix expression):

```
# Add constraints
m.addConstr(A @ x <= rhs, name="c")</pre>
```

### Optimizing the model

Now that the model has been built, the next step is to optimize it:

```
# Optimize model
m.optimize()
```

This routine performs the optimization and populates several internal model attributes (including the status of the optimization, the solution, etc.).

#### Reporting results - attributes

Once the optimization is complete, we can query the values of the attributes. In particular, we can query the varName and x variable attributes to obtain the name and solution value for each variable:

```
print(x.X)
```

We can also query the objVal attribute on the model to obtain the objective value for the current solution:

```
print('Obj: %g' % m.objVal)
```

The names and types of all model, variable, and constraint attributes can be found in the online Python documentation. Type help(GRB.Attr) in the Gurobi Shell for details.

### **Error handling**

Errors in the Gurobi Python interface are handled through the Python exception mechanism. In the example, all Gurobi statements are enclosed inside a try block, and any associated errors would be caught by the except block:

```
except gp.GurobiError as e:
    print('Error code ' + str(e.errno) + ": " + str(e))

except AttributeError:
    print('Encountered an attribute error')
```

### Running the example

```
When you run the example gurobi.sh matrix1.py, you should see the following output:
Using license file /opt/gurobi/gurobi.lic
Set parameter LogFile to value gurobi.log
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Optimize a model with 2 rows, 3 columns and 5 nonzeros
Model fingerprint: 0xbc579ebb
Variable types: 0 continuous, 3 integer (3 binary)
Coefficient statistics:
                  [1e+00, 3e+00]
 Matrix range
 Objective range [1e+00, 2e+00]
 Bounds range
                  [1e+00, 1e+00]
                  [1e+00, 4e+00]
 RHS range
Found heuristic solution: objective 2.0000000
Presolve removed 2 rows and 3 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
Explored 0 nodes (0 simplex iterations) in 0.00 seconds
Thread count was 1 (of 4 available processors)
Solution count 2: 3
Optimal solution found (tolerance 1.00e-04)
Best objective 3.000000000000e+00, best bound 3.00000000000e+00, gap 0.0000%
[1. 0. 1.]
Obj: 3
```

# 12.3 Python Dictionary Example

In order to provide a gentle introduction to our interfaces, the examples so far have demonstrated only very basic capabilities. We will now attempt to demonstrate some of the power of our Python interface by describing a more complex example. This example is intended to capture most of the common ingredients of large, complex optimization models. Implementing this same example in another API would most likely have required hundreds of lines of code (ours is around 70 lines of Python code).

We'll need to present a few preliminaries before getting to the example itself. You'll need to learn a bit about the Python language, and we'll need to describe a few custom classes and functions. Our intent is that you will come away from this section with an appreciation for the power and flexibility of this interface. It can be used to create quite complex models using what we believe

are very concise and natural modeling constructs. Our goal with this interface has been to provide something that feels more like a mathematical modeling language than a programming language API.

If you'd like to dig a bit deeper into the Python language constructs described here, we recommend that you visit the online Python tutorial.

#### Motivation

At the heart of any optimization model lies a set of decision variables. Finding a convenient way to store and access these variables can often represent the main challenge in implementing the model. While the variables in some models map naturally to simple programming language constructs (e.g., x[i] for contiguous integer values i), other models can present a much greater challenge. For example, consider a model that optimizes the flow of multiple different commodities through a supply network. You might have a variable x['Pens', 'Denver', 'New York'] that captures the flow of a manufactured item (pens in this example) from Denver to New York. At the same time, you might not want to have a variable x['Pencils', 'Denver', 'Seattle'], since not all combinations of commodities, source cities, and destination cities represent valid paths through the network. Representing a sparse set of decision variables in a typical programming language can be cumbersome.

To compound the challenge, you typically need to build constraints that involve subsets of these decision variables. For example, in our network flow model you might want to put an upper bound on the total flow that enters a particular city. You could certainly collect the relevant decision variables by iterating over all possible cities and selecting only those variables that capture possible flow from that source city into the desired destination city. However, this is clearly wasteful if not all origin-destination pairs are valid. In a large network problem, the inefficiency of this approach could lead to major performance issues. Handling this efficiently can require complex data structures.

The Gurobi Python interface has been designed to make the issues we've just described quite easy to manage. We'll present a specific example of how this is done shortly. Before we do, though, we'll need to describe a few important Python constructs: lists, tuples, dictionaries, list comprehension, and generator expressions. These are standard Python concepts that are particularly important in our interface. We'll also introduce the tuplelist and tupledict classes, which are custom classes that we've added to the Gurobi Python interface.

A quick reminder: you can consult the online Python documentation for additional information on any of the Python data structures mentioned here.

#### **Lists and Tuples**

The list data structure is central to most Python programs; Gurobi Python programs are no exception. We'll also rely heavily on a similar data structure, the tuple. Tuples are crucial to providing efficient and convenient access to Gurobi decision variables in Gurobi Python programs. The difference between a list and a tuple is subtle but important. We'll discuss it shortly.

Lists and tuples are both just ordered collections of Python objects. A list is created and dis-

played as a comma-separated list of member objects, enclosed in square brackets. A tuple is similar, except that the member objects are enclosed in parenthesis. For example, [1, 2, 3] is a list, while (1, 2, 3) is a tuple. Similarly, ['Pens', 'Denver', 'New York'] is a list, while ('Pens', 'Denver', 'New York') is a tuple.

You can retrieve individual entries from a list or tuple using square brackets and zero-based indices:

```
gurobi> 1 = [1, 2.0, 'abc']
gurobi> t = (1, 2.0, 'abc')
gurobi> print(1[0])
1
gurobi> print(t[1])
2.0
gurobi> print(1[2])
abc
```

What's the difference between a list and a tuple? A tuple is *immutable*, meaning that you can't modify it once it has been created. By contrast, you can add new members to a list, remove members, change existing members, etc. This immutable property allows you to use tuples as indices for dictionaries.

#### **Dictionaries**

A Python dictionary allows you to map arbitrary key values to pieces of data. Any *immutable* Python object can be used as a key: an integer, a floating-point number, a string, or even a tuple. To give an example, the following statements create a dictionary x, and then associates a value 1

```
gurobi> x = {} # creates an empty dictionary
gurobi> x[('Pens', 'Denver', 'New York')] = 1
gurobi> print(x[('Pens', 'Denver', 'New York')])
1
```

with key ('Pens', 'Denver', 'New York')

Python allows you to omit the parenthesis when accessing a dictionary using a tuple, so the following is also valid:

```
gurobi> x = {}
gurobi> x['Pens', 'Denver', 'New York'] = 2
gurobi> print(x['Pens', 'Denver', 'New York'])
2
```

We've stored integers in the dictionary here, but dictionaries can hold arbitrary objects. In particular, they can hold Gurobi decision variables:

```
gurobi> x['Pens', 'Denver', 'New York'] = model.addVar()
gurobi> print(x['Pens', 'Denver', 'New York'])
<gurobi.Var *Awaiting Model Update*>
```

To initialize a dictionary, you can of course simply perform assignments for each relevant key:

```
gurobi> values = {}
gurobi> values['zero'] = 0
gurobi> values['one'] = 1
gurobi> values['two'] = 2
```

You can also use the Python dictionary initialization construct:

```
gurobi> values = { 'zero': 0, 'one': 1, 'two': 2 }
gurobi> print(values['zero'])
0
gurobi> print(values['one'])
1
```

We have included a utility routine in the Gurobi Python interface that simplifies dictionary initialization for a case that arises frequently in mathematical modeling. The multidict function allows you to initialize one or more dictionaries in a single statement. The function takes a dictionary as its argument, where the value associated with each key is a list of length n. The function splits these lists into individual entries, creating n separate dictionaries. The function returns a list. The first result is the list of shared key values, followed by the n individual dictionaries:

```
gurobi> names, lower, upper = multidict({ 'x': [0, 1], 'y': [1, 2], 'z': [0, 3] })
gurobi> print(names)
['x', 'y', 'z']
gurobi> print(lower)
{'x': 0, 'y': 1, 'z': 0}
gurobi> print(upper)
{'x': 1, 'y': 2, 'z': 3}
```

Note that you can also apply this function to a dictionary where each key maps to a scalar value. In that case, the function simply returns the list of keys as the first result, and the original dictionary as the second.

You will see this function in several of our Python examples.

#### List comprehension and generator expressions

List comprehension and generator expressions are important Python features that allows you to do implicit enumeration in a concise fashion. To give a simple example, the following list comprehension builds a list containing the squares of the numbers from 1 through 5:

```
gurobi> [x*x for x in [1, 2, 3, 4, 5]] [1, 4, 9, 16, 25]
```

A generator expression is very similar, but it is used to generate an Iterable (something that can be iterated over). For example, suppose we want to compute the sum of the squares of the numbers from 1 through 5. We could use list comprehension to build the list, and then pass that list to sum. However, it is simpler and more efficient to use a generator expression:

```
gurobi> sum(x*x for x in [1, 2, 3, 4, 5])
```

A generator expression can be used whenever a method accepts an Iterable argument (something that can be iterated over). For example, most Python methods that accept a list argument (the most common type of Iterable) will also accept a generator expression.

Note that there's a Python routine for creating a contiguous list of integers: range. The above would typically be written as follows:

```
gurobi> sum(x*x for x in range(1,6))
```

Details on the range function can be found here.

List comprehension and generator expressions can both contain more than one for clause, and one or more if clauses. The following example builds a list of tuples containing all x,y pairs where x and y are both less than 4 and x is less than y:

```
gurobi> [(x,y) for x in range(4) for y in range(4) if x < y] [(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)]
```

Note that the for statements are executed left-to-right, and values from one can be used in the next, so a more efficient way to write the above is:

```
gurobi> [(x,y) for x in range(4) for y in range(x+1, 4)]
```

Generator expressions are used extensively in our Python examples, primarily in the context of the addConstrs method.

#### The tuplelist class

The next important item we would like to discuss is the tuplelist class. This is a custom sub-class of the Python list class that is designed to allow you to efficiently build sub-lists from a list of tuples. To be more specific, you can use the select method on a tuplelist object to retrieve all tuples that match one or more specified values in specific fields.

Let us give a simple example. We'll begin by creating a simple tuplelist (by passing a list of tuples to the constructor):

```
gurobi> 1 = tuplelist([(1, 2), (1, 3), (2, 3), (2, 4)])
```

To select a sub-list where particular tuple entries match desired values, you specify the desired values as arguments to the select method. The number of arguments to select is equal to the number of entries in the members of the tuplelist (they should all have the same number of entries). You can provide a list argument to indicate that multiple values are acceptable in that position in the tuple, or a '\*' string to indicate that any value is acceptable.

Each tuple in our example contains two entries, so we can perform the following selections:

```
gurobi> print(l.select(1, '*'))
<gurobi.tuplelist (2 tuples, 2 values each):
  ( 1 , 2 )
  ( 1 , 3 )
>
```

```
gurobi> print(l.select('*', 3))
<gurobi.tuplelist (2 tuples, 2 values each):</pre>
(1,3)
(2,3)
gurobi> print(l.select('*', [2, 4]))
<gurobi.tuplelist (2 tuples, 2 values each):</pre>
(1, 2)
(2,4)
gurobi> print(l.select(1, 3))
<gurobi.tuplelist (1 tuples, 2 values each):</pre>
(1,3)
gurobi> print(l.select('*', '*'))
<gurobi.tuplelist (4 tuples, 2 values each):</pre>
(1,2)
(1,3)
(2,3)
(2, 4)
```

You may have noticed that similar results could have been achieved using list comprehension. For example:

```
gurobi> print(1.select(1, '*'))
<gurobi.tuplelist (2 tuples, 2 values each):
    (1 , 2 )
    (1 , 3 )
>
gurobi> print([(x,y) for x,y in 1 if x == 1])
[(1, 2), (1, 3)]
```

The problem is that the latter statement considers every member in the list, which can be quite inefficient for large lists. The **select** method builds internal data structures that make these selections quite efficient.

Note that tuplelist is a sub-class of list, so you can use the standard list methods to access or modify a tuplelist:

```
gurobi> print(1[1])
(1,3)
gurobi> 1 += [(3, 4)]
gurobi> print(1)
<gurobi.tuplelist (5 tuples, 2 values each):
    (1 , 2)
    (1 , 3)
    (2 , 3)
    (2 , 4)
    (3 , 4)
>
```

### The tupledict class

The final important preliminary we would like to discuss is the tupledict class. This is a custom sub-class of the Python dict class that allows you to efficiently work with subsets of Gurobi variable objects. To be more specific, you can use the sum and prod methods on a tupledict object to easily and concisely build linear expressions. The keys for a tupledict are stored as a tuplelist, so the same select syntax can be used to choose subsets of entries. Specifically, by associating a tuple with each Gurobi variable, you can efficiently create expressions that contain a subset of matching variables. For example, using the sum method on a tupledict object, you could easily build an expression that captures the sum over all Gurobi variables for which the first field of the corresponding tuple is equal to 3 (using x.sum(3, '\*')).

While you can directly build your own tupledict, the Gurobi interface provides an addVars method that adds one Gurobi decision variable to the model for each tuple in the input argument(s) and returns the result as a tupledict. Let us give a simple example. We'll begin by constructing a list of tuples, and then we'll create a set of Gurobi variables that are indexed using this list:

```
gurobi> 1 = list([(1, 2), (1, 3), (2, 3), (2, 4)])
gurobi> d = model.addVars(1, name="d")
```

The addVars method will create variables d(1,2), d(1,3), d(2,3), and d(2,4). Note that the name argument is used to name the resulting variables, but it only gives the prefix for the name - the names are subscripted by the tuple keys (so the variables would be named d[1,2], d[1,3], etc.).

You can then use this tupledict to build linear expressions. For example, you could do:

```
gurobi> sum(d.select(1, '*'))
```

The select method returns a list of Gurobi variables where the first field of the associated tuple is 1. The Python sum statement then creates a linear expression that captures the sum of these variables. In this case, that expression would be d(1,2) + d(1,3). Similarly, sum(d.select('\*', 3)) would give d(1,3) + d(2,3). As with a tuplelist, you use a '\*' string to indicate that any value is acceptable in that position in the tuple.

The tupledict class includes a method that simplifies the above. Rather than sum(d.select('\*', 3)), you can use d.sum('\*', 3) instead.

The tupledict class also includes a prod method, for cases where your linear expression has coefficients that aren't all 1.0. Coefficients are provided through a dict argument. They are indexed using the same tuples as the tupledict. For example, given a dict named coeff with two entries: coeff(1,2) = 5 and coeff(2,3) = 7, a call to d.prod(coeff) would give the expression 5 d(1,2) + 7 d(2,3). You can also include a filter, so d.prod(coeff, 2, '\*') would give just 7 d(2,3).

Note that tupledict is a sub-class of dict, so you can use the standard dict methods to access or modify a tupledict:

```
gurobi> print(d[1,3])
<gurobi.Var d[1,3]>
gurobi> d[3, 4] = 0.3
```

```
gurobi> print(d[3, 4])
0.3
gurobi> print(d.values())
dict_values([<gurobi.Var d[1,2]>, 0.3, <gurobi.Var d[1,3]>, <gurobi.Var d[2,3]>, <gurobi.Var d[2,4]>])
```

In our upcoming network flow example, once we've built a tupledict that contains a variable for each valid commodity-source-destination combination on the network (we'll call it flows), we can create a linear expression that captures the total flow on all arcs that empty into a specific destination city as follows:

```
gurobi> inbound = flows.sum('*', '*', 'New York')
```

We now present an example that illustrates the use of all of the concepts discussed so far.

### netflow.py example

Our example solves a multi-commodity flow model on a small network. In the example, two commodities (Pencils and Pens) are produced in two cities (Detroit and Denver), and must be shipped to warehouses in three cities (Boston, New York, and Seattle) to satisfy given demand. Each arc in the transportation network has a cost associated with it, and a total capacity.

This is the complete source code for our example (also available in <installdir>/examples/python/netflow.py)...

```
#!/usr/bin/env python3.7
# Copyright 2019, Gurobi Optimization, LLC
# Solve a multi-commodity flow problem. Two products ('Pencils' and 'Pens')
# are produced in 2 cities ('Detroit' and 'Denver') and must be sent to
# warehouses in 3 cities ('Boston', 'New York', and 'Seattle') to
# satisfy demand ('inflow[h,i]').
# Flows on the transportation network must respect arc capacity constraints
# ('capacity[i,j]'). The objective is to minimize the sum of the arc
# transportation costs ('cost[i,j]').
import gurobipy as gp
from gurobipy import GRB
# Base data
commodities = ['Pencils', 'Pens']
nodes = ['Detroit', 'Denver', 'Boston', 'New York', 'Seattle']
arcs, capacity = gp.multidict({
    ('Detroit', 'Boston'):
    ('Detroit', 'New York'): 80,
    ('Detroit', 'Seattle'): 120,
    ('Denver', 'Boston'):
                             120,
    ('Denver', 'New York'): 120,
    ('Denver', 'Seattle'): 120})
# Cost for triplets commodity-source-destination
```

```
cost = {
    ('Pencils', 'Detroit', 'Boston'): 10,
    ('Pencils', 'Detroit', 'New York'): 20,
    ('Pencils', 'Detroit', 'Seattle'): 60,
    ('Pencils', 'Denver', 'Boston'):
    ('Pencils', 'Denver', 'New York'): 40,
    ('Pencils', 'Denver', 'Seattle'): 30,
              'Detroit', 'Boston'):
    ('Pens',
    ('Pens',
               'Detroit', 'New York'): 20,
    ('Pens',
              'Detroit', 'Seattle'): 80,
              'Denver', 'Boston'):
    ('Pens',
                                        60,
              'Denver', 'New York'): 70,
    ('Pens',
              'Denver', 'Seattle'): 30}
    ('Pens',
# Demand for pairs of commodity-city
inflow = {
    ('Pencils', 'Detroit'):
    ('Pencils', 'Denver'):
                            60,
    ('Pencils', 'Boston'):
                            -50,
    ('Pencils', 'New York'): -50,
    ('Pencils', 'Seattle'): -10,
               'Detroit'): 60,
    ('Pens',
    ('Pens',
               'Denver'):
               'Boston'):
                           -40,
    ('Pens',
    ('Pens',
              'New York'): -30,
              'Seattle'): -30}
    ('Pens',
# Create optimization model
m = gp.Model('netflow')
# Create variables
flow = m.addVars(commodities, arcs, obj=cost, name="flow")
# Arc-capacity constraints
m.addConstrs(
    (flow.sum('*', i, j) <= capacity[i, j] for i, j in arcs), "cap")</pre>
# Equivalent version using Python looping
# for i, j in arcs:
  m.addConstr(sum(flow[h, i, j] for h in commodities) <= capacity[i, j],</pre>
                "cap[%s, %s]" % (i, j))
# Flow-conservation constraints
m.addConstrs(
    (flow.sum(h, '*', j) + inflow[h, j] == flow.sum(h, j, '*')
        for h in commodities for j in nodes), "node")
# Alternate version:
# m.addConstrs(
    (gp.quicksum(flow[h, i, j] for i, j in arcs.select('*', j)) + inflow[h, j] ==
      gp.quicksum(flow[h, j, k] for j, k in arcs.select(j, '*'))
      for h in commodities for j in nodes), "node")
# Compute optimal solution
m.optimize()
```

#### netflow.py example details

Let us now walk through the example, line by line, to understand how it achieves the desired result of computing the optimal network flow. As with the simple Python example presented earlier, this example begins by importing the Gurobi functions and classes:

```
import gurobipy as gp
```

We then create a few lists that contain model data:

```
# Base data
commodities = ['Pencils', 'Pens']
nodes = ['Detroit', 'Denver', 'Boston', 'New York', 'Seattle']

arcs, capacity = gp.multidict({
        ('Detroit', 'Boston'): 100,
        ('Detroit', 'New York'): 80,
        ('Detroit', 'Seattle'): 120,
        ('Denver', 'Boston'): 120,
        ('Denver', 'New York'): 120,
        ('Denver', 'Seattle'): 120}
```

The model works with two commodities (Pencils and Pens), and the network contains 5 nodes and 6 arcs. We initialize commodities and nodes as simple Python lists. We use the Gurobi multidict function to initialize arcs (the list of keys) and capacity (a dictionary).

The model also requires cost data for each commodity-arc pair:

```
# Cost for triplets commodity-source-destination
cost = {
    ('Pencils', 'Detroit', 'Boston'):
    ('Pencils', 'Detroit', 'New York'): 20,
    ('Pencils', 'Detroit', 'Seattle'):
    ('Pencils', 'Denver', 'Boston'):
    ('Pencils', 'Denver',
                          'New York'): 40,
    ('Pencils', 'Denver',
                          'Seattle'):
    ('Pens',
               'Detroit', 'Boston'):
    ('Pens',
               'Detroit', 'New York'): 20,
               'Detroit', 'Seattle'): 80,
    ('Pens',
               'Denver', 'Boston'):
    ('Pens',
                                       60.
    ('Pens',
               'Denver',
                          'New York'): 70,
              'Denver', 'Seattle'):
    ('Pens',
```

Once this dictionary has been created, the cost of moving commodity h from node i to j can be queried as cost[(h,i,j)]. Recall that Python allows you to omit the parenthesis when using a tuple to index a dictionary, so this can be shortened to just cost[h,i,j].

A similar construct is used to initialize node demand data:

```
# Demand for pairs of commodity-city
inflow = {
    ('Pencils', 'Detroit'):
                               50.
    ('Pencils', 'Denver'):
                              60,
    ('Pencils', 'Boston'):
                              -50,
    ('Pencils', 'New York'): -50,
    ('Pencils', 'Seattle'):
                'Detroit'):
    ('Pens',
                              60.
    ('Pens',
                'Denver'):
                              40.
    ('Pens',
               'Boston'):
                              -40,
    ('Pens',
               'New York'): -30,
    ('Pens',
              'Seattle'): -30}
```

### Building a multi-dimensional array of variables

The next step in our example (after creating an empty Model object) is to add variables to the model. The variables are created using addVars, and are returned in a tupledict which we'll call flow:

```
# Create optimization model
m = gp.Model('netflow')

# Create variables
flow = m.addVars(commodities, arcs, obj=cost, name="flow")
```

The first, positional arguments to addVars give the index set. In this case, we'll be indexing flow by commodities and arcs. In other words, flow[c,i,j] will capture the flow of commodity c from node i to node j. Note that flow only contains variables for source, destination pairs that are present in arcs.

### Arc capacity constraints

We begin with a straightforward set of constraints. The sum of the flow variables on an arc must be less than or equal to the capacity of that arc:

```
# Arc-capacity constraints
m.addConstrs(
    (flow.sum('*', i, j) <= capacity[i, j] for i, j in arcs), "cap")</pre>
```

Note that this one statement uses several of the concepts that were introduced earlier in this section.

The first concept used here is the sum method on flow, which is used to create a linear expression over a subset of the variables in the tupledict. In particular, it is summing over all commodities (the '\*' in the first field) associated with an edge between a pair of cities i and j.

The second concept used here is a generator expression, which iterates over all arcs in the network. Specifically, this portion of the statement...

```
for i,j in arcs
```

indicates that we are iterating over every edge in arcs. In each iteration, i and j will be populated using the corresponding values from a tuple in arcs. In a particular iteration, flow.sum('\*',i,j) will be computed using those specific values, as will capacity[i,j].

The third thing to note is that we're passing the result as an argument to addConstrs. This method will create a set of Gurobi constraints, one for each iteration of the generator expression.

The final thing to note is that the last argument gives the base for the constraint name. The addConstrs method will automatically append the corresponding indices for each constraint. Thus, for example, the name of the constraint that limits flow from Denver to Boston will be cap[Denver,Boston].

Note that if you prefer to do your own looping, you could obtain the equivalent behavior with the following loop:

#### Flow conservation constraints

The next set of constraints are the flow conservation constraints. They require that, for each commodity and node, the sum of the flow into the node plus the quantity of external inflow at that node must be equal to the sum of the flow out of the node:

This call to addConstrs is similar to the previous one, although a bit more complex. We invoke the sum method on a tupledict, wrapped inside of a generator expression, to add one linear constraint for each commodity-node pair. In this instance, we call sum twice, and we use a generator expression that contains of a pair of for loops, but the basic concepts remain the same.

#### Results

Once we've added the model constraints, we call optimize and then output the optimal solution:

If you run the example gurobi.sh netflow.py, you should see the following output:

```
Using license file /opt/gurobi/gurobi.lic
Set parameter LogFile to value gurobi.log
Gurobi Optimizer version 9.0.0 build v9.0.0rc0 (linux64)
Optimize a model with 16 rows, 12 columns and 36 nonzeros
Model fingerprint: 0xf10778ba
Coefficient statistics:
                   [1e+00, 1e+00]
 Matrix range
 Objective range [1e+01, 8e+01]
                   [0e+00, 0e+00]
 Bounds range
 RHS range
                   [1e+01, 1e+02]
Presolve removed 16 rows and 12 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
                                                           Time
Iteration
            Objective
                           Primal Inf.
                                           Dual Inf.
                                           2.000000e+01
           5.5000000e+03 0.000000e+00
      0
                                                             0s
Extra one simplex iteration after uncrush
           5.5000000e+03 0.000000e+00
                                           0.000000e+00
                                                             0s
Solved in 1 iterations and 0.00 seconds
Optimal objective 5.500000000e+03
Optimal flows for Pencils:
Detroit -> Boston: 50
Denver -> New York: 50
Denver -> Seattle: 10
Optimal flows for Pens:
Detroit -> Boston: 30
Detroit -> New York: 30
Denver -> Boston: 10
Denver -> Seattle: 30
```

# 12.4 Building and running the examples

Python is an interpreted language, so no explicit compilation step is required to run the examples. For Linux platforms, you can simply type the following in the Gurobi Python example directory (<installdir>/examples/python):

```
gurobi.sh mip1.py
```

If you are a Python user and wish to use Gurobi from within your own Python environment, we recommend that you use the Anaconda Python distribution. Anaconda includes a number of very useful packages, and the Gurobi Anaconda package simplifies the installation process.

You can also install the gurobipy module directly into other Python environments. The steps for doing this depend on your platform. On Linux, you will need to open a terminal window, change your current directory to the Gurobi <installdir> (the directory that contains the file setup.py), and issue the following command:

```
python setup.py install
```

Unless you are using your own private Python installation, you will need to run this command as super-user.

Once gurobipy is successfully installed, you can type python mip1.py (more generally, you can type from gurobipy import \* in your Python environment).

# **MATLAB** Interface

This section describes the Gurobi MATLAB interface. We begin with information on how to set up Gurobi for use within MATLAB. An example of how to use the MATLAB interface follows.

# Setting up Gurobi for MATLAB

To begin, you'll need to tell MATLAB where to find the Gurobi routines. We've provided a script to assist you with this. The Gurobi MATLAB setup script, gurobi\_setup.m, can be found in the <installdir>/matlab directory of your Gurobi installation (the default <installdir> for Gurobi 9.0.0 is /opt/gurobi900/linux64 for Linux). To get started, type the following commands within MATLAB to change to the matlab directory and call gurobi\_setup:

```
>> cd /opt/gurobi900/linux64/matlab
>> gurobi_setup
```

You will need to be careful that the MATLAB binary and the Gurobi package you install both use the same instruction set. You need to install the 64-bit version of MATLAB to use Gurobi.

# **Example**

Let us now turn our attention to an example of using Gurobi to solve a simple MIP model. Our example optimizes the following model:

Note that this is the same model that was modeled and optimized in the C Interface section.

This is the complete source code for our example (also available in <installdir>/examples/matlab/mip1.m)...

```
function mip1()
% Copyright 2019, Gurobi Optimization, LLC
% This example formulates and solves the following simple MIP model:
%
  maximize
%
        x +
             y + 2z
  subject to
%
        x + 2 y + 3 z <= 4
%
        x + y >= 1
        x, y, z binary
names = {'x'; 'y'; 'z'};
model.A = sparse([1 2 3; 1 1 0]);
model.obj = [1 1 2];
```

```
model.rhs = [4; 1];
model.sense = '<>';
model.vtype = 'B';
model.modelsense = 'max';
model.varnames = names;
gurobi_write(model, 'mip1.lp');
params.outputflag = 0;
result = gurobi(model, params);
disp(result);
for v=1:length(names)
    fprintf('%s %d\n', names{v}, result.x(v));
end

fprintf('Obj: %e\n', result.objval);
end
```

# **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

# **Building the model**

The example begins by building an optimization model. The data associated with an optimization model must be stored in a MATLAB struct. Fields in this struct contain the different parts of the model. The most fundamental fields are: The constraint matrix (A), the objective vector (obj), the right-hand side vector (rhs), and the constraint sense vector (sense). Among these, only the constraint matrix is mandatory, and default values are substituted for all other model fields in case they are missing.

The example uses the built-in sparse function to build the constraint matrix A. The Gurobi MATLAB interface only accepts sparse matrices as input. If you have a dense matrix, use sparse to convert it to a sparse matrix before passing it to our interface.

In addition to the fields discussed above, this example sets two more fields: modelsense and vtype. The former is used to indicate the sense of the objective function. The default is minimization, so we've set the field equal to 'max' to indicate that we would like to maximize the specified objective. The vtype field is used to indicate the types of the variables in the model. In our example, all variables are binary ('B'). Note that our interface allows you to specify a scalar value for the sense and vtype arguments. The Gurobi interface will expand that scalar to a constant array of the appropriate length. In this example, the scalar value 'B' will be expanded to an array of length 3, containing one 'B' value for each column of A.

# Modifying Gurobi parameters

The next statements create a struct variable that will be used to modify two Gurobi parameters:

```
params.outputflag = 0;
```

```
params.resultfile = 'mip1.lp';
```

In this example, we set the Gurobi OutputFlag parameter to 0 in order to shut off Gurobi output. We also set the ResultFile parameter to request that Gurobi produce a file as output (in this case, an LP format file that contains the optimization model). The Gurobi MATLAB interface allows you to set as many Gurobi parameters as you like. The field names in the parameter structure simply need to match Gurobi parameter names, and the values of the fields should be set to the desired parameter values. Please consult the Parameters section of the Gurobi Reference Manual for a complete list of all Gurobi parameters.

# Solving the model

The next statement is where the actual optimization occurs:

```
result = gurobi(model, params);
```

We pass the model and the optional list of parameter changes to the gurobi() function. It computes an optimal solution to the specified model and returns the computed result.

# Printing the solution

The gurobi() function returns a struct as its result. This struct contains a number of fields, where each field contains information about the computed solution. The available fields depend on the result of the optimization, the type of model that was solved (LP, QP, QCP, SOCP, or MIP), and the algorithm used to solve the model. The returned struct will always contain a status field, which indicates whether Gurobi was able to compute an optimal solution to the model. You should consult the *Status Codes* section of the Gurobi Reference Manual for a complete list of all possible status codes. If Gurobi was able to find a solution to the model, the return value will also include objval and x fields. The former gives the objective value for the computed solution, and the latter is the computed solution vector (one entry per column of the constraint matrix). For continuous models, we will also return dual information (reduced costs and dual multipliers), and possibly an optimal basis.

In our example, we simply print the optimal objective value (result.objval) and the optimal solution vector (result.x).

# Running the example

The Gurobi MATLAB examples can be found in the <installdir>/examples/matlab/ directory of your Gurobi installation (the default <installdir> for Gurobi 9.0.0 is /opt/gurobi900/linux64 for Linux). To run one of the examples, first change to this directory in MATLAB, then type its name into the MATLAB prompt. For example, to run example mip1, you would say:

```
>> cd /opt/gurobi900/linux64/examples/matlab
>> mip1
```

If Gurobi was successfully set up for use in MATLAB, you should see the following output in the command window:

status: 'OPTIMAL'
versioninfo: [1x1 struct]
runtime: 3.2401e-04

objval: 3

x: [3x1 double]

From all this data we only use the fields objval and x in our example. Please refer to the reference manual for a complete description of all the other output fields.

In order to get more familiar with the Gurobi MATLAB interface, we encourage you to browse through the files in the MATLAB example directory. Often these examples can be used as starting points for your own optimization projects.

This section describes the Gurobi R interface. We begin with information on how to set up Gurobi for use within R. An example of how to use the R interface follows.

# Installing the R Package

To begin, you'll need to install the Gurobi package in R. The R command for doing this is:

```
install.packages('<R-package-file>', repos=NULL)
```

The R package file can be found in the <installdir>/R directory of your Gurobi installation. For a default installation of Gurobi 9.0.0, the command would be:

```
install.packages('/opt/gurobi900/linux64/R/gurobi_9.0-0_R_3.6.1.tar.gz', repos=NULL)
```

You will need to adjust the path to match your installation directory and version.

You will need to be careful that the R binary and the Gurobi package you install both use the same instruction set. To use the Gurobi R package, you will need to use the 64-bit version of R.

If you are using R from RStudio Server, and you get an error indicating that R is unable to load the Gurobi DLL or shared object, you may need to set the rsession-ld-library-path entry in the server config file. Please consult the RStudio documentation for more information.

# **Example**

Let us now turn our attention to an example of using Gurobi to solve a simple MIP model. Our example optimizes the following model:

Note that this is the same model that was modeled and optimized in the C Interface section.

This is the complete source code for our example (also available in <installdir>/examples/R/mip.R)...

```
x, y, z binary
library(gurobi)
model <- list()</pre>
model $ A
                   \leftarrow matrix(c(1,2,3,1,1,0), nrow=2, ncol=3, byrow=T)
model $ obj
                   <-c(1,1,2)
model$modelsense <- 'max'</pre>
                   <-c(4,1)
model$rhs
                   <- c('<', '>')
model$sense
                   <- 'B'
model $ vtype
params <- list(OutputFlag=0)</pre>
result <- gurobi(model, params)</pre>
print('Solution:')
print(result$objval)
print(result$x)
# Clear space
rm(model, result, params)
```

# **Example details**

Let us now walk through the example, line by line, to understand how it achieves the desired result of optimizing the indicated model.

The example begins by importing the Gurobi package (library('gurobi')). R programs that call Gurobi must include this line.

# **Building the model**

The example now builds an optimization model. The data associated with an optimization model must be stored in a single list variable. Named components in this list contain the different parts of the model. The most fundamental named components are: The constraint matrix (A), the objective vector (obj), the right-hand side vector (rhs), and the constraint sense vector (sense). Among these, only the constraint matrix is mandatory, and default values are substituted for all other model fields in case they are missing. A model variable can also include optional components (e.g., the objective sense modelsense).

In our example, we use the built-in R matrix function to build the constraint matrix A. A is stored as a dense matrix here. You can also store A as a sparse matrix, using either the sparse\_triplet\_matrix function from the slam package or the sparseMatrix class from the Matrix package. Sparse input matrices are illustrated in the lp2.R example.

Subsequent statements populate other components of the model variable, including the objective vector, the right-hand side vector, and the constraint sense vector. In each case, we use the built-in  ${\tt c}$  function to initialize the array arguments.

In addition to the mandatory components, this example also sets two optional components: modelsense and vtype. The former is used to indicate the sense of the objective function. The default is minimization, so we've set the component equal to 'max' to indicate that we would like to maximize the specified objective. The vtype component is used to indicate the types of the variables in the model. In our example, all

variables are binary ('B'). Note that our interface allows you to specify a scalar value for any array argument. The Gurobi interface will expand that scalar to a constant array of the appropriate length. In this example, the scalar value 'B' will be expanded to an array of length 3, containing one 'B' value for each column of A.

One important note about default variable bounds: the convention in math programming is that a variable will by default have a lower bound of 0 and an infinite upper bound. If you'd like your variables to have different bounds, you'll need to provide them explicitly.

# **Modifying Gurobi parameters**

The next statement creates a list variable that will be used to modify a Gurobi parameter:

```
params <- list(OutputFlag=0)</pre>
```

In this example, we wish to set the Gurobi OutputFlag parameter to 0 in order to shut off Gurobi output. The Gurobi R interface allows you to pass a list of the Gurobi parameters you would like to change. Please consult the Parameters section of the Gurobi Reference Manual for a complete list of all Gurobi parameters.

# Solving the model

The next statement is where the actual optimization occurs:

```
result <- gurobi(model, params)
```

We pass the model and the optional list of parameter changes to the gurobi() function. It computes an optimal solution to the specified model and returns the computed result.

# Printing the solution

The gurobi() function returns a list as its result. This list contains a number of components, where each component contains information about the computed solution. The available components depend on the result of the optimization, the type of model that was solved (LP, QP, SOCP, or MIP), and the algorithm used to solve the model. This result list will always contain an integer status component, which indicates whether Gurobi was able to compute an optimal solution to the model. You should consult the *Status Codes* section of the Gurobi Reference Manual for a complete list of all possible status codes. If Gurobi was able to find a solution to the model, the return value will also include objval and x components. The former gives the objective value for the computed solution, and the latter is the computed solution vector (one entry per column of the constraint matrix). For continuous models, we will also return dual information (reduced costs and dual multipliers), and possibly an optimal basis.

In our example, we simply print the optimal objective value (result\$objval) and the optimal solution vector (result\$x).

# Running the example

To run one of the R examples provided with the Gurobi distribution, you can use the source command in R. For example, if you are running R from the Gurobi R examples directory, you can say:

```
> source('mip.R')
```

If the Gurobi package was successfully installed, you should see the following output:  $\frac{1}{2}$ 

- [1] "Solution:"
- [1] 3
- [1] 1 0 1

The R example directory <installdir>/examples/R contains a number of examples. We encourage you to browse and modify them in order to become more familiar with the Gurobi R interface.

# **Recommended Reading**

The very basic introduction to mathematical programming and mathematical modeling in this document barely scratches the surface of this very broad and rich field. We've collected a set of recommended books here that provide more information on various aspects of math programming.

If you want more information on the algorithms and mathematics underlying the solution of linear programming problems, we recommend Introduction to Linear Optimization by Bertsimas, Tsitsiklis, and Tsitsiklis, or Linear Programming: Foundations and Extensions by R. Vanderbei. For a detailed treatment of interior-point methods for linear programming, we recommend Primal-Dual Interior-Point Methods by S. Wright.

For more information on the algorithms and mathematics underlying the solution of mixed-integer programming problems, we recommend Integer Programming by L. Wolsey.

For an introduction to the process of creating mathematical programming representations of business problems, we recommend Model Building in Mathematical Programming by H.P. Williams.

# Installing the Anaconda Python distribution

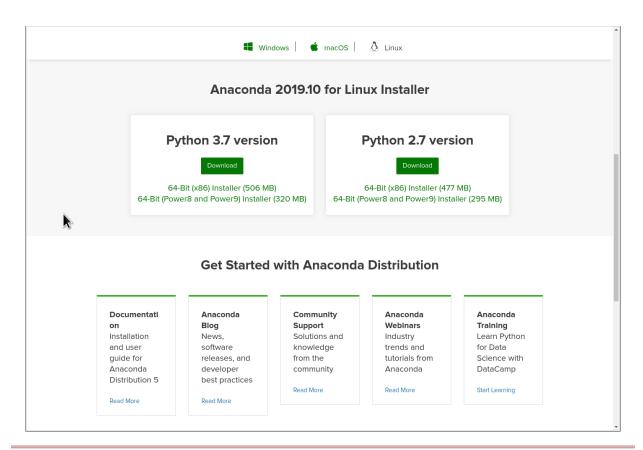
The Gurobi distribution includes a Python interpreter and a basic set of Python modules. While these are sufficient for building and running simple optimization models, they provide just a glimpse of the wealth of tools and modules that are available for Python. This section guides you through the steps involved in installing Anaconda, a widely-used Python platform that includes an Integrated Development Environment (Spyder), a notebook-style interface (Jupyter), and a broad set of Python modules. These tools can significantly increase the interactivity and productivity of your Python model building experience.

Before we begin, we should note that Anaconda isn't your only choice in Python distributions and/or IDEs. Popular alternatives include Canopy, Eric, iep, PyCharm, and PyDev. We won't be covering the details of installing these other options for use with Gurobi, but the Anaconda instructions that follow should provide a good outline for the steps involved. We've found that Anaconda provides a nice balance between power and complexity, but we realize that people may look for different things in their Python environments.

You will also find instructions for installing Anaconda Python at the Get Anaconda page on our website.

#### Step 1: Download and Install Anaconda

The first step is to download and install Anaconda. You can find detailed instructions here. Be sure to check the box Add Anaconda to my PATH environment variable when prompted during the installation process. Otherwise, Step 2 may not work.



Gurobi supports the 64-bit versions of Python 2.7, 3.6, and 3.7. Click on the download button for the version you want. Once the download has been completed, issue the following command in the directory where you have stored the downloaded file:

```
bash Anaconda3-2019.10-Linux-x86_64.sh
```

Once the install is complete, and once you close your current terminal and open a new one, typing python in your terminal window should invoke the Anaconda Python interpreter:

```
>python
Python 3.7.4 |Anaconda, Inc.|...
```

Type quit() in Python to return to the terminal.

#### Step 2: Install Gurobi into Anaconda

The next step is to install the Gurobi package into Anaconda. You do this by first adding the Gurobi channel to your Anaconda channels and then installing the gurobi package from this channel.

From a terminal window issue the following command to add the Gurobi channel to your default search list

```
conda config --add channels http://conda.anaconda.org/gurobi
```

Now issue the following command to install the Gurobi package

```
conda install gurobi
```

You can remove the Gurobi package at any time by issuing the command

conda remove gurobi

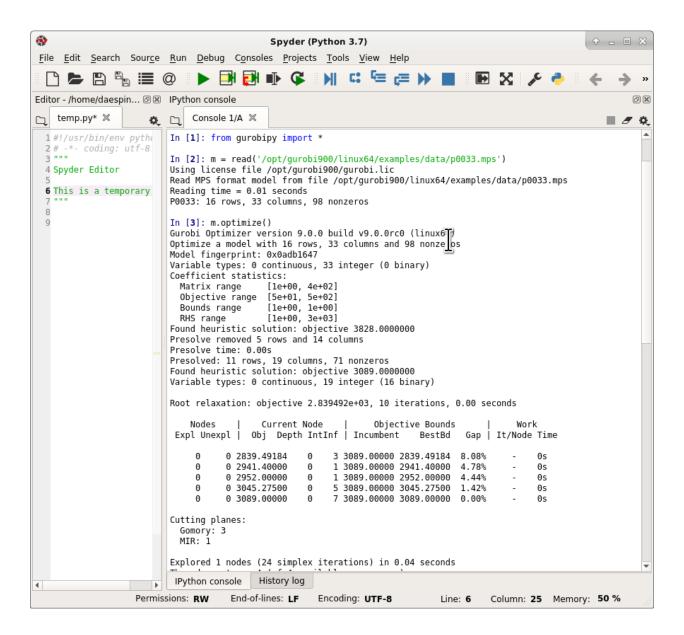
#### Step 3: Install a Gurobi License

The third step is to install a Gurobi license (if you haven't already done so).

You are now ready to use Gurobi from within Anaconda. Your next step is to launch either the Spyder IDE or Jupyter.

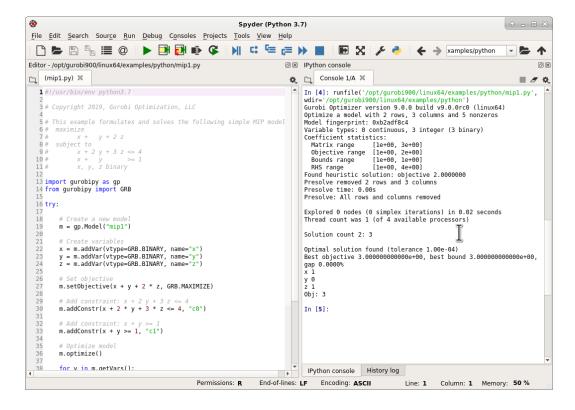
# 16.1 Using the Spyder IDE

You can launch the Spyder Python IDE, which is included in Anaconda Python, simply type spyder in a terminal window. Gurobi Interactive Shell commands can be typed directly into the Spyder Console window:

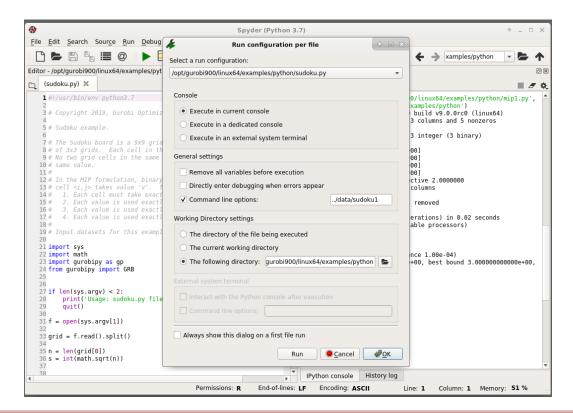


Note that a general-purpose Python IDE like Spyder requires one extra step that isn't required when you launch the Gurobi shell from the gurobi.sh command: you must manually load the Gurobi module by typing from gurobipy import \* (or import gurobipy as gp) before issuing any Gurobi commands.

You can also use Spyder to run any of the Gurobi examples. For example, if you use Open under the File menu to open Gurobi example mip1.py, and then click on the Run icon (the green triangle), you should see:



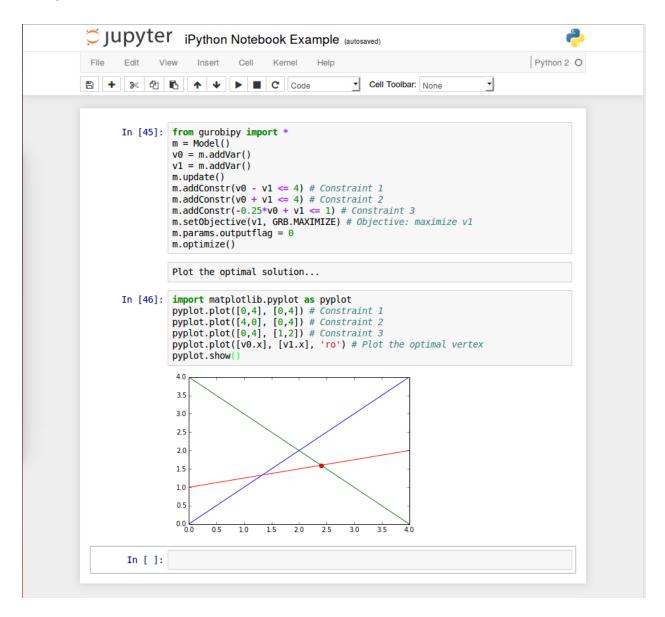
Some Gurobi examples require command-line arguments. Those can be input from the Configure... item of the Run menu. For example, to run the sudoku.py example with file sudoku1 as input ...



Anaconda Python includes all of the Python modules used in the Gurobi examples, but feel free to explore the vast library of additional Python modules. A list of available modules can be found at the PyPI site. You can use the conda command to install additional modules. Type conda into a terminal window with no arguments to get additional information on this command.

# 16.2 Using Jupyter

You can launch Jupyter, which is included in Anaconda Python, simply type jupyter in a terminal window. You can create a new notebook by clicking on the New icon (in the upper right) and choosing one of the Notebook options. Once your new notebook starts, you can type standard Python commands or Gurobi Interactive Shell commands directly into the In window:



Our simple example shows a set of commands that create and solve a simple linear programming model, and then

plot the resulting constraints and the computed optimal vertex.

For those of you who aren't familiar with notebook-style interfaces, they allow you to mix executable code, text, and graphics to create a self-documenting stream of results. Notebooks can be saved and continued later, which make them particularly well suited for prototyping and experimentation.

# File Overview

This section briefly describes the purposes of the more important files in the Gurobi distribution. Note that the list below may not precisely agree with your installation. We've omitted a few less important files. In addition, a few file names depend on the exact version of the Gurobi optimizer that you installed.

The following files and directories are created in your installation directory (typically /opt/gurobi900/linux64 for the 64-bit Linux distribution):

- EULA.pdf Gurobi End User License Agreement PDF format
- ReleaseNotes.html release notes
- bin
  - python2.7 Python shell
  - grbcluster Compute Server command-line tool
  - grb\_ts Gurobi Token Server executable
  - grbgetkey retrieves your Gurobi license key from the Gurobi key server
  - grbprobe probes system details (typically not used)
  - grbtune parameter tuning tool
  - gurobi\_cl simple command-line binary
  - gurobi.env sample parameter initialization file
  - gurobi.sh starts the Gurobi interactive shell

#### • docs

- examples Example Tour HTML (open index.html in this directory)
- examples.pdf Example Tour PDF
- quickstart Quick Start guide HTML (open index.html in this directory)
- quickstart linux.pdf Quick Start guide PDF
- refman Reference Manual HTML (open index.html in this directory)
- refman.pdf Reference Manual PDF
- remoteservices Remote Services Reference Manual HTML (open index.html in this directory)
- remoteservices.pdf Remote Services Reference Manual PDF

#### examples

- build Makefile for C, C++, C#, Java, and Python examples
- c source code for C examples
- c# source code for C# examples
- c++ source code for C++ examples
- data data files for examples
- dot netcore - Makefile and . NET Core App files for C# examples
- java source code for Java examples
- matlab source code for MATLAB examples

- python source code for Python examples
- R source code for R examples
- vb source code for Visual Basic examples (for Windows)

#### • include

- gurobi\_c.h C include file
- gurobi\_c++.h C++ include file
- python2.7 Dummy Python include files (no need to look inside this directory)

#### • lib

- gurobi.jar Java interface
- gurobi-javadoc.jar Javadoc documentation for our Java interface
- gurobi90.netstandard20.dll .NET Standard 2.0 interface
- gurobi90.netstandard20.xml .NET Standard 2.0 interface documentation
- gurobi.py Python startup file
- libgurobi90.so Gurobi library (symbolic link to current version used by all interfaces)
- libgurobi.so.9.0.0 Gurobi native library (used by all interfaces)
- libgurobi90 light.so light Gurobi native library (no support for Compute Server or Instant Cloud)
- libgurobi\_c++.a C++ interface (symbolic link)
- libgurobi\_g++4.2.a C++ interface (when using g++ 4.2)
- libgurobi\_g++5.2.a C++ interface (when using g++ 5.2)
- python2.7 Python files used by the interactive shell and the Python interface (no need to look inside this directory)
- libGurobiJni90.so Java JNI wrapper
- python2.7\_utf16 Python 2.7 files for use with UTF-16 Python versions (no need to look inside this directory)
- python2.7\_utf32 Python 2.7 files for use with UTF-32 Python versions (no need to look inside this directory)
- python3.5\_utf32 Python 3.5 files (no need to look inside this directory)
- python3.6\_utf32 Python 3.6 files (no need to look inside this directory)
- matlab Gurobi MATLAB interface
- R R Gurobi package
- $\bullet\,$  setup.py Python setup file for installing the gurobipy module into your own Python environment
- src
  - build Makefile for Gurobi C++ interface
  - $-\,$  cpp Source for Gurobi C++ interface

The following files and directories are created in your Remote Services installation (typically /opt/gurobi\_server900/linux64 for the 64-bit Linux distribution):

- EULA.pdf Gurobi End User License Agreement PDF format
- ReleaseNotes.html release notes
- bin
  - $-\,$ grb<br/>cluster Compute Server command-line tool
  - grb rs Gurobi Remote Services executable

- grb\_rs.cnf Default configuration file for the Remote Services
- $-\,$  grb\_rs\_aws.cnf Example configuration file for the Remote Services deployment with AWS
- grb\_rsr Gurobi Remote Services Router executable
- grb\_rsr.cnf Default configuration file for the Remote Services Router
- grb\_ts Gurobi Token Server executable
- grbgetkey retrieves your Gurobi license key from the Gurobi key server
- grbprobe probes system details (typically not used)
- data
  - \* files Temporary data files for job execution
  - \* runtimes Worker executables for supported versions
- docs
  - remoteservices Remote Services Reference Manual HTML (open index.html in this directory)
  - remoteservices.pdf Remote Services Reference Manual PDF
- resources Internal resource files for web UI